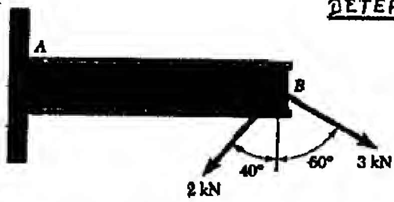
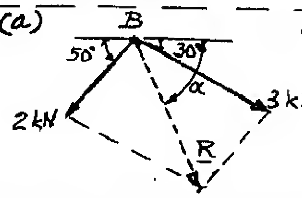
Wang-1
MECE2366

2.1

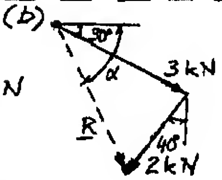


GIVEN: FORCES SHOWN
DETERMINE GRAPHICALLY THEIR RESULTANT, USING
(a) THE PARALLELOGRAM LAW
(b) THE TRIANGLE RULE

(a)



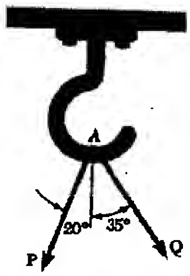
(b)



WE MEASURE: $R = 3.30\ kN$, $\alpha = 66.6°$

$\underline{R} = 3.30\ kN \measuredangle 66.6°$ ◀

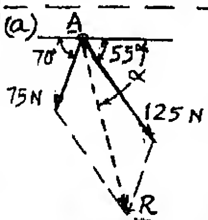
2.2



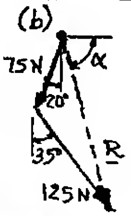
GIVEN: $P = 75\,N$, $Q = 125\,N$

DETERMINE GRAPHICALLY THE RESULTANT OF $\underline{P}$ AND $\underline{Q}$, USING
(a) THE PARALLELOGRAM LAW
(b) THE TRIANGLE RULE

(a)



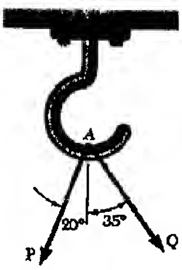
(b)



WE MEASURE: $R = 179\ N$
$\alpha = 75.1°$

$\underline{R} = 179\ N \measuredangle 75.1°$ ◀

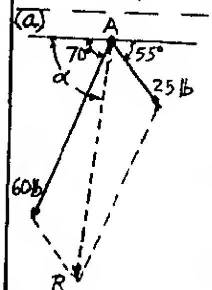
2.3

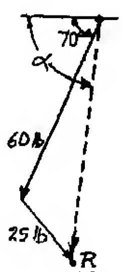


GIVEN: $P = 60\ lb$, $Q = 25\ lb$

DETERMINE GRAPHICALLY THE RESULTANT OF $\underline{P}$ AND $\underline{Q}$, USING
(a) THE PARALLELOGRAM LAW
(b) THE TRIANGLE RULE

(a)

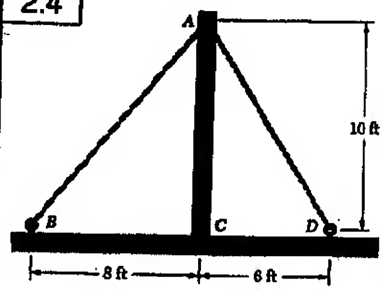




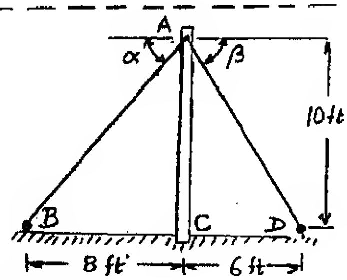
WE MEASURE: $R = 77.1\ lb$
$\alpha = 85.4°$

$\underline{R} = 77.1\ lb \measuredangle 85.4°$ ◀

2.4

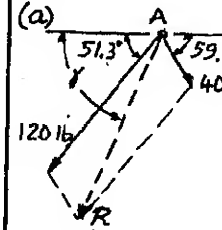


GIVEN:
$T_{AB} = 120\ lb$
$T_{AD} = 40\ lb$
DETERMINE GRAPHICALLY THE RESULTANT AT A, USING
(a) THE PARALLELOGRAM LAW
(b) THE TRIANGLE RULE

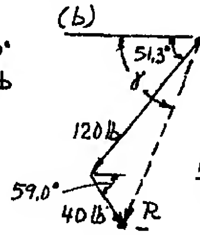


WE MEASURE THE ANGLES α AND β:
$\alpha = 51.3°$
$\beta = 59.0°$

(a)



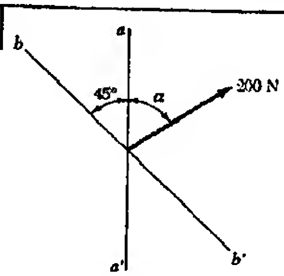
(b)



WE MEASURE:
$R = 139.1\ lb$
$\gamma = 67.0°$

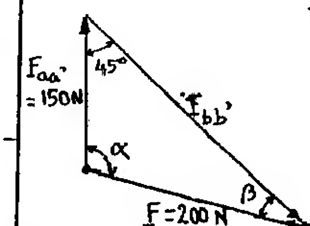
$\underline{R} = 139.1\ lb \measuredangle 67.0°$ ◀

2.5



GIVEN: COMPONENT OF 200-N FORCE ALONG a-a' MUST BE 150 N.
DETERMINE BY TRIGONOMETRY
(a) ANGLE α
(b) COMPONENT ALONG b-b'.

(a) USING TRIANGLE RULE AND LAW OF SINES:



$$\frac{\sin\beta}{150\ N} = \frac{\sin 45°}{200\ N}$$

$$\sin\beta = 0.53033$$

$$\beta = 32.03°$$

$$\alpha + \beta + 45° = 180°$$

$$\alpha = 180° - 45° - 32.03°$$

$$\alpha = 102.97°$$

$$\alpha = 103.0°$$ ◀

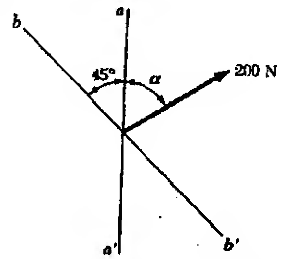
(b) LAW OF SINES:

$$\frac{F_{bb'}}{\sin\alpha} = \frac{200\,N}{\sin 45°}$$

$$F_{bb'} = (200\ N)\frac{\sin 102.97°}{\sin 45°} = 275.63\ N$$

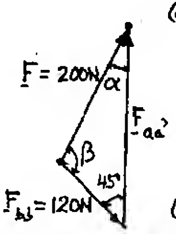
$$F_{bb'} = 276\ N$$ ◀

2.6



GIVEN:
COMPONENT OF 200-N FORCE ALONG b-b' MUST BE 120 N.
DETERMINE BY TRIGONOMETRY
(a) ANGLE α
(b) COMPONENT ALONG a-a'.

(a) USING TRIANGLE RULE AND LAW OF SINES:



$$\frac{\sin\alpha}{120\text{ N}} = \frac{\sin 45^\circ}{200\text{ N}}$$

$\sin\alpha = 0.42426$

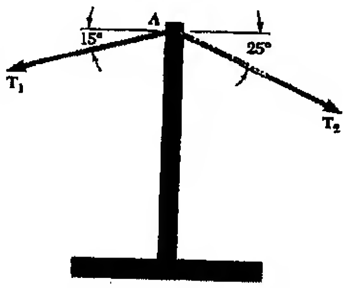
$\alpha = 25.1^\circ$ ◀

(b) $\beta = 180^\circ - 45^\circ - 25.1^\circ = 109.9^\circ$

LAW OF SINES: $\dfrac{F_{aa'}}{\sin\beta} = \dfrac{200\text{ N}}{\sin 45^\circ}$

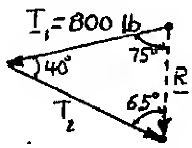
$F_{aa'} = (200\text{ N})\dfrac{\sin 109.9^\circ}{\sin 45^\circ}$ $\qquad F_{aa'} = 266\text{ N}$ ◀

2.7



GIVEN:
RESULTANT R OF T_1 AND T_2 MUST BE VERTICAL AND
$T_1 = 800$ lb
FIND:
(a) T_2
(b) R

TRIANGLE RULE AND LAW OF SINES:



$$\frac{T_1}{\sin 65^\circ} = \frac{T_2}{\sin 75^\circ} = \frac{R}{\sin 40^\circ}$$

$$\frac{800\text{ lb}}{\sin 65^\circ} = \frac{T_2}{\sin 75^\circ} = \frac{R}{\sin 40^\circ}$$

(a) SOLVING FOR T_2:

$T_2 = (800\text{ lb})\dfrac{\sin 75^\circ}{\sin 65^\circ} = 852.6\text{ lb}$

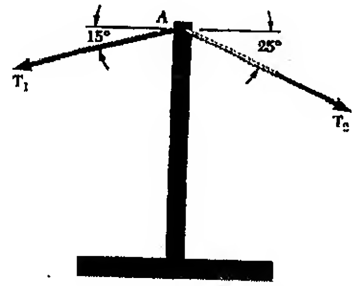
$T_2 = 853\text{ lb}$ ◀

(b) SOLVING FOR R:

$R = (800\text{ lb})\dfrac{\sin 40^\circ}{\sin 65^\circ} = 567.4\text{ lb}$

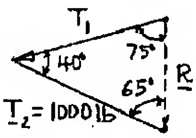
$R = 567\text{ lb}$ ◀

2.8



GIVEN:
RESULTANT R OF T_1 AND T_2 MUST BE VERTICAL AND
$T_2 = 1000$ lb.
FIND:
(a) T_1
(b) R

TRIANGLE RULE AND LAW OF SINES:



$$\frac{T_1}{\sin 65^\circ} = \frac{1000\text{ lb}}{\sin 75^\circ} = \frac{R}{\sin 40^\circ}$$

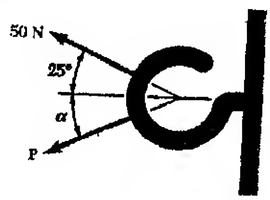
(a) SOLVING FOR T_1:

$T_1 = (1000\text{ lb})\dfrac{\sin 65^\circ}{\sin 75^\circ} = 938.28\text{ lb}$, $T_1 = 938\text{ lb}$ ◀

(b) SOLVING FOR R:

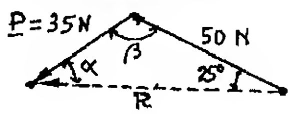
$R = (1000\text{ lb})\dfrac{\sin 40^\circ}{\sin 75^\circ} = 665.46\text{ lb}$, $R = 665\text{ lb}$ ◀

2.9



GIVEN:
RESULTANT R OF THE TWO FORCES MUST BE HORIZONTAL AND
P = 35 N.
FIND:
(a) ANGLE α
(b) R

TRIANGLE RULE:



(a) LAW OF SINES:

$$\frac{\sin\alpha}{50\text{ N}} = \frac{\sin 25^\circ}{35\text{ N}}$$

$\sin\alpha = \dfrac{50\text{ N}}{35\text{ N}}\sin 25^\circ$

$\sin\alpha = 0.60374$, $\alpha = 37.14^\circ$.

$\alpha = 37.1^\circ$ ◀

(b) $\beta = 180^\circ - 25^\circ - 37.14^\circ = 117.86^\circ$

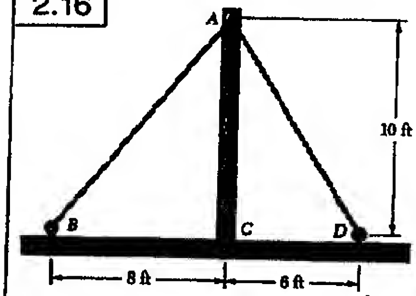
LAW OF SINES:

$$\frac{R}{\sin\beta} = \frac{35\text{ N}}{\sin 25^\circ}$$

$R = (35\text{ N})\dfrac{\sin 117.86^\circ}{\sin 25^\circ} = 73.218\text{ N}$

$R = 73.2\text{ N}$ ◀

2.16



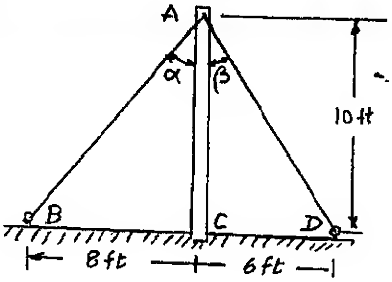
GIVEN:

$T_{AB} = 120$ lb

$T_{AD} = 40$ lb

FIND:

RESULTANT $\underline{R}$ OF THE FORCES EXERTED AT A BY AB AND AD

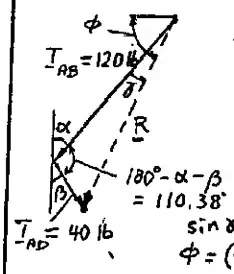


$\tan\alpha = \frac{8}{10}$

$\alpha = 38.66^\circ$

$\tan\beta = \frac{6}{10}$

$\beta = 30.96^\circ$



FROM FORCE TRIANGLE:

LAW OF COSINES:

$R^2 = (120)^2 + (40)^2 - 2(120)(40)\cos 110.38^\circ$

$= 14{,}400 + 1600 - 9600(-0.34824)$

$R^2 = 19{,}343 \qquad R = 139.08$ lb

LAW OF SINES

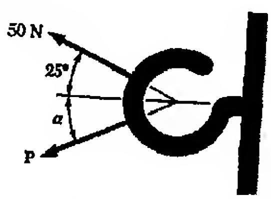
$\frac{\sin\gamma}{40 \text{ lb}} = \frac{\sin 110.38^\circ}{139.08 \text{ lb}}$

$\sin\gamma = 0.26960 \qquad \gamma = 15.64^\circ$

$\phi = (90^\circ - \alpha) + \gamma = 51.34^\circ + 15.64^\circ = 66.98^\circ$

$\underline{R} = 139.1 \text{ lb} \measuredangle 67.0^\circ$

2.17

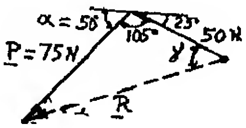


GIVEN:

$P = 75$ N, $\alpha = 50^\circ$

FIND:

RESULTANT $\underline{R}$ OF THE TWO FORCES SHOWN.



FROM FORCE TRIANGLE:

LAW OF COSINES:

$R^2 = (75)^2 + (50)^2 - 2(75)(50)\cos 105^\circ$

$= 5625 + 2500 - 7500(-0.25882)$

$R^2 = 10\,066 \qquad R = 100.33$ N

LAW OF SINES:

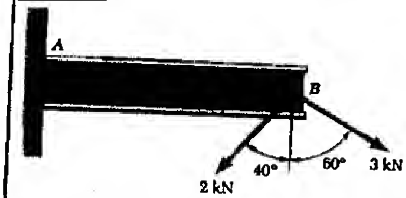
$\frac{\sin\gamma}{75 \text{ N}} = \frac{\sin 105^\circ}{100.33 \text{ N}}$

$\sin\gamma = 0.72206 \qquad \gamma = 46.22^\circ$

$\underline{R}\ \measuredangle = \gamma - 25^\circ = 46.22^\circ - 25^\circ = 21.22^\circ$

$\underline{R} = 100.3 \text{ N} \measuredangle 21.2^\circ$

2.18

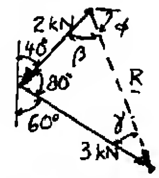


GIVEN:

FORCES SHOWN

FIND:

THEIR RESULTANT



FROM FORCE TRIANGLE:

LAW OF COSINES:

$R^2 = (2)^2 + (3)^2 - 2(2)(3)\cos 80^\circ$

$R^2 = 10.916 \qquad R = 3.304$ kN

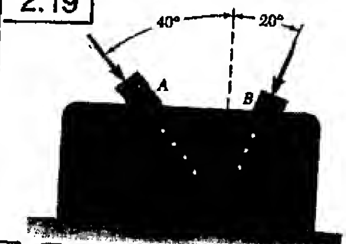
LAW OF SINES:

$\frac{\sin\gamma}{2 \text{ kN}} = \frac{\sin 80^\circ}{3.304 \text{ kN}} \qquad \gamma = 36.59^\circ$

$\beta = 180^\circ - (80^\circ + 36.59^\circ) = 63.41^\circ \qquad \phi = 180^\circ - (\beta + 50^\circ) = 66.59^\circ$

$\underline{R} = 3.30 \text{ kN} \measuredangle 66.6^\circ$

2.19



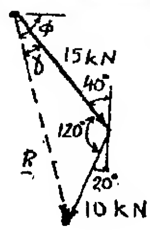
GIVEN:

$F_A = 15$ kN

$F_B = 10$ kN

FIND:

RESULTANT OF FORCES EXERTED ON BRACKET BY MEMBERS A AND B.



FROM FORCE TRIANGLE:

LAW OF COSINES:

$R^2 = (15)^2 + (10)^2 - 2(15)(10)\cos 120^\circ$

$R^2 = 475 \qquad R = 21.794$ kN

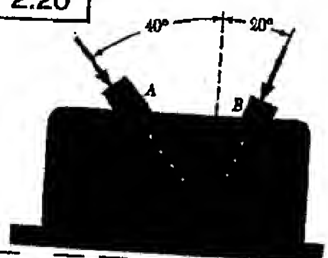
LAW OF SINES:

$\frac{\sin\delta}{10 \text{ kN}} = \frac{\sin 120^\circ}{21.794 \text{ kN}} \qquad \gamma = 23.41^\circ$

$\phi = 50^\circ + \gamma = 50^\circ + 23.41^\circ = 73.41^\circ$

$\underline{R} = 21.8 \text{ kN} \measuredangle 73.4^\circ$

2.20



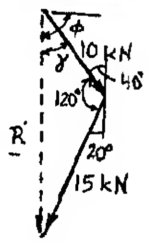
GIVEN:

$F_A = 10$ kN

$F_B = 15$ kN

FIND:

RESULTANT OF FORCES EXERTED ON BRACKET BY MEMBERS A AND B.



FROM FORCE TRIANGLE:

LAW OF COSINES:

$R^2 = (10)^2 + (15)^2 - 2(10)(15)\cos 120^\circ$

$R^2 = 475 \qquad R = 21.794$ kN

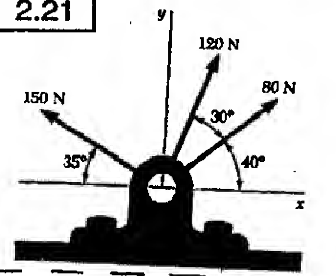
LAW OF SINES:

$\frac{\sin\gamma}{15 \text{ kN}} = \frac{\sin 120^\circ}{21.794 \text{ kN}} \qquad \gamma = 36.59^\circ$

$\phi = 50^\circ + \gamma = 50^\circ + 36.59^\circ = 86.59^\circ$

$\underline{R} = 21.8 \text{ kN} \measuredangle 86.6^\circ$

2.21



GIVEN: MAGNITUDES AND DIRECTIONS OF FORCES

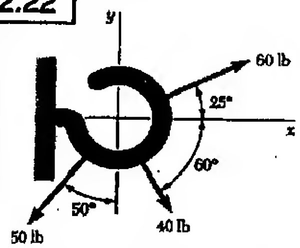
FIND: x AND y COMPONENTS OF THE FORCES.

80-N FORCE: $F_x = +(80\text{ N})\cos 40°$, $F_x = +61.3\text{ N}$ ◀
$F_y = +(80\text{ N})\sin 40°$, $F_y = +51.4\text{ N}$ ◀

120-N FORCE: $F_x = +(120\text{ N})\cos 70°$, $F_x = +41.0\text{ N}$ ◀
$F_y = +(120\text{ N})\sin 70°$, $F_y = +112.8\text{ N}$ ◀

150-N FORCE: $F_x = -(150\text{ N})\cos 35°$, $F_x = -122.9\text{ N}$ ◀
$F_y = +(150\text{ N})\sin 35°$, $F_y = +86.0\text{ N}$ ◀

2.22



GIVEN: MAGNITUDES AND DIRECTIONS OF FORCES

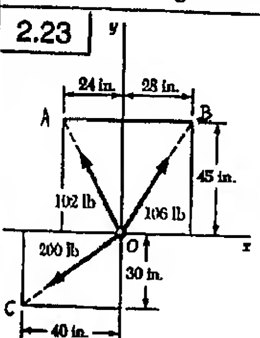
FIND: x AND y COMPONENTS OF THE FORCES.

40-lb FORCE: $F_x = +(40\text{ lb})\cos 60° = +20.00\text{ lb}$, $F_x = +20.0\text{ lb}$ ◀
$F_y = -(40\text{ lb})\sin 60° = -34.64\text{ lb}$, $F_y = -34.6\text{ lb}$ ◀

50-lb FORCE: $F_x = -(50\text{ lb})\sin 50° = -38.30\text{ lb}$, $F_x = -38.3\text{ lb}$ ◀
$F_y = -(50\text{ lb})\cos 50° = -32.14\text{ lb}$, $F_y = -32.1\text{ lb}$ ◀

60-lb FORCE: $F_x = +(60\text{ lb})\cos 25° = +54.38\text{ lb}$, $F_x = +54.4\text{ lb}$ ◀
$F_y = +(60\text{ lb})\sin 25° = +25.36\text{ lb}$, $F_y = +25.4\text{ lb}$ ◀

2.23



GIVEN: FORCES AND DIMENSIONS SHOWN.

FIND: x AND y COMPONENTS OF FORCES.

WE COMPUTE THE FOLLOWING DISTANCES:

$OA = \sqrt{(24)^2 + (45)^2} = 51\text{ in.}$

$OB = \sqrt{(28)^2 + (45)^2} = 53\text{ in.}$

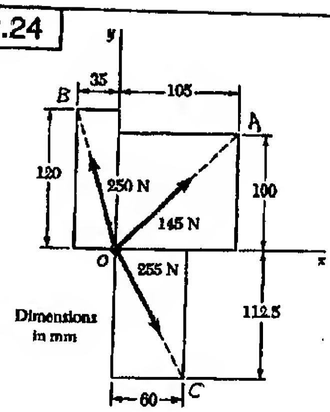
$OC = \sqrt{(40)^2 + (30)^2} = 50\text{ in.}$

102-lb FORCE: $F_x = -(102\text{ lb})\frac{24}{51}$ $F_x = -48.0\text{ lb}$ ◀
$F_y = +(102\text{ lb})\frac{45}{51}$ $F_y = +90.0\text{ lb}$ ◀

106-lb FORCE: $F_x = +(106\text{ lb})\frac{28}{53}$ $F_x = +56.0\text{ lb}$ ◀
$F_y = +(106\text{ lb})\frac{45}{53}$ $F_y = +90.0\text{ lb}$ ◀

200-lb FORCE: $F_x = -(200\text{ lb})\frac{40}{50}$ $F_x = -160.0\text{ lb}$ ◀
$F_y = -(200\text{ lb})\frac{30}{50}$ $F_y = -120.0\text{ lb}$ ◀

2.24



GIVEN: FORCES AND DIMENSIONS SHOWN.

FIND: x- AND y COMPONENTS OF FORCES

145-N FORCE: $OA = \sqrt{(105)^2 + (100)^2} = 145\text{ mm}$

$F_x = +(145\text{ N})\frac{105\text{ mm}}{145\text{ mm}}$ $F_x = +105.0\text{ N}$ ◀

$F_y = +(145\text{ N})\frac{100\text{ mm}}{145\text{ mm}}$ $F_y = +100.0\text{ N}$ ◀

250-N FORCE: $OB = \sqrt{(35)^2 + (120)^2} = 125\text{ mm}$

$F_x = -(250\text{ N})\frac{35\text{ mm}}{125\text{ mm}}$ $F_x = -70.0\text{ N}$ ◀

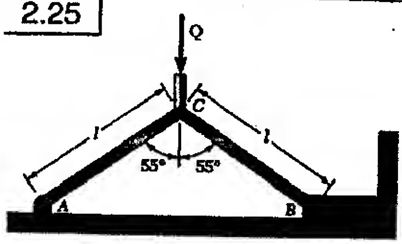
$F_y = +(250\text{ N})\frac{120\text{ mm}}{125\text{ mm}}$ $F_y = +240\text{ N}$ ◀

255-N FORCE: $OC = \sqrt{(60)^2 + (112.5)^2} = 127.5\text{ mm}$

$F_x = +(255\text{ N})\frac{60\text{ mm}}{127.5\text{ mm}}$ $F_x = +120.0\text{ N}$ ◀

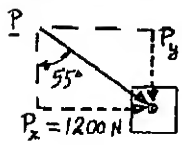
$F_y = -(255\text{ N})\frac{112.5\text{ mm}}{127.5\text{ mm}}$ $F_y = -225\text{ N}$ ◀

2.25



GIVEN:
(1) CB EXERTS FORCE P ON B ALONG CB.
(2) HORIZONTAL COMPONENT OF P IS $P_x = 1200\text{ N}$.

FIND:
(a) MAGNITUDE P
(b) VERT. COMP. P_y



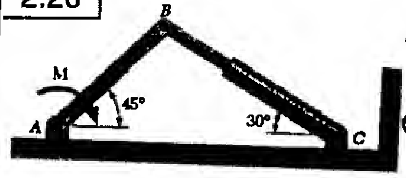
(a) $P_x = P\sin 55°$ $P = \frac{P_x}{\sin 55°} = \frac{1200\text{ N}}{\sin 55°} = 1464.9\text{ N}$

$P = 1465\text{ N}$ ◀

(b) $P_x = P_y \tan 55°$ $P_y = \frac{P_x}{\tan 55°} = \frac{1200\text{ N}}{\tan 55°} = 840.2\text{ N}$

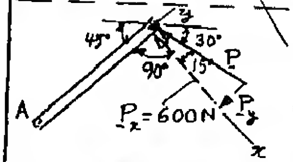
$P_y = 840\text{ N}\downarrow$ ◀

2.26



GIVEN:
(1) FORCE $\underline{P}$ EXERTED BY BC ON AB IS DIRECTED ALONG BC.
(2) COMPONENT OF $\underline{P}$ ⊥ AB IS 600 N

FIND: (a) P
(b) COMP. OF $\underline{P}$ ALONG AB



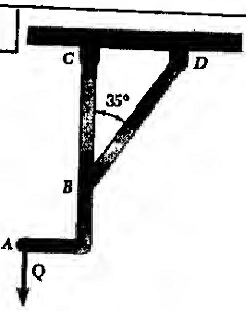
(a) $P = \dfrac{P_x}{\cos 15°} = \dfrac{600\,N}{\cos 15°}$

$P = 621\,N$ ◀

(b) $P_y = P_x \tan 15° = (600\,N)\tan 15°$

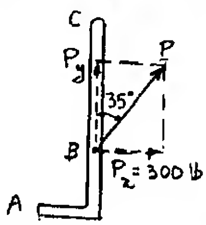
$P_y = 160.8\,N$ ◀

2.27



GIVEN:
(1) FORCE $\underline{P}$ EXERTED BY BD ON ABC IS DIRECTED ALONG BD.
(2) HORIZ. COMPONENT OF $\underline{P}$ IS $P_x = 300$ lb.

FIND:
(a) MAGNITUDE P
(b) VERT. COMP. P_y



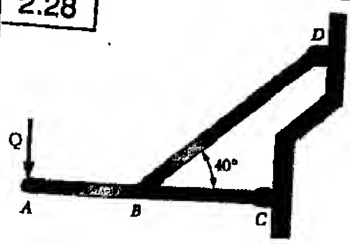
(a) $P = \dfrac{P_x}{\sin 35°} = \dfrac{300\text{ lb}}{\sin 35°}$

$P = 523$ lb ◀

(b) $P_y = \dfrac{P_x}{\tan 35°} = \dfrac{300\text{ lb}}{\tan 35°}$

$P_y = 428$ lb ◀

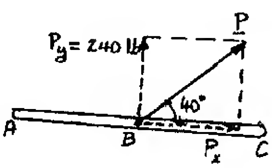
2.28



GIVEN:
(1) FORCE $\underline{P}$ EXERTED BY BD ON ABC IS DIRECTED ALONG BD
(2) VERT. COMPONENT OF $\underline{P}$ IS $P_y = 240$ lb.

FIND:
(a) MAGNITUDE P
(b) HORIZ. COMP. P_x.

(a) $P = \dfrac{P_y}{\sin 40°} = \dfrac{240\text{ lb}}{\sin 40°}$

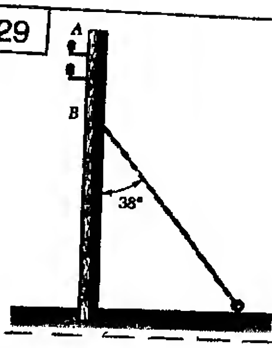


$P = 373$ lb ◀

(b) $P_x = \dfrac{P_y}{\tan 40°} = \dfrac{240\text{ lb}}{\tan 40°}$

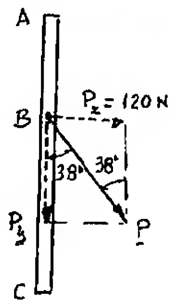
$P_x = 286$ lb ◀

2.29



GIVEN:
(1) FORCE $\underline{P}$ EXERTED BY BD ON POLE IS DIRECTED ALONG BD
(2) COMPONENT OF $\underline{P}$ ⊥ TO AC IS 120 N.

FIND:
(a) MAGNITUDE P
(b) COMPONENT OF $\underline{P}$ ALONG AC.



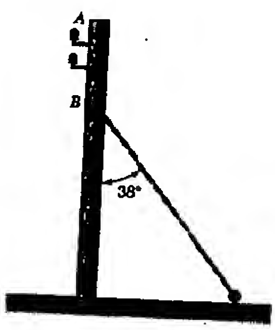
(a) $P = \dfrac{P_x}{\sin 38°} = \dfrac{120\,N}{\sin 38°} = 194.91\,N$

$P = 194.9\,N$ ◀

(b) $P_y = \dfrac{P_x}{\tan 38°} = \dfrac{120\,N}{\tan 38°} = 153.59\,N$

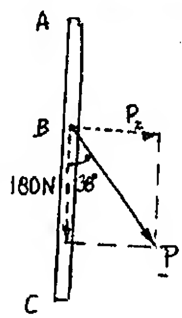
$P_y = 153.6\,N$ ◀

2.30



GIVEN:
(1) FORCE $\underline{P}$ EXERTED BY BD ON POLE IS DIRECTED ALONG BD
(2) COMPONENT OF $\underline{P}$ ALONG AC IS 180 N.

FIND:
(a) MAGNITUDE P
(b) COMPONENT OF $\underline{P}$ ⊥ TO AC.



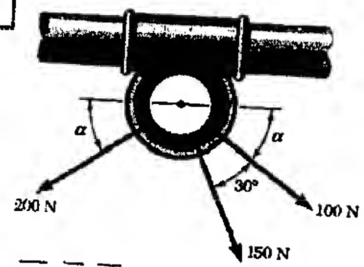
(a) $P = \dfrac{180\,N}{\cos 38°} = 228.4\,N$

$P = 228\,N$ ◀

(b) $P_x = (180\,N)\tan 30° = 140.63\,N$

$P_x = 140.6\,N$ ◀

2.35



GIVEN: $\alpha = 35°$

FIND: RESULTANT OF THE THREE FORCES SHOWN.

100-N FORCE:

$F_x = +(100\ N)\cos 35° = +81.92\ N$, $F_y = -(100\ N)\sin 35° = -57.36\ N$

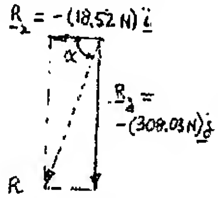
150-N FORCE:

$F_x = +(150\ N)\cos 65° = +63.39\ N$, $F_y = -(150\ N)\sin 65° = -135.95\ N$

200-N FORCE:

$F_x = -(200\ N)\cos 35° = -163.83\ N$, $F_y = -(200\ N)\sin 35° = -114.72\ N$

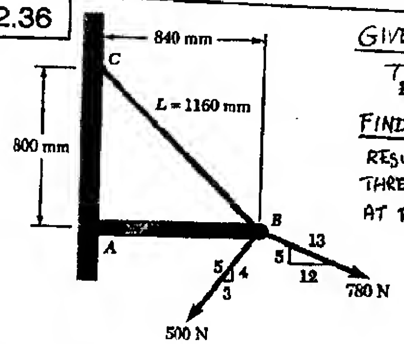
FORCE	x COMP. (N)	y COMP. (N)
100 N	+81.92	−57.36
150 N	+63.39	−135.95
200 N	−163.83	−114.72
	$R_x = -18.52$	$R_y = -308.03$



$\tan\alpha = \dfrac{308.03\ N}{18.52\ N}$ $\alpha = 86.56°$

$R = \dfrac{308.03\ N}{\sin 86.56°} = 308.6\ N$ $R = 309\ N \measuredangle 86.6°$

2.36



GIVEN: $T_{BC} = 725\ N$

FIND: RESULTANT OF THE THREE FORCES EXERTED AT POINT B OF BEAM AB.

FORCE EXERTED BY CABLE BC:

$F_x = -(725\ N)\dfrac{840\ mm}{1160\ mm} = -525\ N$, $F_y = +(725\ N)\dfrac{800\ mm}{1160\ mm} = +500\ N$

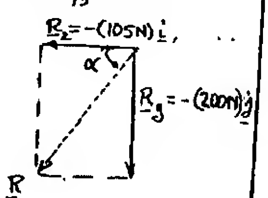
500-N FORCE:

$F_x = -(500\ N)\frac{3}{5} = -300\ N$, $F_y = -(500\ N)\frac{4}{5} = -400\ N$

780-N FORCE:

$F_x = +(780\ N)\frac{12}{13} = +720\ N$, $F_y = -(780\ N)\frac{5}{13} = -300\ N$

FORCE	x COMP. (N)	y COMP. (N)
$T_{BC} = 725\ N$	−525	+500
500 N	−300	−400
780 N	+720	−300
	$R_x = -105$	$R_y = -200$

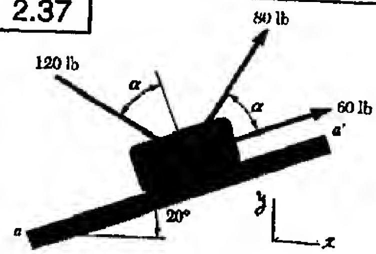


$\tan\alpha = \dfrac{200\ N}{105\ N}$ $\alpha = 62.30°$

$R = \dfrac{200\ N}{\sin 62.30°} = 225.9\ N$

$R = 226\ N \measuredangle 62.3°$

2.37



GIVEN: $\alpha = 40°$

FIND: RESULTANT OF THE THREE FORCES SHOWN

60-lb FORCE:

$F_x = +(60\ lb)\cos 20° = +56.38\ lb$, $F_y = +(60\ lb)\sin 20° = +20.52\ lb$

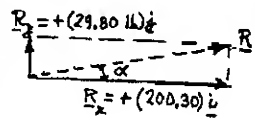
80-lb FORCE:

$F_x = +(80\ lb)\cos 60° = +40.00\ lb$, $F_y = +(80\ lb)\sin 60° = +69.28\ lb$

120-lb FORCE:

$F_x = +(120\ lb)\cos 30° = +103.92\ lb$, $F_y = -(120\ lb)\sin 30° = -60.00\ lb$

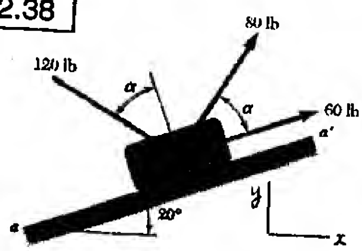
FORCE	X COMP (lb)	y COMP. (lb)
60 lb	+56.38	+20.52
80 lb	+40.00	+69.28
120 lb	+103.92	−60.00
	$R_x = +200.30$	$R_y = +29.80$



$\tan\alpha = \dfrac{29.80\ lb}{200.30\ lb}$ $\alpha = 8.462°$

$R = \dfrac{29.80\ lb}{\sin 8.462°} = 202.51\ lb$ $R = 203\ lb \measuredangle 8.46°$

2.38



GIVEN: $\alpha = 75°$

FIND: RESULTANT OF THE THREE FORCES SHOWN.

60-lb FORCE:

$F_x = +(60\ lb)\cos 20° = +56.38\ lb$, $F_y = +(60\ lb)\sin 20° = +20.52\ lb$

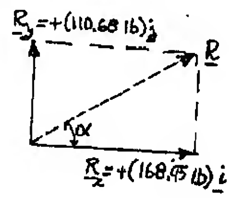
80-lb FORCE:

$F_x = +(80\ lb)\cos 95° = -6.97\ lb$, $F_y = +(80\ lb)\sin 95° = +79.70\ lb$

120-lb FORCE:

$F_x = +(120\ lb)\cos 5° = +119.54\ lb$, $F_y = +(120\ lb)\sin 5° = +10.46\ lb$

FORCE	x COMP. (lb)	y COMP. (lb)
60 lb	+56.38	+20.52
80 lb	−6.97	+79.70
120 lb	+119.54	+10.46
	$R_x = +168.95$	$R_y = +110.68$

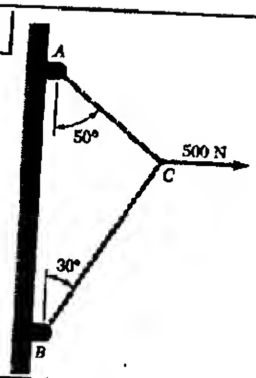


$\tan\alpha = \dfrac{110.68\ lb}{168.95\ lb}$ $\alpha = 33.23°$

$R = \dfrac{110.68\ lb}{\sin 33.23°} = 201.98\ lb$

$R = 202\ lb \measuredangle 33.2°$

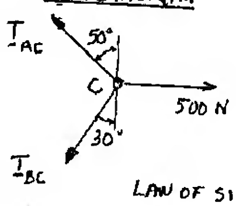
2.43



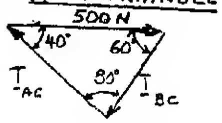
GIVEN: CABLES AC AND BC ARE LOADED AS SHOWN

FIND: (a) TENSION IN AC. (b) TENSION IN BC.

F.B. DIAGRAM



FORCE TRIANGLE

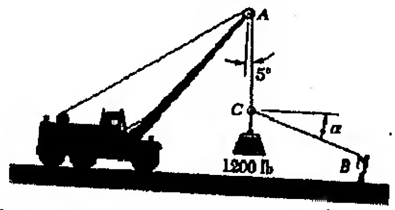


LAW OF SINES: $\dfrac{T_{AC}}{\sin 60°} = \dfrac{T_{BC}}{\sin 40°} = \dfrac{500 N}{\sin 80°}$

(a) $T_{AC} = \dfrac{500 N}{\sin 80°} \sin 60° = 439.7 N$ $\qquad T_{AC} = 440 N$ ◄

(b) $T_{BC} = \dfrac{500 N}{\sin 80°} \sin 40° = 326.4 N$ $\qquad T_{BC} = 326 N$ ◄

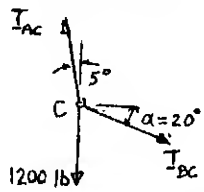
2.45



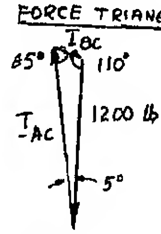
GIVEN: $\alpha = 20°$

FIND: TENSION IN (a) AC (b) BC

F.B. DIAGRAM



FORCE TRIANGLE

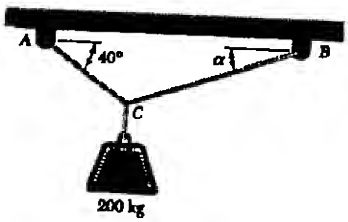


LAW OF SINES; $\dfrac{T_{AC}}{\sin 110°} = \dfrac{T_{BC}}{\sin 5°} = \dfrac{1200 lb}{\sin 65°}$

(a) $T_{AC} = \dfrac{1200 lb}{\sin 65°} \sin 110° = 1244.2 lb$ $\qquad T_{AC} = 1244 lb$ ◄

(b) $T_{BC} = \dfrac{1200 lb}{\sin 65°} \sin 5° = 115.40 lb$ $\qquad T_{BC} = 115.4 lb$ ◄

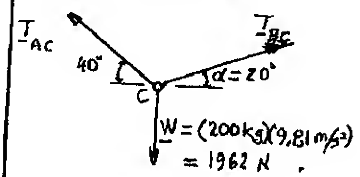
2.44



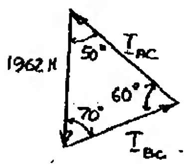
GIVEN (1) CABLES AC AND BC ARE LOADED AS SHOWN (2) $\alpha = 20°$

FIND: TENSION IN (a) AC (b) BC

F.B. DIAGRAM



FORCE TRIANGLE

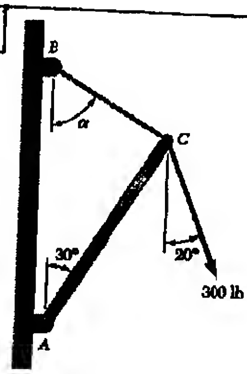


LAW OF SINES; $\dfrac{T_{AC}}{\sin 70°} = \dfrac{T_{BC}}{\sin 50°} = \dfrac{1962 N}{\sin 60°}$

(a) $T_{AC} = \dfrac{1962 N}{\sin 60°} \sin 70° = 2128.9 N$ $\qquad T_{AC} = 2.13 kN$ ◄

(b) $T_{BC} = \dfrac{1962 N}{\sin 60°} \sin 50° = 1735.49 N$ $\qquad T_{BC} = 1.735 kN$ ◄

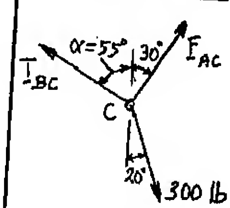
2.46



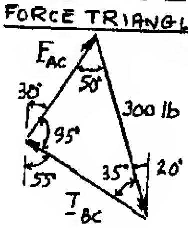
GIVEN: (1) $\alpha = 55°$. (2) BOOM AC EXERTS ON PIN C A FORCE ALONG AC.

FIND: (a) F_{AC} (b) T_{BC}

F.B. DIAGRAM



FORCE TRIANGLE

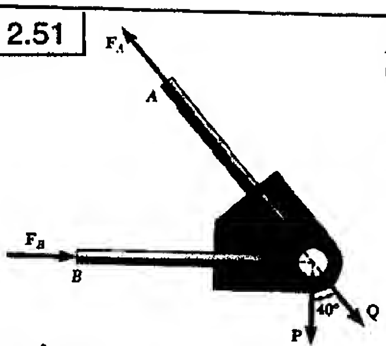


LAW OF SINES: $\dfrac{F_{AC}}{\sin 35°} = \dfrac{T_{BC}}{\sin 50°} = \dfrac{300 lb}{\sin 95°}$

(a) $F_{AC} = \dfrac{300 lb}{\sin 95°} \sin 35° = 172.73 lb$ $\qquad F_{AC} = 172.7 lb$ ◄

(b) $T_{BC} = \dfrac{300 lb}{\sin 95°} \sin 50° = 230.7 lb$ $\qquad T_{BC} = 231 lb$ ◄

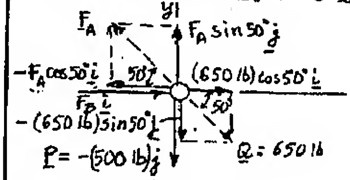
2.51



GIVEN:
(1) CONNECTION IN EQUILIBRIUM UNDER FOUR FORCES
(2) $P = 500$ lb
$Q = 650$ lb

FIND:
F_A AND F_B

FREE-BODY DIAGRAM
RESOLVING THE FORCES INTO x AND y COMPONENTS:



$\mathbf{R} = \mathbf{F}_A + \mathbf{F}_B + \mathbf{P} + \mathbf{Q} = 0$

$-F_A \cos 50^\circ \mathbf{i} + F_A \sin 50^\circ \mathbf{j} + F_B \mathbf{i} - 500\mathbf{j} + 650 \cos 50^\circ \mathbf{i} - 650 \sin 50^\circ \mathbf{j} = 0$

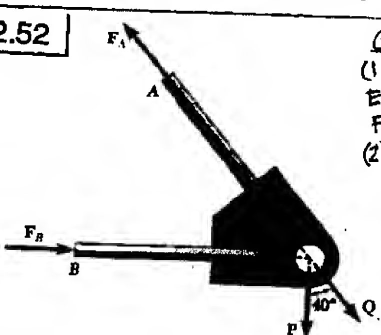
EQUATING TO ZERO THE COEFF. OF $\mathbf{i}$ AND $\mathbf{j}$:

($\mathbf{j}$) $F_A \sin 50^\circ - 500 - 650 \sin 50^\circ = 0$ $\quad F_A = 1303$ lb ◀

($\mathbf{i}$) $-F_A \cos 50^\circ + F_B + 650 \cos 50^\circ = 0$

$F_B = (1303\ \text{lb}) \cos 50^\circ - (650\ \text{lb} \cos 50^\circ)$ $\quad F_B = 420$ lb ◀

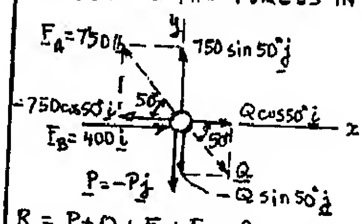
2.52



GIVEN:
(1) CONNECTION IN EQUILIBRIUM UNDER FOUR FORCES
(2) $F_A = 750$ lb
$F_B = 400$ lb

FIND:
P AND Q

FREE-BODY DIAGRAM:
RESOLVING THE FORCES INTO x AND y COMPONENTS



$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{F}_A + \mathbf{F}_B = 0$

$-P\mathbf{j} + Q \cos 50^\circ \mathbf{i} - Q \sin 50^\circ \mathbf{j} - 750 \cos 50^\circ \mathbf{i} + 750 \sin 50^\circ \mathbf{j} + 400\mathbf{i} = 0$

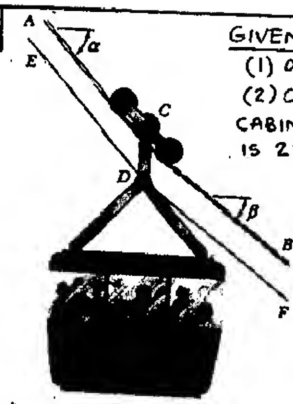
EQUATING TO ZERO THE COEFF. OF $\mathbf{i}$ AND $\mathbf{j}$:

($\mathbf{i}$) $Q \cos 50^\circ - 750 \cos 50^\circ + 400 = 0$ $\quad Q = 127.7$ lb ◀

($\mathbf{j}$) $-P - Q \sin 50^\circ + 750 \sin 50^\circ = 0$

$P = -(127.7\ \text{lb}) \sin 50^\circ + (750\ \text{lb}) \sin 50^\circ$ $\quad P = 477$ lb ◀

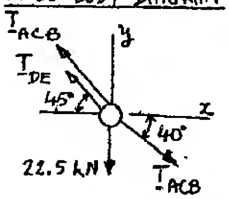
2.53



GIVEN:
(1) $\alpha = 45^\circ$, $\beta = 40^\circ$
(2) COMBINED WEIGHT OF CABIN AND PASSENGERS IS 22.5 kN.
(3) $T_{DF} \approx 0$

FIND:
(a) T_{ACB}
(b) T_{DE}

FREE-BODY DIAGRAM (CABIN CONSIDERED AS PARTICLE)



$\Sigma F_x = 0$:

$T_{ACB} \cos 40^\circ - T_{ACB} \cos 45^\circ - T_{DE} \cos 45^\circ = 0$

$0.05894\, T_{ACB} - 0.7071\, T_{DE} = 0$ (1)

$\Sigma F_y = 0$:

$-T_{ACB} \sin 40^\circ + T_{ACB} \sin 45^\circ + T_{DE} \sin 45^\circ - 22.5 = 0$

$0.06432\, T_{ACB} + 0.7071\, T_{DE} = 22.5$ (2)

(a) ADD (1) AND (2): $0.12326\, T_{ACB} = 22.5$ $\quad T_{ACB} = 182.54$ kN

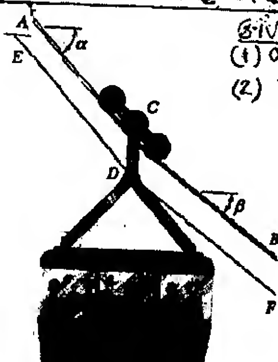
$T_{ACB} = 182.5$ kN ◀

(b) FROM (1): $T_{DE} = \dfrac{0.05894}{0.7071}(182.54)$

$T_{DE} = 15.22$ kN ◀

NOTE: IN PROBS. 2.53 AND 2.54 THE CABIN IS CONSIDERED AS A PARTICLE. IF CONSIDERED AS A RIGID BODY (CHAP. 4) IT WOULD BE FOUND THAT ITS CENTER OF GRAVITY SHOULD BE LOCATED TO THE LEFT OF ℄ FOR GD TO BE VERTICAL.

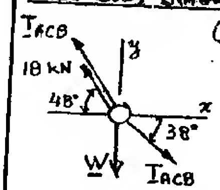
2.54



GIVEN:
(1) $\alpha = 48^\circ$, $\beta = 38^\circ$
(2) $T_{DE} = 18$ kN, $T_{DF} \approx 0$

FIND:
(a) COMBINED WEIGHT OF CABIN, PASSENGERS, AND SUPPORT SYSTEM
(b) T_{ACB}

FREE-BODY DIAGRAM (CABIN CONSIDERED AS PARTICLE)



(b) $\Sigma F_x = 0$:

$T_{ACB} \cos 38^\circ - T_{ACB} \cos 48^\circ - (18\ \text{kN}) \cos 48^\circ = 0$

$0.1189\, T_{ACB} - 12.044\ \text{kN} = 0$

(b) $T_{ACB} = 101.3$ kN ◀

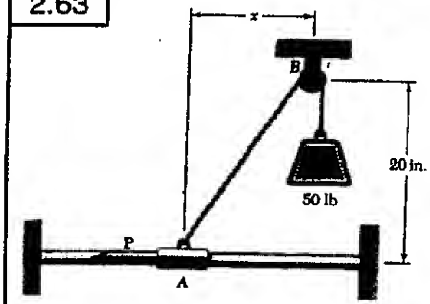
(a) $\Sigma F_y = 0$: $T_{ACB} \sin 48^\circ - T_{ACB} \sin 38^\circ + (18\ \text{kN}) \sin 48^\circ - W = 0$

$W = (101.3\ \text{kN})(\sin 48^\circ - \sin 38^\circ) + (18\ \text{kN}) \sin 48^\circ$

$= 26.29$ kN

(a) $W = 26.3$ kN ◀

2.63



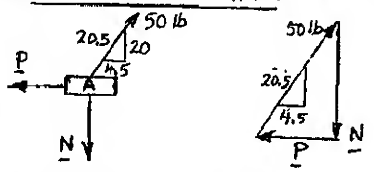
GIVEN: SYSTEM SHOWN IS IN EQUILIBRIUM

FIND P WHEN

(a) $x = 4.5$ in.

(b) $x = 15$ in.

(a) FREE BODY: COLLAR A

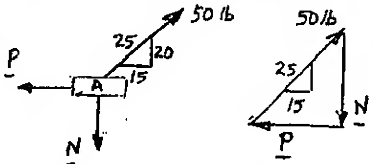


FORCE TRIANGLE

$$\frac{P}{4.5} = \frac{50\text{ lb}}{20.5}$$

$P = 10.98$ lb ◀

(b) FREE BODY: COLLAR A

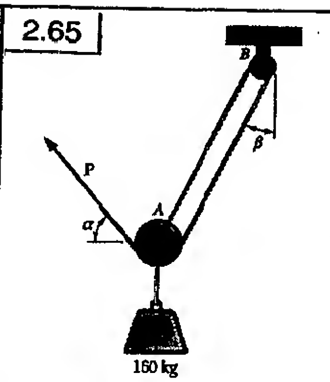


FORCE TRIANGLE

$$\frac{P}{15} = \frac{50\text{ lb}}{25}$$

$P = 30.0$ lb ◀

2.65

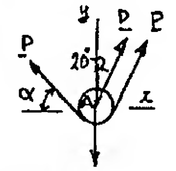


GIVEN: $\beta = 20°$

ALSO: T IS THE SAME IN ALL PORTIONS OF THE ROPE.

FIND: MAGNITUDE AND DIRECTION OF P

FREE BODY: PULLEY A



$W = (160\text{ kg})(9.81\text{ m/s}^2) = 1569.6$ N

$\Sigma F_x = 0$: $2P\sin 20° - P\cos\alpha = 0$

$\cos\alpha = 2\sin 20°$ $\quad \alpha = \pm 46.84°$

FOR $\alpha = +46.84°$:

$\Sigma F_y = 0$: $2P\cos 20° + P\sin 46.84° - 1569.6\text{ N} = 0$

$$P = \frac{1569.6\text{ N}}{2.609} = 601.6\text{ N}$$

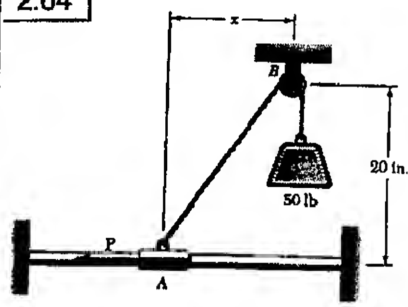
$P = 602$ N ∠ 46.8° ◀

FOR $\alpha = -46.84°$: $\Sigma F_y = 0$: $2P\cos 20° + P\sin(-46.84°) - 1569.6\text{N} = 0$

$$P = \frac{1569.6\text{ N}}{1.1499} = 1364.9\text{ N}$$

$P = 1365$ N ∠ 46.8° ◀

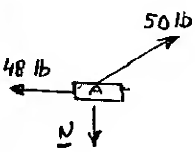
2.64



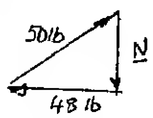
GIVEN: SYSTEM SHOWN IS IN EQUILIBRIUM WITH $P = 48$ lb.

FIND: x

FREE BODY: COLLAR A



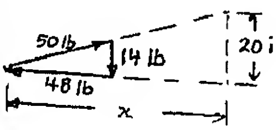
FORCE TRIANGLE



$N^2 = (50)^2 - (48)^2 = 196$

$N = 14.00$ lb

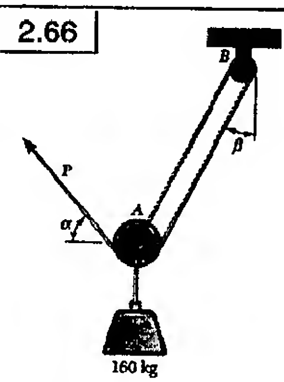
SIMILAR TRIANGLES:



$$\frac{x}{20\text{ in.}} = \frac{48\text{ lb}}{14\text{ lb}}$$

$x = 68.6$ in. ◀

2.66



GIVEN: $\alpha = 40°$

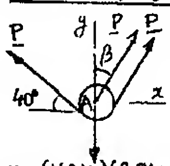
ALSO: T IS THE SAME IN ALL PORTIONS OF THE ROPE.

FIND:

(a) ANGLE β

(b) MAGNITUDE OF P.

FREE BODY: PULLEY A



$W = (160\text{ kg})(9.81\text{ m/s}^2) = 1569.6$ N

$\Sigma F_x = 0$; $2P\sin\beta - P\cos 40° = 0$

(a) $\sin\beta = \frac{1}{2}\cos 40°$ $\quad \beta = 22.52°$

$\beta = 22.5°$ ◀

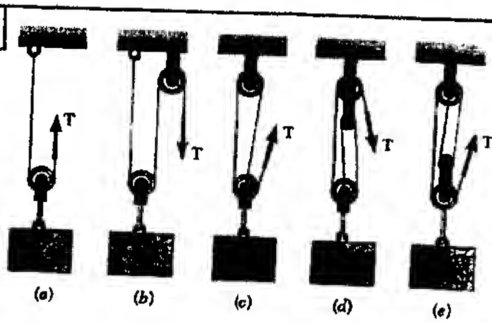
(b) $\Sigma F_y = 0$:

$P\sin 40° + 2P\cos 22.52° - 1569.6\text{ N} = 0$

$$P = \frac{1569.6\text{ N}}{2.4903} = 630.3\text{ N}$$

$P = 630$ N ◀

2.67

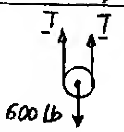


GIVEN: 600-lb CRATE SUPPORTED BY ONE OF THE ROPE-AND-PULLEY ARRANGEMENTS SHOWN.

FIND: TENSION IN THE ROPE FOR EACH ARRANGEMENT.

FREE-BODY : PULLEY

(a)

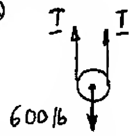


$\Sigma F_y = 0:$

$2T - 600\ \text{lb} = 0$

$T = 300\ \text{lb}$ ◀

(b)

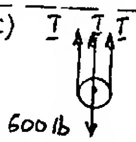


$\Sigma F_y = 0:$

$2T - 600\ \text{lb} = 0$

$T = 300\ \text{lb}$ ◀

(c)

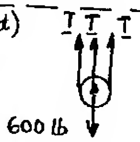


$\Sigma F_y = 0:$

$3T - 600\ \text{lb} = 0$

$T = 200\ \text{lb}$ ◀

(d)

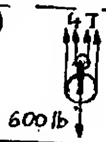


$\Sigma F_y = 0:$

$3T - 600\ \text{lb} = 0$

$T = 200\ \text{lb}$ ◀

(e)



$\Sigma F_y = 0:$

$4T - 600\ \text{lb} = 0$

$T = 150\ \text{lb}$ ◀

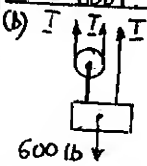
2.68

GIVEN: ASSUME THAT IN PARTS b AND d OF PROB. 2.67 THE FREE END OF THE ROPE IS ATTACHED TO THE CRATE.

FIND: TENSION IN ROPE.

FREE BODY: PULLEY AND CRATE

(b)

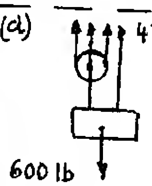


$\Sigma F_y = 0:$

$3T - 600\ \text{lb} = 0$

$T = 200\ \text{lb}$ ◀

(d)



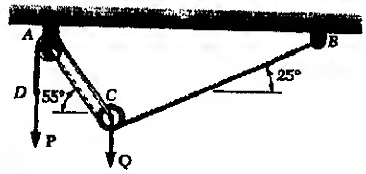
$\Sigma F_y = 0:$

$4T - 600\ \text{lb} = 0$

$T = 150\ \text{lb}$ ◀

2.69

PULLEY C CAN ROLL ON CABLE ACB.



GIVEN:

$P = 750\ \text{N}$

FIND:

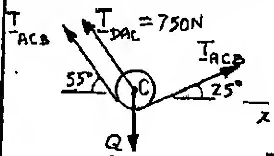
(a) T_{ACB}

(b) Q

NOTE: (1) THE TENSION IS THE SAME IN BOTH PORTIONS OF CABLE ACB.

(2) THE TENSION IN CABLE DAC IS EQUAL TO P.

FREE BODY: PULLEY C



(a) $\Sigma F_x = 0:$

$T_{ACB} \cos 25° - T_{ACB} \cos 55° - (750\ \text{N}) \cos 55° = 0$

$T_{ACB}(\cos 25° - \cos 55°) = 750 \cos 55°$

$T_{ACB} = (750\ \text{N}) \dfrac{0.5736}{0.3327}$ $\quad T_{ACB} = 1293\ \text{N}$ ◀

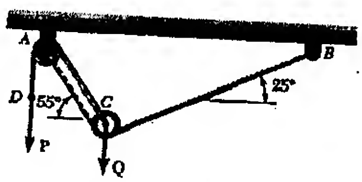
(b) $\Sigma F_y = 0: \ (T_{ACB} + T_{DAC}) \sin 55° + T_{ACB} \sin 25° - Q = 0$

$Q = (1293\ \text{N} + 750\ \text{N}) \sin 55° + (1293\ \text{N}) \sin 25° = 2220.0\ \text{N}$

$Q = 2220\ \text{N}$ ◀

2.70

PULLEY C CAN ROLL ON CABLE ACB.



GIVEN:

$Q = 1800\ \text{N}$

FIND:

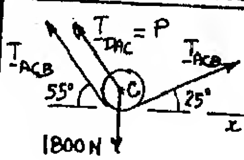
(a) T_{ACB}

(b) P

NOTE: (1) THE TENSION IS THE SAME IN BOTH PORTIONS OF CABLE ACB.

(2) THE TENSION IN CABLE DAC IS EQUAL TO P.

FREE BODY: PULLEY C



$\Sigma F_x = 0:$

$T_{ACB} \cos 25° - T_{ACB} \cos 55° - P \cos 55° = 0$

$P = T_{ACB} \dfrac{\cos 25° - \cos 55°}{\cos 55°}$

$P = 0.5801\ T_{ACB}$ $\quad (1)$

$\Sigma F_y = 0: \ (T_{ACB} + P) \sin 55° + T_{ACB} \sin 25° - 1800\ \text{N} = 0$ $\quad (2)$

(a) SUBSTITUTE FOR P FROM (1) INTO (2):

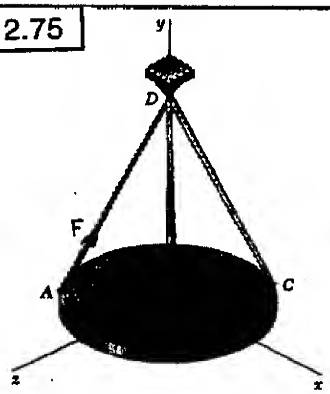
$(1.5801 \sin 55° + \sin 25°)\, T_{ACB} = 1800\ \text{N}$

$T_{ACB} = 1048.4\ \text{N}$ $\quad T_{ACB} = 1048\ \text{N}$ ◀

(b) CARRY INTO (1):

$P = 0.5801\,(1048.4\ \text{N})$ $\quad P = 608\ \text{N}$ ◀

2.75



GIVEN:
(1) WIRES FORM 30° ANGLES WITH VERTICAL
(2) FORCE EXERTED BY AD ON PLATE HAS COMPONENT $F_x = 110.3$ N.

FIND:
(a) TENSION IN AD
(b) ANGLES $\theta_x, \theta_y, \theta_z$ THAT FORCE EXERTED AT A FORMS WITH THE COORDINATE AXES.

(a) $F_x = F\sin 30°\sin 50° = 110.3\text{ N}$ (GIVEN)

$F = \dfrac{110.3\text{ N}}{\sin 30°\sin 50°} = 287.97\text{ N}$ $\qquad F = 288\text{ N}$ ◀

(b) $\cos\theta_x = \dfrac{F_x}{F} = \dfrac{110.3\text{ N}}{287.97\text{ N}} = 0.3830$ $\qquad \theta_x = 67.5°$ ◀

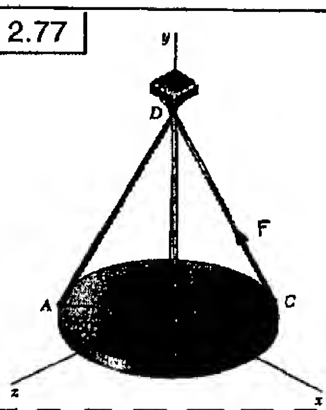
$F_y = F\cos 30°$, $\cos\theta_y = \dfrac{F_y}{F} = \cos 30°$. Thus: $\theta_y = 30.0°$ ◀

$F_z = -F\sin 30°\cos 50°$
$= -(287.97\text{ N})\sin 30°\cos 50° = -92.552\text{ N}$

$\cos\theta_z = \dfrac{F_z}{F} = \dfrac{-92.552\text{ N}}{287.97\text{ N}} = -0.3214$

$\theta_z = 108.7°$ ◀

2.77



GIVEN:
(1) WIRES FORM 30° ANGLES WITH VERTICAL
(2) TENSION IN CD IS 60 lb.

FIND:
(a) COMPONENTS OF FORCE EXERTED AT C.
(b) ANGLES $\theta_x, \theta_y, \theta_z$ THAT FORCE FORMS WITH THE COORDINATE AXES.

(a) $F_x = -(60\text{ lb})\sin 30°\cos 60°$ $\qquad F_x = -15.00\text{ lb}$ ◀

$F_y = (60\text{ lb})\cos 30° = 51.96\text{ lb}$ $\qquad F_y = +52.0\text{ lb}$ ◀

$F_z = (60\text{ lb})\sin 30°\sin 60° = 25.98\text{ lb}$ $\qquad F_z = +26.0\text{ lb}$ ◀

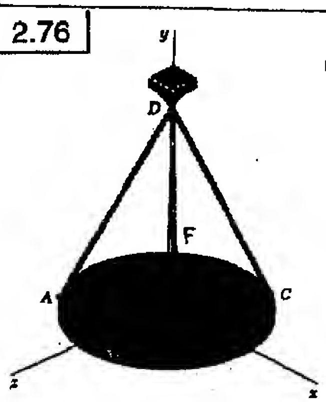
(b) $\cos\theta_x = \dfrac{F_x}{F} = \dfrac{-15.00\text{ lb}}{60\text{ lb}} = -0.2500,$ $\qquad \theta_x = 104.5°$ ◀

$\cos\theta_y = \dfrac{F_y}{F} = \dfrac{+51.96\text{ lb}}{60\text{ lb}} = 0.8660,$ $\qquad \theta_y = 30.0°$ ◀

$\cos\theta_z = \dfrac{F_z}{F} = \dfrac{+25.98\text{ lb}}{60\text{ lb}} = 0.9330$ $\qquad \theta_z = 64.3°$ ◀

NOTE: VALUE OBTAINED FOR θ_y CHECKS WITH GIVEN DATA.

2.76



GIVEN:
(1) WIRES FORM 30° ANGLES WITH VERTICAL
(2) FORCE EXERTED BY BD ON PLATE HAS COMPONENT $F_z = -32.14$ N.

FIND:
(a) TENSION IN BD
(b) ANGLES $\theta_x, \theta_y, \theta_z$ THAT FORCE EXERTED AT B FORMS WITH THE COORDINATE AXES.

(a) $F_z = -F\sin 30°\cos 40° = -32.14\text{ N}$ (GIVEN)

$F = \dfrac{32.14\text{ N}}{\sin 30°\sin 40°} = 100.0\text{ N}$ $\qquad F = 100.0\text{ N}$ ◀

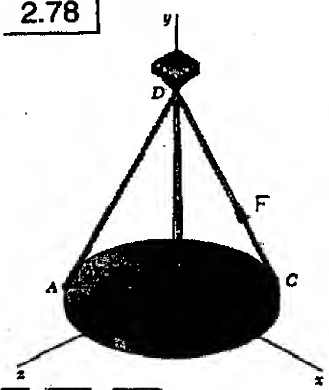
(b) $F_x = -F\sin 30°\cos 40°$
$= -(100.0\text{ N})\sin 30°\cos 40° = -38.30\text{ N}$

$\cos\theta_x = \dfrac{F_x}{F} = \dfrac{-38.30\text{ N}}{100.0\text{ N}} = -0.3830$ $\qquad \theta_x = 112.5°$ ◀

$F_y = F\cos 30°$, $\cos\theta_y = \dfrac{F_y}{F} = \cos 30°$, Thus: $\theta_y = 30.0°$ ◀

$\cos\theta_z = \dfrac{F_z}{F} = \dfrac{-32.14\text{ N}}{100.0\text{ N}} = -0.3214$ $\qquad \theta_z = 108.7°$ ◀

2.78



GIVEN:
(1) WIRES FORM 30° ANGLES WITH VERTICAL.
(2) FORCE EXERTED BY CD ON PLATE HAS COMPONENT $F_x = -20.0$ lb

FIND:
(a) TENSION IN CD.
(b) ANGLES $\theta_x, \theta_y, \theta_z$ THAT FORCE EXERTED AT C FORMS WITH THE COORDINATE AXES.

(a) $F_x = -F\sin 30°\cos 60° = -20.0\text{ lb}$ (GIVEN)

$F = \dfrac{20.0\text{ lb}}{\sin 30°\cos 60°} = 80.0\text{ lb}$ $\qquad F = 80.0\text{ lb}$ ◀

(b) $\cos\theta_x = \dfrac{F_x}{F} = \dfrac{-20.0\text{ lb}}{80.0\text{ lb}} = -0.2500$ $\qquad \theta_x = 104.5°$ ◀

$F_y = F\cos 30°$, $\cos\theta_y = \dfrac{F_y}{F} = \cos 30°$. Thus: $\theta_y = 30.0°$ ◀

$F_z = F\sin 30°\sin 60°$
$= (80.0\text{ lb})\sin 30°\sin 60° = 34.641\text{ lb}$

$\cos\theta_z = \dfrac{F_z}{F} = \dfrac{34.641\text{ lb}}{80.0\text{ lb}} = 0.4330$ $\qquad \theta_z = 64.3°$ ◀

2.79 GIVEN: $\mathbf{F} = (260\text{ N})\mathbf{i} - (320\text{ N})\mathbf{j} + (800\text{ N})\mathbf{k}$

FIND: MAGNITUDE AND DIRECTION OF $\mathbf{F}$

$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(260)^2 + (320)^2 + (800)^2}$, $F = 900\text{ N}$ ◀

$\cos\theta_x = \dfrac{F_x}{F} = \dfrac{260\text{ N}}{900\text{ N}} = 0.2889$ $\theta_x = 73.2°$ ◀

$\cos\theta_y = \dfrac{F_y}{F} = \dfrac{-320\text{ N}}{900\text{ N}} = -0.3556$ $\theta_y = 110.8°$ ◀

$\cos\theta_z = \dfrac{F_z}{F} = \dfrac{800\text{ N}}{900\text{ N}} = 0.8889$ $\theta_z = 27.3°$ ◀

2.80 GIVEN: $\mathbf{F} = (320\text{ N})\mathbf{i} + (400\text{ N})\mathbf{j} - (250\text{ N})\mathbf{k}$

FIND: MAGNITUDE AND DIRECTION OF $\mathbf{F}$.

$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(320)^2 + (400)^2 + (250)^2}$, $F = 570\text{ N}$ ◀

$\cos\theta_x = \dfrac{F_x}{F} = \dfrac{320\text{ N}}{570\text{ N}} = 0.5614$ $\theta_x = 55.8°$ ◀

$\cos\theta_y = \dfrac{F_y}{F} = \dfrac{400\text{ N}}{570\text{ N}} = 0.7018$ $\theta_y = 45.4°$ ◀

$\cos\theta_z = \dfrac{F_z}{F} = \dfrac{-250\text{ N}}{570\text{ N}} = -0.4386$ $\theta_z = 116.0°$ ◀

2.81 GIVEN: FORCE WITH $\theta_x = 69.3°$, $\theta_z = 57.9°$ AND $F_y = -174.0$ lb.

FIND: (a) θ_y, (b) F_x, F_z, AND F.

(a) TO DETERMINE θ_y WE USE THE RELATION

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$ $\cos^2\theta_y = 1 - \cos^2\theta_x - \cos^2\theta_z$

SINCE $F_y < 0$, WE MUST HAVE $\cos\theta_y < 0$, THUS:

$\cos\theta_y = -\sqrt{1 - \cos^2 69.3° - \cos^2 57.9°} = -0.7699$, $\theta_y = 140.3°$ ◀

(b) $F = \dfrac{F_y}{\cos\theta_y} = \dfrac{-174.0\text{ lb}}{-0.7699} = 226.0\text{ lb}$ $F = 226\text{ lb}$ ◀

$F_x = F\cos\theta_x = (226.0\text{ lb})\cos 69.3°$ $F_x = 79.9\text{ lb}$ ◀

$F_z = F\cos\theta_z = (226.0\text{ lb})\cos 57.9°$ $F_z = 120.1\text{ lb}$ ◀

2.82 GIVEN: FORCE WITH $\theta_x = 70.9°$, $\theta_y = 144.9°$ AND $F_z = -52.0$ lb

FIND: (a) θ_z, (b) F_x, F_y, AND F.

(a) TO DETERMINE θ_z WE USE THE RELATION

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$, $\cos^2\theta_z = 1 - \cos^2\theta_x - \cos^2\theta_y$

SINCE $F_z < 0$, WE MUST HAVE $\cos\theta_z < 0$. THUS:

$\cos\theta_z = -\sqrt{1 - \cos^2 70.9° - \cos^2 144.9°} = -0.4728$, $\theta_z = 118.2°$ ◀

(b) $F = \dfrac{F_z}{\cos\theta_z} = \dfrac{-52.0\text{ lb}}{-0.4728} = 110.0\text{ lb}$ $F = 110\text{ lb}$ ◀

$F_x = F\cos\theta_x = (110.0\text{ lb})\cos 70.9°$ $F_x = 36.0\text{ lb}$ ◀

$F_y = F\cos\theta_y = (110.0\text{ lb})\cos 144.9°$ $F_y = -90.0\text{ lb}$ ◀

2.83 GIVEN: $F = 230\text{ N}$, $\theta_x = 32.5°$, $F_y = -60\text{ N}$, $F_z > 0$

FIND: (a) F_x AND F_z, (b) θ_y AND θ_z

(a) $F_x = F\cos\theta_x = (230\text{ N})\cos 32.5°$ $F_x = 194.0\text{ N}$ ◀

$F^2 = F_x^2 + F_y^2 + F_z^2$: $(230\text{ N})^2 = (194.0\text{ N})^2 + (-60\text{ N})^2 + F_z^2$

$F_z = +\sqrt{(230)^2 - (194)^2 - (60)^2}$ $F_z = +108.0\text{ N}$ ◀

(b) $\cos\theta_y = F_y/F = -60/230 = -0.2609$ $\theta_y = 105.1°$ ◀

$\cos\theta_z = F_z/F = 108/230 = +0.4696$ $\theta_z = 62.0°$ ◀

2.84 GIVEN: $F = 210\text{ N}$, $F_x = 80\text{ N}$, $\theta_z = 151.2°$, $F_y < 0$

FIND: (a) F_y AND F_z, (b) θ_x AND θ_y.

(a) $F_z = F\cos\theta_z = (210\text{ N})\cos 151.2°$ $F_z = -184.0\text{ N}$ ◀

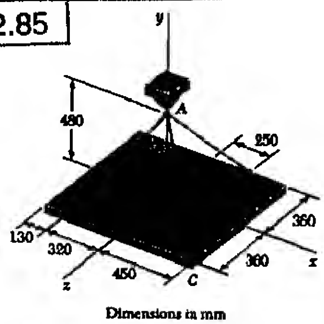
$F^2 = F_x^2 + F_y^2 + F_z^2$: $(210\text{ N})^2 = (80\text{ N})^2 + F_y^2 + (-184.0\text{ N})^2$

$F_y = -\sqrt{(210)^2 - (80)^2 - (184.0)^2}$ $F_y = -62.0\text{ N}$ ◀

(b) $\cos\theta_x = F_x/F = 80/210 = +0.3810$ $\theta_x = 67.6°$ ◀

$\cos\theta_y = F_y/F = -62.0/210 = -$ $\theta_y = 107.2°$ ◀

2.85



GIVEN: TENSION IN CABLE AB IS 408 N.

FIND: COMPONENTS OF FORCE EXERTED ON PLATE AT B.

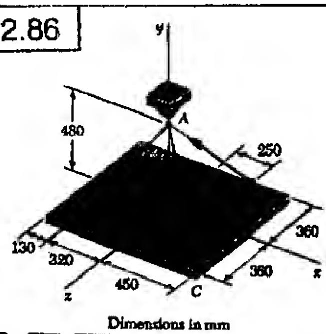
$\overrightarrow{BA} = 320\mathbf{i} + 480\mathbf{j} - 360\mathbf{k}$ $BA = \sqrt{(320)^2 + (480)^2 + (360)^2} = 680$

$\mathbf{F} = F\boldsymbol{\lambda}_{BA} = F\dfrac{\overrightarrow{BA}}{BA} = \dfrac{408\text{ N}}{680\text{ mm}}[(320\text{ mm})\mathbf{i} + (480\text{ mm})\mathbf{j} - (360\text{ mm})\mathbf{k}]$

$\mathbf{F} = (192\text{ N})\mathbf{i} + (288\text{ N})\mathbf{j} - (216\text{ N})\mathbf{k}$

$F_x = +192\text{ N}$, $F_y = +288\text{ N}$, $F_z = -216\text{ N}$ ◀

2.86



GIVEN: TENSION IN CABLE AD IS 429 N.

FIND: COMPONENTS OF FORCE EXERTED ON PLATE AT D.

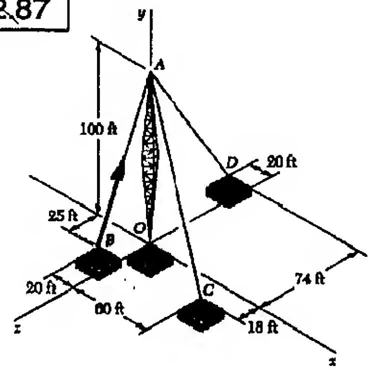
$\overrightarrow{DA} = -250\mathbf{i} + 480\mathbf{j} + 360\mathbf{k}$ $DA = \sqrt{(250)^2 + (480)^2 + (360)^2} = 650$

$\mathbf{F} = F\boldsymbol{\lambda}_{DA} = F\dfrac{\overrightarrow{DA}}{DA} = \dfrac{429\text{ N}}{650\text{ mm}}[(-250\text{ mm})\mathbf{i} + (480\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}]$

$\mathbf{F} = -(165\text{ N})\mathbf{i} + (316.8\text{ N})\mathbf{j} + (237.6\text{ N})\mathbf{k}$

$F_x = -165\text{ N}$, $F_y = +317\text{ N}$, $F_z = +238\text{ N}$ ◀

2.87



GIVEN: TENSION IN WIRE AB IS 525 lb.

FIND: COMPONENTS OF FORCE EXERTED ON BOLT B BY WIRE AB.

$\overrightarrow{BA} = (20\text{ ft})\underline{i} + (100\text{ ft})\underline{j} - (25\text{ ft})\underline{k}$ $\quad BA = \sqrt{(20)^2 + (100)^2 + (25)^2}$

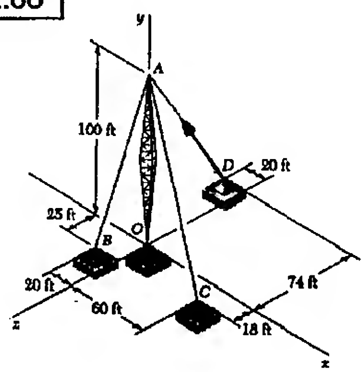
$BA = 105\text{ ft}$

$\underline{F} = F\underline{\lambda}_{BA} = F\dfrac{\overrightarrow{BA}}{BA} = \dfrac{525\text{ lb}}{105\text{ ft}}[(20\text{ ft})\underline{i} + (100\text{ ft})\underline{j} - (25\text{ ft})\underline{k}]$

$\underline{F} = (100\text{ lb})\underline{i} + (500\text{ lb})\underline{j} - (125\text{ lb})\underline{k}$

$F_x = +100\text{ lb}, \quad F_y = +500\text{ lb}, \quad F_z = -125\text{ lb}$ ◀

2.88



GIVEN: TENSION IN WIRE AD IS 315 lb.

FIND: COMPONENTS OF FORCE EXERTED ON BOLT D BY WIRE AD.

$\overrightarrow{DA} = (20\text{ ft})\underline{i} + (100\text{ ft})\underline{j} + (74\text{ ft})\underline{k}$

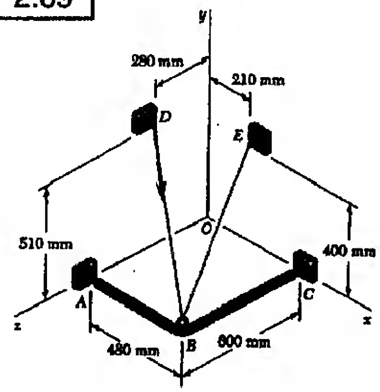
$DA = \sqrt{(20)^2 + (100)^2 + (74)^2} = 126\text{ ft}$

$\underline{F} = F\lambda_{DA} = F\dfrac{\overrightarrow{DA}}{DA} = \dfrac{315\text{ lb}}{126\text{ ft}}[(20\text{ ft})\underline{i} + (100\text{ ft})\underline{j} + (74\text{ ft})\underline{k}]$

$\underline{F} = (50\text{ lb})\underline{i} + (250\text{ lb})\underline{j} + (185\text{ lb})\underline{k}$

$F_x = +50\text{ lb}, \quad F_y = +250\text{ lb}, \quad F_z = +185\text{ lb}$ ◀

2.89



GIVEN: TENSION IN CABLE DBE IS 385 N.

FIND: COMPONENTS OF FORCE EXERTED BY CABLE ON D.

$\overrightarrow{DB} = (480\text{ mm})\underline{i} - (510\text{ mm})\underline{j} + (320\text{ mm})\underline{k}$

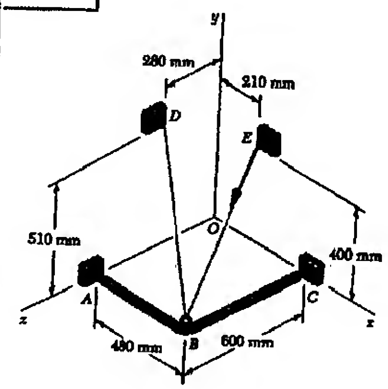
$DB = \sqrt{(480)^2 + (510)^2 + (320)^2} = 770\text{ mm}$

$\underline{F} = F\underline{\lambda}_{DB} = F\dfrac{\overrightarrow{DB}}{DB} = \dfrac{385\text{ N}}{770\text{ mm}}[(480\text{ mm})\underline{i} - (510\text{ mm})\underline{j} + (320\text{ mm})\underline{k}]$

$\underline{F} = (240\text{ N})\underline{i} - (255\text{ N})\underline{j} + (160\text{ N})\underline{k}$

$F_x = +240\text{ N}, \quad F_y = -255\text{ N}, \quad F_z = +160\text{ N}$ ◀

2.90



GIVEN: TENSION IN CABLE DBE IS 385 N.

FIND: COMPONENTS OF FORCE EXERTED BY CABLE ON E.

$\overrightarrow{EB} = (270\text{ mm})\underline{i} - (400\text{ mm})\underline{j} + (600\text{ mm})\underline{k}$

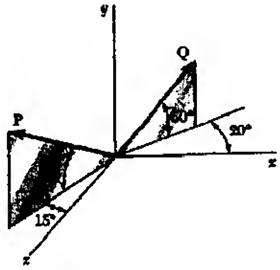
$EB = \sqrt{(270)^2 + (400)^2 + (600)^2} = 770\text{ mm}$

$\underline{F} = F\underline{\lambda}_{EB} = F\dfrac{\overrightarrow{EB}}{EB} = \dfrac{385\text{ N}}{770\text{ mm}}[(270\text{ mm})\underline{i} - (400\text{ mm})\underline{j} + (600\text{ mm})\underline{k}]$

$\underline{F} = (135\text{ N})\underline{i} - (200\text{ N})\underline{j} + (300\text{ N})\underline{k}$

$F_x = +135\text{ N}, \quad F_y = -200\text{ N}, \quad F_z = +300\text{ N}$ ◀

2.91



GIVEN: $P = 300\ N$, $Q = 400\ N$

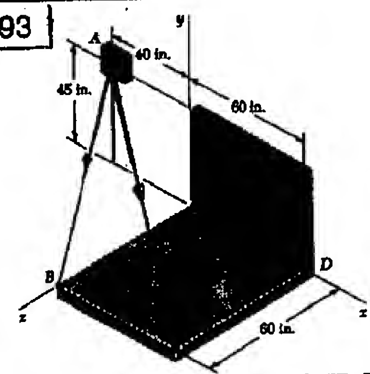
FIND: MAGNITUDE AND DIRECTION OF RESULTANT OF $\underline{P}$ AND $\underline{Q}$.

FORCE $\underline{P}$:
$P_x = -(300\ N)\cos 30^\circ \sin 15^\circ = -67.24\ N$
$P_y = +(300\ N)\sin 30^\circ = +150.00\ N$
$P_z = +(300\ N)\cos 30^\circ \cos 15^\circ = +250.95\ N$
$\underline{P} = -(67.24\ N)\underline{i} + (150.00\ N)\underline{j} + (250.95\ N)\underline{k}$

FORCE $\underline{Q}$:
$Q_x = +(400N)\cos 50^\circ \cos 20^\circ = +241.61\ N$
$Q_y = +(400N)\sin 50^\circ = +306.42\ N$
$Q_z = -(400N)\cos 50^\circ \sin 20^\circ = -87.94\ N$
$\underline{Q} = +(241.61\ N)\underline{i} + (306.42\ N)\underline{j} - (87.94\ N)\underline{k}$

RESULTANT:
$\underline{R} = \underline{P} + \underline{Q} = (174.37\ N)\underline{i} + (456.42\ N)\underline{j} + (163.01\ N)\underline{k}$
$R = \sqrt{(174.37)^2 + (456.42)^2 + (163.01)^2} = 515.07\ N$, $R = 515\ N$
$\cos\theta_x = R_x/R = (174.37\ N)/(515.07\ N) = 0.3385$, $\theta_x = 70.2^\circ$
$\cos\theta_y = R_y/R = (456.42\ N)/(515.07\ N) = 0.8861$, $\theta_y = 27.6^\circ$
$\cos\theta_z = R_z/R = (163.01\ N)/(515.07\ N) = 0.3165$, $\theta_z = 71.5^\circ$

2.93



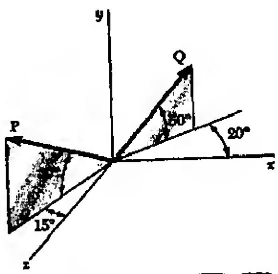
GIVEN: $T_{AB} = 425\ lb$, $T_{AC} = 510\ lb$

FIND: MAGNITUDE AND DIRECTION OF RESULTANT OF FORCES AT A.

$\overrightarrow{AB} = (40\ in.)\underline{i} - (45\ in.)\underline{j} + (60\ in.)\underline{k}$ $\quad AB = \sqrt{(40)^2 + (45)^2 + (60)^2} = 85\ in.$
$\overrightarrow{AC} = (100\ in.)\underline{i} - (45\ in.)\underline{j} + (60\ in.)\underline{k}$ $\quad AC = \sqrt{(100)^2 + (45)^2 + (60)^2} = 125\ in.$

$\underline{F}_{AB} = F_{AB}\underline{\lambda}_{AB} = F_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{425\ lb}{85\ in.}[(40\ in.)\underline{i} - (45\ in.)\underline{j} + (60\ in.)\underline{k}]$
$\underline{F}_{AB} = (200\ lb)\underline{i} - (225\ lb)\underline{j} + (300\ lb)\underline{k}$
$\underline{F}_{AC} = F_{AC}\underline{\lambda}_{AC} = F_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{510\ lb}{125\ in.}[(100\ in.)\underline{i} - (45\ in.)\underline{j} + (60\ in.)\underline{k}]$
$\underline{F}_{AC} = (408\ lb)\underline{i} - (183.6\ lb)\underline{j} + (244.8\ lb)\underline{k}$
$\underline{R} = \underline{F}_{AB} + \underline{F}_{AC} = (608\ lb)\underline{i} - (408.6\ lb)\underline{j} + (544.8\ lb)\underline{k}$, $R = 912.92\ lb$
$R = 913\ lb$
$\cos\theta_x = R_x/R = 608/912.92 = 0.6660$ $\quad \theta_x = 48.2^\circ$
$\cos\theta_y = R_y/R = -408.6/912.92 = -0.4476$ $\quad \theta_y = 116.6^\circ$
$\cos\theta_z = R_z/R = 544.8/912.92 = 0.5968$ $\quad \theta_z = 53.4^\circ$

2.92



GIVEN: $P = 400\ N$, $Q = 300\ N$

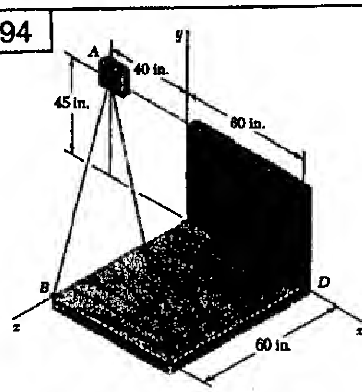
FIND: MAGNITUDE AND DIRECTION OF RESULTANT OF $\underline{P}$ AND $\underline{Q}$.

FORCE $\underline{P}$:
$P_x = -(400\ N)\cos 30^\circ \sin 15^\circ = -89.66\ N$
$P_y = +(400\ N)\sin 30^\circ = +200.00\ N$
$P_z = +(400\ N)\cos 30^\circ \cos 15^\circ = +334.61\ N$
$\underline{P} = -(89.66\ N)\underline{i} + (200.00\ N)\underline{j} + (334.61\ N)\underline{k}$

FORCE $\underline{Q}$:
$Q_x = +(300\ N)\cos 50^\circ \cos 20^\circ = +181.21\ N$
$Q_y = +(300\ N)\sin 50^\circ = +229.81\ N$
$Q_z = -(300\ N)\cos 50^\circ \sin 20^\circ = -65.95\ N$
$\underline{Q} = (181.21\ N)\underline{i} + (229.81\ N)\underline{j} - (65.95\ N)\underline{k}$

RESULTANT:
$\underline{R} = \underline{P} + \underline{Q} = (91.55\ N)\underline{i} + (429.81\ N)\underline{j} + (268.66\ N)\underline{k}$
$R = \sqrt{(91.55)^2 + (429.81)^2 + (268.66)^2} = 515.07\ N$, $R = 515\ N$
$\cos\theta_x = R_x/R = (91.55\ N)/(515.07\ N) = 0.1777$ $\quad \theta_x = 79.8^\circ$
$\cos\theta_y = R_y/R = (429.81\ N)/(515.07\ N) = 0.8345$ $\quad \theta_y = 33.4^\circ$
$\cos\theta_z = R_z/R = (268.66\ N)/(515.07\ N) = 0.5216$ $\quad \theta_z = 58.6^\circ$

2.94



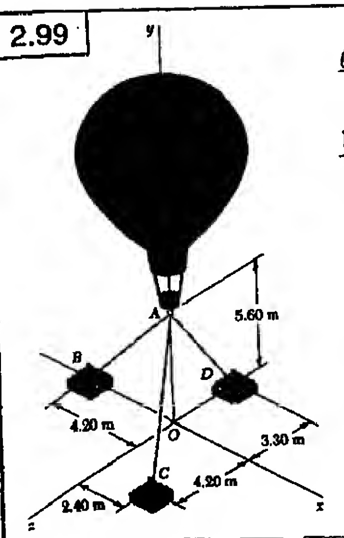
GIVEN: $T_{AB} = 510\ lb$, $T_{AC} = 425\ lb$

FIND: MAGNITUDE AND DIRECTION OF RESULTANT OF FORCES AT A.

$\overrightarrow{AB} = (40\ in.)\underline{i} - (45\ in.)\underline{j} + (60\ in.)\underline{k}$ $\quad AB = \sqrt{(40)^2 + (45)^2 + (60)^2} = 85\ in.$
$\overrightarrow{AC} = (100\ in.)\underline{i} - (45\ in.)\underline{j} + (60\ in.)\underline{k}$ $\quad AC = \sqrt{(100)^2 + (45)^2 + (60)^2} = 125\ in.$

$\underline{F}_{AB} = F_{AB}\underline{\lambda}_{AB} = F_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{510\ lb}{85\ in.}[(40\ in.)\underline{i} - (45\ in.)\underline{j} + (60\ in.)\underline{k}]$
$\underline{F}_{AB} = (240\ lb)\underline{i} - (270\ lb)\underline{j} + (360\ lb)\underline{k}$
$\underline{F}_{AC} = F_{AC}\underline{\lambda}_{AC} = F_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{425\ lb}{125\ in.}[(100\ in.)\underline{i} - (45\ in.)\underline{j} + (60\ in.)\underline{k}]$
$\underline{F}_{AC} = (340\ lb)\underline{i} - (153\ lb)\underline{j} + (204\ lb)\underline{k}$
$\underline{R} = \underline{F}_{AB} + \underline{F}_{AC} = (580\ lb)\underline{i} - (423\ lb)\underline{j} + (564\ lb)\underline{k}$, $R = 912.92\ lb$
$R = 913\ lb$
$\cos\theta_x = R_x/R = 580/912.92 = 0.6353$ $\quad \theta_x = 50.6^\circ$
$\cos\theta_y = R_y/R = -423/912.92 = -0.4633$ $\quad \theta_y = 117.6^\circ$
$\cos\theta_z = R_z/R = 564/912.92 = 0.6178$ $\quad \theta_z = 51.8^\circ$

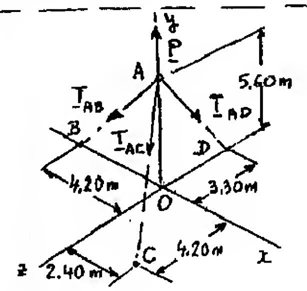
2.99



GIVEN: $T_{AB} = 259$ N

FIND: VERTICAL FORCE $\underline{P}$ EXERTED AT A BY THE BALLOON.

FIG. P 2.99, P 2.100, P 2.101, AND P 2.102



FREE BODY: A

FORCES APPLIED AT A ARE $\underline{T}_{AB}$, $\underline{T}_{AC}$, $\underline{T}_{AD}$, AND $\underline{P}$, WHERE $\underline{P} = P\underline{j}$. TO EXPRESS THE OTHER FORCES IN TERMS OF THE UNIT VECTORS, WE WRITE

$\vec{AB} = -(4.20\text{m})\underline{i} - (5.60\text{m})\underline{j}$ $\quad AB = 7.00$ m

$\vec{AC} = (2.40\text{m})\underline{i} - (5.60\text{m})\underline{j} + (4.20\text{m})\underline{k}$, $\quad AC = 7.40$ m

$\vec{AD} = -(5.60\text{m})\underline{j} - (3.30\text{m})\underline{k}$, $\quad AD = 6.50$ m

$$\underline{T}_{AB} = T_{AB}\underline{\lambda}_{AB} = T_{AB}\frac{\vec{AB}}{AB} = (-0.6\,\underline{i} - 0.8\,\underline{j})T_{AB}$$

$$\underline{T}_{AC} = T_{AC}\underline{\lambda}_{AC} = T_{AC}\frac{\vec{AC}}{AC} = \left(\frac{24}{74}\underline{i} - \frac{56}{74}\underline{j} + \frac{42}{74}\underline{k}\right)T_{AC}$$

$$\underline{T}_{AD} = T_{AD}\underline{\lambda}_{AD} = T_{AD}\frac{\vec{AD}}{AD} = \left(-\frac{56}{65}\underline{j} - \frac{33}{65}\underline{k}\right)T_{AD}$$

EQUILIBRIUM CONDITION:

$$\Sigma \underline{F} = 0: \quad \underline{T}_{AB} + \underline{T}_{AC} + \underline{T}_{AD} + P\underline{j} = 0$$

SUBSTITUTING THE EXPRESSIONS OBTAINED FOR $\underline{T}_{AB}$, $\underline{T}_{AC}$, AND $\underline{T}_{AD}$ AND FACTORING $\underline{i}$, $\underline{j}$, AND $\underline{k}$:

$$\left(-0.6\,T_{AB} + \frac{24}{74}T_{AC}\right)\underline{i} + \left(-0.8\,T_{AB} - \frac{56}{74}T_{AC} - \frac{56}{65}T_{AD} + P\right)\underline{j} + \left(\frac{42}{74}T_{AC} - \frac{33}{65}T_{AD}\right)\underline{k} = 0$$

EQUATING TO ZERO THE COEFFICIENTS OF $\underline{i}, \underline{j}, \underline{k}$:

$(\underline{i})$ $\quad -0.6\,T_{AB} + \frac{24}{74}T_{AC} = 0 \qquad (1)$

$(\underline{j})$ $\quad -0.8\,T_{AB} - \frac{56}{74}T_{AC} - \frac{56}{65}T_{AD} + P = 0 \qquad (2)$

$(\underline{k})$ $\quad \frac{42}{74}T_{AC} - \frac{33}{65}T_{AD} = 0 \qquad (3)$

CONTINUED

2.99 CONTINUED

MAKING $T_{AB} = 259$ N IN EQ.(1) AND SOLVING FOR T_{AC}:

$T_{AC} = \frac{74}{24}(0.6)(259\text{ N}) \qquad T_{AC} = 479.15$ N

CARRYING INTO EQ.(3) AND SOLVING FOR T_{AD}:

$T_{AD} = \frac{65}{33}\,\frac{42}{74}(479.15\text{ N}) \qquad T_{AD} = 535.66$ N

SUBSTITUTING FOR T_{AB}, T_{AC}, T_{AD} INTO (2) AND SOLVING FOR P:

$P = 0.8(259\text{N}) + \frac{56}{74}(479.15\text{N}) + \frac{56}{65}(535.66\text{N}) = 1031.3$ N

$\underline{P} = 1031\text{ N}\uparrow$

2.100

(SEE FIGURE ON LEFT)

GIVEN: $T_{AC} = 444$ N

FIND: VERTICAL FORCE $\underline{P}$ EXERTED AT A BY THE BALLOON

SEE LEFT-HAND COLUMN FOR DERIVATION OF EQS.(1),(2),(3)

MAKING $T_{AC} = 444$ N IN EQS. (1) AND (3) AND SOLVING FOR T_{AB} AND T_{AD}:

$T_{AB} = \frac{24}{0.6(74)}(444\text{ N})$ $\qquad T_{AD} = \frac{65}{33}\,\frac{42}{74}(444\text{ N})$

$T_{AB} = 240$ N $\qquad T_{AD} = 496.36$ N

SUBSTITUTING FOR T_{AB}, T_{AC}, T_{AD} INTO (2) AND SOLVING FOR P:

$P = 0.8(240\text{ N}) + \frac{56}{74}(444\text{ N}) + \frac{56}{65}(496.36\text{N}) = 955.6$ N

$\underline{P} = 956\text{ N}\uparrow$

2.101

(SEE FIGURE ON UPPER LEFT)

GIVEN: $T_{AD} = 481$ N

FIND: VERTICAL FORCE $\underline{P}$ EXERTED AT A BY THE BALLOON.

SEE LEFT-HAND COLUMN FOR DERIVATION OF EQS.(1),(2),(3).

MAKING $T_{AD} = 481$N IN EQ.(3) AND SOLVING FOR T_{AC}:

$T_{AC} = \frac{74}{42}\,\frac{33}{65}(481\text{ N}) \qquad T_{AC} = 430.26$ N

CARRYING INTO EQ.(1) AND SOLVING FOR T_{AB}:

$T_{AB} = \frac{24}{0.6(74)}(430.26\text{ N}) \qquad T_{AB} = 232.57$ N

SUBSTITUTING FOR T_{AB}, T_{AC}, T_{AD} INTO (2) AND SOLVING FOR P:

$P = 0.8(232.57\text{ N}) + \frac{56}{74}(430.26\text{N}) + \frac{56}{65}(481\text{N}) = 926.06$ N

$\underline{P} = 926\text{ N}\uparrow$

2.102

(SEE FIGURE ON UPPER LEFT)

GIVEN: BALLOON EXERTS FORCE $\underline{P} = 800\text{N}\uparrow$ AT A.

FIND: TENSION IN EACH CABLE

SEE LEFT-HAND COLUMN FOR DERIVATION OF EQS.(1),(2),(3).

FROM EQ.(1): $T_{AB} = \frac{24}{0.6(74)}T_{AC} \qquad T_{AB} = 0.54054\,T_{AC}$

FROM EQ.(3): $T_{AD} = \frac{65}{33}\,\frac{42}{74}T_{AC} \qquad T_{AD} = 1.1179\,T_{AC}$

SUBSTITUTE FOR T_{AB} AND T_{AD} INTO EQ.(2):

$-0.8(0.54054\,T_{AC}) - \frac{56}{74}T_{AC} - \frac{56}{65}(1.1179\,T_{AC}) + P = 0$

$2.1523\,T_{AC} = P \qquad T_{AC} = \frac{800\text{N}}{2.1523} \qquad T_{AC} = 371.69$ N

SUBSTITUTE INTO EXPRESSIONS FOR T_{AB} AND T_{AD}:

$T_{AB} = 0.54054(371.69\text{ N}) = 200.91$ N

$T_{AD} = 1.1179(371.69\text{ N}) = 415.51$ N

$T_{AB} = 201$ N, $T_{AC} = 372$ N, $T_{AD} = 416$ N

2.111 CONTINUED

WE REPEAT THE LAST EQS:

$$-160\text{ lb}+\frac{60}{118}T_{AC}-\frac{20}{126}T_{AD}=0 \qquad (1)$$

$$-800\text{ lb}-\frac{100}{118}T_{AC}-\frac{100}{126}T_{AD}+P=0 \qquad (2)$$

$$200\text{ lb}+\frac{18}{118}T_{AC}-\frac{74}{126}T_{AD}=0 \qquad (3)$$

MULTIPLY EQ.(1) BY −3, EQ. (3) BY 10, AND ADD:

$$2480\text{ lb}-\frac{680}{126}T_{AD}=0 \qquad T_{AD}=459.529\text{ lb}$$

SUBSTITUTE INTO (1) AND SOLVE FOR T_{AC}:

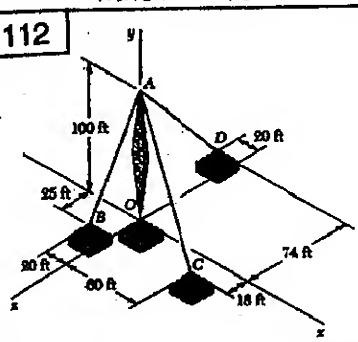
$$T_{AC}=\frac{118}{60}\left(160+\frac{20}{126}\times 459.529\right) \qquad T_{AC}=458.118\text{ lb}$$

SUBSTITUTE FOR THE TENSIONS IN (2) AND SOLVE FOR P:

$$P=800\text{ lb}+\frac{100}{118}(458.118\text{ lb})+\frac{100}{126}(459.529\text{ lb})=1552.94\text{ lb}$$

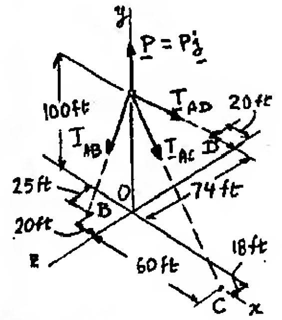
WEIGHT OF PLATE $= P = 1553$ lb ◀

2.112



GIVEN:
$T_{AC}=590$ lb

FIND:
VERTICAL FORCE $\underline{P}$ EXERTED BY TOWER ON PIN A.



FREE BODY: A

$\Sigma F=0$:

$$T_{AB}+T_{AC}+T_{AD}+P\underline{j}=0$$

$$\overrightarrow{AB}=-20\underline{i}-100\underline{j}+25\underline{k} \qquad AB=105\text{ ft}$$

$$\overrightarrow{AC}=60\underline{i}-100\underline{j}+18\underline{k} \qquad AC=118\text{ ft}$$

$$\overrightarrow{AD}=-20\underline{i}-100\underline{j}-74\underline{k} \qquad AD=126\text{ ft}$$

WE WRITE

$$T_{AB}=T_{AB}\lambda_{AB}=T_{AB}\frac{\overrightarrow{AB}}{AB}=\left(-\frac{4}{21}\underline{i}-\frac{20}{21}\underline{j}+\frac{5}{21}\underline{k}\right)T_{AB}$$

$$T_{AC}=T_{AC}\lambda_{AC}=T_{AC}\frac{\overrightarrow{AC}}{AC}=\left(\frac{60}{118}\underline{i}-\frac{100}{118}\underline{j}+\frac{18}{118}\underline{k}\right)T_{AC}$$

$$T_{AD}=T_{AD}\lambda_{AD}=T_{AD}\frac{\overrightarrow{AD}}{AD}=\left(-\frac{20}{126}\underline{i}-\frac{100}{126}\underline{j}-\frac{74}{126}\underline{k}\right)T_{AD}$$

SUBSTITUTING INTO THE EQ. $\Sigma F=0$ AND FACTORING $\underline{i},\underline{j},\underline{k}$:

$$\left(-\frac{4}{21}T_{AB}+\frac{60}{118}T_{AC}-\frac{20}{126}T_{AD}\right)\underline{i}$$

$$+\left(-\frac{20}{21}T_{AB}-\frac{100}{118}T_{AC}-\frac{100}{126}T_{AD}+P\right)\underline{j}$$

$$+\left(\frac{5}{21}T_{AB}+\frac{18}{118}T_{AC}-\frac{74}{126}T_{AD}\right)\underline{k}=0$$

SETTING THE COEFF. OF $\underline{i},\underline{j},\underline{k}$ EQUAL TO ZERO:

$$\underline{i}:\ -\frac{4}{21}T_{AB}+\frac{60}{118}T_{AC}-\frac{20}{126}T_{AD}=0 \qquad (1)$$

$$\underline{j}:\ -\frac{20}{21}T_{AB}-\frac{100}{118}T_{AC}-\frac{100}{126}T_{AD}+P=0 \qquad (2)$$

$$\underline{k}:\ \frac{5}{21}T_{AB}+\frac{18}{118}T_{AC}-\frac{74}{126}T_{AD}=0 \qquad (3)$$

CONTINUED

2.112 CONTINUED

MAKING $T_{AC}=590$ lb IN EQS. (1), (2), AND (3):

$$-\frac{4}{21}T_{AB}-\frac{20}{126}T_{AD}+300\text{ lb}=0 \qquad (1')$$

$$-\frac{20}{21}T_{AB}-\frac{100}{126}T_{AD}-500\text{ lb}+P=0 \qquad (2')$$

$$\frac{5}{21}T_{AB}-\frac{74}{126}T_{AD}+90\text{ lb}=0 \qquad (3')$$

MULTPLY EQ.(1') BY 5, EQ.(3') BY 4, AND ADD:

$$-\frac{396}{126}T_{AD}+1860\text{ lb}=0 \qquad T_{AD}=591.818\text{ lb}$$

SUBSTITUTE INTO (1') AND SOLVE FOR T_{AB}:

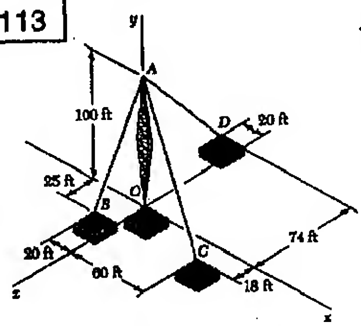
$$T_{AB}=\frac{21}{4}\left(300\text{ lb}-\frac{20}{126}\times 591.818\text{ lb}\right) \qquad T_{AB}=1081.82\text{ lb}$$

SUBSTITUTE FOR THE TENSIONS IN (2') AND SOLVE FOR P:

$$P=500\text{ lb}+\frac{20}{21}(1081.82\text{ lb})+\frac{100}{126}(591.818\text{ lb})=2000.00\text{ lb}$$

WEIGHT OF PLATE $= P = 2000$ lb ◀

2.113



GIVEN:
TOWER EXERTS ON A AN UPWARD VERTICAL FORCE $\underline{P}$ OF 1800 lb.

FIND:
TENSION IN EACH WIRE.

SEE COLUMN ON THE LEFT FOR DERIVATION OF EQS. (1), (2), AND (3). MAKING $P=1800$ lb IN EQ. (2), WE HAVE

$$-\frac{4}{21}T_{AB}+\frac{60}{118}T_{AC}-\frac{20}{126}T_{AD}=0 \qquad (1)$$

$$-\frac{20}{21}T_{AB}-\frac{100}{118}T_{AC}-\frac{100}{126}T_{AD}+1800\text{ lb}=0 \qquad (2)$$

$$\frac{5}{21}T_{AB}+\frac{18}{118}T_{AC}-\frac{74}{126}T_{AD}=0 \qquad (3)$$

MULTIPLY (1) BY −74, (3) BY 20, AND ADD:

$$\frac{396}{21}T_{AB}-\frac{4080}{118}T_{AC}=0 \qquad T_{AC}=0.545378\,T_{AB} \qquad (4)$$

SUBSTITUTE INTO (1):

$$\left[-\frac{4}{21}+\frac{60}{118}(0.545378)\right]T_{AB}-\frac{20}{126}T_{AD}=0$$

$$0.0868347\,T_{AB}-\frac{20}{126}T_{AD}=0 \qquad T_{AD}=0.547059\,T_{AB} \qquad (5)$$

SUBSTITUTE FOR T_{AC} AND T_{AD} INTO (2) AND SOLVE FOR T_{AB}:

$$-\frac{20}{21}T_{AB}-\frac{100}{118}(0.545378\,T_{AB})-\frac{100}{126}(0.547059\,T_{AB})+1800\text{ lb}=0$$

$$1.84814\,T_{AB}=1800\text{ lb} \qquad T_{AB}=973.636\text{ lb} \qquad (6)$$

$T_{AB}=974$ lb ◀

SUBSTITUTING FROM (6) INTO (4):

$$T_{AC}=0.545378\,(973.636\text{ lb})=531.000\text{ lb}$$

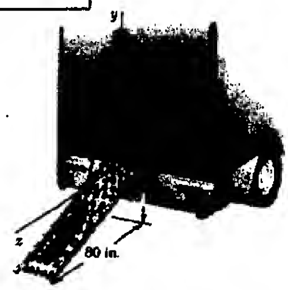
$T_{AC}=531$ lb ◀

SUBSTITUTING FROM (6) INTO (5):

$$T_{AD}=0.547059\,(973.636\text{ lb})=532.637\text{ lb}$$

$T_{AD}=533$ lb ◀

2.119



GIVEN:

(1) 200-lb COUNTERWEIGHT IS IN EQUILIBRIUM UNDER FORCES EXERTED BY ROPES AND FORCE PERPENDICULAR TO CHUTE.

(2) COORDINATES OF A, B, C ARE

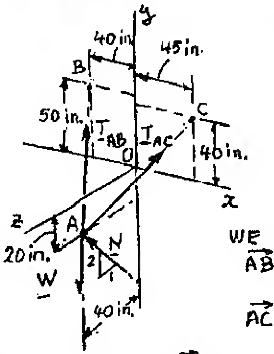
A (0, −20 in., 40 in.)

B (−40 in., 50 in., 0)

C (45 in., 40 in., 0)

FIND:

TENSION IN EACH ROPE.



FREE BODY: COUNTERWEIGHT

$\Sigma \underline{F} = 0$:

$$\underline{T}_{AB} + \underline{T}_{AC} + \underline{W} + \underline{N} = 0$$

WHERE

$$\underline{W} = -(200\text{ lb})\,\underline{j}$$

$$\underline{N} = \left(\tfrac{2}{\sqrt{5}}\underline{j} + \tfrac{1}{\sqrt{5}}\underline{k}\right) N$$

WE NOTE THAT

$$\vec{AB} = -(40\text{ in.})\underline{i} + (70\text{ in.})\underline{j} - (40\text{ in.})\underline{k}$$

$$AB = 90\text{ in.}$$

$$\vec{AC} = (45\text{ in.})\underline{i} + (60\text{ in.})\underline{j} - (40\text{ in.})\underline{k}$$

$$AC = 85\text{ in.}$$

THUS: $\underline{T}_{AB} = T_{AB}\dfrac{\vec{AB}}{AB} = \left(-\tfrac{4}{9}\underline{i} + \tfrac{7}{9}\underline{j} - \tfrac{4}{9}\underline{k}\right)T_{AB}$

$$\underline{T}_{AC} = T_{AC}\frac{\vec{AC}}{AC} = \left(\tfrac{9}{17}\underline{i} + \tfrac{12}{17}\underline{j} - \tfrac{8}{17}\underline{k}\right)T_{AC}$$

SUBSTITUTE FOR $\underline{T}_{AB}, \underline{T}_{AC}, \underline{N}$, AND $\underline{W}$ INTO $\Sigma\underline{F} = 0$ AND FACTOR $\underline{i}, \underline{j}, \underline{k}$:

$$\left(-\tfrac{4}{9}T_{AB} + \tfrac{9}{17}T_{AC}\right)\underline{i} + \left(\tfrac{7}{9}T_{AB} + \tfrac{12}{17}T_{AC} + \tfrac{2}{\sqrt{5}}N - 200\text{ lb}\right)\underline{j}$$
$$+\left(-\tfrac{4}{9}T_{AB} - \tfrac{8}{17}T_{AC} + \tfrac{1}{\sqrt{5}}N\right)\underline{k} = 0$$

EQUATING TO ZERO THE COEFFICIENTS OF $\underline{i}, \underline{j}, \underline{k}$:

$\underline{i}$: $-\tfrac{4}{9}T_{AB} + \tfrac{9}{17}T_{AC} = 0$ (1)

$\underline{j}$: $\tfrac{7}{9}T_{AB} + \tfrac{12}{17}T_{AC} + \tfrac{2}{\sqrt{5}}N - 200\text{ lb} = 0$ (2)

$\underline{k}$: $-\tfrac{4}{9}T_{AB} - \tfrac{8}{17}T_{AC} + \tfrac{1}{\sqrt{5}}N = 0$ (3)

MULTIPLY (3) BY −2 AND ADD (2):

$\tfrac{15}{9}T_{AB} + \tfrac{28}{17}T_{AC} - 200\text{ lb} = 0$ (4)

MULTIPLY (1) BY 15, (4) BY 4, AND ADD:

$\tfrac{247}{17}T_{AC} - 800\text{ lb} = 0$ $\qquad T_{AC} = 55.061\text{ lb}$ (5)

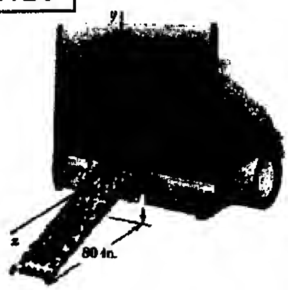
SUBSTITUTE FROM (5) INTO (1) AND SOLVE FOR T_{AB}:

$$T_{AB} = \tfrac{9}{4}\cdot\tfrac{9}{17}(55.061\text{ lb}) = 65.587\text{ lb}$$

THE TENSIONS IN THE ROPES ARE

$$T_{AB} = 65.6\text{ lb}, \quad T_{AC} = 55.1\text{ lb} \quad ◀$$

2.120



GIVEN:

(1) 200-lb COUNTERWEIGHT IS IN EQUILIBRIUM UNDER FORCES EXERTED BY THE TWO WORKERS SHOWN, BY A THIRD WORKER WHO PUSHES WITH $\underline{P} = -(40\text{ lb})\underline{i}$, AND A FORCE PERPENDICULAR TO THE CHUTE.

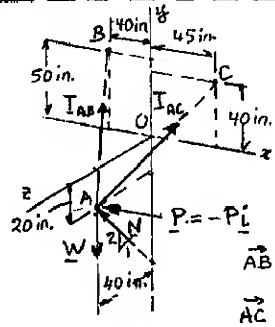
(2) COORDINATES OF A, B, C ARE

A (0, −20 in., 40 in.)

B (−40 in., 50 in., 0)

C (45 in., 40 in., 0)

FIND: TENSION IN ROPES AB AND AC.



FREE BODY: COUNTERWEIGHT

$\Sigma \underline{F} = 0$:

$$\underline{T}_{AB} + \underline{T}_{AC} + \underline{W} + \underline{P} + \underline{N} = 0$$

WHERE

$$\underline{W} = -(200\text{ lb})\underline{j}$$

$$\underline{P} = -(40\text{ lb})\underline{i}$$

$$\underline{N} = \left(\tfrac{2}{\sqrt{5}}\underline{j} + \tfrac{1}{\sqrt{5}}\underline{k}\right)N$$

WE NOTE THAT

$$\vec{AB} = -(40\text{ in.})\underline{i} + (70\text{ in.})\underline{j} - (40\text{ in.})\underline{k}$$

$$AB = 90\text{ in.}$$

$$\vec{AC} = (45\text{ in.})\underline{i} + (60\text{ in.})\underline{j} - (40\text{ in.})\underline{k}$$

$$AC = 85\text{ in.}$$

THUS: $\underline{T}_{AB} = T_{AB}\dfrac{\vec{AB}}{AB} = \left(-\tfrac{4}{9}\underline{i} + \tfrac{7}{9}\underline{j} - \tfrac{4}{9}\underline{k}\right)T_{AB}$

$$\underline{T}_{AC} = T_{AC}\frac{\vec{AC}}{AC} = \left(\tfrac{9}{17}\underline{i} + \tfrac{12}{17}\underline{j} - \tfrac{8}{17}\underline{k}\right)T_{AC}$$

SUBSTITUTE FOR $\underline{T}_{AB}, \underline{T}_{AC}, \underline{N}, \underline{P}$, AND $\underline{W}$ INTO $\Sigma\underline{F} = 0$ AND FACTOR $\underline{i}, \underline{j}, \underline{k}$:

$$\left(-\tfrac{4}{9}T_{AB} + \tfrac{9}{17}T_{AC} - 40\text{ lb}\right)\underline{i} + \left(\tfrac{7}{9}T_{AB} + \tfrac{12}{17}T_{AC} + \tfrac{2}{\sqrt{5}}N - 200\text{ lb}\right)\underline{j}$$
$$+\left(-\tfrac{4}{9}T_{AB} - \tfrac{8}{17}T_{AC} + \tfrac{1}{\sqrt{5}}N\right)\underline{k} = 0$$

EQUATING TO ZERO THE COEFFICIENTS OF $\underline{i}, \underline{j}, \underline{k}$:

$\underline{i}$: $-\tfrac{4}{9}T_{AB} + \tfrac{9}{17}T_{AC} - 40\text{ lb} = 0$ (1)

$\underline{j}$: $\tfrac{7}{9}T_{AB} + \tfrac{12}{17}T_{AC} + \tfrac{2}{\sqrt{5}}N - 200\text{ lb} = 0$ (2)

$\underline{k}$: $-\tfrac{4}{9}T_{AB} - \tfrac{8}{17}T_{AC} + \tfrac{1}{\sqrt{5}}N = 0$ (3)

MULTIPLY (3) BY −2 AND ADD (2):

$\tfrac{15}{9}T_{AB} + \tfrac{28}{17}T_{AC} - 200\text{ lb} = 0$ (4)

MULTIPLY (1) BY 15, (4) BY 4, AND ADD:

$\tfrac{247}{17}T_{AC} - 1400\text{ lb} = 0$ $\qquad T_{AC} = 96.3563\text{ lb}$ (5)

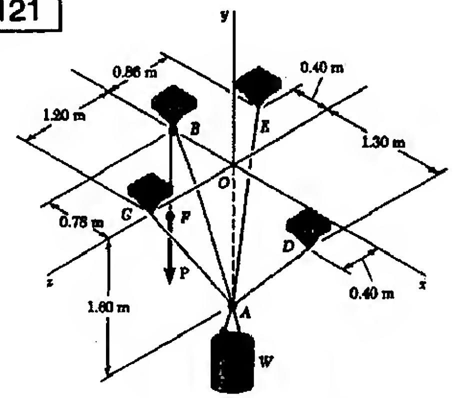
SUBSTITUTE FROM (5) INTO (1) AND SOLVE FOR T_{AB}:

$$T_{AB} = \tfrac{9}{4}\left[\tfrac{9}{17}(96.3563\text{ lb}) - 40\text{ lb}\right] = 24.777\text{ lb}$$

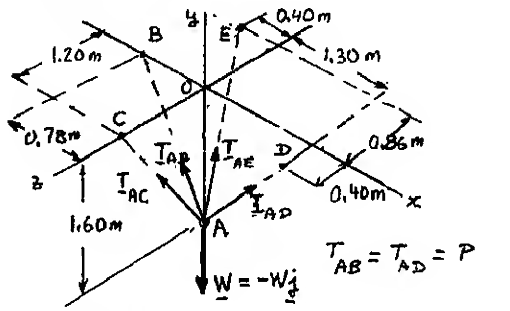
THE TENSIONS IN THE ROPES ARE

$$T_{AB} = 24.8\text{ lb}, \quad T_{AC} = 96.4\text{ lb} \quad ◀$$

2.121



GIVEN: CONTAINER OF WEIGHT $W = 1000$ N IS SUSPENDED FROM RING A. CABLES AC AND AE ARE ATTACHED TO RING. CABLE FBAD PASSES THROUGH RING AND OVER PULLEY B.
FIND: MAGNITUDE OF FORCE $\underline{P}$.



FREE BODY: RING A

$$\Sigma \underline{F} = 0: \quad \underline{T}_{AB} + \underline{T}_{AC} + \underline{T}_{AD} + \underline{T}_{AE} - W\underline{j} = 0$$

WE HAVE

$\vec{AB} = -(0.78\text{m})\underline{i} + (1.60\text{m})\underline{j}$ $\qquad AB = 1.78$ m

$\vec{AC} = (1.60\text{ m})\underline{j} + (1.20\text{m})\underline{k}$ $\qquad AC = 2.00$ m

$\vec{AD} = (1.30\text{m})\underline{i} + (1.60\text{m})\underline{j} + (0.40\text{ m})\underline{k}$ $\qquad AD = 2.10$ m

$\vec{AE} = -(0.40\text{m})\underline{i} + (1.60\text{m})\underline{j} - (0.86\text{m})\underline{k}$ $\qquad AE = 1.86$ m

$$\underline{T}_{AB} = P\underline{\lambda}_{AB} = P\frac{\vec{AB}}{AB} = \left(-\frac{0.78}{1.78}\underline{i} + \frac{1.6}{1.78}\underline{j}\right)P$$

$$\underline{T}_{AC} = T_{AC}\underline{\lambda}_{AC} = T_{AC}\frac{\vec{AC}}{AC} = (0.8\underline{j} + 0.6\underline{k})T_{AC}$$

$$\underline{T}_{AD} = P\underline{\lambda}_{AD} = P\frac{\vec{AD}}{AD} = \left(\frac{1.3}{2.1}\underline{i} + \frac{1.6}{2.1}\underline{j} + \frac{0.4}{2.1}\underline{k}\right)P$$

$$\underline{T}_{AE} = T_{AE}\underline{\lambda}_{AE} = T_{AE}\frac{\vec{AE}}{AE} = \left(-\frac{0.4}{1.86}\underline{i} + \frac{1.6}{1.86}\underline{j} - \frac{0.86}{1.86}\underline{k}\right)T_{AE}$$

SUBSTITUTING FOR THE TENSIONS IN $\Sigma \underline{F} = 0$ AND FACTORING $\underline{i}, \underline{j}, \underline{k}$:

$$\left(-\frac{0.78}{1.78}P + \frac{1.3}{2.1}P - \frac{0.4}{1.86}T_{AE}\right)\underline{i}$$
$$+\left(\frac{1.6}{1.78}P + 0.8T_{AC} + \frac{1.6}{2.1}P + \frac{1.6}{1.86}T_{AE} - W\right)\underline{j}$$
$$+\left(0.6T_{AC} + \frac{0.4}{2.1}P - \frac{0.86}{1.86}T_{AE}\right)\underline{k} = 0$$

EQUATING TO ZERO THE COEFFICIENTS OF $\underline{i}, \underline{j}, \underline{k}$, WE OBTAIN AFTER REDUCTIONS:

CONTINUED

2.121 CONTINUED

$\underline{i}$: $0.180845P - 0.215054T_{AE}$ $\qquad$ (1)

$\underline{j}$: $1.66078P + 0.8T_{AC} + 0.860215T_{AE} - W = 0$ $\qquad$ (2)

$\underline{k}$: $0.190476P + 0.6T_{AC} - 0.462366T_{AE} = 0$ $\qquad$ (3)

SOLVING (1) FOR T_{AE}: $\qquad T_{AE} = 0.840931\,P$

CARRYING INTO EQS. (2) AND (3):

$2.38416P + 0.8T_{AC} - W = 0$ $\qquad$ (4)

$-0.198342P + 0.6T_{AC} = 0$ $\qquad$ (5)

MULTIPLY (4) BY 3, (5) BY −4, AND ADD:

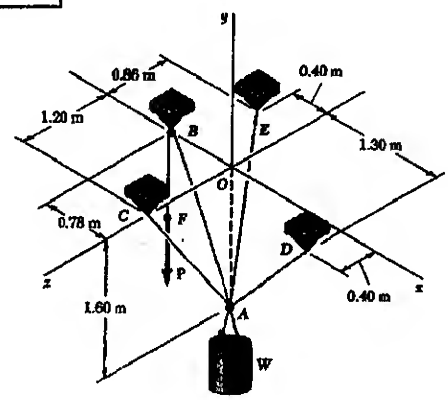
$7.94585P - 3W = 0$

MAKING $W = 1000$ N:

$7.94585P - 3000\text{ N} = 0 \qquad P = 377.556$ N

$P = 378$ N ◀

2.122



GIVEN:

(1) CONTAINER IS SUSPENDED FROM RING A. CABLES AC AND AE ARE ATTACHED TO RING. CABLE FBAD PASSES TROUGH RING AND OVER PULLEY B.

(2) $T_{AC} = 150$ N.

FIND:

(a) MAGNITUDE OF FORCE $\underline{P}$

(b) WEIGHT W OF CONTAINER

SEE SOLUTION OF PROB. 2.121 LEADING TO EQS. (4) AND (5):

$2.38416P + 0.8T_{AC} - W = 0$ $\qquad$ (4)

$-0.198342P + 0.6T_{AC} = 0$ $\qquad$ (5)

(a) MAKE $T_{AC} = 150$ N IN EQ. (5):

$-0.198342P + 0.6(150\text{ N}) = 0$

$P = 453.762$ N

$P = 454$ N ◀

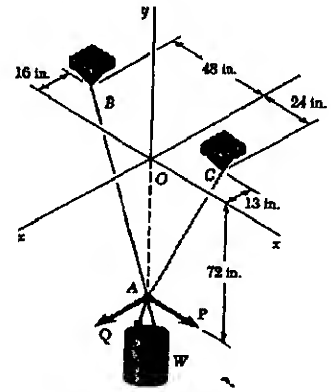
(b) CARRY THE VALUES OF T_{AC} AND P INTO EQ. (4):

$2.38416(453.762\text{ N}) + 0.8(150\text{ N}) - W = 0$

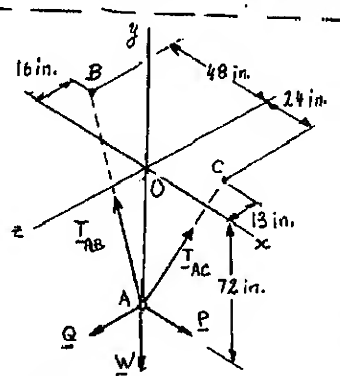
$W = 1201.84$ N

$W = 1202$ N ◀

2.123



GIVEN: CONTAINER OF WEIGHT $W = 270\ lb$ IS SUSPENDED FROM RING A.
CABLE BAC PASSES THROUGH RING A.
FIND: P AND Q FOR EQUILIBRIUM POSITION SHOWN.



FREE BODY: RING A

$\Sigma \underline{F} = 0$:

$\underline{T}_{AB} + \underline{T}_{AC} + \underline{P} + \underline{Q} + \underline{W} = 0$

WHERE $\underline{P} = P\underline{i}$

$\underline{Q} = Q\underline{k}$

$\underline{W} = -W\underline{j}$

$\underline{T}_{AB} = T\underline{\lambda}_{AB}$

$\underline{T}_{AC} = T\underline{\lambda}_{AC}$

(SAME TENSION T IN BOTH PORTIONS OF CABLE)

WE HAVE

$\overrightarrow{AB} = -(48 in.)\underline{i} + (72 in.)\underline{j} - (16 in.)\underline{k}$ $\qquad AB = 88\ in$

$\overrightarrow{AC} = (24 in.)\underline{i} + (72 in.)\underline{j} - (13 in.)\underline{k}$ $\qquad AC = 77\ in$

$$\underline{T}_{AB} = T\underline{\lambda}_{AB} = T\frac{\overrightarrow{AB}}{AB} = \left(-\frac{6}{11}\underline{i} + \frac{9}{11}\underline{j} - \frac{2}{11}\underline{k}\right)T$$

$$\underline{T}_{AC} = T\underline{\lambda}_{AC} = T\frac{\overrightarrow{AC}}{AC} = \left(\frac{24}{77}\underline{i} + \frac{72}{77}\underline{j} - \frac{13}{77}\underline{k}\right)T$$

SUBSTITUTING FOR $\underline{T}_{AB}$, $\underline{T}_{AC}$, $\underline{P}$, $\underline{Q}$, AND $\underline{W}$ INTO $\Sigma\underline{F} = 0$ AND FACTORING $\underline{i}, \underline{j}, \underline{k}$:

$$\left(-\frac{6}{11}T + \frac{24}{77}T + P\right)\underline{i} + \left(\frac{9}{11}T + \frac{72}{77}T - W\right)\underline{j} + \left(-\frac{2}{11}T - \frac{13}{77}T + Q\right)\underline{k} = 0$$

SETTING THE COEFFICIENTS OF $\underline{i}, \underline{j}, \underline{k}$ EQUAL TO ZERO AND REDUCING:

$\underline{i}$: $-\frac{18}{77}T + P = 0 \qquad (1)$

$\underline{j}$: $\frac{135}{77}T - W = 0 \qquad (2)$

$\underline{k}$: $-\frac{27}{77}T + Q = 0 \qquad (3)$

MAKING $W = 270\ lb$ IN EQ. (2) AND SOLVING FOR T:

$T = \frac{77}{135}(270\ lb) = 154.0\ lb$

SUBSTITUTING FOR T IN EQS. (1) AND (3), WE OBTAIN

$P = 36.0\ lb, \quad Q = 54.0\ lb$ ◀

2.124

(SEE FIGURE ON THE LEFT)

GIVEN: (1) $Q = 36\ lb$.

(2) CABLE BAC PASSES THROUGH RING A.

FIND: W AND P.

SEE SOLUTION AT LEFT FOR DERIVATION OF EQS. (1), (2), (3).

MAKING $Q = 36\ lb$ IN EQ. (3):

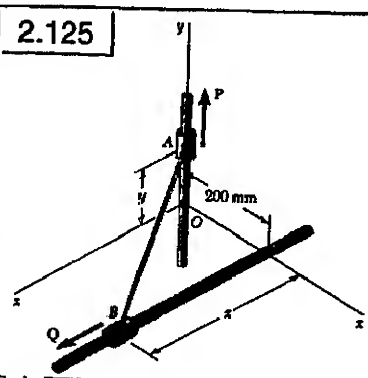
$-\frac{27}{77}T + 36\ lb = 0 \qquad T = \frac{77}{27}(36\ lb) \qquad T = 102.667\ lb$

SUBSTITUTING FOR T IN EQS. (1) AND (2):

$-\frac{18}{77}(102.667\ lb) + P = 0 \qquad P = 24.0\ lb$ ◀

$\frac{135}{77}(102.667\ lb) - W = 0 \qquad W = 180.0\ lb$ ◀

2.125



GIVEN:

(1) COLLARS A AND B CONNECTED BY WIRE OF LENGTH 525 mm

(2) $\underline{P} = (341\ N)\underline{j}$

(3) $y = 155\ mm$

FIND:

(a) T_{AB}

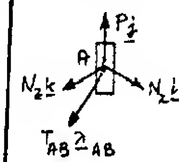
(b) Q FOR EQUILIBRIUM

$(AB)^2 = x^2 + y^2 + z^2$: $\quad (525\ mm)^2 = (200\ mm)^2 + (155\ mm)^2 + z^2$

$z = 460\ mm$

$\overrightarrow{AB} = (200\ mm)\underline{i} - (155\ mm)\underline{j} + (460\ mm)\underline{k} \qquad AB = 525\ mm$

$$\underline{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{200}{525}\underline{i} - \frac{155}{525}\underline{j} + \frac{460}{525}\underline{k}$$



(a) FREE BODY: COLLAR A:

$\Sigma\underline{F} = 0$:

$N_x\underline{i} + P\underline{j} + N_z\underline{k} + T_{AB}\underline{\lambda}_{AB} = 0$

SUBSTITUTING FOR $\underline{\lambda}_{AB}$ AND SETTING THE COEFF. OF $\underline{j}$ EQUAL TO ZERO:

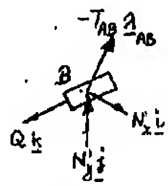
$P + \left(-\frac{155}{525}T_{AB}\right) = 0$

MAKING $P = 341\ N$ AND SOLVING FOR T_{AB}:

$T_{AB} = \frac{525}{155}(341\ N) \qquad T_{AB} = 1155\ N$ ◀

(b) FREE BODY: COLLAR B

$\Sigma\underline{F} = 0$



$N'_x\underline{i} + N'_y\underline{j} + Q\underline{k} - T_{AB}\underline{\lambda}_{AB} = 0$

SUBSTITUTING FOR $\underline{\lambda}_{AB}$ AND SETTING THE COEFF. OF $\underline{k}$ EQUAL TO ZERO:

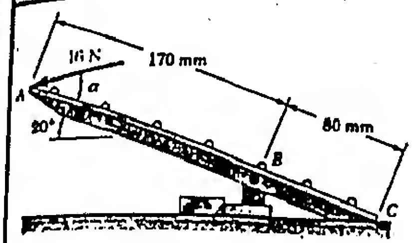
$Q - \left(\frac{460}{525}T_{AB}\right) = 0$

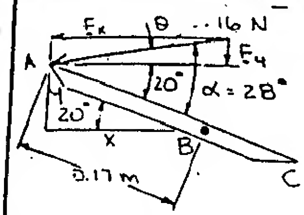
MAKING $T_{AB} = 1155\ N$ AND SOLVING FOR Q:

$Q = \frac{460}{525}(1155\ N) \qquad Q = 1012\ N$ ◀

3.1

GIVEN: $\alpha = 28°$
FIND: MOMENT OF FORCE ABOUT B (RESOLVE FORCE INTO HORIZONTAL AND VERTICAL COMPONENTS)





FIRST NOTE THAT

$\theta = 28° - 20° = 8°$

THEN

$F_x = (16\text{ N})\cos 8°$
$= 15.8443\text{ N}$
$F_y = (16\text{ N})\sin 8°$
$= 2.2268\text{ N}$

AND $x = (0.17\text{ m})\cos 20° = 0.159748\text{ m}$
$y = (0.17\text{ m})\sin 20° = 0.058143\text{ m}$

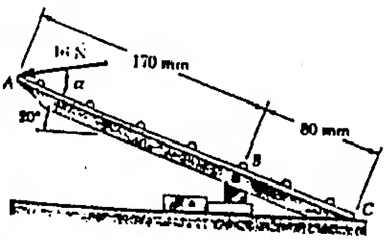
NOTING THAT THE DIRECTION OF THE MOMENT OF EACH FORCE COMPONENT ABOUT B IS COUNTERCLOCKWISE, HAVE

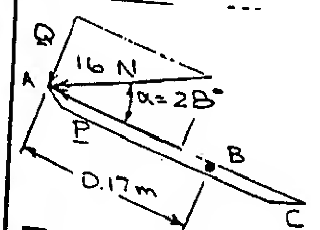
$M_B = xF_y + yF_x$
$= (0.159748\text{ m})(2.2268\text{ N})$
$+ (0.058143\text{ m})(15.8443\text{ N})$
$= 1.277\text{ N}$

OR $\underline{M}_B = 1.277\text{ N}\cdot\text{m}$ ◀

3.2

GIVEN: $\alpha = 28°$
FIND: MOMENT OF FORCE ABOUT B (RESOLVE FORCE INTO COMPONENTS PARALLEL AND PERPENDICULAR TO ABC)





FIRST RESOLVE THE 16-N FORCE INTO COMPONENTS $\underline{P}$ AND $\underline{Q}$, WHERE

$Q = (16\text{ N})\sin 28°$
$= 7.5115\text{ N}$

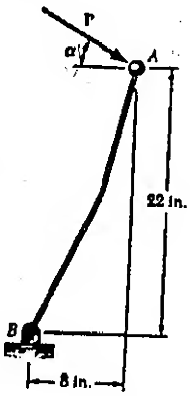
THEN.. $M_B = r_{A/B}Q$

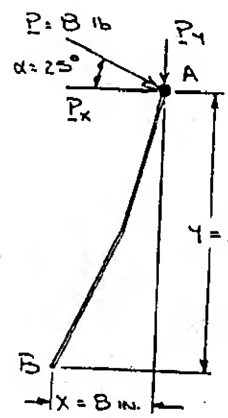
$= (0.17\text{ m})(7.5115\text{ N})$
$= 1.277\text{ N}\cdot\text{m}$

OR $\underline{M}_B = 1.277\text{ N}\cdot\text{m}$ ◀

3.3

GIVEN: $P = 8\text{ lb}$, $\alpha = 25°$
FIND: MOMENT OF FORCE ABOUT B





FIRST NOTE..

$P_x = (8\text{ lb})\cos 25° = 7.2505\text{ lb}$
$P_y = (8\text{ lb})\sin 25° = 3.3809\text{ lb}$

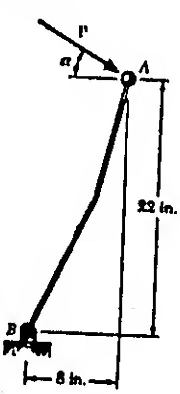
NOTING THAT THE DIRECTION OF THE MOMENT OF EACH FORCE COMPONENT ABOUT B IS CLOCKWISE, HAVE

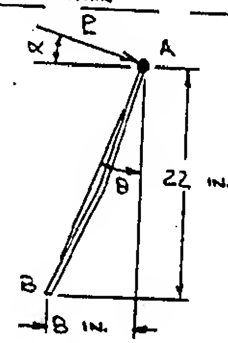
$M_B = -xP_y - yP_x$
$= -(8\text{ in.})(3.3809\text{ lb})$
$-(22\text{ in.})(7.2505\text{ lb})$
$= -186.6\text{ lb}\cdot\text{in.}$

OR $\underline{M}_B = 186.6\text{ lb}\cdot\text{in.}$ ◀

3.4

GIVEN: $M_B = 210\text{ lb}\cdot\text{in.}$
FIND: $(\underline{P})_{MIN}$





FOR $\underline{P}$ TO BE MINIMUM, IT MUST BE PERPENDICULAR TO THE LINE JOINING POINTS A AND B. THUS,

$\alpha = \theta$
$= \tan^{-1}\frac{8}{22}$
$= 19.98°$

AND $M_B = dP_{MIN}$

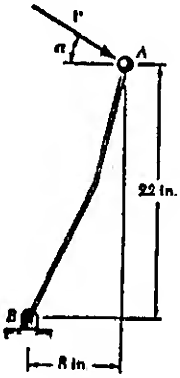
WHERE $d = r_{A/B} = \sqrt{(8\text{ in.})^2 + (22\text{ in.})^2}$
$= 23.409\text{ in.}$

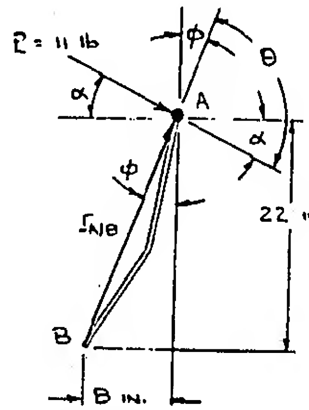
THEN.. $P_{MIN} = \dfrac{210\text{ lb}\cdot\text{in.}}{23.409\text{ in.}}$
$= 8.97\text{ lb}$

$\underline{P}_{MIN} = 8.97\text{ lb} \measuredangle 19.98°$ ◀

3.5

GIVEN: $P = 11$ lb, $M_B = 250$ lb·in.
FIND: α





BY DEFINITION.. $M_B = r_{A/B} P \sin\theta$

WHERE $\theta = \alpha + (90° - \phi)$

AND $\phi = \tan^{-1}\frac{8}{22} = 19.9831°$

ALSO.. $r_{A/B} = \sqrt{(8\text{ in.})^2 + (22\text{ in.})^2} = 23.409$ in.

THEN.. $250\text{ lb·in} = (23.409\text{ in.})(11\text{ lb}) \times \sin(\alpha + 90° - 19.9831°)$

OR $\sin(\alpha + 70.0169°) = 0.97088$

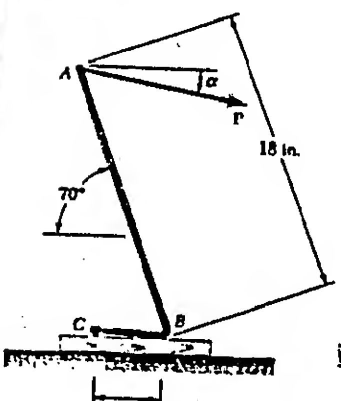
OR $\alpha + 70.0169° = 76.1391°$

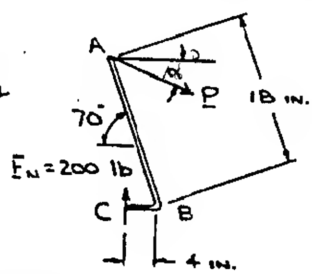
AND $\alpha + 70.0169° = 103.861°$

$\alpha = 6.12°,\ 33.8°$ ◄

3.6

GIVEN: $F_N = 200$ lb ↑
FIND:
(a) MOMENT M_B OF F_N ABOUT B
(b) P GIVEN M_B AND $\alpha = 10°$
(c) P_{MIN} GIVEN M_B





(a) HAVE $M_B = r_{C/B} F_N$

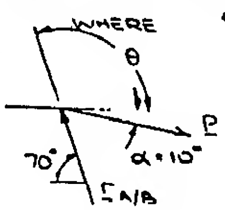
$= (4\text{ in.})(200\text{ lb})$

$= 800$ lb·in.

OR $M_B = 800$ lb·in. ⤸ ◄

(b) BY DEFINITION $M_B = r_{A/B} P \sin\theta$

WHERE $\theta = 10° + (180° - 70°) = 120°$



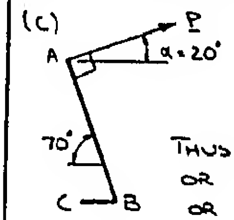
THEN $800\text{ lb·in.} = (18\text{ in.}) \times P \sin 120°$

OR $P = 51.3$ lb ◄

(CONTINUED)

3.6 , CONTINUED

(c)



FOR P TO BE MINIMUM, IT MUST BE PERPENDICULAR TO THE LINE JOINING POINTS A AND B. THUS, P MUST BE DIRECTED AS SHOWN.

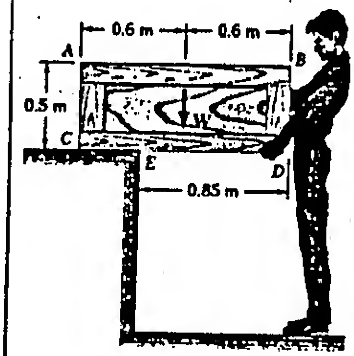
THUS.. $M_B = dP_{MIN}$ $\quad d = r_{A/B}$

OR $800\text{ lb·in.} = (18\text{ in.})P_{MIN}$

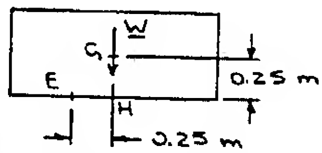
OR $P_{MIN} = 44.4$ lb

$P_{MIN} = 44.4$ lb ∠20° ◄

3.7



GIVEN: MASS m OF CRATE = 80 kg
FIND:
(a) MOMENT M_E OF WEIGHT W ABOUT E
(b) $(F_B)_{MIN}$ GIVEN $-M_E$



FIRST NOTE..
$W = mg$
$= (80\text{ kg})(9.81\frac{m}{s^2})$
$= 784.8$ N

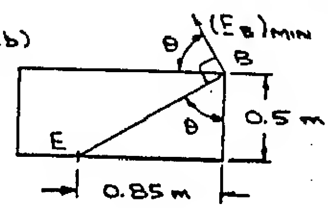
(a) HAVE $M_E = r_{H/E} W$

$= (0.25\text{ m})(784.8\text{ N})$

$= 196.2$ N·m

OR $M_E = 196.2$ N·m ⤸ ◄

(b)



FOR F_B TO BE MINIMUM, IT MUST BE PERPENDICULAR TO THE LINE JOINING POINTS B AND E. THEN, WITH F_B DIRECTED AS SHOWN, HAVE

$(-M_E) = r_{B/E}(F_B)_{MIN}$

WHERE $r_{B/E} = \sqrt{(0.85\text{ m})^2 + (0.5\text{ m})^2}$

$= 0.98615$ m

THEN $196.2\text{ N·m} = (0.98615\text{ m})(F_B)_{MIN}$

OR $(F_B)_{MIN} = 199.0$ N

ALSO -- $\tan\theta = \frac{0.85\text{ m}}{0.5\text{ m}}$

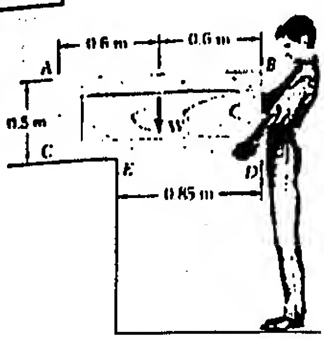
OR $\theta = 59.5°$

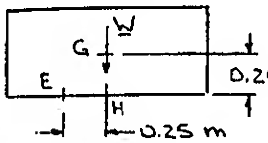
$(F_B)_{MIN} = 199.0$ N ⦣ 59.5° ◄

3.8

GIVEN: MASS m OF CRATE = 80 kg

FIND:
(a) MOMENT M_E OF WEIGHT $\underline{W}$ ABOUT E
(b) $(F_A)_{MIN}$ GIVEN $-M_E$
(c) $(E_{VERTICAL})_{MIN}$ GIVEN $-M_E$



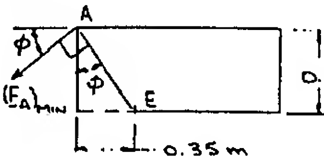


FIRST NOTE...
$$W = mg = (80\ \text{kg})(9.81\ \tfrac{m}{s^2}) = 784.8\ \text{N}$$

(a) HAVE $\quad M_E = r_{H/E} W$
$$= (0.25\ m)(784.8\ N)$$
$$= 196.2\ N\cdot m$$

OR $\quad \underline{M}_E = 196.2\ N\cdot m \circlearrowright$ ◀

(b)



FOR F_A TO BE MINIMUM, IT MUST BE PERPENDICULAR TO THE LINE JOINING POINTS A AND E. THEN, WITH F_A DIRECTED AS SHOWN, HAVE
$$(-M_E) = r_{A/E}(F_A)_{MIN}$$

WHERE $\quad r_{A/E} = \sqrt{(0.35\ m)^2 + (0.5\ m)^2} = 0.610\,33\ m$

THEN $\quad 196.2\ N\cdot m = (0.610\,33\ m)(F_A)_{MIN}$

OR $\quad (F_A)_{MIN} = 321\ N$

ALSO... $\quad \tan\phi = \dfrac{0.35\ m}{0.5\ m}$ $\quad$ OR $\phi = 35.0°$

$$(F_A)_{MIN} = 321\ N \measuredangle\ 35.0° \quad ◀$$

(c) FOR $E_{VERTICAL}$ TO BE MINIMUM, THE PERPENDICULAR DISTANCE FROM ITS LINE OF ACTION TO POINT E MUST BE MAXIMUM. THUS, APPLY $(E_{VERTICAL})_{MIN}$ AT POINT D, AND THEN
$$(-M_E) = r_{D/E}(F_{VERTICAL})_{MIN}$$

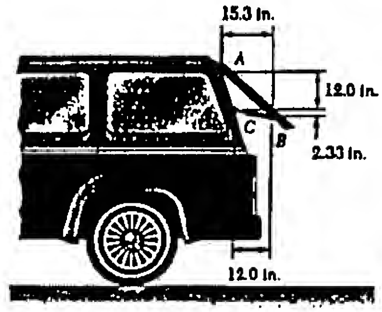
$$196.2\ N\cdot m = (0.85\ m)(F_{VERTICAL})_{MIN}$$

OR $\quad (E_{VERTICAL})_{MIN} = 231\ N\uparrow$ AT POINT D ◀

3.9

GIVEN: $F_{CB} = 125$ lb

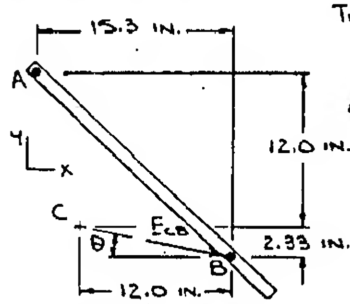
FIND: MOMENT OF F_{CB} ABOUT A



FIRST NOTE.. $d_{CB} = \sqrt{(12\ in.)^2 + (2.33\ in.)^2}$
$$= 12.2241\ in.$$

THEN $\quad \cos\theta = \dfrac{12.0}{12.2241}$

$\sin\theta = \dfrac{2.33}{12.2241}$



AND $\quad \underline{F}_{CB} = F_{CB}\cos\theta\,\underline{i} - F_{CB}\sin\theta\,\underline{j}$
$$= \frac{125\ lb}{12.2241}(12.0\,\underline{i} - 2.33\,\underline{j})$$

NOW.. $\quad \underline{M}_A = \underline{r}_{B/A} \times \underline{F}_{CB}$

WHERE $\quad \underline{r}_{B/A} = (15.3\ in.)\underline{i} - (14.33\ in.)\underline{j}$

THEN.. $\quad \underline{M}_A = [(15.3\ in.)\underline{i} - (14.33\ in.)\underline{j}] \times \dfrac{125\ lb}{12.2241}(12.0\,\underline{i} - 2.33\,\underline{j})$

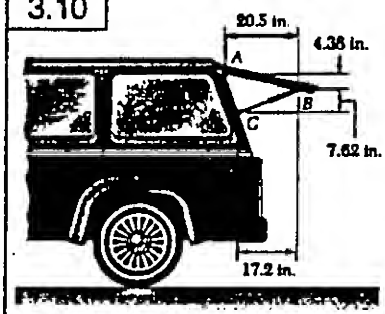
$$= -(364.54\ lb\cdot in.)\underline{k} + (1758.41\ lb\cdot in.)\underline{k}$$
$$= (1393.87\ lb\cdot in.)\underline{k}$$
$$= (116.2\ lb\cdot ft)\underline{k}$$

$$\underline{M}_A = 116.2\ lb\cdot ft \circlearrowleft \quad ◀$$

3.10

GIVEN: $F_{CB} = 125$ lb

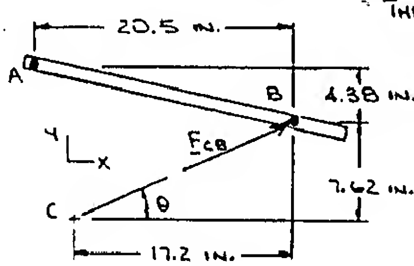
FIND: MOMENT OF F_{CB} ABOUT A



FIRST NOTE .. $d_{CB} = \sqrt{(17.2\ in.)^2 + (7.62\ in.)^2}$
$$= 18.8123\ in.$$

THEN $\quad \cos\theta = \dfrac{17.2}{18.8123}$

$\sin\theta = \dfrac{7.62}{18.8123}$



(CONTINUED)

3.10 CONTINUED

AND $\mathbf{F}_{CB} = F_{CB}\cos\theta\,\mathbf{i} + F_{CB}\sin\theta\,\mathbf{j}$

$= \dfrac{125 \text{ lb}}{18.8123}(17.2\,\mathbf{i} + 7.62\,\mathbf{j})$

NOW.. $\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{F}_{CB}$

WHERE $\mathbf{r}_{B/A} = (20.5 \text{ in.})\mathbf{i} - (4.38 \text{ in.})\mathbf{j}$

THEN. $\mathbf{M}_A = [(20.5 \text{ in.})\mathbf{i} - (4.38 \text{ in.})\mathbf{j}] \times \dfrac{125 \text{ lb}}{18.8123}(17.2\,\mathbf{i} + 7.62\,\mathbf{j})$

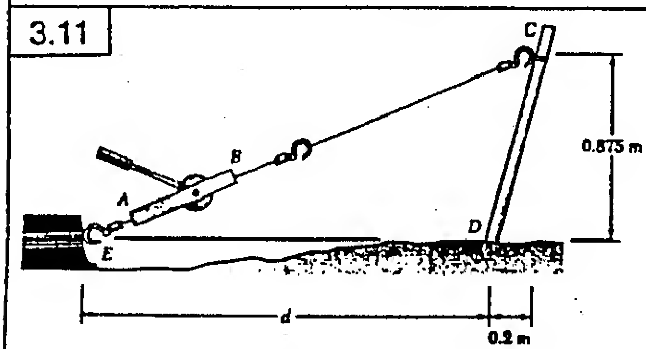
$= (1037.95 \text{ lb·in.})\mathbf{k} + (500.58 \text{ lb·in.})\mathbf{k}$

$= (1538.53 \text{ lb·in.})\mathbf{k}$

$= (128.2 \text{ lb·ft})\mathbf{k}$

$\mathbf{M}_A = 128.2 \text{ lb·ft}$ ↺ ◀

3.11

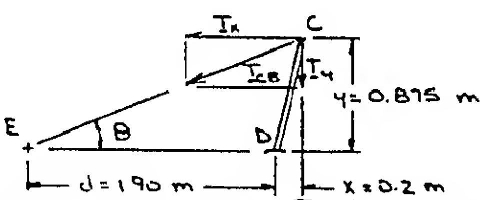


GIVEN: $T_{CB} = 1040$ N, $d = 1.90$ m.

FIND: MOMENT OF T_{CB} ABOUT D; RESOLVE T_{CB} INTO HORIZONTAL AND VERTICAL COMPONENTS APPLIED AT

(a) POINT C

(b) POINT E



FIRST NOTE .. $d_{CE} = \sqrt{(2.1 \text{ m})^2 + (0.875 \text{ m})^2}$

$= 2.275$ m

THEN $\cos\theta = \dfrac{2.1}{2.275} = \dfrac{12}{13}$ $\quad \sin\theta = \dfrac{0.875}{2.275} = \dfrac{5}{13}$

AND $T_x = T_{CB}\cos\theta = (1040 \text{ N})(\tfrac{12}{13}) = 960$ N

$T_y = T_{CB}\sin\theta = (1040 \text{ N})(\tfrac{5}{13}) = 400$ N

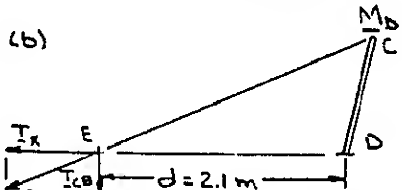
(a) BY OBSERVATION.. $M_D = -xT_y + yT_x$

OR $M_D = (0.2 \text{ m})(400 \text{ N}) + (0.875 \text{ m})(960 \text{ N})$

$= 760$ N·m

$\mathbf{M}_D = 760$ N·m ↺ ◀

(b)



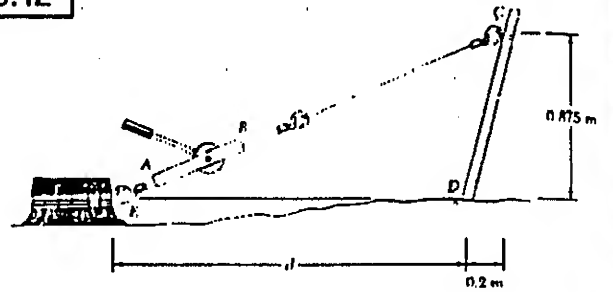
BY OBSERVATION.. $M_D = dT_y$

$= (1.90 \text{ m})(400 \text{ N})$

$= 760$ N·m

$\mathbf{M}_D = 760$ N·m ↺ ◀

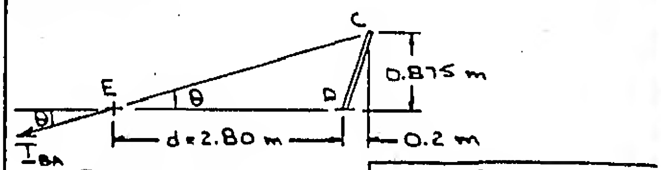
3.12



GIVEN: MOMENT OF T_{BA} ABOUT D = 960 N·m ↺

$d = 2.80$ m

FIND: T_{BA}



FIRST NOTE.. $d_{CE} = \sqrt{(3.0 \text{ m})^2 + (0.875 \text{ m})^2}$

$= 3.125$ m

THEN $\sin\theta = \dfrac{0.875}{3.125} = \dfrac{7}{25}$

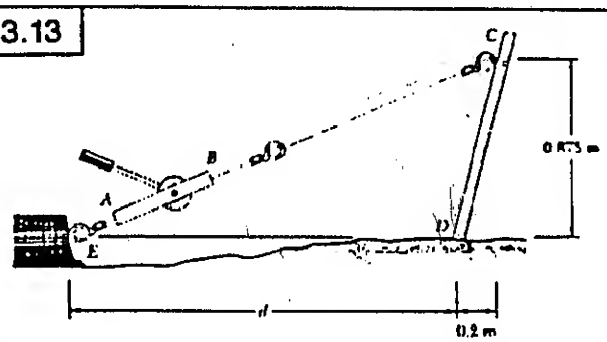
WITH T_{BA} APPLIED AT POINT E, HAVE

$M_D = d(T_{BA}\sin\theta)$

OR $960 \text{ N·m} = (2.80 \text{ m})(T_{BA} \times \tfrac{7}{25})$

OR $T_{BA} = 1224$ N ◀

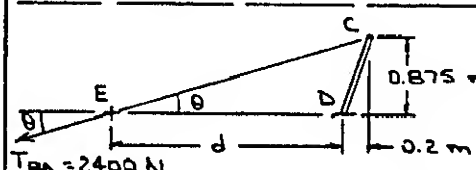
3.13



GIVEN: MOMENT OF T_{BA} ABOUT D = 960 N·m ↺

$(T_{BA})_{MAX} = 2400$ N

FIND: d_{MIN}



WITH T_{BA} APPLIED AT POINT E, HAVE

$M_D = d(T_{BA}\sin\theta)$

WHERE $\sin\theta = \dfrac{0.875}{\sqrt{(d+0.2)^2 + (0.875)^2}}$

THEN.. $960 \text{ N·m} = (d \text{ m})(2400 \text{ N})\left(\dfrac{0.875}{\sqrt{(d+0.2)^2 + (0.875)^2}}\right)$

OR $\sqrt{(d+0.2)^2 + (0.875)^2} = 2.1875\,d$

SQUARING BOTH SIDES OF THE EQUATION..

$d^2 + 0.4d + 0.04 + 0.7656 = 4.7852\,d^2$

OR $3.7852\,d^2 - 0.4d - 0.8056 = 0$

(CONTINUED)

3.13 CONTINUED

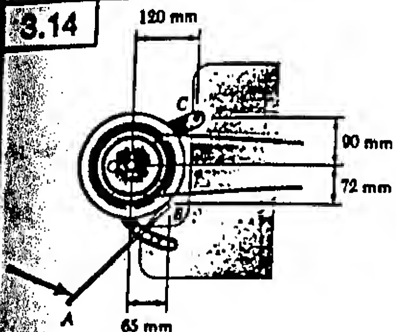
THEN $d = \dfrac{0.4 \pm \sqrt{(-0.4)^2 - 4(3.7852)(-0.8056)}}{2(3.7852)}$

REJECTING THE NEGATIVE ROOT

$d = 0.517$ m

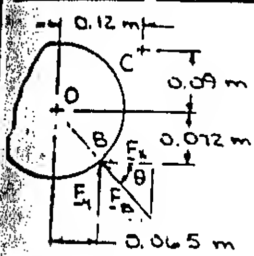
$d = 517$ mm ◀

3.14



GIVEN: $F_B = 485$ N, LINE OF ACTION OF F_B PASSES THROUGH O

FIND: MOMENT OF F_B ABOUT C



FIRST NOTE...

$d_{OB} = \sqrt{(65\text{ mm})^2 + (72\text{ mm})^2}$

$= 97$ mm

THEN $\cos\theta = \frac{65}{97}$

$\sin\theta = \frac{72}{97}$

AND $F_x = F_B \cos\theta = (485\text{ N})(\frac{65}{97}) = 325$ N

$F_y = F_B \sin\theta = (485\text{ N})(\frac{72}{97}) = 360$ N

BY OBSERVATION.. $M_C = -xF_y - yF_x$

WHERE $x = 0.12\text{ m} - 0.065\text{ m} = 0.055$ m

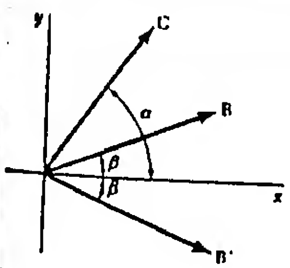
$y = 0.072\text{ m} + 0.09\text{ m} = 0.162$ m

THEN $M_C = -(0.055\text{ m})(360\text{ N}) - (0.162\text{ m})(325\text{ N})$

$= -72.45$ N·m

$\mathbf{M}_C = 72.5$ N·m ↻ ◀

3.15



GIVEN: VECTORS $\mathbf{B}$, $\mathbf{B}'$ AND $\mathbf{C}$

PROVE: $\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha+\beta) + \frac{1}{2}\sin(\alpha-\beta)$

FIRST NOTE.. $\mathbf{B} = B(\cos\beta\,\mathbf{i} + \sin\beta\,\mathbf{j})$

$\mathbf{B}' = B(\cos\beta\,\mathbf{i} - \sin\beta\,\mathbf{j})$

$\mathbf{C} = C(\cos\alpha\,\mathbf{i} + \sin\alpha\,\mathbf{j})$

BY DEFINITION.. $|\mathbf{B}\times\mathbf{C}| = BC\sin(\alpha-\beta)$ (1)

$|\mathbf{B}'\times\mathbf{C}| = BC\sin(\alpha+\beta)$ (2)

(CONTINUED)

3.15 CONTINUED

NOW $\mathbf{B}\times\mathbf{C} = B(\cos\beta\,\mathbf{i} + \sin\beta\,\mathbf{j}) \times C(\cos\alpha\,\mathbf{i} + \sin\alpha\,\mathbf{j})$

$= BC(\cos\beta\sin\alpha - \sin\beta\cos\alpha)\mathbf{k}$ (3)

AND $\mathbf{B}'\times\mathbf{C} = B(\cos\beta\,\mathbf{i} - \sin\beta\,\mathbf{j}) \times C(\cos\alpha\,\mathbf{i} + \sin\alpha\,\mathbf{j})$

$= BC(\cos\beta\sin\alpha + \sin\beta\cos\alpha)\mathbf{k}$ (4)

EQUATING THE RIGHT-HAND SIDES OF EQS. (1) AND (2) TO THE MAGNITUDES OF THE RIGHT-HAND SIDES OF EQS. (3) AND (4), RESPECTIVELY, YIELDS..

$BC\sin(\alpha-\beta) = BC(\cos\beta\sin\alpha - \sin\beta\cos\alpha)$ (5)

$BC\sin(\alpha+\beta) = BC(\cos\beta\sin\alpha + \sin\beta\cos\alpha)$ (6)

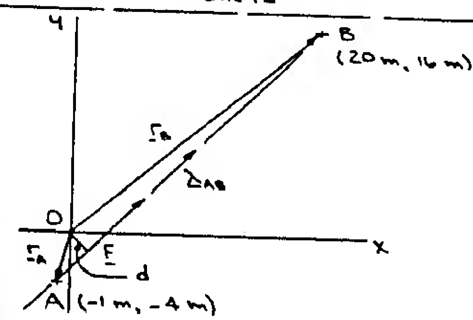
$(5)+(6) \Rightarrow \sin(\alpha-\beta) + \sin(\alpha+\beta) = 2\cos\beta\sin\alpha$

OR $\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha+\beta) + \frac{1}{2}\sin(\alpha-\beta)$ ◀

3.16

GIVEN: POINTS (20 m, 16 m) AND (−1 m, −4 m)

FIND: PERPENDICULAR DISTANCE d FROM THE ORIGIN TO THE LINE DRAWN THROUGH THE POINTS



FIRST NOTE.. $d_{AB} = \sqrt{[20\text{ m} - (-1\text{ m})]^2 + [16\text{ m} - (-4\text{ m})]^2}$

$= 29$ m

NOW ASSUME THAT A FORCE $\mathbf{F}$, OF MAGNITUDE F, ACTS AT POINT A AND IS DIRECTED FROM A TO B. THEN

$\mathbf{F} = F\lambda_{AB}$ (F IN N)

WHERE $\lambda_{AB} = \dfrac{\mathbf{r}_B - \mathbf{r}_A}{d_{AB}}$

$= \frac{1}{29}(21\,\mathbf{i} + 20\,\mathbf{j})$

BY DEFINITION.. $M_O = |\mathbf{r}_A \times \mathbf{F}| = dF$

WHERE $\mathbf{r}_A = -(1\text{ m})\mathbf{i} - (4\text{ m})\mathbf{j}$

THEN $\mathbf{M}_O = [-(1\text{ m})\mathbf{i} - (4\text{ m})\mathbf{j}] \times \frac{F}{29}(21\,\mathbf{i} + 20\,\mathbf{j})(\text{N})$

$= \frac{F}{29}[-(20)\mathbf{k} + (84)\mathbf{k}]$ N·m

$= (\frac{64}{29}F\text{ N·m})\mathbf{k}$

FINALLY.. $(\frac{64}{29}F)\text{N·m} = d(F\text{ N})$

OR $d = \frac{64}{29}$ m

$d = 2.21$ m ◀

3.17

GIVEN: VECTORS $\mathbf{A}$ AND $\mathbf{B}$

FIND: UNIT VECTOR $\boldsymbol{\lambda}$ NORMAL TO THE PLANE DEFINED BY $\mathbf{A}$ AND $\mathbf{B}$ WHEN

(a) $\mathbf{A} = \mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$
$\mathbf{B} = 4\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$

(b) $\mathbf{A} = 3\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$
$\mathbf{B} = -2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$

BY DEFINITION, THE VECTOR $\mathbf{A} \times \mathbf{B}$ IS NORMAL TO THE PLANE DEFINED BY $\mathbf{A}$ AND $\mathbf{B}$. THUS,

$$\boldsymbol{\lambda} = \frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$$

(a) HAVE

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -5 \\ 4 & -7 & -5 \end{vmatrix}$$

$$= (-10 - 35)\mathbf{i} + (-20 + 5)\mathbf{j} + (-7 - 8)\mathbf{k}$$

$$= -45\mathbf{i} - 15\mathbf{j} - 15\mathbf{k}$$

THEN $|\mathbf{A} \times \mathbf{B}| = 15\sqrt{(-3)^2 + (-1)^2 + (-1)^2}$

$$= 15\sqrt{11}$$

$$\therefore \boldsymbol{\lambda} = \frac{1}{\sqrt{11}}(-3\mathbf{i} - \mathbf{j} - \mathbf{k}) \quad ◀$$

(b) HAVE

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -3 & 2 \\ -2 & 6 & -4 \end{vmatrix}$$

$$= (12 - 12)\mathbf{i} + (-4 + 12)\mathbf{j} + (18 - 6)\mathbf{k}$$

$$= 8\mathbf{j} + 12\mathbf{k}$$

THEN $|\mathbf{A} \times \mathbf{B}| = 4\sqrt{(2)^2 + (3)^2}$

$$= 4\sqrt{13}$$

$$\therefore \boldsymbol{\lambda} = \frac{1}{\sqrt{13}}(2\mathbf{j} + 3\mathbf{k}) \quad ◀$$

3.18

GIVEN: ADJACENT SIDES $\mathbf{P}$ AND $\mathbf{Q}$ OF A PARALLELOGRAM

FIND: AREA OF PARALLELOGRAM WHEN

(a) $\mathbf{P} = -7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$
$\mathbf{Q} = 2\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$

(b) $\mathbf{P} = 6\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$
$\mathbf{Q} = -2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$

HAVE.. AREA $A = |\mathbf{P} \times \mathbf{Q}|$

(a)

$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -7 & 3 & -3 \\ 2 & 2 & 5 \end{vmatrix}$$

$$= (15 + 6)\mathbf{i} + (-6 + 35)\mathbf{j} + (-14 - 6)\mathbf{k}$$

$$= 21\mathbf{i} + 29\mathbf{j} - 20\mathbf{k}$$

THEN $A = \sqrt{(20)^2 + (29)^2 + (-20)^2}$ $\qquad A = 41.0$ ◀

(b)

$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -5 & -2 \\ -2 & 5 & -1 \end{vmatrix}$$

$$= (5 + 10)\mathbf{i} + (4 + 6)\mathbf{j} + (30 - 10)\mathbf{k}$$

$$= 15\mathbf{i} + 10\mathbf{j} + 20\mathbf{k}$$

THEN $A = 5\sqrt{(3)^2 + (2)^2 + (4)^2}$ $\qquad A = 26.9$ ◀

3.19

GIVEN: FORCE $\mathbf{F} = 6\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ ACTING AT POINT A

FIND: MOMENT OF $\mathbf{F}$ ABOUT ORIGIN O WHEN

(a) $\mathbf{r}_A = -2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$

(b) $\mathbf{r}_A = 5\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$

(c) $\mathbf{r}_A = -9\mathbf{i} - 6\mathbf{j} + 1.5\mathbf{k}$

BY DEFINITION $\quad \mathbf{M}_O = \mathbf{r}_A \times \mathbf{F}$

(a) HAVE.. $\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 6 & 3 \\ 6 & 4 & -1 \end{vmatrix}$

$$= (-6 - 12)\mathbf{i} + (18 - 2)\mathbf{j} + (-8 - 36)\mathbf{k}$$

$$= -18\mathbf{i} + 16\mathbf{j} - 44\mathbf{k} \quad ◀$$

(b) HAVE.. $\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -3 & 7 \\ 6 & 4 & -1 \end{vmatrix}$

$$= (3 - 28)\mathbf{i} + (42 + 5)\mathbf{j} + (20 + 18)\mathbf{k}$$

$$= -25\mathbf{i} + 47\mathbf{j} + 38\mathbf{k} \quad ◀$$

(c) HAVE.. $\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -9 & -6 & 1.5 \\ 6 & 4 & -1 \end{vmatrix}$

$$= (6 - 6)\mathbf{i} + (9 - 9)\mathbf{j} + (-36 + 36)\mathbf{k}$$

$$= 0 \quad ◀$$

NOTE: THE ANSWER TO PART C IS AS EXPECTED SINCE $\mathbf{r}_A$ AND $\mathbf{F}$ ARE PROPORTIONAL (THUS, THEIR LINES OF ACTION ARE PARALLEL).

3.20

GIVEN: FORCE $\mathbf{F} = 2\mathbf{i} - 7\mathbf{j} - 3\mathbf{k}$ ACTING AT POINT A

FIND: MOMENT OF $\mathbf{F}$ ABOUT ORIGIN O WHEN

(a) $\mathbf{r}_A = 4\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$

(b) $\mathbf{r}_A = -8\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

(c) $\mathbf{r}_A = \mathbf{i} - 3.5\mathbf{j} - 1.5\mathbf{k}$

BY DEFINITION $\quad \mathbf{M}_O = \mathbf{r}_A \times \mathbf{F}$

(a) HAVE.. $\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -3 & -5 \\ 2 & -7 & -3 \end{vmatrix}$

$$= (9 - 35)\mathbf{i} + (-10 + 12)\mathbf{j} + (-28 + 6)\mathbf{k}$$

$$= -26\mathbf{i} + 2\mathbf{j} - 22\mathbf{k} \quad ◀$$

(b) HAVE.. $\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & -2 & 1 \\ 2 & -7 & -3 \end{vmatrix}$

$$= (6 + 7)\mathbf{i} + (2 - 24)\mathbf{j} + (56 + 4)\mathbf{k}$$

$$= 13\mathbf{i} - 22\mathbf{j} + 60\mathbf{k} \quad ◀$$

(c) HAVE.. $\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3.5 & -1.5 \\ 2 & -7 & -3 \end{vmatrix}$

$$= (10.5 - 10.5)\mathbf{i} + (-3 + 3)\mathbf{j} + (-7 + 7)\mathbf{k}$$

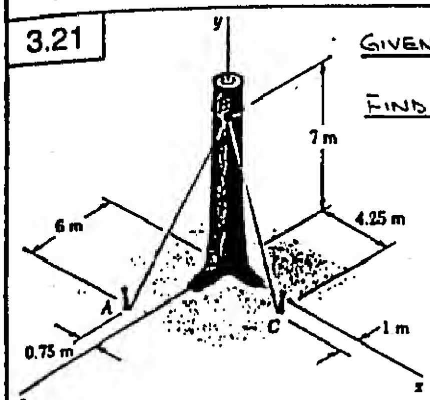
$$= 0 \quad ◀$$

(CONTINUED)

3.20 CONTINUED

NOTE: THE ANSWER TO PART C IS AS EXPECTED SINCE $\mathbf{r}_A$ AND $\mathbf{E}$ ARE PROPORTIONAL (THUS, THEIR LINES OF ACTION ARE PARALLEL).

3.21



GIVEN: $T_{BA} = 555$ N, $T_{BC} = 660$ N

FIND: MOMENT OF $(\mathbf{T}_{BA} + \mathbf{T}_{BC})$ ABOUT O

FIRST NOTE.. $d_{BA} = \sqrt{(-0.75)^2 + (-7)^2 + (6)^2} = 9.25$ m

$d_{BC} = \sqrt{(4.25)^2 + (-7)^2 + (1)^2} = 8.25$ m

NOW.. $\mathbf{T}_{BA} = \dfrac{T_{BA}}{d_{BA}}\overrightarrow{BA} = \dfrac{555\text{ N}}{9.25}(-0.75\mathbf{i} - 7\mathbf{j} + 6\mathbf{k})$

$= -(45\text{ N})\mathbf{i} - (420\text{ N})\mathbf{j} + (360\text{ N})\mathbf{k}$

AND $\mathbf{T}_{BC} = \dfrac{T_{BC}}{d_{BC}}\overrightarrow{BC} = \dfrac{660\text{ N}}{8.25}(4.25\mathbf{i} - 7\mathbf{j} + \mathbf{k})$

$= (340\text{ N})\mathbf{i} - (560\text{ N})\mathbf{j} + (80\text{ N})\mathbf{k}$

THEN $\mathbf{R} = \mathbf{T}_{BA} + \mathbf{T}_{BC}$

$= -(295\text{ N})\mathbf{i} - (980\text{ N})\mathbf{j} + (440\text{ N})\mathbf{k}$

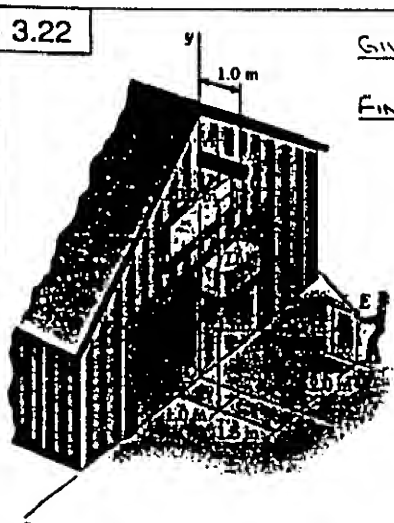
FINALLY.. $\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{R}$ WHERE $\mathbf{r}_{B/O} = (7\text{ m})\mathbf{j}$

$= (7\text{ m})\mathbf{j} \times [-(295\text{ N})\mathbf{i} - (980\text{ N})\mathbf{j} + (440\text{ N})\mathbf{k}]$

$= (3080\text{ N}\cdot\text{m})\mathbf{i} - (2065\text{ N}\cdot\text{m})\mathbf{k}$

$\mathbf{M}_O = (3080\text{ N}\cdot\text{m})\mathbf{i} - (2070\text{ N}\cdot\text{m})\mathbf{k}$ ◀

3.22



GIVEN: MASS m OF BALE = 26 kg

FIND: MOMENT ABOUT A OF RESULTANT FORCE EXERTED ON THE PULLEY BY THE ROPE

(CONTINUED)

3.22 CONTINUED

FIRST NOTE.. $T_{CD} = T_{CE} = W_{BALE} = mg$ WHERE $m = 26$ kg $g = 9.81\ \frac{m}{s^2}$

NOW.. $d_{CE} = \sqrt{(1.5)^2 + (-6.0)^2 + (-2.0)^2} = 6.5$ m

THEN $\mathbf{T}_{CE} = \dfrac{T_{CE}}{d_{CE}}\overrightarrow{CE} = \dfrac{26g}{6.5}(1.5\mathbf{i} - 6\mathbf{j} - 2\mathbf{k})$

$= g(6\mathbf{i} - 24\mathbf{j} - 8\mathbf{k})$ (N)

ALSO.. $\mathbf{T}_{CD} = -(26g)\mathbf{j}$ (N)

NOW.. $\mathbf{R} = \mathbf{T}_{CD} + \mathbf{T}_{CE}$

$= g(6\mathbf{i} - 50\mathbf{j} - 8\mathbf{k})$ (N)

AND $\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{R}$

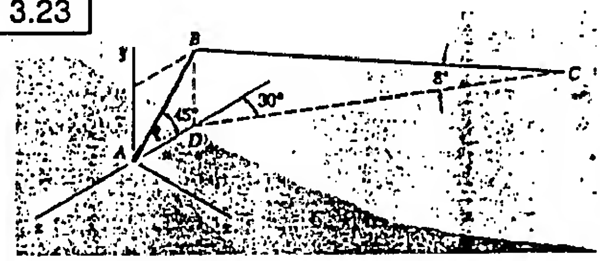
WHERE $\mathbf{r}_{C/A} = (1\text{ m})\mathbf{i} - (0.3\text{ m})\mathbf{j}$

THEN.. $\mathbf{M}_A = 9.81\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -0.3 & 0 \\ 6 & -50 & -8 \end{vmatrix}$

$= 9.81[2.4\mathbf{i} + 8\mathbf{j} + (-50 + 1.8)\mathbf{k}]$

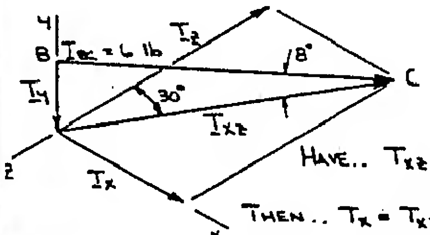
OR $\mathbf{M}_A = (23.5\text{ N}\cdot\text{m})\mathbf{i} + (78.5\text{ N}\cdot\text{m})\mathbf{j} - (473\text{ N}\cdot\text{m})\mathbf{k}$ ◀

3.23



GIVEN: $d_{AB} = 6$ ft, $T_{BC} = 6$ lb

FIND: MOMENT ABOUT A OF $\mathbf{T}_{BC}$ AT B



HAVE.. $T_{xz} = (6\text{ lb})\cos 8° = 5.9416$ lb

THEN.. $T_x = T_{xz}\sin 30° = 2.9708$ lb

$T_y = -T_{BC}\sin 8° = -0.83504$ lb

$T_z = -T_{xz}\cos 30° = -5.1456$ lb

NOW.. $\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BC}$

WHERE $\mathbf{r}_{B/A} = (6\sin 45°)\mathbf{j} - (6\cos 45°)\mathbf{k}$

$= \dfrac{6\text{ ft}}{\sqrt{2}}(\mathbf{j} - \mathbf{k})$

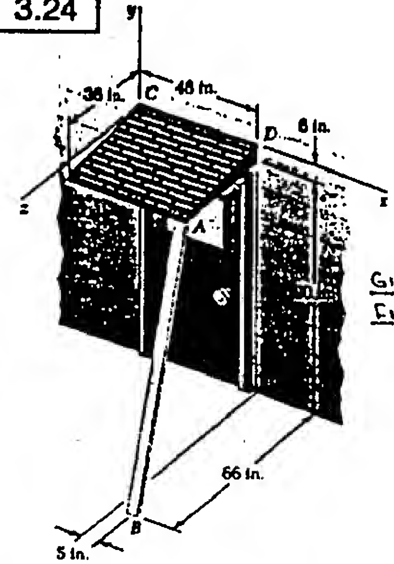
THEN $\mathbf{M}_A = \dfrac{6}{\sqrt{2}}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & -1 \\ 2.9708 & -0.83504 & -5.1456 \end{vmatrix}$

$= \dfrac{6}{\sqrt{2}}(-5.1456 - 0.83504)\mathbf{i}$

$- \dfrac{6}{\sqrt{2}}(2.9708)\mathbf{j} - \dfrac{6}{\sqrt{2}}(-2.9708)\mathbf{k}$

OR $\mathbf{M}_A = -(25.4\text{ lb}\cdot\text{ft})\mathbf{i} - (12.60\text{ lb}\cdot\text{ft})\mathbf{j} - (12.60\text{ lb}\cdot\text{ft})\mathbf{k}$ ◀

3.24



GIVEN: $T_{BA} = 57$ lb

FIND: MOMENT ABOUT C OF $\mathbf{T}_{BA}$ AT A

FIRST NOTE.. $d_{BA} = \sqrt{(-5)^2+(90)^2+(-30)^2} = 95$ in.

THEN $\mathbf{T}_{BA} = \dfrac{T_{BA}}{d_{BA}}\overrightarrow{BA} = \dfrac{57\text{ lb}}{95}(-5\mathbf{i}+90\mathbf{j}-30\mathbf{k})$

$= 3[-(1\text{ lb})\mathbf{i}+(18\text{ lb})\mathbf{j}-(6\text{ lb})\mathbf{k}]$

NOW.. $\mathbf{M}_C = \mathbf{r}_{A/C}\times\mathbf{T}_{BA}$

WHERE $\mathbf{r}_{A/C} = (48\text{ in.})\mathbf{i}-(6\text{ in.})\mathbf{j}+(36\text{ in.})\mathbf{k}$

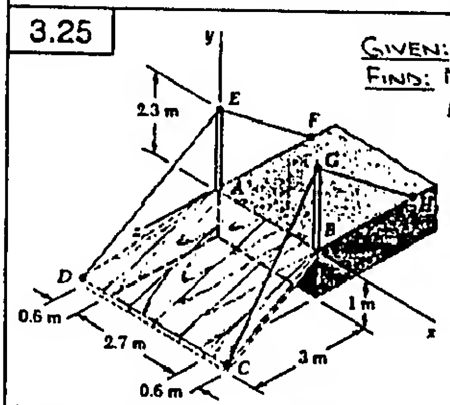
THEN $\mathbf{M}_C = (6)(3)\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 8 & -1 & 6\\ -1 & 18 & -6\end{vmatrix}$

$= 18[(6-108)\mathbf{i}+(-6+48)\mathbf{j}+(144-1)\mathbf{k}]$

$= -(1836\text{ lb·in.})\mathbf{i}+(756\text{ lb·in.})\mathbf{j}+(2574\text{ lb·in.})\mathbf{k}$

OR $\mathbf{M}_C = -(153.0\text{ lb·ft})\mathbf{i}+(63.0\text{ lb·ft})\mathbf{j}+(215\text{ lb·ft})\mathbf{k}$ ◀

3.25



GIVEN: $T_{DE} = T_{CG} = 810$ N

FIND: MOMENT ABOUT A OF

(a) $\mathbf{T}_{DE}$ AT D

(b) $\mathbf{T}_{CG}$ AT C

FIRST NOTE: $d_{DE} = \sqrt{(0.6)^2+(3.3)^2+(-3)^2} = 4.5$ m

$d_{CG} = \sqrt{(-0.6)^2+(3.3)^2+(-3)^2} = 4.5$ m

THEN $\mathbf{T}_{DE} = \dfrac{T_{DE}}{d_{DE}}\overrightarrow{DE} = \dfrac{810\text{ N}}{4.5}(0.6\mathbf{i}+3.3\mathbf{j}-3\mathbf{k})$

$= 54[(2\text{ N})\mathbf{i}+(11\text{ N})\mathbf{j}-(10\text{ N})\mathbf{k}]$

SIMILARLY, $\mathbf{T}_{CG} = 54[-(2\text{ N})\mathbf{i}+(11\text{ N})\mathbf{j}-(10\text{ N})\mathbf{k}]$

(a) NOW.. $\mathbf{M}_A = \mathbf{r}_{E/A}\times\mathbf{T}_{DE}$ WHERE $\mathbf{r}_{E/A} = (2.3\text{ m})\mathbf{j}$

$= 2.3\mathbf{j}\times 54(2\mathbf{i}+11\mathbf{j}-10\mathbf{k})$

OR $\mathbf{M}_A = -(1242\text{ N·m})\mathbf{i}-(248\text{ N·m})\mathbf{k}$ ◀

(CONTINUED)

3.25 CONTINUED

(b) NOW.. $\mathbf{M}_A = \mathbf{r}_{C/A}\times\mathbf{T}_{CG}$

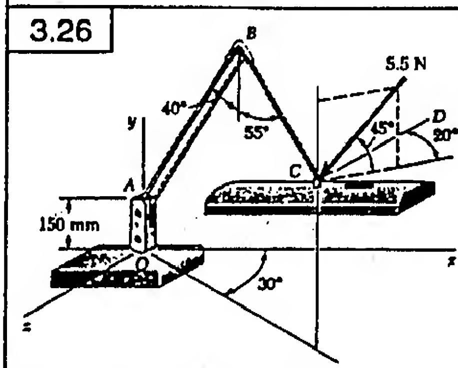
WHERE $\mathbf{r}_{C/A} = (2.7\text{ m})\mathbf{i}+(2.3\text{ m})\mathbf{j}$

THEN.. $\mathbf{M}_A = 54\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2.7 & 2.3 & 0\\ -2 & 11 & -10\end{vmatrix}$

$= 54[-23\mathbf{i}+27\mathbf{j}+(29.7+4.6)\mathbf{k}]$

OR $\mathbf{M}_A = -(1242\text{ N·m})\mathbf{i}+(1458\text{ N·m})\mathbf{j}+(1852\text{ N·m})\mathbf{k}$ ◀

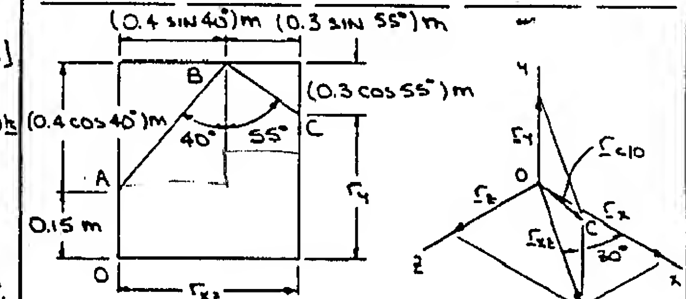
3.26



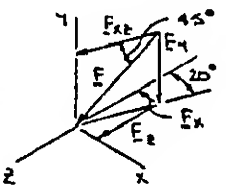
GIVEN: AB = 0.4 m

BC = 0.3 m

FIND: MOMENT OF FORCE ABOUT O



HAVE.. $\mathbf{r}_{C/O} = [(0.4\sin 40°+0.3\sin 55°)\cos 30°]\mathbf{i}$
$+[0.15+0.4\cos 40°-0.3\cos 55°]\mathbf{j}$
$+[(0.4\sin 40°+0.3\sin 55°)\sin 30°]\mathbf{k}$
$= (0.43549\text{ m})\mathbf{i}+(0.28434\text{ m})\mathbf{j}+(0.25143\text{ m})\mathbf{k}$



ALSO.. $\mathbf{F} = 5.5(-\cos 45°\sin 20°\,\mathbf{i}-\sin 45°\,\mathbf{j}+\cos 45°\cos 20°\,\mathbf{k})$

$= \dfrac{5.5\text{ N}}{\sqrt{2}}(-\sin 20°\,\mathbf{i}-\mathbf{j}+\cos 20°\,\mathbf{k})$

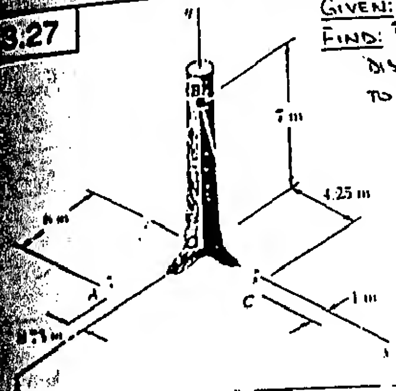
NOW.. $\mathbf{M}_O = \mathbf{r}_{C/O}\times\mathbf{F}$

$= \dfrac{5.5}{\sqrt{2}}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0.43549 & 0.28434 & 0.25143\\ -\sin 20° & -1 & \cos 20°\end{vmatrix}$

$= \dfrac{5.5}{\sqrt{2}}[(0.28434\cos 20°+0.25143)\mathbf{i}$
$+(-0.25143\sin 20°-0.43549\cos 20°)\mathbf{j}$
$+(-0.43549+0.28434\sin 20°)\mathbf{k}]$

OR $\mathbf{M}_O = (2.02\text{ N·m})\mathbf{i}-(1.926\text{ N·m})\mathbf{j}-(1.315\text{ N·m})\mathbf{k}$ ◀

3.27



GIVEN: $\underline{T}_{BA}$, $T_{BA} = 555$ N

FIND: PERPENDICULAR DISTANCE FROM O TO CABLE AB

FROM THE SOLUTION TO PROBLEM 3.21

$\underline{T}_{BA} = -(45\text{ N})\underline{i} - (420\text{ N})\underline{j} + (360\text{ N})\underline{k}$

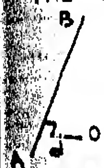
NOW.. $\underline{M}_O = \underline{r}_{B/O} \times \underline{T}_{BA}$ $\quad \underline{r}_{B/O} = (7\text{ m})\underline{j}$

$= 7\underline{j} \times (-45\underline{i} - 420\underline{j} + 360\underline{k})$

$= 7[(360\text{ N}\cdot\text{m})\underline{i} + (45\text{ N}\cdot\text{m})\underline{k}]$

THEN.. $M_O = 7\sqrt{(360)^2 + (45)^2}$

$= 2539.6$ N·m

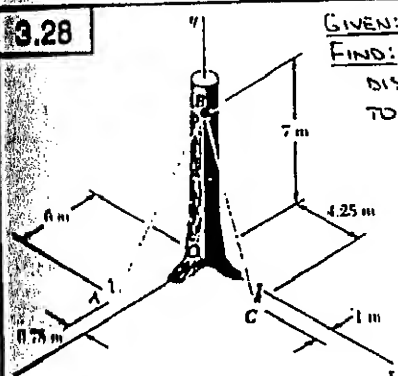


ALSO.. $M_O = dT_{BA}$

OR 2539.6 N·m $= d \cdot 555$ N

OR $d = 4.58$ m ◀

3.28



GIVEN: $\underline{T}_{BC}$, $T_{BC} = 660$ N

FIND: PERPENDICULAR DISTANCE FROM O TO CABLE BC

FROM THE SOLUTION TO PROBLEM 3.21

$\underline{T}_{BC} = (340\text{ N})\underline{i} - (560\text{ N})\underline{j} + (80\text{ N})\underline{k}$

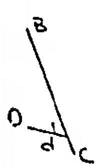
NOW.. $\underline{M}_O = \underline{r}_{B/O} \times \underline{T}_{BC}$ $\quad \underline{r}_{B/O} = (7\text{ m})\underline{j}$

$= 7\underline{j} \times (340\underline{i} - 560\underline{j} + 80\underline{k})$

$= 7[(80\text{ N}\cdot\text{m})\underline{i} - (340\text{ N}\cdot\text{m})\underline{k}]$

THEN.. $M_O = 7\sqrt{(80)^2 + (-340)^2}$

$= 2445.0$ N·m

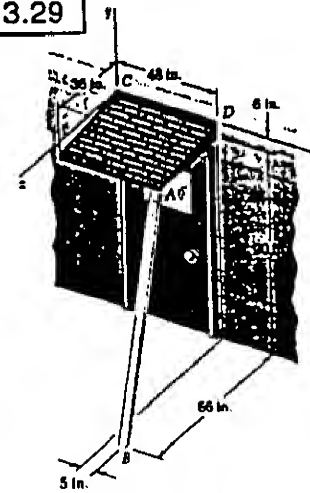


ALSO.. $M_O = dT_{BC}$

OR 2445.0 N·m $= d \cdot 660$ N

OR $d = 3.70$ m ◀

3.29



GIVEN: $\underline{T}_{BA}$, $T_{BA} = 57$ lb

FIND: PERPENDICULAR DISTANCE FROM D TO A LINE THROUGH A AND B

FROM THE SOLUTION TO PROBLEM 3.24

$\underline{T}_{BA} = 3[-(1\text{ lb})\underline{i} + (18\text{ lb})\underline{j} - (6\text{ lb})\underline{k}]$

NOW.. $\underline{M}_D = \underline{r}_{A/D} \times \underline{T}_{BA}$

WHERE $\underline{r}_{A/D} = -(6\text{ in.})\underline{j} + (36\text{ in.})\underline{k}$

$= 6[-(1\text{ in.})\underline{j} + (6\text{ in.})\underline{k}]$

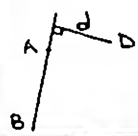
THEN.. $\underline{M}_D = (3)(6)\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -1 & 6 \\ -1 & 18 & -6 \end{vmatrix}$

$= 18[(6-108)\underline{i} - 6\underline{j} - \underline{k}]$

$= 18[(-102\text{ lb}\cdot\text{in.})\underline{i} - (6\text{ lb}\cdot\text{in.})\underline{j} - (1\text{ lb}\cdot\text{in.})\underline{k}]$

AND $M_D = 18\sqrt{(-102)^2 + (-6)^2 + (-1)^2}$

$= 1839.26$ lb·in.

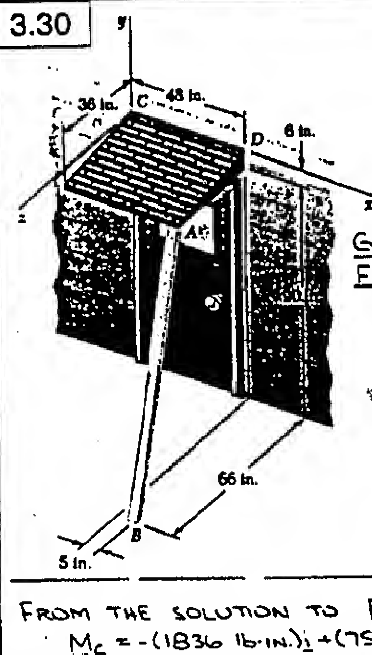


ALSO.. $M_D = dT_{BA}$

OR 1839.26 lb·in. $= d \cdot 57$ lb

OR $d = 32.3$ in. ◀

3.30



GIVEN: $\underline{M}_C$, $T_{BA} = 57$ lb

FIND: PERPENDICULAR DISTANCE FROM C TO A LINE THROUGH A AND B

FROM THE SOLUTION TO PROBLEM 3.24

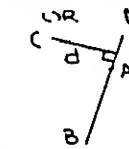
$\underline{M}_C = -(1836\text{ lb}\cdot\text{in.})\underline{i} + (756\text{ lb}\cdot\text{in.})\underline{j} + (2574\text{ lb}\cdot\text{in.})\underline{k}$

THEN $M_C = \sqrt{(-1836)^2 + (756)^2 + (2574)^2}$

(CONTINUED)

ALSO. $M_A = dT_{CG}$

3.30 CONTINUED



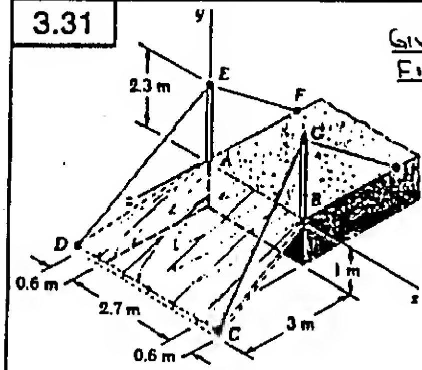
OR $M_C = 3250.8$ lb·in.

ALSO.. $M_C = dT_{BA}$

OR 3250.8 lb·in. $= d \cdot 57$ lb

OR $d = 57.0$ in. ◄

3.31



GIVEN: $\underline{M}_A$, $T_{DE} = 810$ N

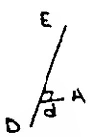
FIND: PERPENDICULAR DISTANCE FROM A TO CABLE DE

FROM THE SOLUTION TO PROBLEM 3.25(a)

$\underline{M}_A = -(1242 \text{ N·m})\underline{i} - (248 \text{ N·m})\underline{k}$

THEN $M_A = \sqrt{(-1242)^2 + (-248)^2}$

$= 1266.52$ N·m

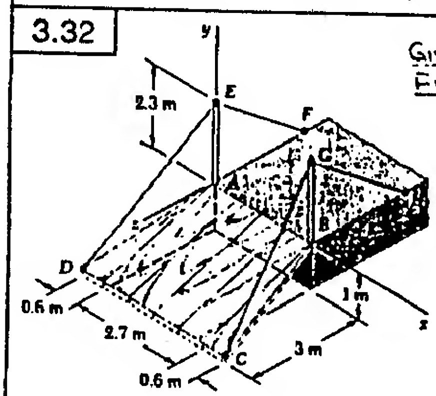


ALSO.. $M_A = dT_{DE}$

OR 1266.52 N·m $= d \cdot 810$ N

OR $d = 1.564$ m ◄

3.32



GIVEN: $\underline{M}_A$, $T_{CG} = 810$ N

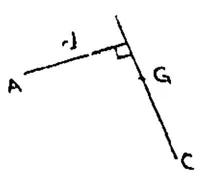
FIND: PERPENDICULAR DISTANCE FROM A TO A LINE THROUGH C AND G

FROM THE SOLUTION TO PROBLEM 3.25(b)

$\underline{M}_A = -(1242 \text{ N·m})\underline{i} + (1458 \text{ N·m})\underline{j} + (1852 \text{ N·m})\underline{k}$

THEN $M_A = \sqrt{(-1242)^2 + (1458)^2 + (1852)^2}$

$= 2664.3$ N·m

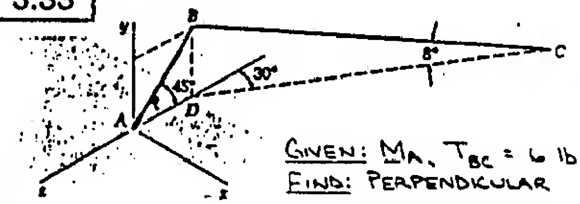


ALSO. $M_A = dT_{CG}$

OR 2664.3 N·m $= d \cdot 810$ N

OR $d = 3.29$ m ◄

3.33



GIVEN: $\underline{M}_A$, $T_{BC} = 6$ lb

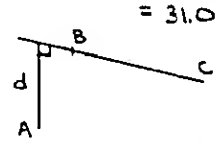
FIND: PERPENDICULAR DISTANCE FROM A TO A LINE THROUGH C AND G

FROM THE SOLUTION TO PROBLEM 3.23

$\underline{M}_A = -(25.4 \text{ lb·ft})\underline{i} - (12.60 \text{ lb·ft})\underline{j} - (12.60 \text{ lb·ft})\underline{k}$

THEN $M_A = \sqrt{(-25.4)^2 + (-12.60)^2 + (-12.60)^2}$

$= 31.027$ lb·ft

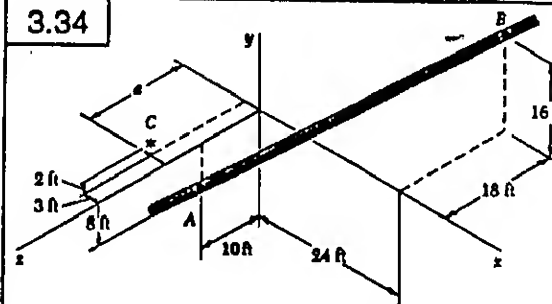


ALSO.. $M_A = dT_{BC}$

OR 31.027 lb·ft $= d \cdot 6$ lb

OR $d = 5.17$ ft ◄

3.34



GIVEN: SECTION OF PIPELINE

FIND: a SO THAT PERPENDICULAR DISTANCE d FROM C TO A LINE THROUGH A AND B IS A MINIMUM

FIRST NOTE.. $d_{AB} = \sqrt{(24-0)^2 + [16-(-8)]^2 + (-18-10)^2}$

$= 44$ ft

NOW ASSUME THAT A FORCE $\underline{F}$, OF MAGNITUDE F, ACTS AT POINT A AND IS DIRECTED FROM A TO B. THEN

$\underline{F} = F\underline{\lambda}_{AB}$ (F IN lb)

$= \frac{F}{44}(24\underline{i} + 24\underline{j} - 28\underline{k})$

$= \frac{F}{11}(6\underline{i} + 6\underline{j} - 7\underline{k})$

BY DEFINITION.. $M_C = |\underline{r}_{A/C} \times \underline{F}| = dF$

WHERE $\underline{r}_{A/C} = (3 \text{ ft})\underline{i} - (10 \text{ ft})\underline{j} + [(10-a)\text{ft}]\underline{k}$

THEN

$$\underline{M}_C = \frac{F}{11}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & -10 & (10-a) \\ 6 & 6 & -7 \end{vmatrix}$$

$$= \frac{F}{11}\{[70 - 6(10-a)]\underline{i} + [6(10-a) + 21]\underline{j} + [18 + 60]\underline{k}\}$$

$$= \frac{F}{11}\{[(10+6a) \text{ lb·ft}]\underline{i} + [(81-6a) \text{ lb·ft}]\underline{j} + [78 \text{ lb·ft}]\underline{k}\}$$

THEN.. $\left(\frac{F}{11}\right)^2[(10+6a)^2 + (81-6a)^2 + (78)^2] = (dF)^2$

OR $d^2 = \frac{1}{121}[(10+6a)^2 + (81-6a)^2 + (78)^2]$ (ft)^2

FINALLY.. $\frac{d(d^2)}{da} = \frac{1}{121}[2(6)(10+6a) + 2(-6)(81-6a)] = 0$

(CONTINUED)

3.34 CONTINUED

OR $(10+6a)-(81-6a)=0$

SO THAT FOR d_{MIN} $\qquad a = 5.92$ ft ◀

3.35

GIVEN: $\underline{P} = 4\underline{i} + 3\underline{j} - 2\underline{k}$

$\underline{Q} = -\underline{i} + 4\underline{j} - 5\underline{k}$

$\underline{S} = \underline{i} + 4\underline{j} + 3\underline{k}$

FIND: $\underline{P}\cdot\underline{Q}$, $\underline{P}\cdot\underline{S}$, $\underline{Q}\cdot\underline{S}$

HAVE.. $\underline{P}\cdot\underline{Q} = (4\underline{i}+3\underline{j}-2\underline{k})\cdot(-\underline{i}+4\underline{j}-5\underline{k})$

$= (4)(-1)+(3)(4)+(-2)(-5)$

OR $\underline{P}\cdot\underline{Q} = 18$ ◀

$\underline{P}\cdot\underline{S} = (4\underline{i}+3\underline{j}-2\underline{k})\cdot(\underline{i}+4\underline{j}+3\underline{k})$

$= (4)(1)+(3)(4)+(-2)(3)$

OR $\underline{P}\cdot\underline{S} = 10$ ◀

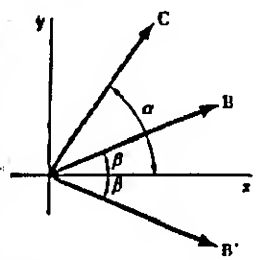
$\underline{Q}\cdot\underline{S} = (-\underline{i}+4\underline{j}-5\underline{k})\cdot(\underline{i}+4\underline{j}+3\underline{k})$

$= (-1)(1)+(4)(4)+(-5)(3)$

OR $\underline{Q}\cdot\underline{S} = 0$ ◀

THUS, $\underline{Q}$ AND $\underline{S}$ ARE PERPENDICULAR

3.36



GIVEN: $\underline{B}$, $\underline{B}'$, AND $\underline{C}$

PROVE: $\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta)$

FIRST NOTE.. $\underline{B} = B(\cos\beta\,\underline{i} + \sin\beta\,\underline{j})$

$\underline{B}' = B(\cos\beta\,\underline{i} - \sin\beta\,\underline{j})$

$\underline{C} = C(\cos\alpha\,\underline{i} + \sin\alpha\,\underline{j})$

BY DEFINITION.. $\underline{B}\cdot\underline{C} = BC\cos(\alpha-\beta)$ $\qquad$ (1)

$\underline{B}'\cdot\underline{C} = BC\cos(\alpha+\beta)$ $\qquad$ (2)

NOW $\underline{B}\cdot\underline{C} = B(\cos\beta\,\underline{i}+\sin\beta\,\underline{j})\cdot C(\cos\alpha\,\underline{i}+\sin\alpha\,\underline{j})$

$= BC(\cos\beta\cos\alpha + \sin\beta\sin\alpha)$ $\qquad$ (3)

AND $\underline{B}'\cdot\underline{C} = B(\cos\beta\,\underline{i} - \sin\beta\,\underline{j})\cdot C(\cos\alpha\,\underline{i} + \sin\alpha\,\underline{j})$

$= BC(\cos\beta\cos\alpha - \sin\beta\sin\alpha)$ $\qquad$ (4)

EQUATING THE RIGHT-HAND SIDES OF EQS. (1) AND (2) TO THE RIGHT-HAND SIDES OF EQS. (3) AND (4), RESPECTIVELY, YIELDS

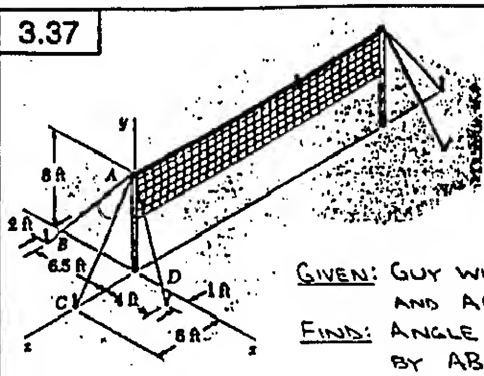
$BC\cos(\alpha-\beta) = BC(\cos\beta\cos\alpha + \sin\beta\sin\alpha)$ (5)

$BC\cos(\alpha+\beta) = BC(\cos\beta\cos\alpha - \sin\beta\sin\alpha)$ (6)

(5)+(6) $\Rightarrow \cos(\alpha-\beta) + \cos(\alpha+\beta) = 2\cos\beta\cos\alpha$

OR $\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta)$ ◀

3.37



GIVEN: GUY WIRES AB AND AC

FIND: ANGLE θ FORMED BY AB AND AC

FIRST NOTE.. $AB = \sqrt{(-6.5)^2 + (-8)^2 + (2)^2}$

$= 10.5$ ft

$AC = \sqrt{(0)^2 + (-8)^2 + (6)^2}$

$= 10$ ft

AND $\vec{AB} = -(6.5\text{ ft})\underline{i} - (8\text{ ft})\underline{j} + (2\text{ ft})\underline{k}$

$\vec{AC} = -(8\text{ ft})\underline{j} + (6\text{ ft})\underline{k}$

BY DEFINITION.. $\vec{AB}\cdot\vec{AC} = (AB)(AC)\cos\theta$

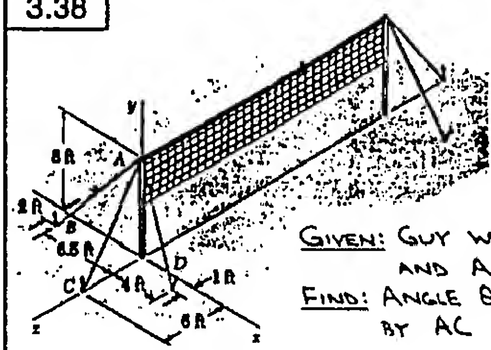
OR $(-6.5\underline{i} - 8\underline{j} + 2\underline{k})\cdot(-8\underline{j} + 6\underline{k}) = (10.5)(10)\cos\theta$

$(-6.5)(0) + (-8)(-8) + (2)(6) = 105\cos\theta$

OR $\cos\theta = 0.723\,81$

OR $\theta = 43.6°$ ◀

3.38



GIVEN: GUY WIRES AC AND AD

FIND: ANGLE θ FORMED BY AC AND AD

FIRST NOTE.. $AC = \sqrt{(0)^2 + (-8)^2 + (6)^2}$

$= 10$ ft

$AD = \sqrt{(4)^2 + (-8)^2 + (1)^2}$

$= 9$ ft

AND $\vec{AC} = -(8\text{ ft})\underline{j} + (6\text{ ft})\underline{k}$

$\vec{AD} = (4\text{ ft})\underline{i} - (8\text{ ft})\underline{j} + (1\text{ ft})\underline{k}$

BY DEFINITION.. $\vec{AC}\cdot\vec{AD} = (AC)(AD)\cos\theta$

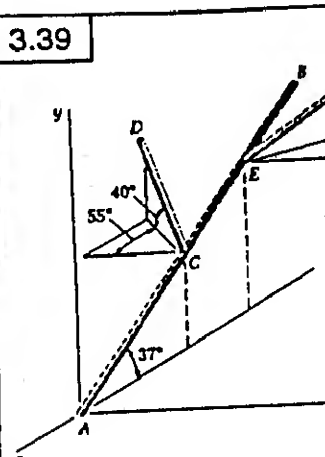
OR $(-8\underline{j} + 6\underline{k})\cdot(4\underline{i} - 8\underline{j} + \underline{k}) = (10)(9)\cos\theta$

$(0)(4) + (-8)(-8) + (6)(1) = 90\cos\theta$

OR $\cos\theta = 0.777\,78$

OR $\theta = 38.9$ ◀

3.39

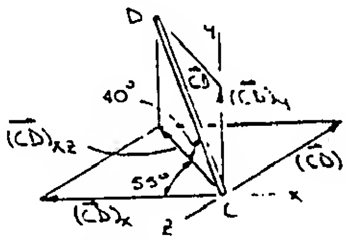


GIVEN: PIPELINES AB AND CD

FIND: ANGLE θ FORMED BY AB AND CD

FIRST NOTE. $\overrightarrow{AB} = AB(\sin 37^\circ\,\mathbf{j} - \cos 37^\circ\,\mathbf{k})$

$\overrightarrow{CD} = CD(-\cos 40^\circ \cos 55^\circ\,\mathbf{i} + \sin 40^\circ\,\mathbf{j} - \cos 40^\circ \sin 55^\circ\,\mathbf{k})$



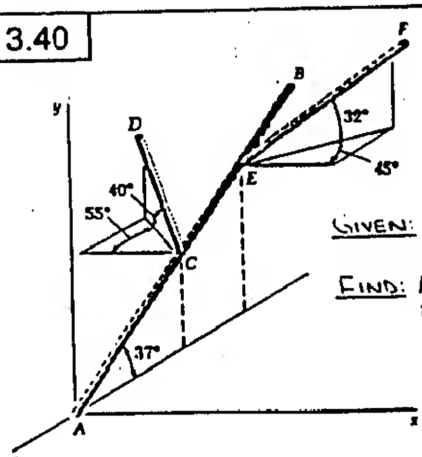
NOW.. $\overrightarrow{AB} \cdot \overrightarrow{CD} = (AB)(CD)\cos\theta$

OR $AB(\sin 37^\circ\,\mathbf{j} - \cos 37^\circ\,\mathbf{k}) \cdot CD(-\cos 40^\circ \cos 55^\circ\,\mathbf{i} + \sin 40^\circ\,\mathbf{j} - \cos 40^\circ \sin 55^\circ\,\mathbf{k}) = (AB)(CD)\cos\theta$

OR $\cos\theta = (\sin 37^\circ)(\sin 40^\circ) + (-\cos 37^\circ)(-\cos 40^\circ \sin 55^\circ)$
$= 0.88799$

OR $\theta = 27.4^\circ$ ◀

3.40

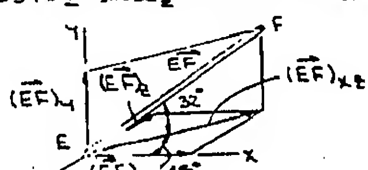


GIVEN: PIPELINES AB AND CD

FIND: ANGLE θ FORMED BY AB AND CD

FIRST NOTE.. $\overrightarrow{AB} = AB(\sin 37^\circ\,\mathbf{j} - \cos 37^\circ\,\mathbf{k})$

$\overrightarrow{EF} = EF(\cos 32^\circ \cos 45^\circ\,\mathbf{i} + \sin 32^\circ\,\mathbf{j} - \cos 32^\circ \sin 45^\circ\,\mathbf{k})$



(CONTINUED)

3.40 CONTINUED

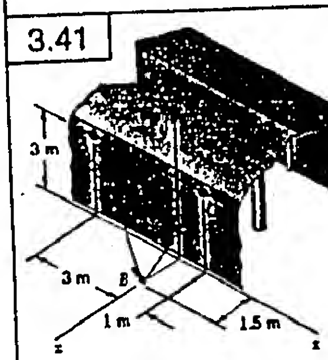
NOW $\overrightarrow{AB} \cdot \overrightarrow{EF} = (AB)(EF)\cos\theta$

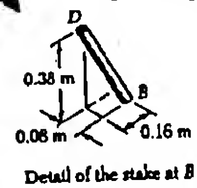
OR $AB(\sin 37^\circ\,\mathbf{j} - \cos 37^\circ\,\mathbf{k}) \cdot EF(\cos 32^\circ \cos 45^\circ\,\mathbf{i} + \sin 32^\circ\,\mathbf{j} - \cos 32^\circ \sin 45^\circ\,\mathbf{k}) = (AB)(EF)\cos\theta$

OR $\cos\theta = (\sin 37^\circ)(\sin 32^\circ) + (-\cos 37^\circ)(-\cos 32^\circ \sin 45^\circ)$
$= 0.79782$

OR $\theta = 37.1^\circ$ ◀

3.41





Detail of the stake at B

GIVEN: STAKE BD AND ROPE AB, $T_{BA} = 540\text{N}$

FIND: (a) ANGLE θ FORMED BY AB AND BD

(b) $(T_{BA})_{BD}$

FIRST NOTE.. $BA = \sqrt{(-3)^2 + (3)^2 + (-1.5)^2} = 4.5\text{ m}$

$BD = \sqrt{(-0.08)^2 + (0.38)^2 + (0.16)^2}$
$= 0.42\text{ m}$

THEN $\mathbf{T}_{BA} = \dfrac{T_{BA}}{4.5}(-3\mathbf{i} + 3\mathbf{j} - 1.5\mathbf{k})$

$= \dfrac{T_{BA}}{3}(-2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$

$\boldsymbol{\lambda}_{BD} = \dfrac{\overrightarrow{BD}}{BD} = \dfrac{1}{0.42}(-0.08\mathbf{i} + 0.38\mathbf{j} + 0.16\mathbf{k})$

$= \dfrac{1}{21}(-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k})$

(a) HAVE $\mathbf{T}_{BA} \cdot \boldsymbol{\lambda}_{BD} = T_{BA}\cos\theta$

OR $\dfrac{T_{BA}}{3}(-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \cdot \dfrac{1}{21}(-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k}) = T_{BA}\cos\theta$

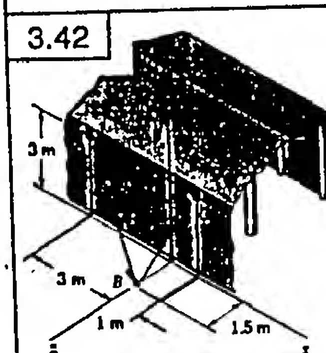
OR $\cos\theta = \frac{1}{63}[(-2)(-4) + (2)(19) + (-1)(8)]$
$= 0.60317$

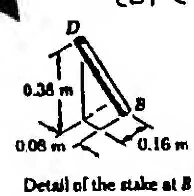
OR $\theta = 52.9^\circ$ ◀

(b) HAVE $(T_{BA})_{BD} = \mathbf{T}_{BA} \cdot \boldsymbol{\lambda}_{BD}$
$= T_{BA}\cos\theta$
$= (540\text{ N})(0.60317)$

OR $(T_{BA})_{BD} = 326\text{ N}$ ◀

3.42





Detail of the stake at B

GIVEN: STAKE BD AND ROPE BC, $T_{BC} = 490$

FIND: (a) ANGLE θ FORMED BY BD AND BC

(b) $(T_{BC})_{BD}$

FIRST NOTE.. $BC = \sqrt{(1)^2 + (3)^2 + (-1.5)^2} = 3.5\text{ m}$

(CONTINUED)

3.42 CONTINUED

$BD = \sqrt{(-0.08)^2+(0.38)^2+(0.16)^2}$
$= 0.42 \text{ m}$

THEN $\mathbf{T}_{BC} = \frac{T_{BC}}{3.5}(\mathbf{i}+3\mathbf{j}-1.5\mathbf{k})$
$= \frac{T_{BC}}{7}(2\mathbf{i}+6\mathbf{j}-3\mathbf{k})$

$\boldsymbol{\lambda}_{BD} = \frac{\vec{BD}}{BD} = \frac{1}{0.42}(-0.08\mathbf{i}+0.38\mathbf{j}+0.16\mathbf{k})$
$= \frac{1}{21}(-4\mathbf{i}+19\mathbf{j}+8\mathbf{k})$

(a) HAVE $\mathbf{T}_{BC}\cdot\boldsymbol{\lambda}_{BD} = T_{BC}\cos\theta$

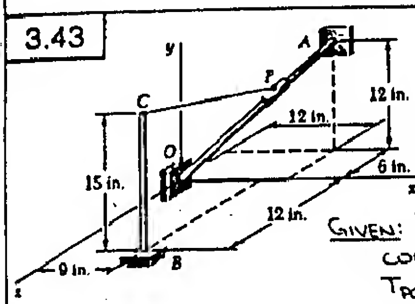
OR $\frac{T_{BC}}{7}(2\mathbf{i}+6\mathbf{j}-3\mathbf{k})\cdot\frac{1}{21}(-4\mathbf{i}+19\mathbf{j}+8\mathbf{k}) = T_{BC}\cos\theta$

OR $\cos\theta = \frac{1}{147}[(2)(-4)+(6)(19)+(-3)(8)]$
$= 0.55782$

OR $\theta = 56.1^\circ$ ◀

(b) HAVE $(T_{BC})_{BD} = \mathbf{T}_{BC}\cdot\boldsymbol{\lambda}_{BD}$
$= T_{BC}\cos\theta$
$= (490 \text{ N})(0.55782)$
OR $(T_{BC})_{BD} = 273 \text{ N}$ ◀

3.43



GIVEN: ROD OA AND CORD PC, OP = 6 IN., $T_{PC} = 3$ lb

FIND: (a) ANGLE θ FORMED BY OA AND PC
(b) $(T_{PC})_{OA}$

FIRST NOTE.. $OA = \sqrt{(12)^2+(12)^2+(-6)^2} = 18$ IN.

THEN.. $\boldsymbol{\lambda}_{OA} = \frac{\vec{OA}}{OA} = \frac{1}{18}(12\mathbf{i}+12\mathbf{j}-6\mathbf{k})$
$= \frac{1}{3}(2\mathbf{i}+2\mathbf{j}-\mathbf{k})$

NOW OP = 6 IN. $\Rightarrow$ $OP = \frac{1}{3}(OA)$

$\therefore$ THE COORDINATES OF POINT P ARE (4 IN., 4 IN., −2 IN.)

SO THAT $\vec{PC} = (5 \text{ in.})\mathbf{i} + (11 \text{ in.})\mathbf{j} + (14 \text{ in.})\mathbf{k}$

AND $PC = \sqrt{(5)^2+(11)^2+(14)^2} = \sqrt{342}$ IN.

(a) HAVE.. $\vec{PC}\cdot\boldsymbol{\lambda}_{OA} = (PC)\cos\theta$

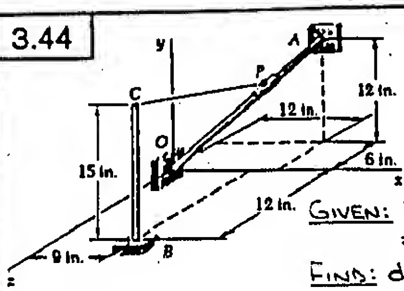
OR $(5\mathbf{i}+11\mathbf{j}+14\mathbf{k})\cdot\frac{1}{3}(2\mathbf{i}+2\mathbf{j}-\mathbf{k}) = \sqrt{342}\cos\theta$

OR $\cos\theta = \frac{1}{3\sqrt{342}}[(5)(2)+(11)(2)+(14)(-1)]$
$= 0.32444$

OR $\theta = 71.1^\circ$ ◀

(b) HAVE.. $(T_{PC})_{OA} = \mathbf{T}_{PC}\cdot\boldsymbol{\lambda}_{OA}$
$= (T_{PC}\boldsymbol{\lambda}_{PC})\cdot\boldsymbol{\lambda}_{OA}$
$= T_{PC}\frac{\vec{PC}}{PC}\cdot\boldsymbol{\lambda}_{OA}$
$= T_{PC}\cos\theta$
$= (3 \text{ lb})(0.32444)$
OR $(T_{PC})_{OA} = 0.973$ lb ◀

3.44



GIVEN: ROD OA AND POINT P

FIND: d_{OP} SO THAT $\overline{OA}$ AND $\overline{PC}$ ARE PERPENDICULAR

FIRST NOTE.. $OA = \sqrt{(12)^2+(12)^2+(-6)^2} = 18$ IN.

THEN.. $\boldsymbol{\lambda}_{OA} = \frac{\vec{OA}}{OA} = \frac{1}{18}(12\mathbf{i}+12\mathbf{j}-6\mathbf{k})$
$= \frac{1}{3}(2\mathbf{i}+2\mathbf{j}-\mathbf{k})$

LET THE COORDINATES OF POINT P BE (x IN., y IN., z IN.). THEN
$\vec{PC} = [(9-x)\text{ in.}]\mathbf{i} + [(15-y)\text{ in.}]\mathbf{j} + [(12-z)\text{ in.}]\mathbf{k}$

ALSO, $\vec{OP} = d_{OP}\boldsymbol{\lambda}_{OA}$
$= \frac{d_{OP}}{3}(2\mathbf{i}+2\mathbf{j}-\mathbf{k})$

AND $\vec{OP} = (x \text{ in.})\mathbf{i} + (y \text{ in.})\mathbf{j} + (z \text{ in.})\mathbf{k}$

$\therefore$ $x = \frac{2}{3}d_{OP}$ $y = \frac{2}{3}d_{OP}$ $z = -\frac{1}{3}d_{OP}$

THE REQUIREMENT THAT $\overline{OA}$ AND $\overline{PC}$ BE PERPENDICULAR IMPLIES THAT

$\boldsymbol{\lambda}_{OA}\cdot\vec{PC} = 0$

OR $\frac{1}{3}(2\mathbf{i}+2\mathbf{j}-\mathbf{k})\cdot[(9-x)\mathbf{i}+(15-y)\mathbf{j}+(12-z)\mathbf{k}] = 0$

OR $(2)(9-\frac{2}{3}d_{OP})+(2)(15-\frac{2}{3}d_{OP})+(-1)[12-(-\frac{1}{3}d_{OP})] = 0$

OR $d_{OP} = 12$ IN ◀

3.45

GIVEN: VECTORS $\mathbf{P}$, $\mathbf{Q}$, AND $\mathbf{S}$

FIND: VOLUME OF THE PARALLELOGRAM DEFINED BY $\mathbf{P}$, $\mathbf{Q}$, AND $\mathbf{S}$ WHEN

(a) $\mathbf{P} = 4\mathbf{i}-3\mathbf{j}+2\mathbf{k}$
$\mathbf{Q} = -2\mathbf{i}-5\mathbf{j}+\mathbf{k}$
$\mathbf{S} = 7\mathbf{i}+\mathbf{j}-\mathbf{k}$

(b) $\mathbf{P} = 5\mathbf{i}-\mathbf{j}+6\mathbf{k}$
$\mathbf{Q} = 2\mathbf{i}+3\mathbf{j}+\mathbf{k}$
$\mathbf{S} = -3\mathbf{i}-2\mathbf{j}+4\mathbf{k}$

AS EXPLAINED IN SEC. 3.10, THE VOLUME V OF THE PARALLELOGRAM IS GIVEN BY
$V = |\mathbf{P}\cdot(\mathbf{Q}\times\mathbf{S})|$

(a) HAVE
$$\mathbf{P}\cdot(\mathbf{Q}\times\mathbf{S}) = \begin{vmatrix} 4 & -3 & 2 \\ -2 & -5 & 1 \\ 7 & 1 & -1 \end{vmatrix}$$
$= 20-21-4+70+6-4$
$= 67$

$\therefore$ $V = 67$ ◀

(b) HAVE
$$\mathbf{P}\cdot(\mathbf{Q}\times\mathbf{S}) = \begin{vmatrix} 5 & -1 & 6 \\ 2 & 3 & 1 \\ -3 & -2 & 4 \end{vmatrix}$$
$= 60+3-24+54+8+10$
$= 111$

$\therefore$ $V = 111$ ◀

3.46

GIVEN: $\underline{P} = 3\underline{i} - \underline{j} + \underline{k}$

$\underline{Q} = 4\underline{i} + Q_y\underline{j} - 2\underline{k}$

$\underline{S} = 2\underline{i} - 2\underline{j} + 2\underline{k}$

FIND: Q_y SO THAT $\underline{P}$, $\underline{Q}$, AND $\underline{S}$ ARE COPLANAR

IF $\underline{P}$, $\underline{Q}$, AND $\underline{S}$ ARE COPLANAR, THEN $\underline{P}$ MUST BE PERPENDICULAR TO $(\underline{Q} \times \underline{S})$.

$\therefore \; \underline{P} \cdot (\underline{Q} \times \underline{S}) = 0$

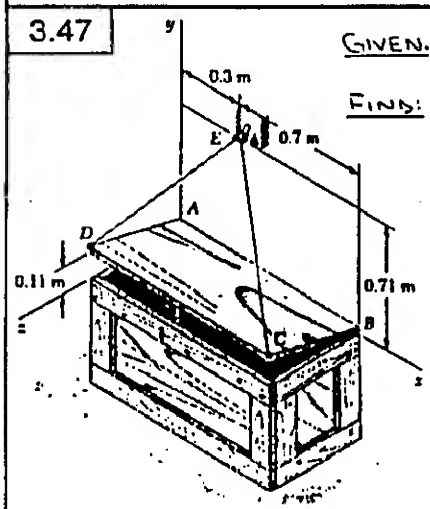
(OR, THE VOLUME OF THE PARALLELOGRAM DEFINED BY $\underline{P}$, $\underline{Q}$, AND $\underline{S}$ IS ZERO). THEN

$$\begin{vmatrix} 3 & -1 & 1 \\ 4 & Q_y & -2 \\ 2 & -2 & 2 \end{vmatrix} = 0$$

OR $6Q_y + 4 - 8 - 2Q_y + 8 - 12 = 0$

OR $Q_y = 2$

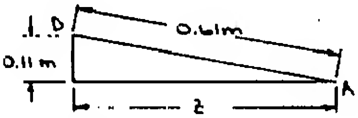
3.47



GIVEN: 0.61×1.00-m LID, $T_{DE} = 66$ N

FIND: M_x, M_y, M_z OF $\underline{T}_{DE}$ AT D

FIRST NOTE.. $z = \sqrt{(0.61)^2 - (0.11)^2}$
$= 0.60$ m



THEN $d_{DE} = \sqrt{(0.3)^2 + (0.6)^2 + (-0.6)^2} = 0.9$ m

AND $\underline{T}_{DE} = \frac{66\text{ N}}{0.9}(0.3\underline{i} + 0.6\underline{j} - 0.6\underline{k})$

$= 22[(1\text{ N})\underline{i} + (2\text{ N})\underline{j} - (2\text{ N})\underline{k}]$

NOW.. $\underline{M}_A = \underline{r}_{D/A} \times \underline{T}_{DE}$

WHERE $\underline{r}_{D/A} = (0.11\text{ m})\underline{j} + (0.60\text{ m})\underline{k}$

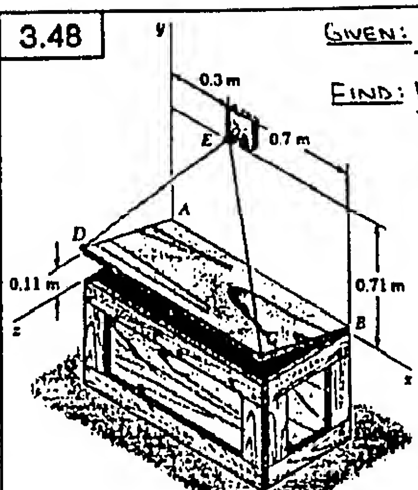
THEN.. $\underline{M}_A = 22 \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0.11 & 0.60 \\ 1 & 2 & -2 \end{vmatrix}$

$= 22[(-0.22 - 1.20)\underline{i} + 0.60\underline{j} - 0.11\underline{k}]$

$= -(31.24\text{ N·m})\underline{i} + (13.20\text{ N·m})\underline{j} - (2.42\text{ N·m})\underline{k}$

$\therefore M_x = -31.2$ N·m, $M_y = 13.20$ N·m, $M_z = -2.42$ N·m

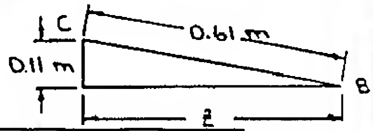
3.48



GIVEN: 0.61×1.00-m LID, $T_{CE} = 66$ N

FIND: M_x, M_y, M_z OF $\underline{T}_{CE}$ AT C

FIRST NOTE.. $z = \sqrt{(0.61)^2 - (0.11)^2}$
$= 0.60$ m



THEN $d_{CE} = \sqrt{(-0.7)^2 + (0.6)^2 + (-0.6)^2} = 1.1$ m

AND $\underline{T}_{CE} = \frac{66\text{ N}}{1.1}(-0.7\underline{i} + 0.6\underline{j} - 0.6\underline{k})$

$= 6[-(7\text{ N})\underline{i} + (6\text{ N})\underline{j} - (6\text{ N})\underline{k}]$

NOW.. $\underline{M}_A = \underline{r}_{E/A} \times \underline{T}_{CE}$

WHERE $\underline{r}_{E/A} = (0.3\text{ m})\underline{i} + (0.71\text{ m})\underline{j}$

THEN.. $\underline{M}_A = 6 \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.3 & 0.71 & 0 \\ -7 & 6 & -6 \end{vmatrix}$

$= 6[-4.26\underline{i} + 1.8\underline{j} + (1.8 + 4.97)\underline{k}]$

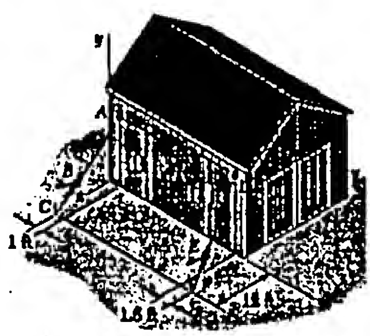
$= -(25.56\text{ N·m})\underline{i} + (10.80\text{ N·m})\underline{j} + (40.62\text{ N·m})\underline{k}$

$\therefore M_x = -25.6$ N·m, $M_y = 10.80$ N·m, $M_z = 40.6$ N·m

3.49 and 3.50

GIVEN: T_{AB}, M_x OF $\underline{T}_{AB}$ (AT A) AND $\underline{T}_{DE}$ (AT D) $= 4728$ lb·ft

FIND: T_{DE}



FIRST NOTE.. $d_{AC} = \sqrt{(-1)^2 + (-12)^2 + (12)^2} = 17$ ft

$d_{DF} = \sqrt{(1.5)^2 + (-14)^2 + (12)^2} = 18.5$ ft

THEN.. $\underline{T}_{AB} = \frac{T_{AB}}{17}(-\underline{i} - 12\underline{j} + 12\underline{k})$ (lb)

$\underline{T}_{DE} = \frac{T_{DE}}{18.5}(1.5\underline{i} - 14\underline{j} + 12\underline{k})$ (lb)

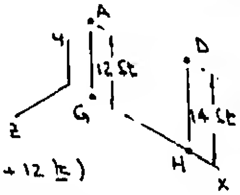
NOW.. $M_x = \underline{i} \cdot (\underline{r}_{A/G} \times \underline{T}_{AB}) + \underline{i} \cdot (\underline{r}_{D/H} \times \underline{T}_{DE})$

(CONTINUED)

3.49 and 3.50 CONTINUED

WHERE $\underline{r}_{A/G} = (12\text{ ft})\underline{j}$

$\underline{r}_{D/H} = (14\text{ ft})\underline{j}$



THEN.. $M_x = \underline{i}\cdot[12\underline{j}\times\frac{T_{AB}}{17}(-\underline{i}-12\underline{j}+12\underline{k})$

$+\underline{i}\cdot[14\underline{j}\times\frac{T_{DE}}{18.5}(1.5\underline{i}-14\underline{j}+12\underline{k})$

$=\frac{144}{17}T_{AB}+\frac{168}{18.5}T_{DE}\quad(\text{lb}\cdot\text{ft})\quad(1)$

3.49 SUBSTITUTING INTO EQ. (1) WITH

$T_{AB} = 255\text{ lb}\qquad M_x = 4728\text{ lb}\cdot\text{ft}$

HAVE $4728 = \frac{144}{17}(255)+\frac{168}{18.5}T_{DE}$

OR $T_{DE} = 283\text{ lb}$ ◀

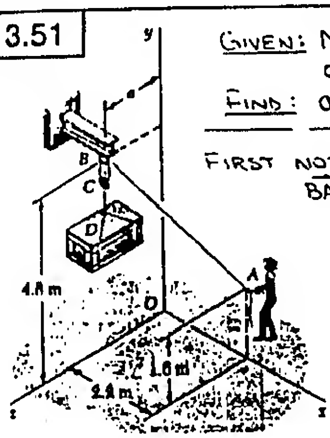
3.50 SUBSTITUTING INTO EQ. (1) WITH

$T_{AB} = 306\text{ lb}\qquad M_x = 4728\text{ lb}\cdot\text{ft}$

HAVE $4728 = \frac{144}{17}(306)+\frac{168}{18.5}T_{DE}$

OR $T_{DE} = 235\text{ lb}$ ◀

3.51



GIVEN: $M_y = 120\text{ N}\cdot\text{m},\ M_z = -460\text{ N}\cdot\text{m}$ OF $\underline{T}_{BA}$ AT B

FIND: a

FIRST NOTE..

$\overline{BA} = (2.2\text{ m})\underline{i}-(3.2\text{ m})\underline{j}-(a\text{ m})\underline{k}$

NOW.. $\underline{M}_O = \underline{r}_{A/O}\times\underline{T}_{BA}$

WHERE

$\underline{r}_{A/O} = (2.2\text{ m})\underline{i}+(1.6\text{ m})\underline{j}$

$\underline{T}_{BA} = \frac{T_{BA}}{d_{BA}}(2.2\underline{i}-3.2\underline{j}-a\underline{k})\ (\text{N})$

THEN..

$$\underline{M}_O = \frac{T_{BA}}{d_{BA}}\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ 2.2 & 1.6 & 0\\ 2.2 & -3.2 & -a\end{vmatrix}$$

$$=\frac{T_{BA}}{d_{BA}}\{-1.6a\underline{i}+2.2a\underline{j}+[(2.2)(-3.2)-(1.6)(2.2)]\underline{k}\}$$

THUS.. $M_y = 2.2\frac{T_{BA}}{d_{BA}}a\ (\text{N}\cdot\text{m})$

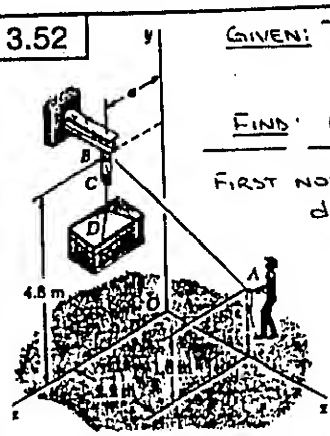
$M_z = -10.56\frac{T_{BA}}{d_{BA}}\ (\text{N}\cdot\text{m})$

THEN.. FORMING THE RATIO $\frac{M_y}{M_z}$..

$$\frac{120\text{ N}\cdot\text{m}}{-460\text{ N}\cdot\text{m}} = \frac{2.2\frac{T_{BA}}{d_{BA}}a\ (\text{N}\cdot\text{m})}{-10.56\frac{T_{BA}}{d_{BA}}\ (\text{N}\cdot\text{m})}$$

OR $a = 1.252\text{ m}$ ◀

3.52



GIVEN: $T_{BA} = 195\text{ N}$,

$M_y = 132\text{ N}\cdot\text{m}$ OF $\underline{T}_{BA}$ AT B

FIND: a

FIRST NOTE..

$d_{BA} = \sqrt{(2.2)^2+(-3.2)^2+(-a)^2}$

$=\sqrt{15.08+a^2}\text{ m}$

AND

$\underline{T}_{BA} = \frac{195\text{ N}}{d_{BA}}(2.2\underline{i}-3.2\underline{j}-a\underline{k})$

NOW $M_y = \underline{j}\cdot(\underline{r}_{A/O}\times\underline{T}_{BA})$

WHERE $\underline{r}_{A/O} = (2.2\text{ m})\underline{i}+(1.6\text{ m})\underline{j}$

$$\text{THEN..}\quad M_y = \frac{195}{d_{BA}}\begin{vmatrix}0 & 1 & 0\\ 2.2 & 1.6 & 0\\ 2.2 & -3.2 & -a\end{vmatrix}$$

$$=\frac{195}{d_{BA}}(2.2a)\quad(\text{N}\cdot\text{m})$$

SUBSTITUTING FOR M_y AND d_{BA}..

$$132\text{ N}\cdot\text{m} = \frac{195}{\sqrt{15.08+a^2}}(2.2a)$$

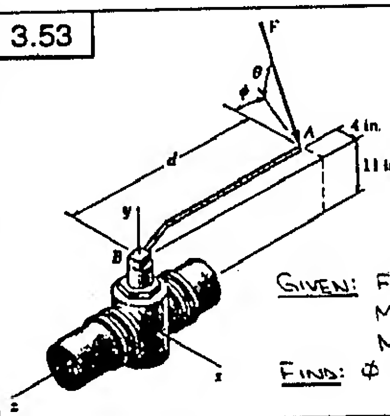
OR $0.30769\sqrt{15.08+a^2} = a$

SQUARING BOTH SIDES OF THE EQUATION..

$0.094675(15.08+a^2) = a^2$

OR $a = 1.256\text{ m}$ ◀

3.53



GIVEN: $F = 70\text{ lb},\ \theta = 25°$,

$M_x = -61\text{ lb}\cdot\text{ft}$,

$M_z = -43\text{ lb}\cdot\text{ft}$

FIND: ϕ AND d

HAVE.. $\underline{M}_O = \underline{r}_{A/O}\times\underline{F}$

WHERE $\underline{r}_{A/O} = -(4\text{ in.})\underline{i}+(11\text{ in.})\underline{j}-(d\text{ in.})\underline{k}$

AND $\underline{F} = (70\text{ lb})(\cos\theta\cos\phi\,\underline{i}-\sin\theta\,\underline{j}+\cos\theta\sin\phi\,\underline{k})$

$$\text{THEN..}\quad \underline{M}_O = 70\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ -4 & 11 & -d\\ \cos\theta\cos\phi & -\sin\theta & \cos\theta\sin\phi\end{vmatrix}$$

$=70[(11\cos\theta\sin\phi-d\sin\theta)\underline{i}$

$+(-d\cos\theta\cos\phi+4\cos\theta\sin\phi)\underline{j}$

$+(4\sin\theta-11\cos\theta\cos\phi)\underline{k}]\ (\text{lb}\cdot\text{in.})$

NOW CONSIDER THE Z AND X COMPONENTS OF $\underline{M}_O$. HAVE..

M_z: $-43\text{ lb}\cdot\text{ft}\times\frac{12\text{ in.}}{1\text{ ft}} = 70(4\sin25°-11\cos25°\cos\phi)\text{ lb}\cdot\text{in.}$

OR $\cos\phi = 0.90891$

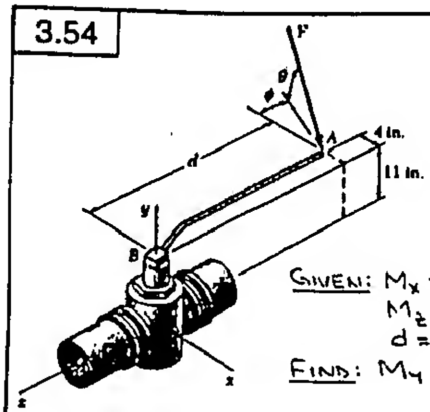
OR $\phi = 24.637°$ $\qquad\phi = 24.6°$ ◀

M_x: $-61\text{ lb}\cdot\text{ft}\times\frac{12\text{ in.}}{1\text{ ft}} = 70(11\cos25°\sin24.637°-d\sin25°)\text{ lb}\cdot\text{in.}$

OR $d = 34.6\text{ in.}$ ◀

EVIDENCE!! PROOF!! It was a woman!

3.54



GIVEN: $M_x = -77$ lb·ft,
$M_z = -81$ lb·ft,
$d = 27$ in.

FIND: M_y

HAVE.. $\underline{M}_O = \underline{r}_{A/O} \times \underline{F}$

WHERE $\underline{r}_{A/O} = -(4 \text{ in.})\underline{i} + (11 \text{ in})\underline{j} - (27 \text{ in.})\underline{k}$

AND $\underline{F} = F(\cos\theta\cos\phi\,\underline{i} - \sin\theta\,\underline{j} + \cos\theta\sin\phi\,\underline{k})$

THEN.. $$\underline{M}_O = F\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -4 & 11 & -27 \\ \cos\theta\cos\phi & -\sin\theta & \cos\theta\sin\phi \end{vmatrix}$$

$$= F[(11\cos\theta\sin\phi - 27\sin\theta)\underline{i} + (-27\cos\theta\cos\phi + 4\cos\theta\sin\phi)\underline{j} + (4\sin\theta - 11\cos\theta\cos\phi)\underline{k}] \text{ (lb·in)}$$

SO THAT
$$M_x = F(11\cos\theta\sin\phi - 27\sin\theta) \quad (1)$$
$$M_y = F(-27\cos\theta\cos\phi + 4\cos\theta\sin\phi) \quad (2)$$
$$M_z = F(4\sin\theta - 11\cos\theta\cos\phi) \quad (3)$$

WHERE M_x, M_y, AND M_z ARE IN lb·in. NOW..

EQ. (1) ⇒ $\cos\theta\sin\phi = \frac{1}{11}\left(\frac{M_x}{F} + 27\sin\theta\right) \quad (4)$

EQ. (3) ⇒ $\cos\theta\cos\phi = \frac{1}{11}\left(4\sin\theta - \frac{M_z}{F}\right) \quad (5)$

SUBSTITUTING EQS. (4) AND (5) INTO EQ. (2) YIELDS

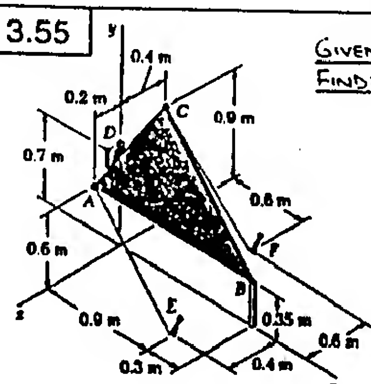
$$M_y = F\left\{-27\left[\frac{1}{11}\left(4\sin\theta - \frac{M_z}{F}\right)\right] + 4\left[\frac{1}{11}\left(\frac{M_x}{F} + 27\sin\theta\right)\right]\right\}$$
$$= \frac{1}{11}(27M_z + 4M_x)$$

NOTING THAT THE RATIOS $\frac{27}{11}$ AND $\frac{4}{11}$ ARE THE RATIOS OF LENGTHS, HAVE..

$$M_y = \frac{27}{11}(-81 \text{ lb·ft}) + \frac{4}{11}(-77 \text{ lb·ft})$$

OR $M_y = -227$ lb·ft

3.55



GIVEN: $T_{AE} = 55$ N

FIND: MOMENT OF T_{AE} AT A ABOUT LINE JOINING D AND B

FIRST NOTE.. $d_{AE} = \sqrt{(0.9)^2 + (-0.6)^2 + (0.2)^2} = 1.1$ m

THEN.. $\underline{T}_{AE} = \frac{55 \text{ N}}{1.1}(0.9\underline{i} - 0.6\underline{j} + 0.2\underline{k})$

$= 5[(9 \text{ N})\underline{i} - (6 \text{ N})\underline{j} + (2 \text{ N})\underline{k}]$

(CONTINUED)

3.55 CONTINUED

ALSO.. $DB = \sqrt{(1.2)^2 + (-0.35)^2 + (0)^2} = 1.25$ m

THEN $\underline{\lambda}_{DB} = \frac{\overrightarrow{DB}}{DB} = \frac{1}{1.25}(1.2\underline{i} - 0.35\underline{j})$

$= \frac{1}{25}(24\underline{i} - 7\underline{j})$

NOW.. $M_{DB} = \underline{\lambda}_{DB} \cdot (\underline{r}_{A/D} \times \underline{T}_{AE})$

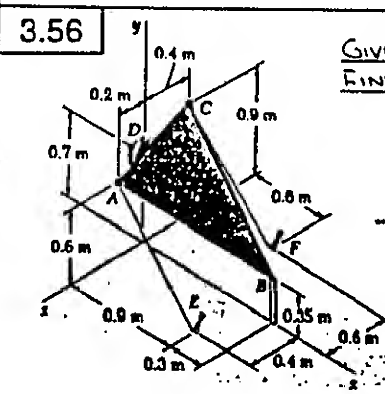
WHERE $\underline{r}_{DA} = -(0.1 \text{ m})\underline{j} + (0.2 \text{ m})\underline{k}$

THEN, $$M_{DB} = \frac{1}{25}(5)\begin{vmatrix} 24 & -7 & 0 \\ 0 & -0.1 & 0.2 \\ 9 & -6 & 2 \end{vmatrix}$$

$= \frac{1}{5}(-4.8 - 12.6 + 28.8)$

OR $M_{DB} = 2.28$ N·m

3.56



GIVEN: $T_{CF} = 33$ N

FIND: MOMENT OF T_{CF} AT C ABOUT LINE JOINING D AND B

FIRST NOTE.. $d_{CF} = \sqrt{(0.6)^2 + (-0.9)^2 + (-0.2)^2} = 1.1$ m

THEN. $\underline{T}_{CF} = \frac{33 \text{ N}}{1.1}(0.6\underline{i} - 0.9\underline{j} - 0.2\underline{k})$

$= 3[(6 \text{ N})\underline{i} - (9 \text{ N})\underline{j} - (2 \text{ N})\underline{k}]$

ALSO.. $DB = \sqrt{(1.2)^2 + (-0.35)^2 + (0)^2} = 1.25$ m

THEN $\underline{\lambda}_{DB} = \frac{\overrightarrow{DB}}{DB} = \frac{1}{1.25}(1.2\underline{i} - 0.35\underline{j})$

$= \frac{1}{25}(24\underline{i} - 7\underline{j})$

NOW.. $M_{DB} = \underline{\lambda}_{DB} \cdot (\underline{r}_{C/D} \times \underline{T}_{CF})$

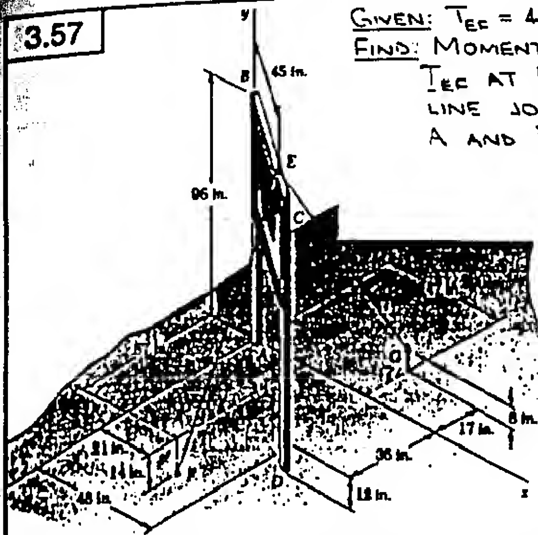
WHERE $\underline{r}_{C/D} = (0.2 \text{ m})\underline{j} - (0.4 \text{ m})\underline{k}$

THEN.. $$M_{DB} = \frac{1}{25}(3)\begin{vmatrix} 24 & -7 & 0 \\ 0 & 0.2 & -0.4 \\ 6 & -9 & -2 \end{vmatrix}$$

$= \frac{3}{25}(-9.6 + 16.8 - 86.4)$

OR $M_{DB} = -9.50$ N·m

3.57



GIVEN: $T_{EF} = 46$ lb

FIND: MOMENT OF T_{EF} AT E ABOUT LINE JOINING A AND D

FIRST NOTE THAT $BC = \sqrt{(48)^2 + (36)^2} = 60$ in. AND THAT $\frac{BE}{BC} = \frac{45}{60} = \frac{3}{4}$ THE COORDINATES OF POINT E ARE THEN $(\frac{3}{4}\cdot 48, 96, \frac{3}{4}\cdot 36)$ OR (36 in., 96 in., 27 in.). THEN..

$d_{EF} = \sqrt{(-15)^2 + (-110)^2 + (30)^2} = 115$ in.

THEN.. $\underline{T}_{EF} = \frac{46 \text{ lb}}{115}(-15\underline{i} - 110\underline{j} + 30\underline{k})$

$= 2[-(3 \text{ lb})\underline{i} - (22 \text{ lb})\underline{j} + (6 \text{ lb})\underline{k}]$

ALSO.. $AD = \sqrt{(48)^2 + (-12)^2 + (36)^2} = 12\sqrt{26}$ in.

THEN $\underline{\lambda}_{AD} = \frac{\vec{AD}}{AD} = \frac{1}{12\sqrt{26}}(48\underline{i} - 12\underline{j} + 36\underline{k})$

$= \frac{1}{\sqrt{26}}(4\underline{i} - \underline{j} + 3\underline{k})$

NOW.. $M_{AD} = \underline{\lambda}_{AD} \cdot (\underline{r}_{E/A} \times \underline{T}_{EF})$

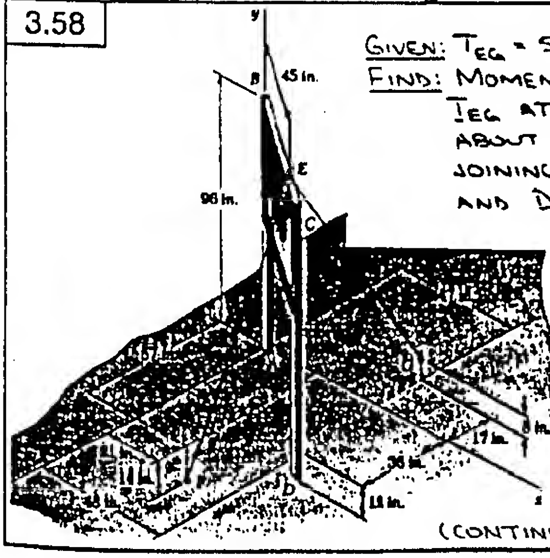
WHERE.. $\underline{r}_{E/A} = (36 \text{ in.})\underline{i} + (96 \text{ in.})\underline{j} + (27 \text{ in.})\underline{k}$

THEN.. $M_{AD} = \frac{1}{\sqrt{26}}(2)\begin{vmatrix} 4 & -1 & 3 \\ 36 & 96 & 27 \\ -3 & -22 & 6 \end{vmatrix}$

$= \frac{2}{\sqrt{26}}(2304 + 81 - 2376 + 864 + 216 + 2376)$

OR $M_{AD} = 1359$ lb·in.

3.58



GIVEN: $T_{EG} = 54$ lb

FIND: MOMENT OF T_{EG} AT E ABOUT LINE JOINING A AND D.

(CONTINUED)

3.58 CONTINUED

FIRST NOTE THAT $BC = \sqrt{(48)^2 + (36)^2} = 60$ in. AND THAT $\frac{BE}{BC} = \frac{45}{60} = \frac{3}{4}$. THE COORDINATES OF POINT E ARE THEN $(\frac{3}{4}\cdot 48, 96, \frac{3}{4}\cdot 36)$ OR (36 in., 96 in., 27 in.). THEN..

$d_E = \sqrt{(11)^2 + (-88)^2 + (-44)^2} = 99$ in.

THEN.. $\underline{T}_{EG} = \frac{54 \text{ lb}}{99}(11\underline{i} - 88\underline{j} - 44\underline{k})$

$= 6[(1 \text{ lb})\underline{i} - (8 \text{ lb})\underline{j} - (4 \text{ lb})\underline{k}]$

ALSO.. $AD = \sqrt{(48)^2 + (-12)^2 + (36)^2} = 12\sqrt{26}$ in.

THEN $\underline{\lambda}_{AD} = \frac{\vec{AD}}{AD} = \frac{1}{12\sqrt{26}}(48\underline{i} - 12\underline{j} + 36\underline{k})$

$= \frac{1}{\sqrt{26}}(4\underline{i} - \underline{j} + 3\underline{k})$

NOW.. $M_{AD} = \underline{\lambda}_{AD} \cdot (\underline{r}_{E/A} \times \underline{T}_{EG})$

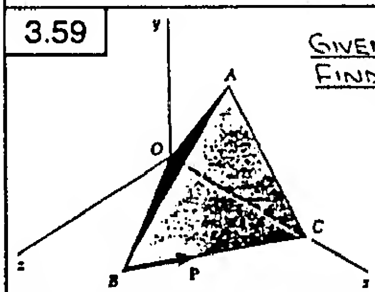
WHERE $\underline{r}_{E/A} = (36 \text{ in.})\underline{i} + (96 \text{ in.})\underline{j} + (27 \text{ in.})\underline{k}$

THEN $M_{AD} = \frac{1}{\sqrt{26}}(6)\begin{vmatrix} 4 & -1 & 3 \\ 36 & 96 & 27 \\ 1 & -8 & -4 \end{vmatrix}$

$= \frac{6}{\sqrt{26}}(-1536 - 27 - 864 - 288 - 144 + 864)$

OR $M_{AD} = -2350$ lb·in.

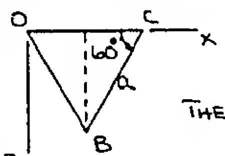
3.59



GIVEN: TETRAHEDRON, $\underline{P}$

FIND: MOMENT OF $\underline{P}$ ABOUT EDGE OA

FIRST CONSIDER TRIANGLE OBC. WITH THE LENGTH OF THE SIDES OF THE TRIANGLE EQUAL TO a, HAVE..



$\vec{BC} = a\cos 60°\,\underline{i} - a\sin 60°\,\underline{k}$

THEN $\underline{\lambda}_{BC} = \cos 60°\,\underline{i} - \sin 60°\,\underline{k}$

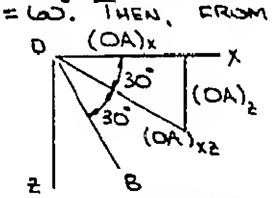
$= \frac{1}{2}(\underline{i} - \sqrt{3}\,\underline{k})$

AND $\underline{P} = P\underline{\lambda}_{BC} = \frac{P}{2}(\underline{i} - \sqrt{3}\,\underline{k})$

TO DETERMINE $\underline{\lambda}_{OA}$, FIRST OBSERVE THAT $\angle AOC = 60°$. THE PROJECTION OF $\vec{OA}$ ON THE X AXIS IS THEN

$(OA)_x = a\cos 60° = \frac{a}{2}$

ALSO, THE PROJECTION OF $\vec{OA}$ ONTO THE XZ PLANE BISECTS $\angle BOC$, WHERE $\angle BOC = 60°$. THEN, FROM THE SKETCH..



$(OA)_z = (OA)_x \tan 30° = \frac{a}{2\sqrt{3}}$

NOW..

$(OA)^2 = (OA)_x^2 + (OA)_y^2 + (OA)_z^2$

SUBSTITUTING...

$a^2 = (\frac{a}{2})^2 + (OA)_y^2 + (\frac{a}{2\sqrt{3}})^2$

OR $(OA)_y = a\sqrt{\frac{2}{3}}$

THEN.. $\vec{OA} = \frac{a}{2}\underline{i} + a\sqrt{\frac{2}{3}}\,\underline{j} + \frac{a}{2\sqrt{3}}\underline{k}$

SO THAT $\underline{\lambda}_{OA} = \frac{1}{2}\underline{i} + \sqrt{\frac{2}{3}}\,\underline{j} + \frac{1}{2\sqrt{3}}\underline{k}$

(CONTINUED)

3.59 CONTINUED

FINALLY.. $M_{OA} = \lambda_{OA}\cdot(\mathbf{r}_{C/O}\times\mathbf{P})$

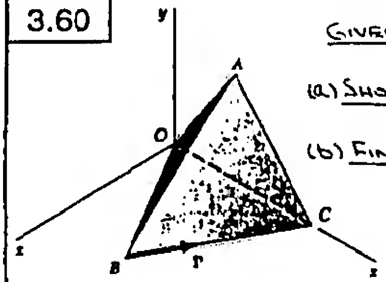
WHERE $\mathbf{r}_{C/O} = a\mathbf{i}$

THEN.. $M_{OA} = a\left(\frac{P}{2}\right)\begin{vmatrix} \frac{1}{2} & \sqrt{\frac{2}{3}} & \frac{1}{2\sqrt{3}} \\ 1 & 0 & 0 \\ 1 & 0 & -\sqrt{3} \end{vmatrix}$

$= \frac{1}{2}aP\left(\sqrt{\frac{2}{3}}\cdot\sqrt{3}\right)$

OR $M_{OA} = \frac{aP}{\sqrt{2}}$

3.60

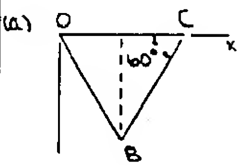


GIVEN: TETRAHEDRON, $\mathbf{P}$, M_{OA} OF $\mathbf{P}$

(a) SHOW: $\overrightarrow{OA}$ AND $\overrightarrow{BC}$ ARE PERPENDICULAR

(b) FIND: PERPENDICULAR DISTANCE BETWEEN $\overrightarrow{OA}$ AND $\overrightarrow{BC}$

(a)



FIRST CONSIDER TRIANGLE OBC. WITH THE LENGTH OF THE SIDES OF THE TRIANGLE EQUAL TO a, HAVE..

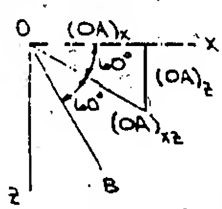
$\overrightarrow{BC} = a\cos 60°\,\mathbf{i} - a\sin 60°\,\mathbf{k}$

$= \frac{a}{2}(\mathbf{i} - \sqrt{3}\,\mathbf{k})$

TO DETERMINE $\overrightarrow{OA}$, FIRST OBSERVE THAT $\measuredangle AOC = 60°$. THE PROJECTION OF $\overrightarrow{OA}$ ON THE X AXIS IS THEN

$(OA)_x = a\cos 60° = \frac{a}{2}$

ALSO, THE PROJECTION OF $\overrightarrow{OA}$ ONTO THE XZ PLANE BISECTS $\measuredangle BOC$, WHERE $\measuredangle BOC = 60°$. THEN, FROM THE SKETCH..



$(OA)_z = (OA)_x\tan 30° = \frac{a}{2\sqrt{3}}$

THEN..

$\overrightarrow{OA} = \frac{a}{2}\mathbf{i} + (OA)_y\,\mathbf{j} + \frac{a}{2\sqrt{3}}\mathbf{k}$

IF $\overrightarrow{BC}$ AND $\overrightarrow{OA}$ ARE PERPENDICULAR,

$\overrightarrow{BC}\cdot\overrightarrow{OA} = 0$

THUS, $\overrightarrow{BC}\cdot\overrightarrow{OA} = \frac{a}{2}(\mathbf{i} - \sqrt{3}\,\mathbf{k})\cdot\left[\frac{a}{2}\mathbf{i} + (OA)_y\mathbf{j} + \frac{a}{2\sqrt{3}}\mathbf{k}\right]$

$= \frac{a}{2}\left[(1)\left(\frac{a}{2}\right) + (0)(OA)_y + (-\sqrt{3})\left(\frac{a}{2\sqrt{3}}\right)\right]$

$= 0$

$\therefore\ \overrightarrow{BC}\cdot\overrightarrow{OA} = 0 \Rightarrow \overrightarrow{BC}$ AND $\overrightarrow{OC}$ ARE PERPENDICULAR

(b) SINCE $\overrightarrow{OA}$ IS PERPENDICULAR TO $\overrightarrow{BC}$, AND THUS TO $\mathbf{P}$, IT FOLLOWS THAT

$M_{OA} = dP$

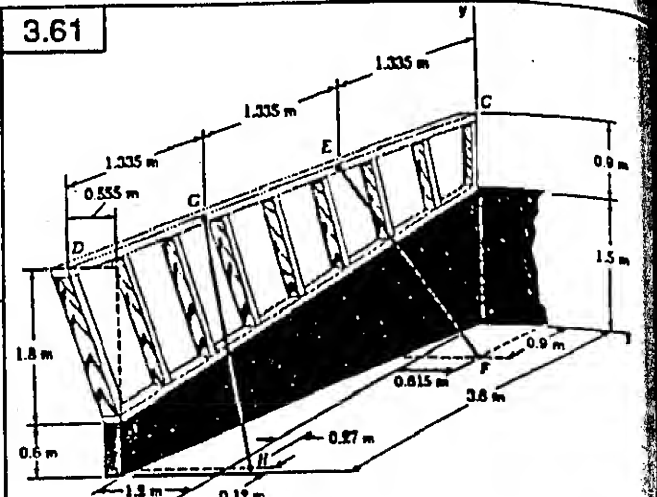
WHERE d IS THE PERPENDICULAR DISTANCE BETWEEN $\overrightarrow{OA}$ AND $\overrightarrow{BC}$ AND FROM THE SOLUTION TO PROBLEM 3.59

$M_{OA} = \frac{1}{\sqrt{2}}aP$

THEN.. $\frac{1}{\sqrt{2}}aP = dP$

OR $d = \frac{a}{\sqrt{2}}$

3.61



GIVEN: $T_{EF} = 63$ N

FIND: MOMENT OF $\mathbf{T}_{EF}$ AT E ABOUT SILL AB

FIRST NOTE THAT $CE = \frac{1}{3}CD$

THEN...

$d_{EF} = \left\{\left[\frac{1}{3}(0.555+1.2)+0.615\right]^2 + (-2.4)^2 + \left[0.9-\left(\frac{1}{3}\times 3.6\right)\right]^2\right\}^{\frac{1}{2}}$

$= \sqrt{(1.2)^2+(-2.4)^2+(-0.3)^2} = 2.7$ m

AND $\mathbf{T}_{EF} = \frac{63\text{ N}}{2.7}(1.2\mathbf{i} - 2.4\mathbf{j} - 0.3\mathbf{k})$

$= 7[(4\text{ N})\mathbf{i} - (8\text{ N})\mathbf{j} - (1\text{ N})\mathbf{k}]$

ALSO.. $AB = \sqrt{(1.2)^2+(0.9)^2+(-3.6)^2} = 3.9$ m

THEN $\lambda_{AB} = \frac{1}{3.9}(1.2\mathbf{i} + 0.9\mathbf{j} - 3.6\mathbf{k})$

$= \frac{1}{13}(4\mathbf{i} + 3\mathbf{j} - 12\mathbf{k})$

NOW.. $M_{AB} = \lambda_{AB}\cdot(\mathbf{r}_{F/B}\times\mathbf{T}_{EF})$

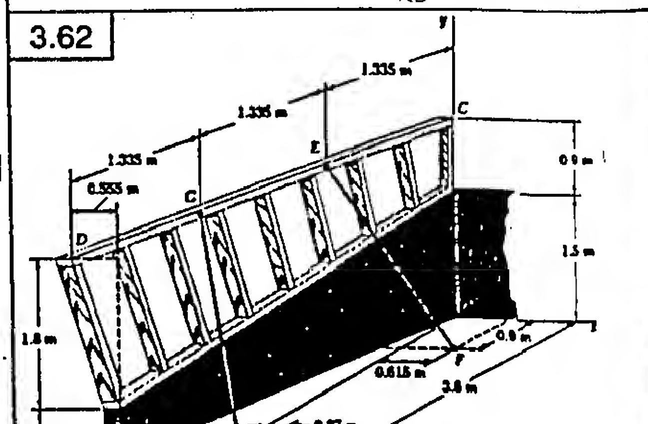
WHERE $\mathbf{r}_{F/B} = (0.615\text{ m})\mathbf{i} - (1.5\text{ m})\mathbf{j} + (0.9\text{ m})\mathbf{k}$

THEN.. $M_{AB} = \frac{1}{13}(7)\begin{vmatrix} 4 & 3 & -12 \\ 0.615 & -1.5 & 0.9 \\ 4 & -8 & -1 \end{vmatrix}$

$= \frac{7}{13}(6 + 10.8 + 59.04 - 72 + 1.845 + 28.8)$

OR $M_{AB} = 18.57$ N·m

3.62



GIVEN: $T_{GH} = 75$ N

FIND: MOMENT OF $\mathbf{T}_{GH}$ AT G ABOUT SILL AB

(CONTINUED)

3.62 CONTINUED

First note that $CG = \frac{2}{3} CD$

Then.. $d_{GH} = \left\{\left[\frac{2}{3}(0.555+1.2) - 0.27\right]^2 + (-2.4)^2 + \left[(3.6-0.12) - \left(\frac{2}{3}\times 3.6\right)\right]^2\right\}^{\frac{1}{2}}$

$= \sqrt{(1.44)^2 + (-2.4)^2 + (1.08)^2} = 3 \text{ m}$

and $T_{GH} = \frac{75\text{ N}}{3}(1.44\mathbf{i} - 2.4\mathbf{j} + 1.08\mathbf{k})$

$= 3[(12\text{ N})\mathbf{i} - (20\text{ N})\mathbf{j} + (9\text{ N})\mathbf{k}]$

Also.. $AB = \sqrt{(1.2)^2 + (0.9)^2 + (-3.6)^2} = 3.9 \text{ m}$

Then.. $\lambda_{AB} = \frac{1}{3.9}(1.2\mathbf{i} + 0.9\mathbf{j} - 3.6\mathbf{k})$

$= \frac{1}{13}(4\mathbf{i} + 3\mathbf{j} - 12\mathbf{k})$

Now.. $M_{AB} = \lambda_{AB} \cdot (\mathbf{r}_{H/A} \times \mathbf{T}_{GH})$

where $\mathbf{r}_{H/A} = (1.47\text{ m})\mathbf{i} - (0.6\text{ m})\mathbf{j} - (0.12\text{ m})\mathbf{k}$

Then.. $M_{AB} = \frac{1}{13}(3)\begin{vmatrix} 4 & 3 & -12 \\ 1.47 & -0.6 & -0.12 \\ 12 & -20 & 9 \end{vmatrix}$

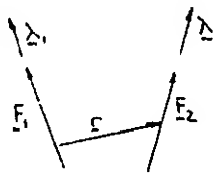
$= \frac{3}{13}(-21.6 - 4.32 + 352.8 - 86.4 - 39.69 - 9.6)$

or $M_{AB} = 44.1 \text{ N}\cdot\text{m}$

3.63

Given: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$, $F_1 = F_2 = F$

Show: M_{λ_1} of $\mathbf{F}_2$ = M_{λ_2} of $\mathbf{F}_1$



First note that

$\mathbf{F}_1 = F\lambda_1 \qquad \mathbf{F}_2 = F\lambda_2$

Now, by definition..

$M_{\lambda_1} = \lambda_1 \cdot (\mathbf{r} \times \mathbf{F}_2)$

$= \lambda_1 \cdot (\mathbf{r} \times \lambda_2)F$

and $M_{\lambda_2} = \lambda_2 \cdot (-\mathbf{r} \times \mathbf{F}_1)$

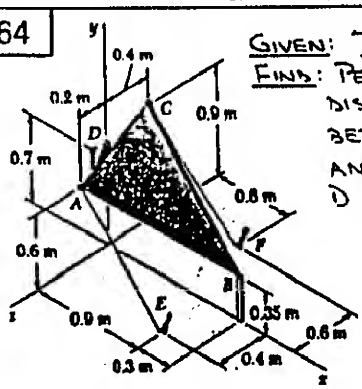
$= \lambda_2 \cdot (-\mathbf{r} \times \lambda_1)F$

Using Eq. (3.39) $\lambda_2 \cdot (-\mathbf{r} \times \lambda_1) = \lambda_1 \cdot (\mathbf{r} \times \lambda_2)$

so that $M_{\lambda_2} = \lambda_1 \cdot (\mathbf{r} \times \lambda_2)F$

$\therefore \quad M_{\lambda_1} = M_{\lambda_2}$

* 3.64



Given: T_{AE}, λ_{DB}, M_{DB}

Find: Perpendicular distance d between cable AE and line joining D and B

From the solution to Problem 3.55..

$T_{AE} = 55 \text{ N}$, $\mathbf{T}_{AE} = 5[(9\text{ N})\mathbf{i} - (6\text{ N})\mathbf{j} + (2\text{ N})\mathbf{k}]$

$M_{DB} = 2.28 \text{ N}\cdot\text{m} \quad \lambda_{DB} = \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$

Based on the discussion of Sec. 3.11, it

(CONTINUED)

3.64 CONTINUED

follows that only the perpendicular component of $\mathbf{T}_{AE}$ will contribute to the moment of $\mathbf{T}_{AE}$ about line $\overrightarrow{DB}$. Now

$(T_{AE})_{\text{parallel}} = \mathbf{T}_{AE} \cdot \lambda_{DB}$

$= 5(9\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}) \cdot \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$

$= \frac{1}{5}[(9)(24) + (-6)(-7)]$

$= 51.6 \text{ N}$

Also.. $\mathbf{T}_{AE} = (\mathbf{T}_{AE})_{\text{parallel}} + (\mathbf{T}_{AE})_{\text{perp.}}$

so that $(T_{AE})_{\text{perp.}} = \sqrt{(55)^2 - (51.6)^2}$

$= 19.0379 \text{ N}$

Since λ_{DB} and $(\mathbf{T}_{AE})_{\text{perp.}}$ are perpendicular, it follows that

$M_{DB} = d(T_{AE})_{\text{perp.}}$

or $2.28 \text{ N}\cdot\text{m} = d \cdot 19.0379 \text{ N}$

or $d = 0.1198 \text{ m}$

Alternative Solution

Let the perpendicular line, drawn from line $\overrightarrow{DB}$ to the line of action of $\mathbf{T}_{AE}$, be represented by

$\mathbf{d} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \qquad x, y, z \text{ in m}$

Now.. $\mathbf{d} \perp \mathbf{T}_{AE} \Rightarrow \mathbf{d} \cdot \mathbf{T}_{AE} = 0$

or $(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot 5(9\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}) = 0$

or $9x - 6y + 2z = 0 \qquad (1)$

and $\mathbf{d} \perp \lambda_{DB} \Rightarrow \mathbf{d} \cdot \lambda_{DB} = 0$

or $(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot \frac{1}{25}(24\mathbf{i} - 7\mathbf{j}) = 0$

or $24x - 7y = 0 \Rightarrow y = \frac{24}{7}x \qquad (2)$

Substituting Eq. (2) into Eq. (1)..

$9x - 6\left(\frac{24}{7}x\right) + 2z = 0 \Rightarrow z = \frac{81}{14}x \qquad (3)$

Now.. $M_{DB} = \lambda_{DB} \cdot (\mathbf{d} \times \mathbf{T}_{AE})$

$= \frac{1}{25}(5)\begin{vmatrix} 24 & -7 & 0 \\ x & y & z \\ 9 & -6 & 2 \end{vmatrix}$

$= \frac{1}{5}(48y - 63z + 14x + 144z)$

$= \frac{1}{5}(48y + 14x + 81z)$

Substituting for M_{DB} and using Eqs. (2) and (3) yields..

$2.28 = \frac{1}{5}\left[48\left(\frac{24}{7}x\right) + 14x + 81\left(\frac{81}{14}x\right)\right]$

or $x = 0.017614 \text{ m}$

and then $(2) \Rightarrow y = 0.060391 \text{ m}$

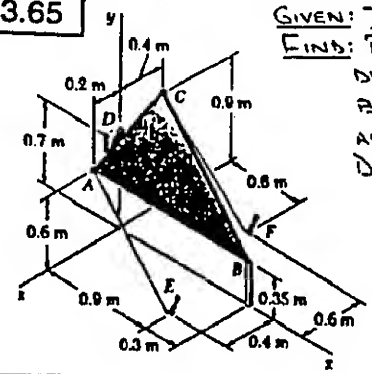
$(3) \Rightarrow z = 0.101909 \text{ m}$

Finally, $d = \sqrt{x^2 + y^2 + z^2}$

$= [(0.017614)^2 + (0.060391)^2 + (0.101909)^2]^{\frac{1}{2}}$

or $d = 0.1198 \text{ m}$

*** 3.65**



GIVEN: T_{CF}, λ_{DB}, M_{DB}
FIND: PERPENDICULAR DISTANCE d BETWEEN CABLE CF AND LINE JOINING D AND B

FROM THE SOLUTION TO PROBLEM 3.56..

$T_{CF} = 33\text{ N}$ $\quad \mathbf{T}_{CF} = 3[(6\text{ N})\mathbf{i} - (9\text{ N})\mathbf{j} - (2\text{ N})\mathbf{k}]$

$|M_{DB}| = 9.50\text{ N}\cdot\text{m}$ $\quad \lambda_{DB} = \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$

BASED ON THE DISCUSSION OF SEC. 3.11, IT FOLLOWS THAT ONLY THE PERPENDICULAR COMPONENT OF $\mathbf{T}_{CF}$ WILL CONTRIBUTE TO THE MOMENT OF $\mathbf{T}_{CF}$ ABOUT LINE $\overrightarrow{DB}$. NOW..

$$(T_{CF})_{PARALLEL} = \mathbf{T}_{CF} \cdot \lambda_{DB}$$
$$= 3(6\mathbf{i} - 9\mathbf{j} - 2\mathbf{k}) \cdot \tfrac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$
$$= \tfrac{3}{25}[(6)(24) + (-9)(-7)]$$
$$= 24.84\text{ N}$$

ALSO.. $\mathbf{T}_{CF} = (\mathbf{T}_{CF})_{PARALLEL} + (\mathbf{T}_{CF})_{PERP.}$

SO THAT $(T_{CF})_{PERP.} = \sqrt{(33)^2 - (24.84)^2}$
$$= 21.725\text{ N}$$

SINCE λ_{DB} AND $(\mathbf{T}_{CF})_{PERP.}$ ARE PERPENDICULAR, IT FOLLOWS THAT

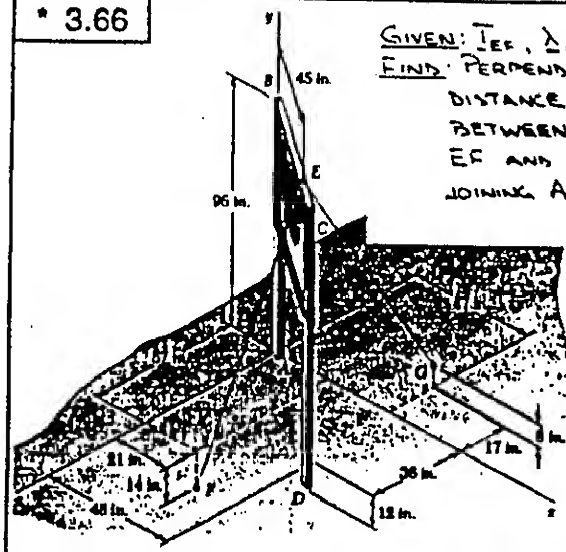
$$|M_{DB}| = d(T_{CF})_{PERP.}$$

OR $\quad 9.50\text{ N}\cdot\text{m} = d \cdot 21.725\text{ N}$

OR $\quad d = 0.437\text{ m}$ ◀

FOR A SECOND METHOD OF SOLUTION, SEE THE SOLUTION TO PROBLEM 3.64.

*** 3.66**



GIVEN: T_{EF}, λ_{AD}, M_{AD}
FIND: PERPENDICULAR DISTANCE d BETWEEN CABLE EF AND LINE JOINING A AND D

FROM THE SOLUTION TO PROBLEM 3.57..

$T_{EF} = 46\text{ lb}$ $\quad \mathbf{T}_{EF} = 2[-(3\text{ lb})\mathbf{i} - (22\text{ lb})\mathbf{j} + (6\text{ lb})\mathbf{k}]$

(CONTINUED)

3.66 CONTINUED

$M_{AD} = 1359\text{ lb}\cdot\text{in.}$ $\quad \lambda_{AD} = \frac{1}{\sqrt{26}}(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$

BASED ON THE DISCUSSION OF SEC. 3.11, IT FOLLOWS THAT ONLY THE PERPENDICULAR COMPONENT OF $\mathbf{T}_{EF}$ WILL CONTRIBUTE TO THE MOMENT OF $\mathbf{T}_{EF}$ ABOUT LINE $\overrightarrow{AD}$. NOW

$$(T_{EF})_{PARALLEL} = \mathbf{T}_{EF} \cdot \lambda_{AD}$$
$$= 2(-3\mathbf{i} - 22\mathbf{j} + 6\mathbf{k}) \cdot \tfrac{1}{\sqrt{26}}(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$$
$$= \tfrac{2}{\sqrt{26}}[(-3)(4) + (-22)(-1) + (6)(3)]$$
$$= 10.9825\text{ lb}$$

ALSO.. $\mathbf{T}_{EF} = (\mathbf{T}_{EF})_{PARALLEL} + (\mathbf{T}_{EF})_{PERP.}$

SO THAT $(T_{EF})_{PERP.} = \sqrt{(46)^2 - (10.9825)^2}$
$$= 44.670\text{ lb}$$

SINCE λ_{AD} AND $(\mathbf{T}_{EF})_{PERP.}$ ARE PERPENDICULAR, IT FOLLOWS THAT

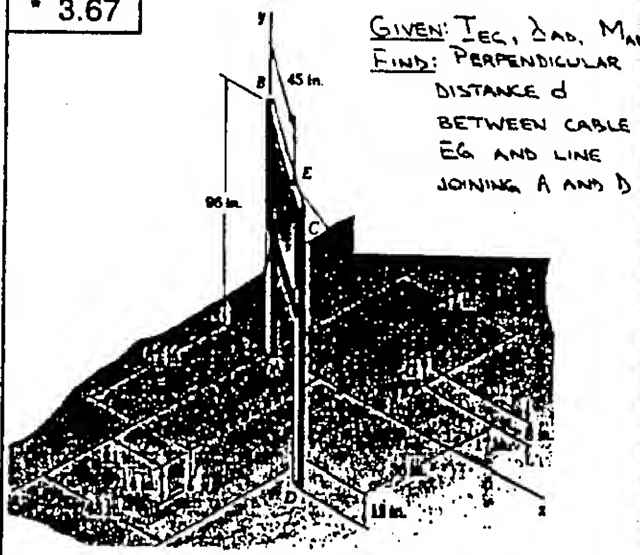
$$M_{AD} = d(T_{EF})_{PERP.}$$

OR $\quad 1359\text{ lb}\cdot\text{in.} = d \cdot 44.670\text{ lb}$

OR $\quad d = 30.4\text{ in.}$ ◀

FOR A SECOND METHOD OF SOLUTION, SEE THE SOLUTION TO PROBLEM 3.64.

*** 3.67**



GIVEN: T_{EG}, λ_{AD}, M_{AD}
FIND: PERPENDICULAR DISTANCE d BETWEEN CABLE EG AND LINE JOINING A AND D

FROM THE SOLUTION TO PROBLEM 3.58..

$T_{EG} = 54\text{ lb}$ $\quad \mathbf{T}_{EG} = 6[(1\text{ lb})\mathbf{i} - (8\text{ lb})\mathbf{j} - (4\text{ lb})\mathbf{k}]$

$|M_{AD}| = 2350\text{ lb}\cdot\text{in.}$ $\quad \lambda_{AD} = \frac{1}{\sqrt{26}}(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$

BASED ON THE DISCUSSION OF SEC. 3.11, IT FOLLOWS THAT ONLY THE PERPENDICULAR COMPONENT OF $\mathbf{T}_{EG}$ WILL CONTRIBUTE TO THE MOMENT OF $\mathbf{T}_{EG}$ ABOUT LINE $\overrightarrow{AD}$. NOW

$$(T_{EG})_{PARALLEL} = \mathbf{T}_{EG} \cdot \lambda_{AD}$$
$$= 6(\mathbf{i} - 8\mathbf{j} - 4\mathbf{k}) \cdot \tfrac{1}{\sqrt{26}}(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$$
$$= \tfrac{6}{\sqrt{26}}[(1)(4) + (-8)(-1) + (-4)(3)]$$
$$= 0$$

THUS, $(T_{EG})_{PERP.} = T_{EG} = 54\text{ lb}$

(CONTINUED)

3.67 CONTINUED

SINCE λ_{AD} AND $(\mathbf{T}_{EG})_{PERP}$ ARE PERPENDICULAR, IT FOLLOWS THAT

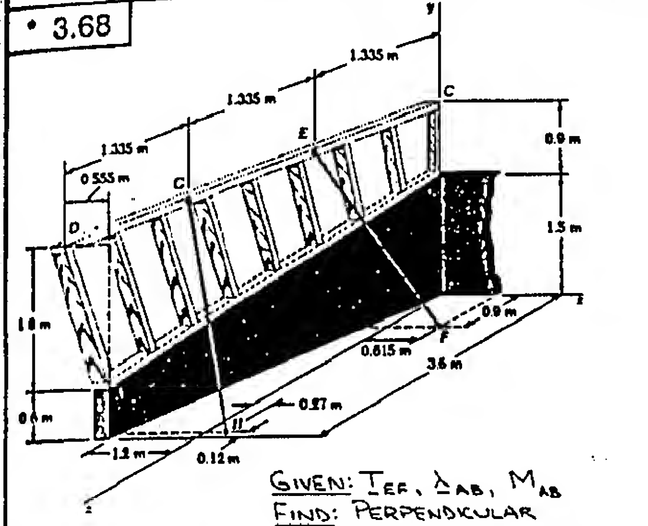
$$|M_{AD}| = d\,(T_{EG})_{PERP}$$

OR $2350 \text{ lb}\cdot\text{in.} = d \times 54 \text{ lb}$

OR $d = 43.5 \text{ in.}$ ◀

FOR A SECOND METHOD OF SOLUTION, SEE THE SOLUTION TO PROBLEM 3.64.

• 3.68



GIVEN: $\mathbf{T}_{EF}$, λ_{AB}, M_{AB}

FIND: PERPENDICULAR DISTANCE d BETWEEN CABLE EF AND SILL AB

FROM THE SOLUTION TO PROBLEM 3.61..

$T_{EF} = 63 \text{ N}$ $\quad \mathbf{T}_{EF} = 7[(4\text{ N})\mathbf{i} - (8\text{ N})\mathbf{j} - (1\text{ N})\mathbf{k}]$

$M_{AB} = 18.57 \text{ N}\cdot\text{m}$ $\quad \lambda_{AB} = \frac{1}{13}(4\mathbf{i} + 3\mathbf{j} - 12\mathbf{k})$

BASED ON THE DISCUSSION OF SEC. 3.11, IT FOLLOWS THAT ONLY THE PERPENDICULAR COMPONENT OF $\mathbf{T}_{EF}$ WILL CONTRIBUTE TO THE MOMENT OF $\mathbf{T}_{EF}$ ABOUT SILL AB. NOW..

$$(T_{EF})_{PARALLEL} = \mathbf{T}_{EF} \cdot \lambda_{AB}$$
$$= 7(4\mathbf{i} - 8\mathbf{j} - \mathbf{k}) \cdot \tfrac{1}{13}(4\mathbf{i} + 3\mathbf{j} - 12\mathbf{k})$$
$$= \tfrac{7}{13}[(4)(4) + (-8)(3) + (-1)(-12)]$$
$$= 2.1538 \text{ N}$$

ALSO.. $\mathbf{T}_{EF} = (\mathbf{T}_{EF})_{PARALLEL} + (\mathbf{T}_{EF})_{PERP}$

SO THAT $\quad (T_{EF})_{PERP} = \sqrt{(63)^2 - (2.1538)^2}$

$$= 62.963 \text{ N}$$

SINCE λ_{AB} AND $(\mathbf{T}_{EF})_{PERP}$ ARE PERPENDICULAR, IT FOLLOWS THAT

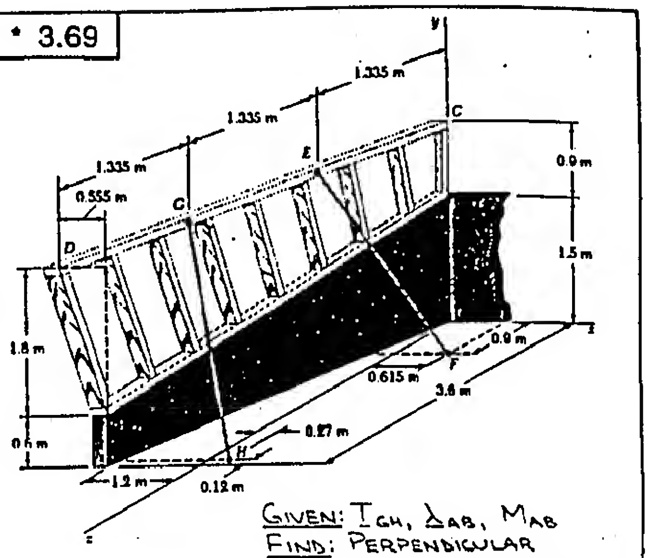
$$M_{AB} = d\,(T_{EF})_{PERP}$$

OR $18.57 \text{ N}\cdot\text{m} = d \cdot 62.963 \text{ N}$

OR $d = 0.295 \text{ m}$ ◀

FOR A SECOND METHOD OF SOLUTION, SEE THE SOLUTION TO PROBLEM 3.64.

* 3.69



GIVEN: $\mathbf{T}_{GH}$, λ_{AB}, M_{AB}

FIND: PERPENDICULAR DISTANCE d BETWEEN CABLE GH AND SILL AB

FROM THE SOLUTION TO PROBLEM 3.62..

$T_{GH} = 75 \text{ N}$ $\quad \mathbf{T}_{GH} = 3[(12\text{ N})\mathbf{i} - (20\text{ N})\mathbf{j} + (9\text{ N})\mathbf{k}]$

$M_{AB} = 44.1 \text{ N}\cdot\text{m}$ $\quad \lambda_{AB} = \frac{1}{13}(4\mathbf{i} + 3\mathbf{j} - 12\mathbf{k})$

BASED ON THE DISCUSSION OF SEC. 3.11, IT FOLLOWS THAT ONLY THE PERPENDICULAR COMPONENT OF $\mathbf{T}_{GH}$ WILL CONTRIBUTE TO THE MOMENT OF $\mathbf{T}_{GH}$ ABOUT SILL AB. NOW..

$$(T_{GH})_{PARALLEL} = \mathbf{T}_{GH} \cdot \lambda_{AB}$$
$$= 3(12\mathbf{i} - 20\mathbf{j} + 9\mathbf{k}) \cdot \tfrac{1}{13}(4\mathbf{i} + 3\mathbf{j} - 12\mathbf{k})$$
$$= \tfrac{3}{13}[(12)(4) + (-20)(3) + (9)(-12)]$$
$$= -27.692 \text{ N}$$

ALSO.. $\mathbf{T}_{GH} = (\mathbf{T}_{GH})_{PARALLEL} + (\mathbf{T}_{GH})_{PERP}$

SO THAT.. $(T_{GH})_{PERP} = \sqrt{(75)^2 - (-27.692)^2}$

$$= 69.700 \text{ N}$$

SINCE λ_{AB} AND $(\mathbf{T}_{GH})_{PERP}$ ARE PERPENDICULAR, IT FOLLOWS THAT

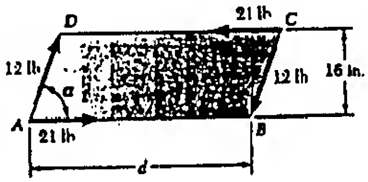
$$M_{AB} = d\,(T_{GH})_{PERP}$$

OR $44.1 \text{ N}\cdot\text{m} = d \cdot 69.700 \text{ N}$

OR $d = 0.633 \text{ m}$ ◀

FOR A SECOND METHOD OF SOLUTION, SEE THE SOLUTION TO PROBLEM 3.64.

3.70



GIVEN: $F_1 = 21 \text{ lb}$

$F_2 = 12 \text{ lb}$

FIND: (a) M_1

(b) $(d_2)_{PERP}$, GIVEN $\mathbf{M}_1 + \mathbf{M}_2 = 0$

(c) α, GIVEN $\mathbf{M}_1 + \mathbf{M}_2 = 72 \text{ lb}\cdot\text{in.}\,\circlearrowright$

$d = 42 \text{ in.}$

(a) HAVE $\quad M_1 = d_1 F_1 \quad$ WHERE $d_1 = 16 \text{ in.}$

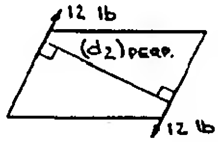
$$= (16 \text{ in.})(21 \text{ lb})$$

OR $\mathbf{M}_1 = 336 \text{ lb}\cdot\text{in.}\,\circlearrowright$ ◀

(CONTINUED)

3.70 CONTINUED

(b)

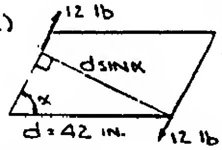


HAVE.. $M_1 + M_2 = 0$

OR $336 \text{ lb}\cdot\text{in.} - (d_2)_{PERP}(12 \text{ lb}) = 0$

OR $(d_2)_{PERP} = 28 \text{ in.}$ ◀

(c)



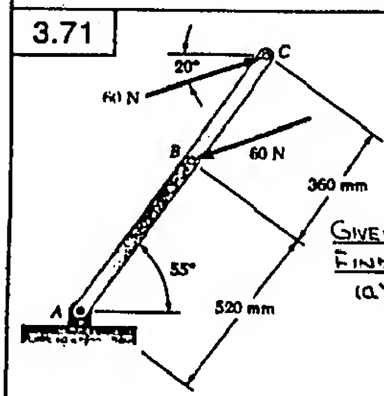
HAVE.. $M_{TOTAL} = M_1 + M_2$

OR $-72 \text{ lb}\cdot\text{in.} = 336 \text{ lb}\cdot\text{in.} - [(42 \text{ in.})(\sin\alpha)](12 \text{ lb})$

OR $\sin\alpha = 0.80952$

OR $\alpha = 54.0°$ ◀

3.71



GIVEN: 60-N FORCES

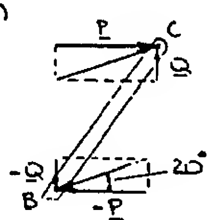
FIND: MOMENT OF COUPLE

(a) BY RESOLVING FORCES INTO HORIZONTAL AND VERTICAL COMPONENTS

(b) BY USING d_{PERP}

(c) BY SUMMING MOMENTS ABOUT A

(a)



EACH 60-N FORCE IS FIRST RESOLVED INTO HORIZONTAL (P) AND VERTICAL (Q) COMPONENTS, WHERE

$P = (60 \text{ N})\cos 20°$

$Q = (60 \text{ N})\sin 20°$

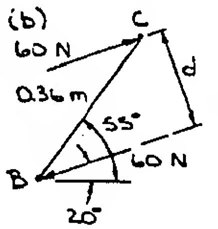
SINCE P AND −P AND Q AND −Q ARE BOTH COUPLES, THE TOTAL MOMENT IS GIVEN BY..

$M = -[(0.36 \text{ m})(\sin 55°)][(60 \text{ N})(\cos 20°)] + [(0.36 \text{ m})(\cos 55°)][(60 \text{ N})(\sin 20°)]$

$= -(0.36)(60)\sin(55° - 20°) \text{ N}\cdot\text{m}$

OR $M = 12.39 \text{ N}\cdot\text{m}$ ↺ ◀

(b)



HAVE.. $M = -dF$

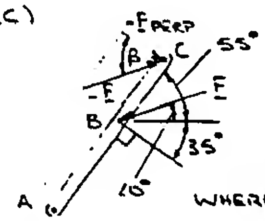
WHERE $d = (0.36 \text{ m})\sin(55° - 20°)$

THEN..

$M = -[(0.36 \text{ m})(\sin 35°)](60 \text{ N})$

OR $M = 12.39 \text{ N}\cdot\text{m}$ ↺ ◀

(c)



SINCE ONLY THE PERPENDICULAR COMPONENTS OF THE FORCES WILL CONTRIBUTE TO THE MOMENT ABOUT A, HAVE..

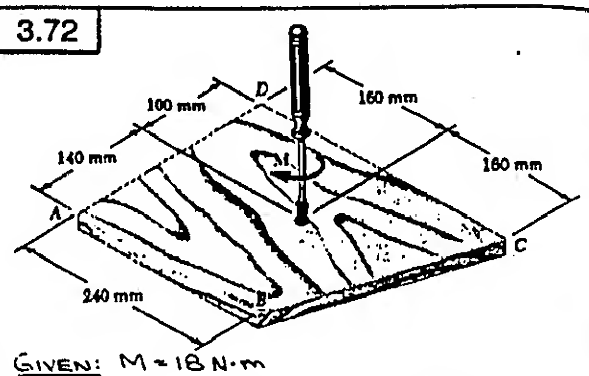
$M_A = r_{B/A} F_{PERP} - r_{C/A} F_{PERP}$

WHERE $F_{PERP} = F\cos\beta = (60 \text{ N})\cos(35° + 20°)$

THEN.. $M_A = (0.52 - 0.88) \text{ m} \cdot (60 \text{ N})\cos 55°$

OR $M = M_A = 12.39 \text{ N}\cdot\text{m}$ ↺ ◀

3.72



GIVEN: $M = 18 \text{ N}\cdot\text{m}$

FIND: TWO SMALLEST FORCES EQUIVALENT TO M AND APPLIED AT

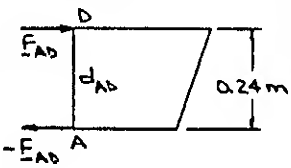
(a) CORNERS A AND D

(b) CORNERS B AND C

(c) ANYWHERE ON THE BLOCK

FIRST NOTE THAT IF THE TWO FORCES ARE TO BE EQUIVALENT TO M, THEY MUST FORM A COUPLE. FURTHER, THE FORCES WILL BE MINIMUM WHEN THEY ARE PERPENDICULAR TO THE LINE JOINING THEIR POINTS OF APPLICATION. THUS, FOR EACH PART OF THE PROBLEM -- $M = dF$

(a) FORCES AT A AND D

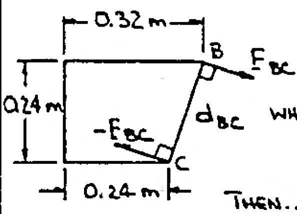


HAVE.. $M = d_{AD} F_{AD}$

OR $18 \text{ N}\cdot\text{m} = 0.24 \text{ m} \cdot F_{AD}$

OR $F_{AD} = 75 \text{ N}$ ◀

(b) FORCES AT B AND C



HAVE.. $M = d_{BC} F_{BC}$

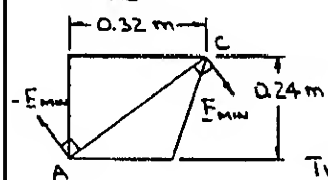
WHERE $d_{BC} = \sqrt{(0.32 - 0.24)^2 + (0.24)^2} = 0.25298 \text{ m}$

THEN.. $18 \text{ N}\cdot\text{m} = 0.25298 \text{ m} \cdot F_{BC}$

OR $F_{BC} = 71.2 \text{ N}$ ◀

(c) F_{MIN}

FOR F_{MIN}, WANT d TO BE MAXIMUM. THUS, $d = d_{AC}$



HAVE.. $M = d_{AC} F_{MIN}$

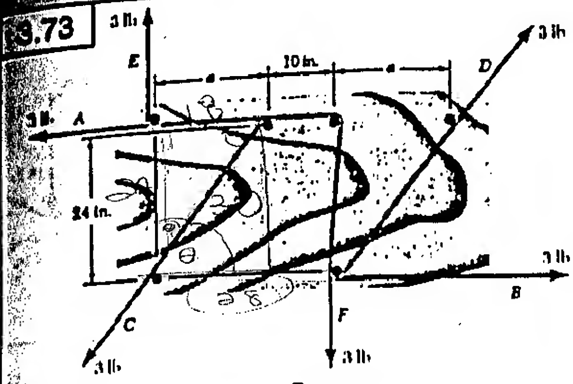
WHERE

$d_{AC} = \sqrt{(0.32)^2 + (0.24)^2} = 0.4 \text{ m}$

THEN.. $18 \text{ N}\cdot\text{m} = 0.4 \text{ m} \cdot F_{MIN}$

OR $F_{MIN} = 45 \text{ N}$ ◀

3.73



GIVEN: $d_{PEG} = 2$ in., $F = 3$ lb, $a = 18$ in

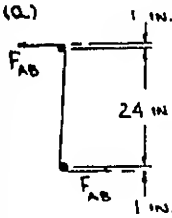
FIND: $\underline{M}$ FOR

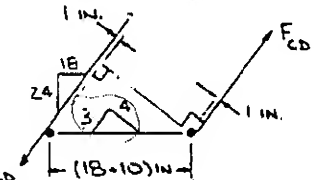
(a) WIRES AB AND CD

(b) WIRES AB, CD, AND EF

IN GENERAL, $M = \Sigma dF$, WHERE d IS THE PERPENDICULAR DISTANCE BETWEEN THE LINES OF ACTION OF THE TWO FORCES ACTING ON A GIVEN WIRE.

(a)



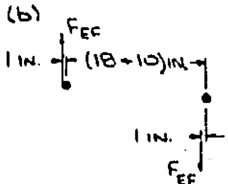


HAVE.. $M = d_{AB}F_{AB} + d_{CD}F_{CD}$

$= (2+24)\text{ in.} \times 3\text{ lb} + (2+\tfrac{4}{5} \times 28)\text{ in.} \times 3\text{ lb}$

78 OR $\underline{M} = 151.2$ lb·in. ◀

(b)



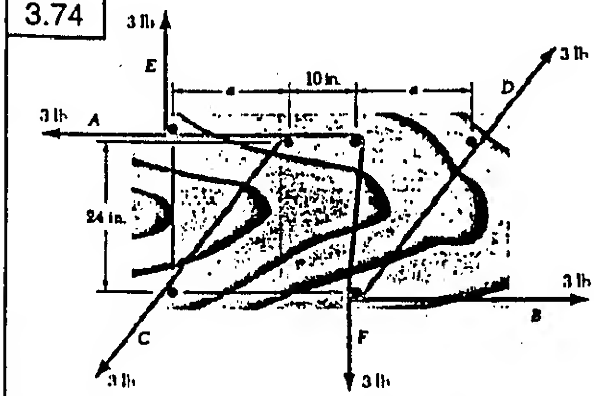
HAVE

$M = [d_{AB}F_{AB} + d_{CD}F_{CD}] + d_{EF}F_{EF}$

$= 151.2$ lb·in. $- 28$ in. $\times 3$ lb

OR $\underline{M} = 67.2$ lb·in. ◀

3.74



GIVEN: $d_{PEG} = 2$ in., $F_{AB} = F_{CD} = 3$ lb, $\underline{M} = 159.6$ lb·in.

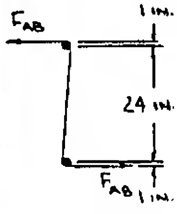
FIND: a_{MIN}

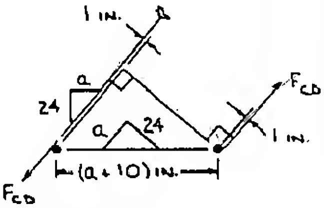
HAVE.. $M = d_{AB}F_{AB} + d_{CD}F_{CD}$

(CONTINUED)

3.74 CONTINUED

WHERE d_{AB} AND d_{CD} ARE THE PERPENDICULAR DISTANCES BETWEEN THE LINES OF ACTION OF THE FORCES ACTING ON WIRES AB AND CD, RESPECTIVELY.





THEN.. $159.6\text{ lb·in.} = (2+24)\text{ in.} \times 3\text{ lb} + \left[2 + \dfrac{24}{\sqrt{24^2+a^2}}(a+10)\right]\text{in.} \times 3\text{ lb}$

OR $25.2 = \dfrac{24(a+10)}{\sqrt{576+a^2}}$

OR $(25.2)^2(576+a^2) = (576)(a+10)^2$

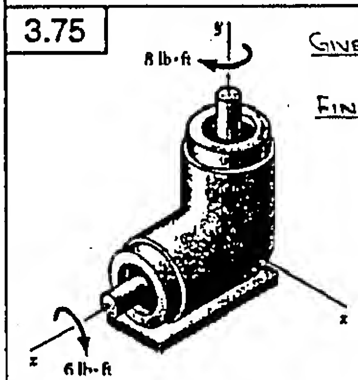
OR $59.04a^2 - 11\,520a + 308\,183 = 0$

OR $a = \dfrac{11\,520 \pm \sqrt{(-11\,520)^2 - 4(59.04)(308\,183)}}{2(59.04)}$

SOLVING YIELDS.. $a = 32.0$ in., $a = 163.1$ in.

TAKING THE SMALLER ROOT.. $a = 32.0$ in. ◀

3.75



GIVEN: $M_1 = 8$ lb·ft

$M_2 = 6$ lb·ft

FIND: $|\underline{M}_1 + \underline{M}_2|$, θ_x, θ_y, θ_z

HAVE.. $\underline{M} = \underline{M}_1 + \underline{M}_2$

$= -(8\text{ lb·ft})\underline{j} - (6\text{ lb·ft})\underline{k}$

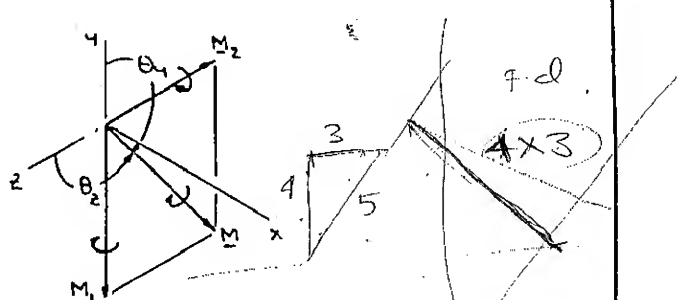
THEN .. $M = \sqrt{(0)^2 + (-8)^2 + (-6)^2}$

OR $M = 10$ lb·ft ◀

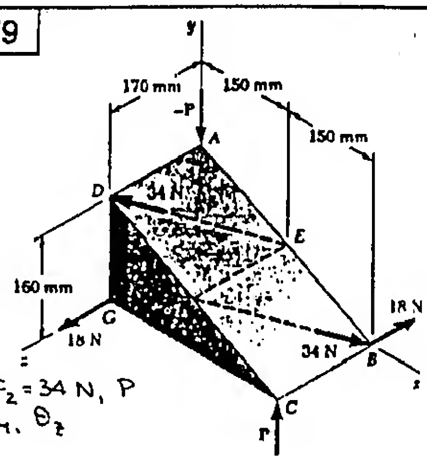
AND

$\cos\theta_x = 0 \quad \cos\theta_y = -\frac{8}{10} \quad \cos\theta_z = -\frac{6}{10}$

OR $\theta_x = 90°$ $\theta_y = 143.1°$ $\theta_z = 126.9°$ ◀



3.76 and 3.79



GIVEN: $F_1 = 18\text{ N}$, $F_2 = 34\text{ N}$, P

FIND: M, θ_x, θ_y, θ_z

HAVE. $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$

WHERE $\mathbf{M}_1 = \mathbf{r}_{C/G} \times \mathbf{F}_1 = (0.3\text{ m})\mathbf{i} \times [-(18\text{ N})\mathbf{k}]$
$= (5.4\text{ N}\cdot\text{m})\mathbf{j}$

$\mathbf{M}_2 = \mathbf{r}_{F/E} \times \mathbf{F}_2$

WHERE $\mathbf{r}_{F/E} = (0.17\text{ m})\mathbf{k}$

AND $d_{FB} = \sqrt{(0.15)^2 + (-0.08)^2 + (-0.17)^2}$
$= 0.17\sqrt{2}\text{ m}$

THEN $\mathbf{F}_2 = \dfrac{34\text{ N}}{0.17\sqrt{2}}(0.15\mathbf{i} - 0.08\mathbf{j} - 0.17\mathbf{k})$
$= \sqrt{2}[(15\text{ N})\mathbf{i} - (8\text{ N})\mathbf{j} - (17\text{ N})\mathbf{k}]$

SO THAT $\mathbf{M}_2 = 0.17\mathbf{k} \times \sqrt{2}(15\mathbf{i} - 8\mathbf{j} - 17\mathbf{k})$
$= \sqrt{2}[(1.36\text{ N}\cdot\text{m})\mathbf{i} + (2.55\text{ N}\cdot\text{m})\mathbf{j}]$

$\mathbf{M}_3 = \mathbf{r}_C \times \mathbf{P} = [(0.3\text{ m})\mathbf{i} + (0.17\text{ m})\mathbf{k}] \times P\mathbf{j}$
$= P(-0.17\mathbf{i} + 0.3\mathbf{k})\quad (\text{N}\cdot\text{m})$

3.76 $P = 0 \quad \therefore \mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$

OR $\mathbf{M} = (5.4\mathbf{j}) + \sqrt{2}(1.36\mathbf{i} + 2.55\mathbf{j})$
$= (1.923\,33\text{ N}\cdot\text{m})\mathbf{i} + (9.0062\text{ N}\cdot\text{m})\mathbf{j}$

THEN.. $M = \sqrt{(1.923\,33)^2 + (9.0062)^2 + (0)^2} = 9.2093\text{ N}\cdot\text{m}$

OR $M = 9.21\text{ N}\cdot\text{m}$

AND $\boldsymbol{\lambda}_{AXIS} = \dfrac{\mathbf{M}}{M} = 0.208\,85\mathbf{i} + 0.977\,95\mathbf{j}$

THEN.. $\cos\theta_x = 0.20885 \quad \cos\theta_y = 0.97795 \quad \cos\theta_z = 0$

SO THAT $\theta_x = 77.9° \quad \theta_y = 12.05° \quad \theta_z = 90°$

3.79 $P = 20\text{ N} \quad \therefore \mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$

OR $\mathbf{M} = (1.923\,33\mathbf{i} + 9.0062\mathbf{j}) + 20(-0.17\mathbf{i} + 0.3\mathbf{k})$
$= -(1.47667\text{ N}\cdot\text{m})\mathbf{i} + (9.0062\text{ N}\cdot\text{m})\mathbf{j} + (6\text{ N}\cdot\text{m})\mathbf{k}$

THEN.. $M = \sqrt{(-1.47667)^2 + (9.0062)^2 + (6)^2} = 10.9221\text{ N}\cdot\text{m}$

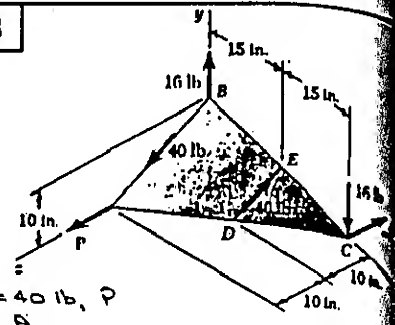
OR $M = 10.92\text{ N}\cdot\text{m}$

AND $\boldsymbol{\lambda}_{AXIS} = \dfrac{\mathbf{M}}{M} = -0.135\,200\mathbf{i} + 0.824\,59\mathbf{j} + 0.549\,34\mathbf{k}$

THEN..
$\cos\theta_x = -0.135\,200 \quad \cos\theta_y = 0.82459 \quad \cos\theta_z = 0.54934$

SO THAT $\theta_x = 97.8° \quad \theta_y = 34.5° \quad \theta_z = 56.7°$

3.77 and 3.78



GIVEN: $F_1 = 16\text{ lb}$, $F_2 = 40\text{ lb}$, P

FIND: M, θ_x, θ_y, θ_z

HAVE.. $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$

WHERE $\mathbf{M}_1 = \mathbf{r}_C \times \mathbf{F}_1 = (30\text{ in.})\mathbf{i} \times [-(16\text{ lb})\mathbf{j}]$
$= -(480\text{ lb}\cdot\text{in.})\mathbf{k}$

$\mathbf{M}_2 = \mathbf{r}_{E/B} \times \mathbf{F}_2$

WHERE $\mathbf{r}_{E/B} = (15\text{ in.})\mathbf{i} - (5\text{ in.})\mathbf{j}$

AND $d_{DE} = \sqrt{(0)^2 + (5)^2 + (-10)^2} = 5\sqrt{5}\text{ in.}$

THEN.. $\mathbf{F}_2 = \dfrac{40\text{ lb}}{5\sqrt{5}}(5\mathbf{j} - 10\mathbf{k})$
$= 8\sqrt{5}[(1\text{ lb})\mathbf{j} - (2\text{ lb})\mathbf{k}]$

SO THAT
$\mathbf{M}_2 = 8\sqrt{5}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 15 & -5 & 0 \\ 0 & 1 & -2 \end{vmatrix}$
$= 8\sqrt{5}[(10\text{ lb}\cdot\text{in.})\mathbf{i} + (30\text{ lb}\cdot\text{in.})\mathbf{j} + (15\text{ lb}\cdot\text{in.})\mathbf{k}]$

$\mathbf{M}_3 = \mathbf{r}_C \times -\mathbf{P} = (30\text{ in.})\mathbf{i} \times (-P)\mathbf{k}$
$= (30P)\mathbf{j}\quad (\text{lb}\cdot\text{in.})$

3.77 $P = 0 \quad \therefore \mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$

OR $\mathbf{M} = -(480)\mathbf{k} + 8\sqrt{5}(10\mathbf{i} + 30\mathbf{j} + 15\mathbf{k})$
$= (178.885\text{ lb}\cdot\text{in.})\mathbf{i} + (536.66\text{ lb}\cdot\text{in})\mathbf{j} - (211.67\text{ lb}\cdot\text{in.})\mathbf{k}$

THEN.. $M = \sqrt{(178.885)^2 + (536.66)^2 + (-211.67)^2}$
$= 603.99\text{ lb}\cdot\text{in.}$

OR $M = 604\text{ lb}\cdot\text{in.}$

AND $\boldsymbol{\lambda}_{AXIS} = \dfrac{\mathbf{M}}{M} = 0.296\,17\mathbf{i} + 0.888\,52\mathbf{j} - 0.350\,45\mathbf{k}$

THEN..
$\cos\theta_x = 0.29617 \quad \cos\theta_y = 0.88852 \quad \cos\theta_z = -0.3504$

SO THAT $\theta_x = 72.8° \quad \theta_y = 27.3° \quad \theta_z = 110.5°$

3.78 $P = 20\text{ lb} \quad \therefore \mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$

OR $\mathbf{M} = -(480)\mathbf{k} + 8\sqrt{5}(10\mathbf{i} + 30\mathbf{j} + 15\mathbf{k}) + (30 \times 20)\mathbf{j}$
$= (178.885\text{ lb}\cdot\text{in.})\mathbf{i} + (1136.66\text{ lb}\cdot\text{in.})\mathbf{j} - (211.67\text{ lb}\cdot\text{in.})\mathbf{k}$

THEN.. $M = \sqrt{(178.885)^2 + (1136.66)^2 + (-211.67)^2}$
$= 1169.96\text{ lb}\cdot\text{in.}$

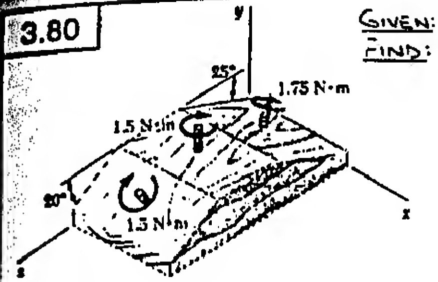
OR $M = 1170\text{ lb}\cdot\text{in.}$

AND $\boldsymbol{\lambda}_{AXIS} = \dfrac{\mathbf{M}}{M} = 0.152\,898\mathbf{i} + 0.971\,54\mathbf{j} - 0.180\,921\mathbf{k}$

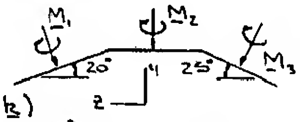
THEN..
$\cos\theta_x = 0.152\,898 \quad \cos\theta_y = 0.971\,54 \quad \cos\theta_z = -0.1809$

SO THAT $\theta_x = 81.2° \quad \theta_y = 13.70° \quad \theta_z = 100.4°$

3.80



GIVEN: M_1, M_2, AND M_3
FIND: $M, \theta_x, \theta_y, \theta_z$



HAVE.. $\underline{M} = \underline{M}_1 + \underline{M}_2 + \underline{M}_3$

OR $\underline{M} = 1.5(-\cos 20° \underline{j} - \sin 20° \underline{k})$
$\quad -1.5\underline{j} + 1.75(-\cos 25° \underline{j} + \sin 25° \underline{k})$
$\quad = -(4.4956 \text{ N·m})\underline{j} + (0.22655 \text{ N·m})\underline{k}$

THEN. $M = \sqrt{(0)^2 + (-4.4956)^2 + (0.22655)^2}$
$\quad = 4.5013$ N·m

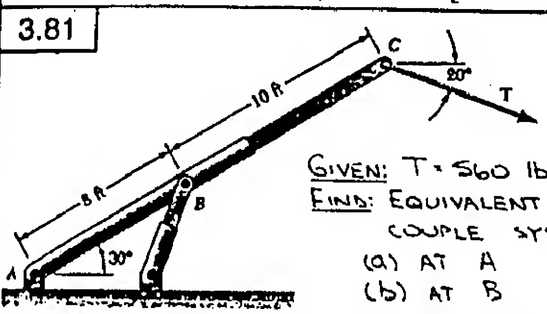
OR $M = 4.50$ N·m

AND $\underline{\lambda}_{AXIS} = \dfrac{\underline{M}}{M} = -0.99873\underline{j} + 0.050330\underline{k}$

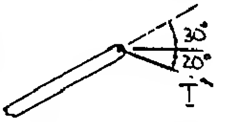
THEN.. $\cos\theta_x = 0 \quad \cos\theta_y = -0.99873 \quad \cos\theta_z = 0.050330$

SO THAT .. $\theta_x = 90° \quad \theta_y = 177.1° \quad \theta_z = 87.1°$

3.81



GIVEN: $T = 560$ lb
FIND: EQUIVALENT FORCE-COUPLE SYSTEM
(a) AT A
(b) AT B



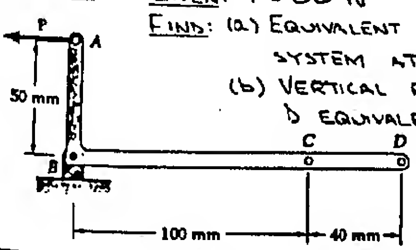
(a) HAVE.. $\underline{F} = 560 \text{ lb} \measuredangle 20°$
AND $M = M_A$
$\quad = -(18 \text{ ft})(560 \text{ lb})\sin 50°$
$\quad = -7720$ lb·ft

∴ THE EQUIVALENT FORCE-COUPLE SYSTEM AT A
IS $\underline{F} = 560 \text{ lb} \measuredangle 20°$, $\underline{M} = 7720$ lb·ft ↻

(b) HAVE.. $\underline{F} = 560 \text{ lb} \measuredangle 20°$
AND $M = M_B = -(10 \text{ ft})(560 \text{ lb})\sin 50°$
$\quad = -4290$ lb·ft

∴ THE EQUIVALENT FORCE-COUPLE SYSTEM AT B
IS $\underline{F} = 560 \text{ lb} \measuredangle 20°$, $\underline{M} = 4290$ lb·ft ↻

3.82



GIVEN: $P = 80$ N
FIND: (a) EQUIVALENT FORCE-COUPLE SYSTEM AT B
(b) VERTICAL FORCES AT C AND D EQUIVALENT TO COUPLE OF PART a

(a) HAVE $\underline{F} = 80$ N ⟵

(CONTINUED)

3.82 CONTINUED

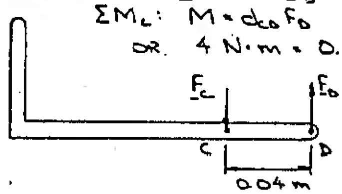
AND $M = M_B = (0.05 \text{ m})(80 \text{ N}) = 4$ N·m

∴ THE EQUIVALENT FORCE-COUPLE SYSTEM AT B
IS $\underline{F} = 80$ N ⟵, $\underline{M} = 4$ N·m ↺

(b) IF THE TWO VERTICAL FORCES ARE TO BE EQUIVALENT TO $\underline{M}$, THEY MUST BE A COUPLE. FURTHER, THE SENSE OF THE MOMENT OF THIS COUPLE MUST BE COUNTERCLOCKWISE.

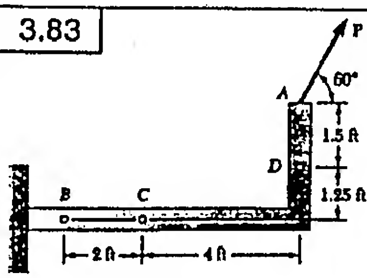
THEN.. WITH F_C AND F_D ACTING AS SHOWN, HAVE

ΣM_C: $M = d_{CD} F_D$
OR $4 \text{ N·m} = 0.04 \text{ m} \times F_D$



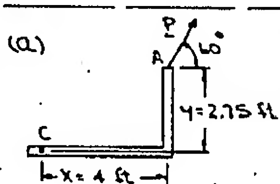
OR $\underline{F}_D = 100$ N ↑
AND $\underline{F}_C = 100$ N ↓

3.83



GIVEN: $P = 160$ lb
FIND: (a) EQUIVALENT FORCE-COUPLE SYSTEM AT C
(b) EQUIVALENT SYSTEM (F_B, F_D), WHERE F_B IS VERTICAL

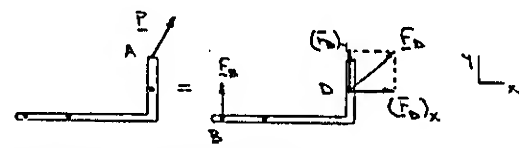
(a)



HAVE .. $\underline{F} = 160 \text{ lb} \measuredangle 60°$
AND $M = M_C = xP_y - yP_x$
OR $M = (4 \text{ ft})(160 \text{ lb})\sin 60° - (2.75 \text{ ft})(160 \text{ lb})\cos 60°$
$\quad = 334.26$ lb·ft

∴ THE EQUIVALENT FORCE-COUPLE SYSTEM AT C IS.. $\underline{F} = 160 \text{ lb} \measuredangle 60°$, $\underline{M} = 334$ lb·ft ↺

(b) REQUIRE



THEN FOR EQUIVALENCE..

ΣF_x: $(160 \text{ lb})\cos 60° = (F_D)_x$
OR $(F_D)_x = 80$ lb

ΣM_A: $0 = (1.5 \text{ ft})(80 \text{ lb}) - (6 \text{ ft})F_B = 0$
OR $\underline{F}_B = 20$ lb ↑

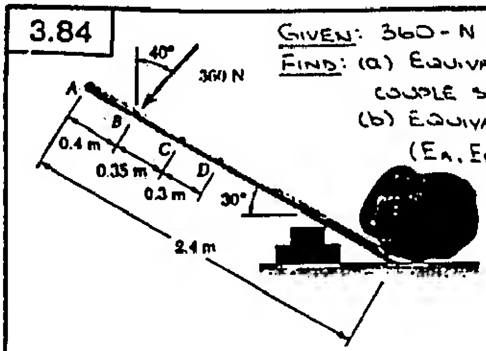
ΣF_y: $(160 \text{ lb})\sin 60° = 20 \text{ lb} + (F_D)_y$
OR $(F_D)_y = 118.564$ lb

THEN.. $F_D = \sqrt{(80)^2 + (118.564)^2}$
$\quad = 143.0$ lb

$\tan\theta = \dfrac{118.564}{80}$
OR $\theta = 56.0°$

∴ $\underline{F}_D = 143.0 \text{ lb} \measuredangle 56.0°$

3.84

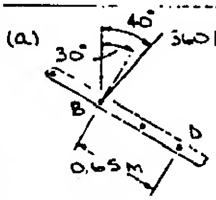


GIVEN: 360-N FORCE

FIND: (a) EQUIVALENT FORCE-COUPLE SYSTEM AT D

(b) EQUIVALENT SYSTEM $(\underline{F}_A, \underline{F}_D)$, WHERE $\underline{F}_A$ IS VERTICAL

(a)



HAVE $\underline{F} = 360\ \text{N} \measuredangle 50°$

AND $M = M_D$

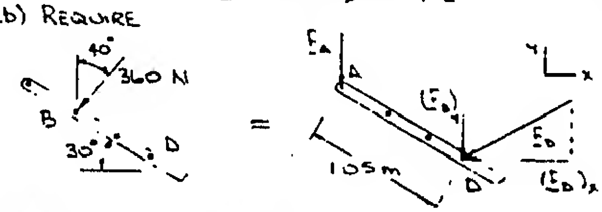
$= (0.65\ \text{m})(360\ \text{N})\cos 10°$

$= 230.45\ \text{N}\cdot\text{m}$

$\therefore$ THE EQUIVALENT FORCE-COUPLE SYSTEM AT D IS

$\underline{F} = 360\ \text{N} \measuredangle 50°,\ \underline{M} = 230\ \text{N}\cdot\text{m}$

(b) REQUIRE



THEN FOR EQUIVALENCE ..

ΣM_D: $M = (d_{AD}\cos 30°)F_A$

OR $230.45\ \text{N}\cdot\text{m} = (1.05\ \text{m})(\cos 30°)F_A$

OR $F_A = 253.43\ \text{N}$

THEN.. $\underline{F}_A = 253\ \text{N} \downarrow$

ΣF_x: $-(360\ \text{N})\sin 40° = (F_D)_x$

OR $(F_D)_x = -231.40\ \text{N}$

ΣF_y: $-(360\ \text{N})\cos 40° = -253.43\ \text{N} - (F_D)_y$

OR $(F_D)_y = -22.35\ \text{N}$

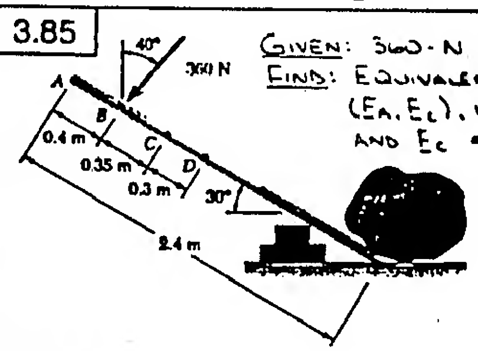
THEN.. $F_D = \sqrt{(-231.40)^2 + (-22.35)^2}$ $\qquad \tan\theta = \frac{-22.35}{-231.40}$

$= 232\ \text{N}$

OR $\theta = 5.52°$

$\therefore \underline{F}_D = 232\ \text{N} \measuredangle 5.52°$

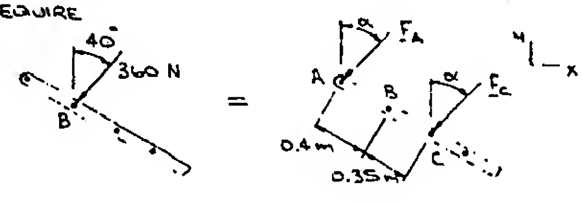
3.85



GIVEN: 360-N FORCE

FIND: EQUIVALENT SYSTEM $(\underline{F}_A, \underline{F}_C)$, WHERE $\underline{F}_A$ AND $\underline{F}_C$ ARE PARALLEL

REQUIRE



(CONTINUED)

3.85 CONTINUED

THEN FOR EQUIVALENCE --

ΣF_x: $-360\sin 40° = -F_A\sin\alpha - F_C\sin\alpha \qquad (1)$

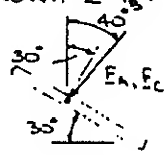
ΣF_y: $-360\cos 40° = -F_A\cos\alpha - F_C\cos\alpha \qquad (2)$

FORMING $\frac{(1)}{(2)}$ .. $\frac{-360\sin 40°}{-360\cos 40°} = \frac{-(F_A + F_C)\sin\alpha}{-(F_A + F_C)\cos\alpha}$

SIMPLIFYING YIELDS $\alpha = 40°$

AND THEN $F_A + F_C = 360\ \text{N} \qquad (3)$

NOW.. ΣM_B: $0 = (0.4\ \text{m})F_A\cos 10° - (0.35\ \text{m})F_C\cos 10°$



OR $F_A = \frac{7}{8}F_C \qquad (4)$

SUBSTITUTING FOR F_A IN EQ. (3)..

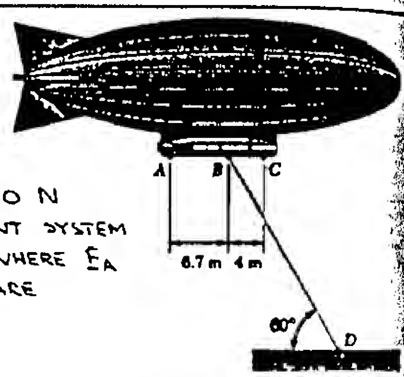
$\frac{7}{8}F_C + F_C = 360$

OR $F_C = 192\ \text{N}$

AND THEN $F_A = 168\ \text{N}$

$\therefore \underline{F}_A = 168\ \text{N} \measuredangle 50°,\ \underline{F}_C = 192\ \text{N} \measuredangle 50°$

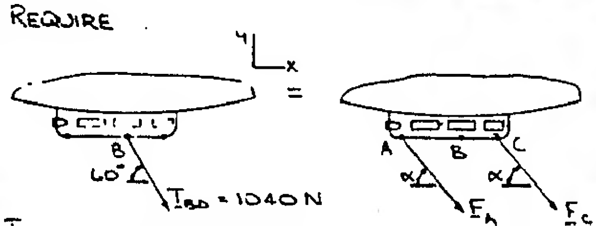
3.86



GIVEN: $T_{BD} = 1040\ \text{N}$

FIND: EQUIVALENT SYSTEM $(\underline{F}_A, \underline{F}_C)$, WHERE $\underline{F}_A$ AND $\underline{F}_C$ ARE PARALLEL

REQUIRE



THEN FOR EQUIVALENCE --

ΣF_x: $1040\cos 60° = F_A\cos\alpha + F_C\cos\alpha \qquad (1)$

ΣF_y: $-1040\sin 60° = -F_A\sin\alpha - F_C\sin\alpha \qquad (2)$

FORMING $\frac{(1)}{(2)}$ .. $\frac{1040\cos 60°}{-1040\sin 60°} = \frac{(F_A + F_C)\cos\alpha}{-(F_A + F_C)\sin\alpha}$

SIMPLIFYING YIELDS $\alpha = 60°$

AND THEN $F_A + F_C = 1040\ \text{N} \qquad (3)$

NOW.. ΣM_B: $0 = (6.7\ \text{m})F_A\sin 60° - (4\ \text{m})F_C\sin 60°$

SUBSTITUTING FOR F_C FROM EQ. (3)...

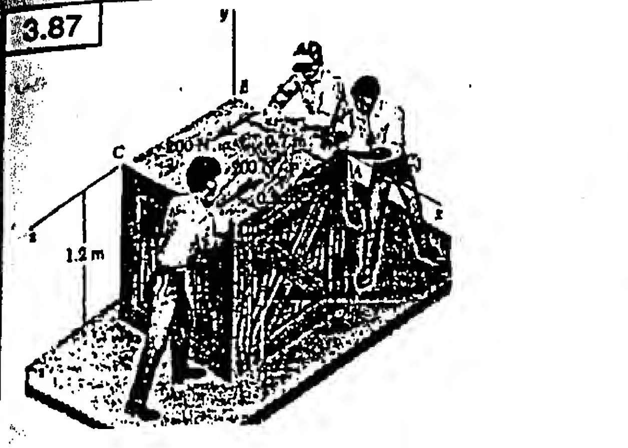
$6.7F_A - 4(1040 - F_A) = 0$

OR $F_A = 388.79\ \text{N}$

AND THEN $F_C = 651.21\ \text{N}$

$\therefore \underline{F}_A = 389\ \text{N} \measuredangle 60°,\ \underline{F}_C = 651\ \text{N} \measuredangle 60°$

3.87



GIVEN: $1 \times 1 \times 1.2$-m CRATE

FIND: (a) EQUIVALENT FORCE-COUPLE SYSTEM AT A IF $P = 240$ N

(b) SINGLE EQUIVALENT FORCE AND POINT OF APPLICATION ON SIDE AB

(c) P IF THREE FORCES ARE EQUIVALENT TO A SINGLE FORCE AT B

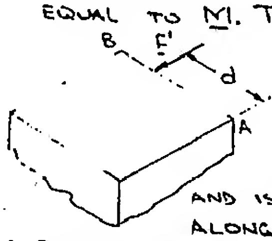
(a) SINCE THE TWO 200-N FORCES FORM A COUPLE, THE THREE FORCES ARE EQUIVALENT TO A FORCE $\underline{F}$ AND A COUPLE VECTOR $\underline{M}$, WHERE

$$\underline{F} = (240\ \text{N})\underline{k}$$

AND $M = (0.7 - 0.2)\text{m} \times 200\ \text{N} = 100\ \text{N}\cdot\text{m}$

$\therefore$ THE EQUIVALENT FORCE-COUPLE SYSTEM AT A IS.. $\underline{F} = (240\ \text{N})\underline{k}$, $\underline{M} = (100\ \text{N}\cdot\text{m})\underline{j}$ ◀

(b) THE SINGLE EQUIVALENT FORCE F' IS EQUAL TO $(240\ \text{N})\underline{k}$ AND IS APPLIED ALONG AB SO THAT ITS MOMENT ABOUT A IS EQUAL TO $\underline{M}$. THUS



$$M = dF'$$

OR $100\ \text{N}\cdot\text{m} = d(240\ \text{N})$

OR $d = 0.417$ m

$\therefore$ $\underline{F}' = (240\ \text{N})\underline{k}$ ◀

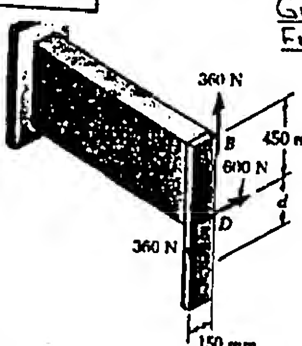
AND IS APPLIED 0.417 m FROM A ALONG SIDE AB ◀

(c) FOR THIS CASE, $d = 1$ m. THEN..

$M = dP$

OR $100\ \text{N}\cdot\text{m} = (1\ \text{m})P$ OR $P = 100$ N ◀

3.88



GIVEN: FORCE-COUPLE SYSTEM

FIND: (a) SINGLE EQUIVALENT FORCE $\underline{F}$ AT C, DISTANCE d

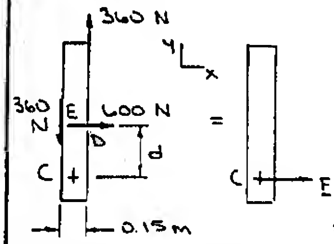
(b) $\underline{F}$ AND d IF THE DIRECTIONS OF THE TWO 360-N FORCES ARE REVERSED

(CONTINUED)

3.88 CONTINUED

(a) HAVE $F = 600$ N

REQUIRE



FOR EQUIVALENCE..

ΣM_C: $(0.15\ \text{m})(360\ \text{N}) - d(600\ \text{N}) = 0$

OR $d = 0.090$ m

$\therefore$ $\underline{F} = (600\ \text{N})\underline{i}$ ◀

AND $d = 90$ mm BELOW POINTS D AND E ◀

(b) THE ONLY EFFECT OF REVERSING THE DIRECTIONS OF THE TWO 360-N FORCES WILL BE TO CHANGE THE SENSE OF THE MOMENT OF THE COUPLE. THUS

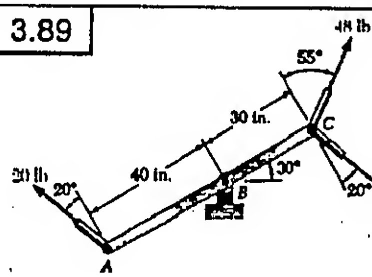
$\underline{F} = (600\ \text{N})\underline{i}$ ◀

AND ΣM_C: $-(0.15\ \text{m})(360\ \text{N}) - d(600\ \text{N}) = 0$

OR $d = -0.090$ m

$\therefore$ $d = 90$ mm ABOVE POINTS D AND E ◀

3.89

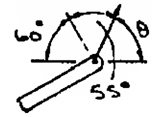


GIVEN: FORCE-COUPLE SYSTEM

FIND: (a) EQUIVALENT FORCE-COUPLE SYSTEM AT B

(b) SINGLE EQUIVALENT FORCE, POINT OF APPLICATION

(a) FIRST NOTE THAT THE TWO 20-lb FORCES FORM A COUPLE. THEN



$\underline{F} = 48\ \text{lb} \measuredangle \theta$

WHERE $\theta = 180° - (60° + 55°) = 65°$

AND $M = \Sigma M_B$

$= (30\ \text{in.})(48\ \text{lb})\cos 55° - (70\ \text{in.})(20\ \text{lb})\cos 20°$

$= -489.62\ \text{lb}\cdot\text{in.}$

$\therefore$ THE EQUIVALENT FORCE-COUPLE SYSTEM AT B IS $\underline{F} = 48\ \text{lb} \measuredangle 65°$, $\underline{M} = 490\ \text{lb}\cdot\text{in.}$ ◀

(b) THE SINGLE EQUIVALENT FORCE $\underline{F}'$ IS EQUAL TO $\underline{F}$. FURTHER, SINCE THE SENSE OF $\underline{M}$ IS CLOCKWISE, $\underline{F}'$ MUST BE APPLIED BETWEEN A AND B. FOR EQUIVALENCE..

ΣM_B: $M = -aF' \cos 55°$

WHERE a IS THE DISTANCE FROM B TO THE POINT OF APPLICATION OF $\underline{F}'$. THEN..

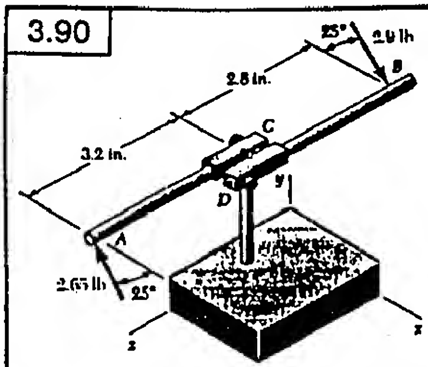
$-489.62\ \text{lb}\cdot\text{in} = -a(48\ \text{lb})\cos 55°$

OR $a = 17.78$ in.

$\therefore$ $\underline{F}' = 48\ \text{lb} \measuredangle 65°$ ◀

AND IS APPLIED TO THE LEVER 17.78 in. TO THE LEFT OF PIN B ◀

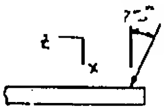
3.90



GIVEN: APPLIED FORCES

FIND: SINGLE EQUIVALENT FORCE, POINT OF APPLICATION

FIRST TRANSFER THE 2.65-lb FORCE AT A TO B. THE RESULTING FORCE-COUPLE SYSTEM ($\underline{F}$, $\underline{M}$) AT B IS THEN --



$F = (2.9 - 2.65)\text{ lb} = 0.25\text{ lb}$

AND $M = M_B = (6\text{ in.})(2.65\text{ lb})\cos 25°$

OR $\underline{M} = -(14.4103\text{ lb}\cdot\text{in.})\underline{j}$

THE SINGLE EQUIVALENT FORCE $\underline{F}'$ IS EQUAL TO $\underline{F}$. FURTHER, FOR EQUIVALENCE

ΣM_B: $M = aF'\cos 25°$

WHERE a IS THE DISTANCE FROM B TO THE POINT OF APPLICATION OF $\underline{F}'$. SINCE $\underline{M}$ ACTS IN THE $-\underline{j}$ DIRECTION, $\underline{F}'$ WOULD HAVE TO BE APPLIED TO THE RIGHT OF B. THEN..

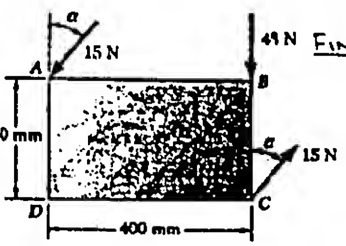
$-14.4103\text{ lb}\cdot\text{in.} = -a(0.25\text{ lb})\cos 25°$

OR $a = 63.6$ in.

$\therefore\ \underline{F}' = (0.25\text{ lb})(\cos 25°\underline{i} + \sin 25°\underline{k})$ ◀

AND IS APPLIED ON AN EXTENSION OF HANDLE BD AT A DISTANCE OF 63.6 IN. TO THE RIGHT OF B. ◀

3.91



GIVEN: FORCE-COUPLE SYSTEM

FIND: (a) MAGNITUDE OF SINGLE EQUIVALENT FORCE $\underline{F}'$ AND ITS LINE OF ACTION FOR $\alpha = 40°$

(b) α IF LINE OF ACTION OF $\underline{F}'$ INTERSECTS CD 300 mm TO THE RIGHT OF D

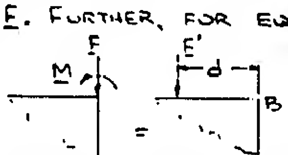
(a) THE GIVEN FORCE-COUPLE SYSTEM ($\underline{F}$, $\underline{M}$) AT B IS $\underline{F} = 48\text{ N}\downarrow$

AND $M = \Sigma M_B$

$= (0.4\text{ m})(15\text{ N})\cos 40° + (0.24\text{ m})(15\text{ N})\sin 40°$

OR $\underline{M} = 6.9103\text{ N}\cdot\text{m}$ ↻

THE SINGLE EQUIVALENT FORCE $\underline{F}'$ IS EQUAL TO $\underline{F}$. FURTHER, FOR EQUIVALENCE..



ΣM_B: $M = dF'$

OR $6.9103\text{ N}\cdot\text{m} = d \times 48\text{ N}$

OR $d = 0.14396$ m

$\therefore\ F' = 48$ N ◀

AND THE LINE OF ACTION OF $\underline{F}'$ INTERSECTS LINE AB 144 mm TO THE RIGHT OF A. ◀

(CONTINUED)

3.91 CONTINUED

(b) FOLLOWING THE SOLUTION TO PART a BUT WITH $d = 0.1$ m AND α UNKNOWN, HAVE

ΣM_B: $(0.4\text{ m})(15\text{ N})\cos\alpha + (0.24\text{ m})(15\text{ N})\sin\alpha$
$= (0.1\text{ m})(48\text{ N})$

OR $5\cos\alpha + 3\sin\alpha = 4$

REARRANGING AND SQUARING.. $25\cos^2\alpha = (4 - 3\sin\alpha)^2$

USING $\cos^2\alpha = 1 - \sin^2\alpha$ AND EXPANDING..

$25(1 - \sin^2\alpha) = 16 - 24\sin\alpha + 9\sin^2\alpha$

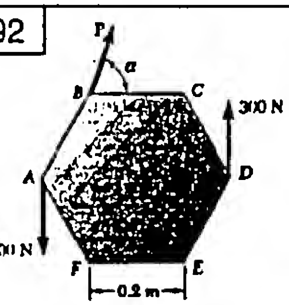
OR $34\sin^2\alpha - 24\sin\alpha - 9 = 0$

THEN $\sin\alpha = \dfrac{24 \pm \sqrt{(-24)^2 - 4(34)(-9)}}{2(34)}$

OR $\sin\alpha = 0.97686$ $\quad$ $\sin\alpha = -0.27098$

OR $\alpha = 77.7°$ $\quad$ $\alpha = -15.72°$ ◀

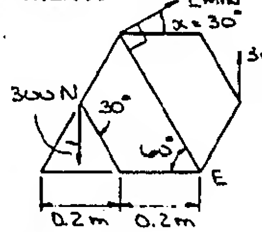
3.92



GIVEN: FORCE-COUPLE SYSTEM ($\underline{P}$, $\underline{M}$)

FIND: $\underline{P}_{MIN}$ SO THAT ($\underline{P}$, $\underline{M}$) IS EQUIVALENT TO A SINGLE FORCE AT E

FROM THE STATEMENT OF THE PROBLEM, IT FOLLOWS THAT $\Sigma M_E = 0$ FOR THE GIVEN FORCE-COUPLE SYSTEM. FURTHER, FOR $\underline{P}_{MIN}$, MUST REQUIRE THAT $\underline{P}$ BE PERPENDICULAR TO $\underline{r}_{B/E}$. THEN..

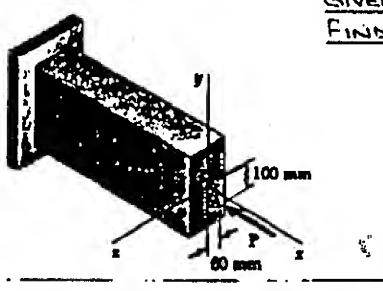


ΣM_E: $(0.2\sin 30° + 0.2)\text{m} \times 300\text{ N}$
$+ (0.2\text{ m})\sin 30° \times 300\text{ N}$
$- (0.4\text{ m})P_{MIN} = 0$

OR $P_{MIN} = 300$ N

$\therefore\ \underline{P}_{MIN} = 300\text{ N} \measuredangle 30°$ ◀

3.93



GIVEN: $P = 1220$ N

FIND: EQUIVALENT FORCE-COUPLE SYSTEM ($\underline{F}$, $\underline{M}$) AT G

HAVE $\underline{P} = -(1220\text{ N})\underline{i}$

NOW.. $\underline{M} = \underline{M}_G$

$= \underline{r}_{A/G} \times \underline{P}$

$= [-(0.1\text{ m})\underline{j} - (0.06\text{ m})\underline{k}] \times [-(1220\text{ N})\underline{i}]$

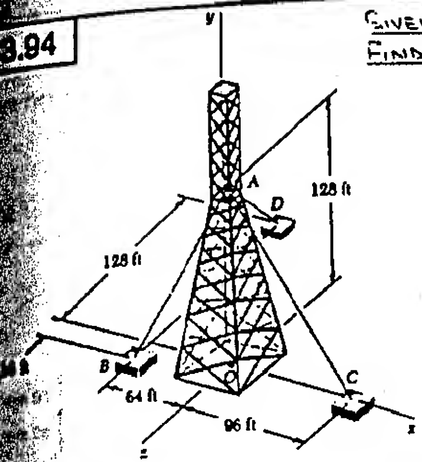
$= (73.2\text{ N}\cdot\text{m})\underline{j} - (122\text{ N}\cdot\text{m})\underline{k}$

$\therefore$ THE EQUIVALENT FORCE-COUPLE SYSTEM AT G IS..

$\underline{F} = -(1220\text{ N})\underline{i}$ ◀

$\underline{M} = (73.2\text{ N}\cdot\text{m})\underline{j} - (122\text{ N}\cdot\text{m})\underline{k}$ ◀

3.94



GIVEN: $T_{AB} = 288$ lb

FIND: EQUIVALENT FORCE-COUPLE SYSTEM $(\underline{F}, \underline{M})$ AT O

HAVE.. $d_{AB} = \sqrt{(-64)^2 + (-128)^2 + (16)^2} = 144$ ft

THEN $\underline{T}_{AB} = \frac{288 \text{ lb}}{144}(-64\underline{i} - 128\underline{j} + 16\underline{k})$

$= (32 \text{ lb})(-4\underline{i} - 8\underline{j} + \underline{k})$

NOW.. $\underline{M} = \underline{M}_O = \underline{r}_{A/O} \times \underline{T}_{AB}$

$= 128\underline{j} \times 32(-4\underline{i} - 8\underline{j} + \underline{k})$

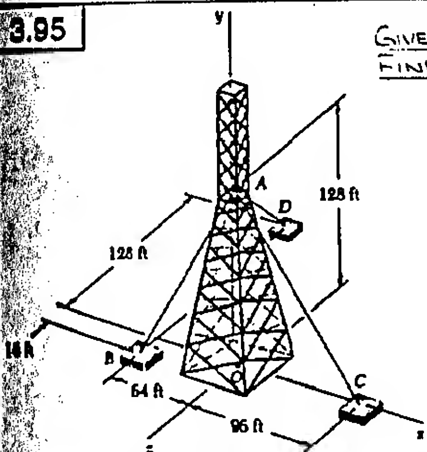
$= (4096 \text{ lb}\cdot\text{ft})\underline{i} + (16{,}384 \text{ lb}\cdot\text{ft})\underline{k}$

$\therefore$ THE EQUIVALENT FORCE-COUPLE SYSTEM AT O IS..

$\underline{F} = -(128 \text{ lb})\underline{i} - (256 \text{ lb})\underline{j} + (32 \text{ lb})\underline{k}$

$\underline{M} = (4.10 \text{ kip}\cdot\text{ft})\underline{i} + (16.38 \text{ kip}\cdot\text{ft})\underline{k}$

3.95



GIVEN: $T_{AD} = 270$ lb

FIND: EQUIVALENT FORCE-COUPLE SYSTEM $(\underline{F}, \underline{M})$ AT O

HAVE.. $d_{AD} = \sqrt{(-64)^2 + (-128)^2 + (-128)^2} = 192$ ft

THEN.. $\underline{T}_{AD} = \frac{270 \text{ lb}}{192}(-64\underline{i} - 128\underline{j} - 128\underline{k})$

$= (90 \text{ lb})(-\underline{i} - 2\underline{j} - 2\underline{k})$

NOW.. $\underline{M} = \underline{M}_O = \underline{r}_{A/O} \times \underline{T}_{AD}$

$= 128\underline{j} \times 90(-\underline{i} - 2\underline{j} - 2\underline{k})$

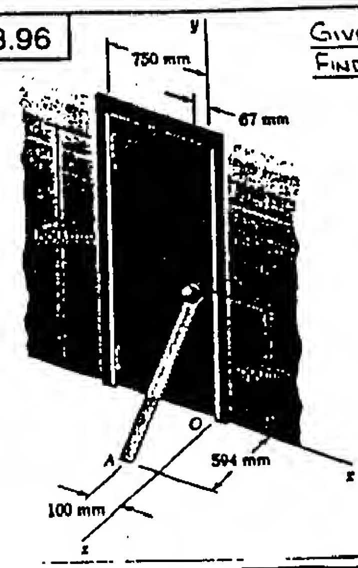
$= -(23{,}040 \text{ lb}\cdot\text{ft})\underline{i} + (11{,}520 \text{ lb}\cdot\text{ft})\underline{k}$

$\therefore$ THE EQUIVALENT FORCE-COUPLE SYSTEM AT O IS..

$\underline{F} = -(90 \text{ lb})\underline{i} - (180 \text{ lb})\underline{j} - (180 \text{ lb})\underline{k}$

$\underline{M} = -(23.0 \text{ kip}\cdot\text{ft})\underline{i} + (11.52 \text{ kip}\cdot\text{ft})\underline{k}$

3.96



GIVEN: $F_{AB} = 175$ N

FIND: EQUIVALENT FORCE-COUPLE SYSTEM $(\underline{F}, \underline{M})$ AT C

HAVE $d_{AB} = \sqrt{(33)^2 + (990)^2 + (-594)^2} = 1155$ mm

THEN $\underline{F}_{AB} = \frac{175 \text{ N}}{1155}(33\underline{i} + 990\underline{j} - 594\underline{k})$

$= (5 \text{ N})(\underline{i} + 30\underline{j} - 18\underline{k})$

NOW.. $\underline{M} = \underline{M}_C = \underline{r}_{B/C} \times \underline{F}_{AB}$

WHERE $\underline{r}_{B/C} = (0.683 \text{ m})\underline{i} - (0.860 \text{ m})\underline{j}$

THEN.. $\underline{M} = 5 \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.683 & -0.860 & 0 \\ 1 & 30 & -18 \end{vmatrix}$

$= 5\{[(-0.860)(-18)]\underline{i} + [-(0.683)(-18)]\underline{j} + [(0.683)(30) - (-0.860)(1)]\underline{k}\}$

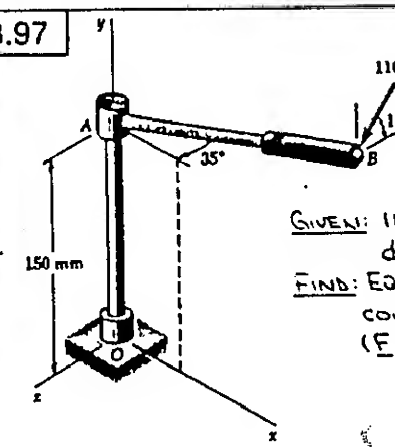
$= (77.4 \text{ N}\cdot\text{m})\underline{i} + (61.47 \text{ N}\cdot\text{m})\underline{j} + (106.75 \text{ N}\cdot\text{m})\underline{k}$

$\therefore$ THE EQUIVALENT FORCE-COUPLE SYSTEM AT C IS.

$\underline{F} = (5 \text{ N})\underline{i} + (150 \text{ N})\underline{j} - (90 \text{ N})\underline{k}$

$\underline{M} = (77.4 \text{ N}\cdot\text{m})\underline{i} + (61.5 \text{ N}\cdot\text{m})\underline{j} + (106.8 \text{ N}\cdot\text{m})\underline{k}$

3.97



GIVEN: 110-N FORCE $\underline{P}$

$d_{AB} = 220$ mm

FIND: EQUIVALENT FORCE-COUPLE SYSTEM $(\underline{F}, \underline{M})$ AT O

HAVE.. $\underline{P} = (110 \text{ N})(-\sin 15^\circ \underline{j} + \cos 15^\circ \underline{k})$

NOW.. $\underline{M} = \underline{M}_O = \underline{r}_{B/O} \times \underline{P}$

WHERE $\underline{r}_{B/O} = (0.22 \text{ m})\cos 35^\circ \underline{i} + (0.15 \text{ m})\underline{j} - (0.22 \text{ m})\sin 35^\circ \underline{k}$

THEN.. $\underline{M} = 110 \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.22\cos 35^\circ & 0.15 & -0.22\sin 35^\circ \\ 0 & -\sin 15^\circ & \cos 15^\circ \end{vmatrix}$

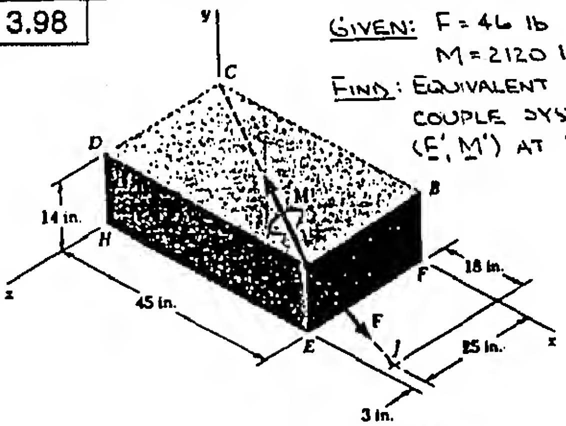
(CONTINUED)

3.97 CONTINUED

OR $\underline{M} = 110\{[(0.15)(\cos 15°) - (-0.22 \sin 35°)(-\sin 15°)]\underline{i}$
$+ [-(0.22 \cos 35°)(\cos 15°)]\underline{j}$
$+ [(0.22 \cos 35°)(-\sin 15°)\underline{k}]$
$= (12.345\ N\cdot m)\underline{i} - (19.148\ N\cdot m)\underline{j} - (5.131\ N\cdot m)\underline{k}$

$\therefore$ THE EQUIVALENT FORCE-COUPLE SYSTEM AT O IS..
$\underline{F} = (110\ N)(-\sin 15°\ \underline{j} + \cos 15°\ \underline{k})$
$= -(28.5\ N)\underline{j} + (106.3\ N)\underline{k}$
$\underline{M} = (12.35\ N\cdot m)\underline{i} - (19.15\ N\cdot m)\underline{j} - (5.13\ N\cdot m)\underline{k}$

3.98



GIVEN: $F = 46\ lb$
$M = 2120\ lb\cdot in.$
FIND: EQUIVALENT FORCE-COUPLE SYSTEM $(\underline{F}', \underline{M}')$ AT H

HAVE.. $d_{AJ} = \sqrt{(18)^2 + (-14)^2 + (-3)^2} = 23\ in.$

THEN. $\underline{F} = \frac{46\ lb}{23}(18\underline{i} - 14\underline{j} - 3\underline{k})$
$= (36\ lb)\underline{i} - (28\ lb)\underline{j} - (6\ lb)\underline{k}$

ALSO.. $d_{AC} = \sqrt{(-45)^2 + (0)^2 + (-28)^2} = 53\ in.$

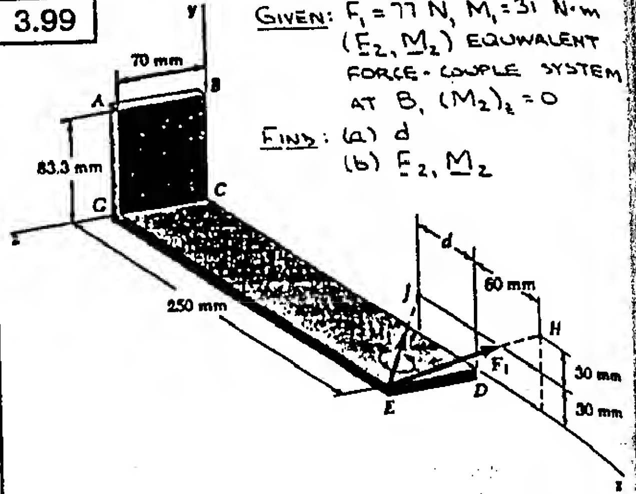
THEN $\underline{M} = \frac{2120\ lb\cdot in.}{53}(-45\underline{i} - 28\underline{k})$
$= -(1800\ lb\cdot in.)\underline{i} - (1120\ lb\cdot in.)\underline{k}$

NOW. $\underline{M}' = \underline{M} + \underline{r}_{A/H} \times \underline{F}$
WHERE $\underline{r}_{A/H} = (45\ in.)\underline{i} + (14\ in.)\underline{j}$

THEN. $\underline{M}' = (-1800\underline{i} - 1120\underline{k}) + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 45 & 14 & 0 \\ 36 & -28 & -6 \end{vmatrix}$
$= (-1800\underline{i} - 1120\underline{k}) + \{[(14)(-6)]\underline{i}$
$+ [-(45)(-6)]\underline{j}$
$+ [(45)(-28) - (14)(36)]\underline{k}\}$
$= (-1800 - 84)\underline{i} + (270)\underline{j}$
$+ (-1120 - 1764)\underline{k}$
$= -(1884\ lb\cdot in)\underline{i} + (270\ lb\cdot in)\underline{j}$
$- (2884\ lb\cdot in)\underline{k}$
$= -(157\ lb\cdot ft)\underline{i} + (22.5\ lb\cdot ft)\underline{j}$
$- (240\ lb\cdot ft)\underline{k}$

$\therefore$ THE EQUIVALENT FORCE-COUPLE SYSTEM AT H IS
$\underline{F}' = (36\ lb)\underline{i} - (28\ lb)\underline{j} - (6\ lb)\underline{k}$
$\underline{M}' = -(157\ lb\cdot ft)\underline{i} + (22.5\ lb\cdot ft)\underline{j} - (240\ lb\cdot ft)\underline{k}$

3.99



GIVEN: $F_1 = 77\ N$, $M_1 = 31\ N\cdot m$
$(\underline{F}_2, \underline{M}_2)$ EQUIVALENT FORCE-COUPLE SYSTEM AT B, $(M_2)_z = 0$
FIND: (a) d
(b) $\underline{F}_2, \underline{M}_2$

HAVE.. $d_{EH} = \sqrt{(60)^2 + (60)^2 + (-70)^2} = 110\ mm$

THEN $\underline{F}_1 = \frac{77\ N}{110}(60\underline{i} + 60\underline{j} - 70\underline{k})$
$= (42\ N)\underline{i} + (42\ N)\underline{j} - (49\ N)\underline{k}$

ALSO.. $d_{EJ} = \sqrt{(-d)^2 + (30)^2 + (-70)^2}\ mm$

AND $\underline{M}_1 = \frac{31\ N\cdot m}{d_{EJ}}[-(d)\underline{i} + (30\ mm)\underline{j} - (70\ mm)\underline{k}]$

(a) HAVE.. $\underline{M}_2 = \underline{M}_1 + \underline{r}_{H/B} \times \underline{F}_1$ (1)
WHERE $\underline{r}_{H/B} = (0.31\ m)\underline{i} - (0.0233\ m)\underline{j}$

THEN.. $\underline{r}_{H/B} \times \underline{F}_1 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.31 & -0.0233 & 0 \\ 42 & 42 & -49 \end{vmatrix}$
$= [(-0.0233)(-49)]\underline{i} + [-(0.31)(-49)]\underline{j}$
$+ [(0.31)(42) - (-0.0233)(42)]\underline{k}$
$= (1.1417\ N\cdot m)\underline{i} + (15.19\ N\cdot m)\underline{j}$
$+ (13.9986\ N\cdot m)\underline{k}$

EQ. (1) CAN THEN BE EXPRESSED AS
$(M_2)_x\underline{i} + (M_2)_y\underline{j} = \frac{31\ N\cdot m}{d_{EJ}}[-(d)\underline{i} + (30\ mm)\underline{j}$
$- (70\ mm)\underline{k}]$
$+ [(1.1417\ N\cdot m)\underline{i} + (15.19\ N\cdot m)\underline{j}$
$+ (13.9986\ N\cdot m)\underline{k}]$ (2)

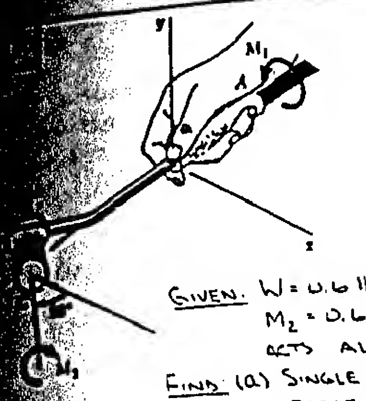
EQUATING THE $\underline{k}$ COEFFICIENTS..
$0 = \frac{31\ N\cdot m}{d_{EJ}}(-70\ mm) + 13.9986\ N\cdot m$

THEN.. $d_{EJ}^2 = \left(\frac{31}{13.9986}\cdot 70\ mm\right)^2 = [(-d)^2 + (30)^2 + (-70)^2]\ mm^2$
OR $d = 135.018\ mm$ $\qquad d = 135.0\ mm$

(b) FIRST NOTE $d_{EJ} = \sqrt{(-135.018)^2 + (30)^2 + (-70)^2}$
$= 155.016\ mm$

USING EQ. (2), $\underline{M}_2$ IS THEN..
$\underline{M}_2 = \left(-\frac{31 \times 135.018}{155.016} + 1.1417\right)\underline{i}$
$+ \left(\frac{31 \times 30}{155.016} + 15.19\right)\underline{j}$
$= -(25.859\ N\cdot m)\underline{i} + (21.189\ N\cdot m)\underline{j}$

$\therefore$ $\underline{F}_2 = (42\ N)\underline{i} + (42\ N)\underline{j} - (49\ N)\underline{k}$
$\underline{M}_2 = -(25.9\ N\cdot m)\underline{i} + (21.2\ N\cdot m)\underline{j}$



GIVEN: $W = 0.6$ lb, $M_1 = 0.68$ lb·in, $M_2 = 0.65$ lb·in, $\underline{W}$ acts along y axis

FIND: (a) Single equivalent force $\underline{F}$

(b) Point where line of action of $\underline{F}$ intersects xz plane

---

[A]ssume that the given force $\underline{W}$ and [couple]s $\underline{M}_1$ and $\underline{M}_2$ act at the origin.

$$\underline{W} = -W\underline{j}$$

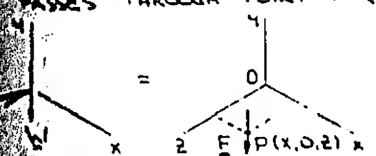
$$\underline{M} = \underline{M}_1 + \underline{M}_2$$

$$= -(M_2 \cos 25°)\underline{i} + (M_1 - M_2 \sin 25°)\underline{k}$$

[Note] that since $\underline{W}$ and $\underline{M}$ are [perp]endicular, it follows that they can be [repl]aced with a single equivalent force.

[(a)] Have.. $\underline{F} = \underline{W}$ or $\underline{F} = -(0.6 \text{ lb})\underline{j}$

[(b)] Assume that the line of action of $\underline{F}$ passes through point $P\,(x, 0, z)$. Then



[For] equivalence..

$$\underline{M} = \underline{r}_P \times \underline{F}$$

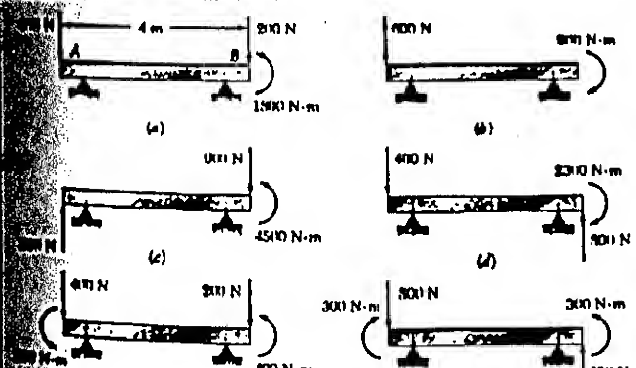
$$[-](0.65 \cos 25°)\underline{i} + (0.68 - 0.65 \sin 25°)\underline{k} = (x\underline{i} + z\underline{k}) \times (-0.6)\underline{j}$$

[Equa]ting the $\underline{i}$ and $\underline{k}$ coefficients

[$\underline{i}$:] $-0.65 \cos 25° = 0.6z$ or $z = -0.982$ in

[$\underline{k}$:] $0.68 - 0.65 \sin 25° = -0.6x$ or $x = -0.675$ in.

[3].101 and 3.102



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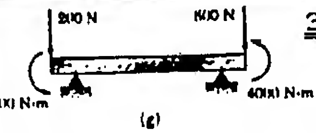
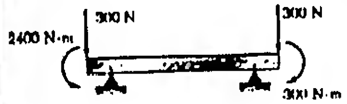
3.101 and 3.102 CONTINUED



(g)



(h)

3.101

GIVEN: Applied loads and couples

FIND: (a) Equivalent force-couple $(\underline{R}, \underline{M})$ at A for each loading

(b) Equivalent loadings

(a) Have...

a. $R_a = \Sigma F = -400 - 200$ or $\underline{R}_a = 600 \text{ N} \downarrow$

$M_a = \Sigma M_A = 1800 \text{ N·m} - (4 \text{ m})(200 \text{ N})$ or $\underline{M}_a = 1000 \text{ N·m} \circlearrowright$

b. $R_b = \Sigma F = -600$ or $\underline{R}_b = 600 \text{ N} \downarrow$

$M_b = \Sigma M_A = -900 \text{ N·m}$ or $\underline{M}_b = 900 \text{ N·m} \circlearrowright$

c. $R_c = \Sigma F = 300 - 900$ or $\underline{R}_c = 600 \text{ N} \downarrow$

$M_c = \Sigma M_A = 4500 \text{ N·m} - (4 \text{ m})(900 \text{ N})$ or $\underline{M}_c = 900 \text{ N·m} \circlearrowleft$

d. $R_d = \Sigma F = -400 + 800$ or $\underline{R}_d = 400 \text{ N} \uparrow$

$M_d = \Sigma M_A = -2300 \text{ N·m} + (4 \text{ m})(800 \text{ N})$ or $\underline{M}_d = 900 \text{ N·m} \circlearrowleft$

e. $R_e = \Sigma F = -400 - 200$ or $\underline{R}_e = 600 \text{ N} \downarrow$

$M_e = \Sigma M_A = 200 \text{ N·m} + 400 \text{ N·m} - (4 \text{ m})(200 \text{ N})$ or $\underline{M}_e = 200 \text{ N·m} \circlearrowright$

f. $R_f = \Sigma F = -800 + 200$ or $\underline{R}_f = 600 \text{ N} \downarrow$

$M_f = \Sigma M_A = -300 \text{ N·m} + 300 \text{ N·m} + (4 \text{ m})(200 \text{ N})$ or $\underline{M}_f = 800 \text{ N·m} \circlearrowleft$

g. $R_g = \Sigma F = -200 - 800$ or $\underline{R}_g = 1000 \text{ N} \downarrow$

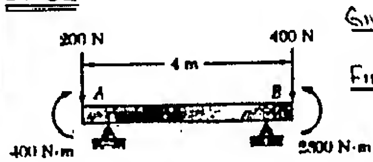
$M_g = \Sigma M_A = 200 \text{ N·m} + 4000 \text{ N·m} - (4 \text{ m})(800 \text{ N})$ or $\underline{M}_g = 1000 \text{ N·m} \circlearrowleft$

h. $R_h = \Sigma F = -300 - 300$ or $\underline{R}_h = 600 \text{ N} \downarrow$

$M_h = \Sigma M_A = 2400 \text{ N·m} - 300 \text{ N·m} - (4 \text{ m})(300 \text{ N})$ or $\underline{M}_h = 900 \text{ N·m} \circlearrowleft$

(b) ∴ Loadings (c) and (h) are equivalent

3.102



GIVEN: Applied loads and couples

FIND: Loading of Prob. 3.101 equivalent to the given loading

First replace the given loading with an equivalent force-couple system $(\underline{R}, \underline{M})$ at A. Thus..

$$R = \Sigma F = -200 - 400$$

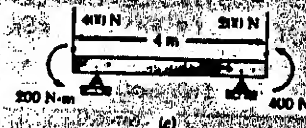
or $\underline{R} = 600 \text{ N} \downarrow$

and $M = \Sigma M_A = -400 \text{ N·m} + 2800 \text{ N·m} - (4 \text{ m})(400 \text{ N})$

or $\underline{M} = 800 \text{ N·m} \circlearrowleft$

∴ The given loading is equivalent to loading (f) of Prob. 3.101.

3.103



GIVEN: APPLIED FORCES AND COUPLES

FIND: SINGLE EQUIVALENT FORCE $\underline{R}$ AND DISTANCE d FROM A TO LINE OF ACTION OF $\underline{R}$

FOR EACH LOADING, FIRST DETERMINE THE EQUIVALENT FORCE-COUPLE SYSTEM ($\underline{\ }$, $\underline{M}$) AT A

(a)

NOW... $R = \Sigma F_y = -600$ OR $\underline{R} = 600\text{ N}\downarrow$ ◀

AND $M = \Sigma M_A = -900\text{ N·m}$

THEN FOR EQUIVALENCE...

ΣM_A: $-900\text{ N·m} = -d(600\text{ N})$

OR $d = 1.5\text{ m}$ ◀

(b)

NOW... $R = \Sigma F_y = -400 + 800$ OR $\underline{R} = 400\text{ N}\uparrow$ ◀

AND $M = \Sigma M_A = -2300\text{ N·m} + (4\text{ m})(800\text{ N})$

$= 900\text{ N·m}$

THEN FOR EQUIVALENCE...

ΣM_A: $900\text{ N·m} = d(400\text{ N})$

OR $d = 2.25\text{ m}$ ◀

(c)

NOW... $R = \Sigma F_y = -400 - 200$ OR $\underline{R} = 600\text{ N}\downarrow$ ◀

AND $M = \Sigma M_A = 200\text{ N·m} - 400\text{ N·m} - (4\text{ m})(200\text{ N})$

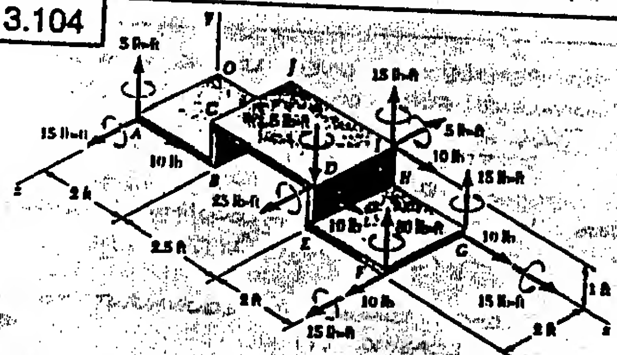
$= -200\text{ N·m}$

THEN FOR EQUIVALENCE...

ΣM_A: $-200\text{ N·m} = d(600\text{ N})$

OR $d = 0.333\text{ m}$ ◀

3.104



GIVEN: FIVE FORCE-COUPLE SYSTEMS

FIND: WHICH OF THE SYSTEMS IS EQUIVALENT TO $\underline{F} = (10\text{ lb})\underline{i}$, $\underline{M} = (15\text{ lb·ft})\underline{j} + (15\text{ lb·ft})\underline{k}$ AT O

(CONTINUED)

3.104 CONTINUED

FIRST NOTE THAT THE FORCE-COUPLE SYSTEM AT F CANNOT BE EQUIVALENT BECAUSE OF THE DIRECTION OF THE FORCE [THE FORCE OF THE OTHER FOUR SYSTEMS IS $(10\text{ lb})\underline{i}$]. NEXT MOVE EACH OF THE SYSTEMS TO THE ORIGIN O, THE FORCES REMAIN UNCHANGED.

A: $\underline{M}_A = \Sigma \underline{M}_O = (5\text{ lb·ft})\underline{j} - (15\text{ lb·ft})\underline{k} + (2\text{ ft})\underline{k} \times (10\text{ lb})\underline{i}$

$= (25\text{ lb·ft})\underline{j} + (15\text{ lb·ft})\underline{k}$

D: $\underline{M}_D = \Sigma \underline{M}_O = -(5\text{ lb·ft})\underline{j} + (25\text{ lb·ft})\underline{k}$

$+ [(4.5\text{ ft})\underline{i} + (1\text{ ft})\underline{j} + (2\text{ ft})\underline{k}] \times (10\text{ lb})\underline{i}$

$= (15\text{ lb·ft})\underline{j} + (15\text{ lb·ft})\underline{k}$

G: $\underline{M}_G = \Sigma \underline{M}_O = (15\text{ lb·ft})\underline{i} + (15\text{ lb·ft})\underline{j}$

I: $\underline{M}_I = \Sigma \underline{M}_I = (15\text{ lb·ft})\underline{j} - (5\text{ lb·ft})\underline{k}$

$- [(4.5\text{ ft})\underline{i} + (1\text{ ft})\underline{j}] \times (10\text{ lb})\underline{i}$

$= (15\text{ lb·ft})\underline{j} - (15\text{ lb·ft})\underline{k}$

∴ THE EQUIVALENT FORCE-COUPLE SYSTEM IS THE SYSTEM AT CORNER D ◀

3.105

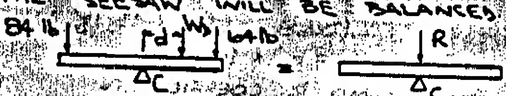
GIVEN: $W_A = 84\text{ lb}$, $W_B = 64\text{ lb}$

FIND: POSITION OF THIRD CHILD D SO THAT RESULTANT OF THE WEIGHTS PASSES THROUGH C WHEN

(a) $W_D = 60\text{ lb}$

(b) $W_D = 52\text{ lb}$

FROM THE STATEMENT OF THE PROBLEM, IT FOLLOWS THAT THE THREE WEIGHTS ARE EQUIVALENT TO A SINGLE FORCE AT C; THAT, THE SEESAW WILL BE BALANCED. THEN:



AND ΣM_C: $(6\text{ ft})(84\text{ lb}) - d(W_D\text{ lb}) - (6\text{ ft})(64\text{ lb}) = 0$

OR $d = \dfrac{120}{W_D}$ (ft)

(a) $W_D = 60\text{ lb}$: $d = \dfrac{120}{60} = 2\text{ ft}$

∴ THE THIRD CHILD SHOULD SIT 2 ft TO THE RIGHT OF C. ◀

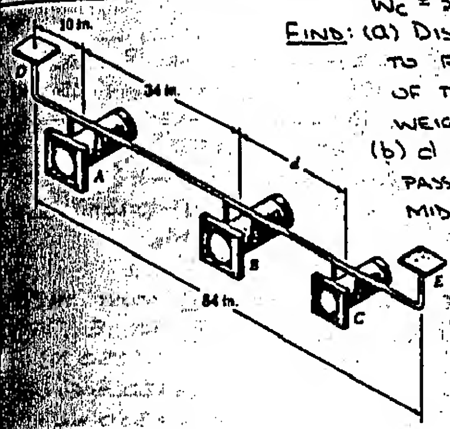
(b) $W_D = 52\text{ lb}$: $d = \dfrac{120}{52} = 2.31\text{ ft}$

∴ THE THIRD CHILD SHOULD SIT 2.31 ft TO THE RIGHT OF C. ◀

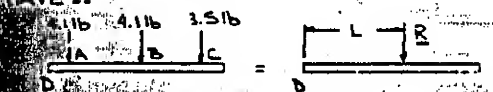
3.106

GIVEN: $W_A = W_B = 4.1$ lb
$W_C = 3.5$ lb

FIND: (a) DISTANCE FROM D TO RESULTANT $\underline{R}$ OF THE THREE WEIGHTS IF $d = 25$ in
(b) d SO THAT $\underline{R}$ PASSES THROUGH MIDPOINT OF PIPE



HAVE..



FOR EQUIVALENCE..

$\Sigma F_y: -4.1 - 4.1 - 3.5 = -R$ OR $\underline{R} = 11.7$ lb ↓

$\Sigma M_D: -(10 \text{ in.})(4.1 \text{ lb}) - (44 \text{ in.})(4.1 \text{ lb})$
$-[(44+d) \text{ in.}](3.5 \text{ lb}) = -(L \text{ in.})(11.7 \text{ lb})$

OR $375.4 + 3.5d = 11.7L$ (d, L IN IN.)

(a) $d = 25$ in.

HAVE $375.4 + 3.5(25) = 11.7L$ OR $L = 39.6$ in.

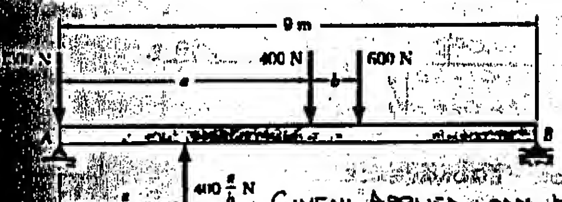
THE RESULTANT PASSES THROUGH A POINT 39.6 IN. TO THE RIGHT OF D. ◀

(b) $L = 42$ in.

HAVE $375.4 + 3.5d = 11.7(42)$

OR $d = 33.1$ in. ◀

3.107

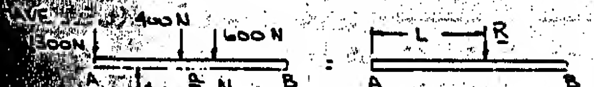


GIVEN: APPLIED LOADS, $b = 1.5$ m, LOADS ARE EQUIVALENT TO A SINGLE FORCE $\underline{R}$

FIND: (a) a SO THAT DISTANCE L FROM A TO $\underline{R}$ IS MAXIMUM

(b) R AND POINT OF APPLICATION ON THE BEAM

HAVE (a)



FOR EQUIVALENCE..

$\Sigma F_y: -1300 + 400\frac{a}{b} - 400 - 600 = -R$

OR $R = (2300 - 400\frac{a}{b})$ N (1)

$\Sigma M_A: \frac{a}{2}(400\frac{a}{b}) - a(400) - (a+b)(600) = -LR$

(CONTINUED)

3.107 CONTINUED

OR $L = \dfrac{1000a + 600b - 200\frac{a^2}{b}}{2300 - 400\frac{a}{b}}$

THEN WITH $b = 1.5$ m $\therefore$ $L = \dfrac{10a + 9 - \frac{4}{3}a^2}{23 - \frac{8}{3}a}$ (2)

WHERE a, L ARE IN m

(a) FIND VALUE OF a TO MAXIMIZE L.

$$\frac{dL}{da} = \frac{(10 - \frac{8}{3}a)(23 - \frac{8}{3}a) - (10a + 9 - \frac{4}{3}a^2)(-\frac{8}{3})}{(23 - \frac{8}{3}a)^2}$$

OR $230 - \frac{184}{3}a - \frac{80}{3}a + \frac{64}{9}a^2 + \frac{80}{3}a + 24 - \frac{32}{9}a^2 = 0$

OR $16a^2 - 276a + 1143 = 0$

THEN.. $a = \dfrac{276 \pm \sqrt{(-276)^2 - 4(16)(1143)}}{2(16)}$

OR $a = 10.3435$ m AND $a = 6.9065$ m

SINCE $AB = 9$ m, a MUST BE LESS THAN 9 m

$\therefore$ $a = 6.91$ m ◀

(b) USING EQ. (1)..

$R = 2300 - 400\dfrac{6.9065}{1.5}$

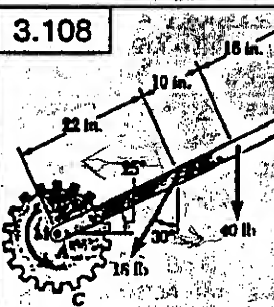
OR $R = 458$ N ◀

AND USING EQ. (2)..

$L = \dfrac{10(6.9065) + 9 - \frac{4}{3}(6.9065)^2}{23 - \frac{8}{3}(6.9065)} = 3.16$ m

$\therefore$ $\underline{R}$ IS APPLIED 3.16 m TO THE RIGHT OF A. ◀

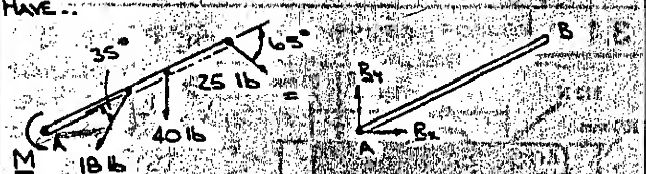
3.108



GIVEN: APPLIED FORCES AND COUPLE $\underline{M}$

FIND: SINGLE EQUIVALENT FORCE $\underline{R}$ AT A AND $\underline{M}$

HAVE..



FOR EQUIVALENCE..

$\Sigma F_x: -18 \sin 30° + 25 \cos 40° = R_x$

OR $R_x = 10.1511$ lb

$\Sigma F_y: -18 \cos 30° - 40 - 25 \sin 40° = R_y$

OR $R_y = -71.658$ lb

THEN.. $R = \sqrt{(10.1511)^2 + (71.658)^2}$ $\tan\theta = \dfrac{71.658}{10.1511}$

$= 72.4$ lb

OR $\theta = 81.9°$

$\therefore$ $\underline{R} = 72.4$ lb ∠ 81.9° ◀

ALSO.. $\Sigma M_A: M - (22 \text{ in.})(18 \text{ lb})\sin 35° - (32 \text{ in.})(40 \text{ lb})\cos 25°$
$- (48 \text{ in.})(25 \text{ lb})\sin 65° = 0$

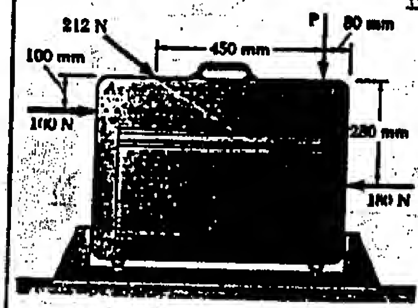
OR $M = 2474.8$ lb·in.

OR $M = 206$ lb·ft ◀

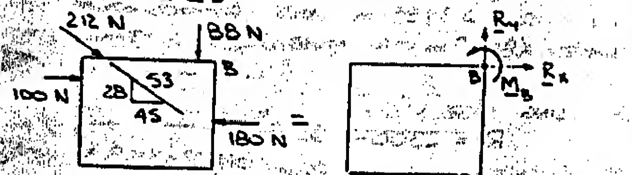
3.109

GIVEN: $P = 88\text{ N}$

FIND: (a) RESULTANT $\underline{R}$ OF THE APPLIED FORCES (b) POINTS WHERE THE LINE OF ACTION OF $\underline{R}$ INTERSECTS SIDES OF THE SUITCASE



(a) FIRST DETERMINE THE EQUIVALENT FORCE-COUPLE SYSTEM $(\underline{R}, \underline{M}_B)$ AT B. HAVE..



THEN FOR EQUIVALENCE..

$\Sigma F_x: \; 100 + \frac{45}{53}(212) - 180 = R_x$ OR $R_x = 100\text{ N}$

$\Sigma F_y: \; -\frac{28}{53}(212) - 88 = R_y$ OR $R_y = -200\text{ N}$

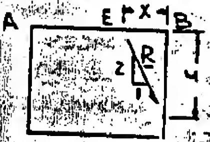
$\therefore \; \underline{R} = (100\text{ N})\underline{i} - (200\text{ N})\underline{j}$

OR $\underline{R} = 224\text{ N} \measuredangle 63.4°$ ◀

(b) ALSO.. $\Sigma M_B: (0.1\text{ m})(100\text{ N}) + (0.53\text{ m})(\frac{28}{53} \cdot 212\text{ N}) + (0.08\text{ m})(88\text{ N}) - (0.28\text{ m})(180\text{ N}) = M_B$

OR $M_B = 26\text{ N}\cdot\text{m}$ ↻

THE SINGLE EQUIVALENT FORCE $\underline{R}$ MUST THEN ACT AS INDICATED. THEN WITH $\underline{R}$ AT E..



$\Sigma M_B: \; 26\text{ N}\cdot\text{m} = x(200\text{ N})$

OR $x = 130\text{ mm}$

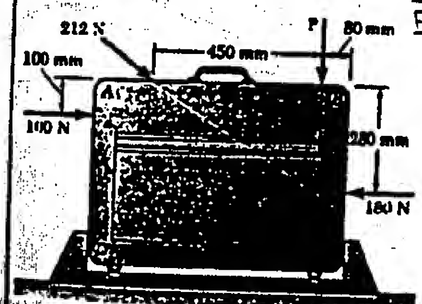
NOW $\frac{y}{x} = \frac{2}{1} \Rightarrow y = 260\text{ mm}$

$\therefore$ THE LINE OF ACTION OF $\underline{R}$ INTERSECTS TOP AB 130 mm TO THE LEFT OF B AND INTERSECTS SIDE BC 260 mm BELOW B. ◀

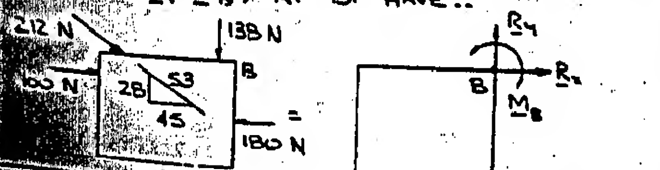
3.110

GIVEN: $P = 138\text{ N}$

FIND: (a) RESULTANT $\underline{R}$ OF THE APPLIED FORCES (b) POINTS WHERE THE LINE OF ACTION OF $\underline{R}$ INTERSECTS SIDES OF THE SUITCASE



(a) FIRST DETERMINE THE EQUIVALENT FORCE-COUPLE SYSTEM $(\underline{R}, \underline{M}_B)$ AT B. HAVE..



3.110 CONTINUED

THEN FOR EQUIVALENCE..

$\Sigma F_x: \; 100 + \frac{45}{53}(212) - 180 = R_x$ OR $R_x = 100\text{ N}$

$\Sigma F_y: \; -\frac{28}{53}(212) - 138 = R_y$ OR $R_y = -250\text{ N}$

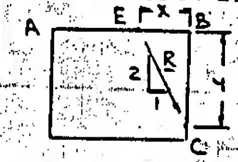
$\therefore \; \underline{R} = (100\text{ N})\underline{i} - (250\text{ N})\underline{j}$

OR $\underline{R} = 269\text{ N} \measuredangle 68.2°$ ◀

(b) ALSO.. $\Sigma M_B: (0.1\text{ m})(100\text{ N}) + (0.53\text{ m})(\frac{28}{53} \cdot 212\text{ N}) + (0.08\text{ m})(138\text{ N}) - (0.28\text{ m})(180\text{ N}) = M_B$

OR $M_B = 30\text{ N}\cdot\text{m}$ ↻

THE SINGLE EQUIVALENT FORCE $\underline{R}$ MUST THEN ACT AS INDICATED. THEN WITH $\underline{R}$ AT E..



$\Sigma M_B: \; 30\text{ N}\cdot\text{m} = x(250\text{ N})$

OR $x = 120\text{ mm}$

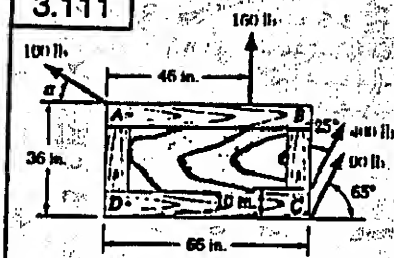
NOW $\frac{y}{x} = \frac{5}{2} \Rightarrow y = 300\text{ mm}$

$\therefore$ THE LINE OF ACTION OF $\underline{R}$ INTERSECTS TOP AB 120 mm TO THE LEFT OF B AND INTERSECTS SIDE BC 300 mm BELOW B. ◀

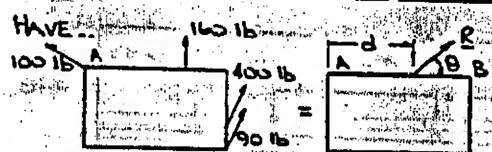
3.111

GIVEN: APPLIED FORCES ARE EQUIVALENT TO A SINGLE FORCE $\underline{R}$ APPLIED ALONG AB

FIND: (a) $\underline{R}$ AND DISTANCE FROM A TO ITS POINT OF APPLICATION IF $\alpha = 30°$ (b) α IF $\underline{R}$ IS AT B



HAVE..



(a) FOR EQUIVALENCE..

$\Sigma F_x: \; -100\cos 30° + 400\cos 65° + 90\cos 65° = R_x$

OR $R_x = 120.480\text{ lb}$

$\Sigma F_y: \; 100\sin\alpha + 160 + 400\sin 65° + 90\sin 65° = R_y$

OR $R_y = (604.09 + 100\sin\alpha)\text{ lb}$ (1)

WITH $\alpha = 30°$.. $R_y = 654.09\text{ lb}$

THEN.. $R = \sqrt{(120.480)^2 + (654.09)^2} = 665\text{ lb}$ $\quad \tan\theta = \frac{654.09}{120.480}$

OR $\theta = 79.6°$

ALSO.. $\Sigma M_A: (46\text{ in.})(160\text{ lb}) + (66\text{ in.})(400\text{ lb})\sin 65° + (26\text{ in.})(400\text{ lb})\cos 65° + (66\text{ in.})(90\text{ lb})\sin 65° + (36\text{ in.})(90\text{ lb})\cos 65° = d(654.09\text{ lb})$

OR.. $\Sigma M_A = 42{,}435\text{ lb}\cdot\text{in}$ AND $d = 64.9\text{ in.}$

$\therefore \; \underline{R} = 665\text{ lb} \measuredangle 79.6°$ ◀

AND $\underline{R}$ IS APPLIED 64.9 in. TO THE RIGHT OF A. ◀

(b) HAVE.. $d = 66\text{ in.}$

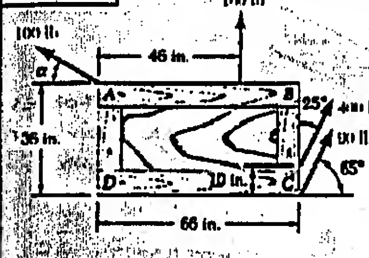
THEN.. $\Sigma M_A: \; 42{,}435\text{ lb}\cdot\text{in} = (66\text{ in})R_y$

OR $R_y = 642.95\text{ lb}$

USING EQ (1) $642.95 = 604.09 + 100\sin\alpha$

OR $\alpha = 22.9°$ ◀

3.112

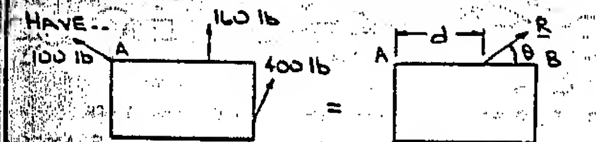


GIVEN: 90-lb FORCE REMOVED, APPLIED FORCES ARE EQUIVALENT TO A SINGLE FORCE $\mathbf{R}$ APPLIED ALONG AB

FIND: (a) $\mathbf{R}$ AND DISTANCE FROM A TO ITS POINT OF APPLICATION IF $\alpha = 30°$

(b) α IF $\mathbf{R}$ IS AT B

HAVE..



(a) FOR EQUIVALENCE..

$\Sigma F_x: -100\cos 30° + 400\cos 65° = R_x$

OR $R_x = 82.445$ lb

$\Sigma F_y: 100\sin\alpha + 160 + 400\sin 65° = R_y$

OR $R_y = (522.52 + 100\sin\alpha)$ lb ... (1)

WITH $\alpha = 30°$ .. $R_y = 572.52$ lb.

THEN.. $R = \sqrt{(82.445)^2 + (572.52)^2}$ $\tan\theta = \dfrac{572.52}{82.445}$

$= 578$ lb

OR $\theta = 81.8°$

ALSO.. ΣM_A: (46 in.)(160 lb) + (66 in.)(400 lb) sin 65° + (26 in.)(400 lb) cos 65° = d(572.52 lb)

OR $\Sigma M_A = 35{,}682$ lb·in. AND $d = 62.3$ in.

$\therefore\ \mathbf{R} = 578$ lb $\measuredangle 81.8°$ ◀

AND $\mathbf{R}$ IS APPLIED 62.3 in. TO THE RIGHT OF A. ◀

(b) HAVE $d = 66$ in.

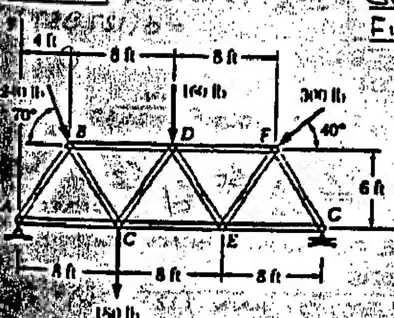
THEN.. ΣM_A: 35,682 lb·in. = (66 in.)R_y

OR $R_y = 540.64$ lb

USING EQ. (1).. $540.64 = 522.52 + 100\sin\alpha$

OR $\alpha = 10.44°$ ◀

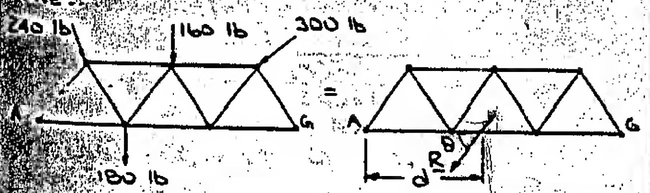
3.113



GIVEN: APPLIED FORCES

FIND: SINGLE EQUIVALENT FORCE $\mathbf{R}$ AND POINT WHERE ITS LINE OF ACTION INTERSECTS A LINE DRAWN THROUGH AG

HAVE..



(CONTINUED)

3.113 CONTINUED

FOR EQUIVALENCE..

ΣF_x: $240\cos 70° - 300\cos 40° = R_x$

OR $R_x = -147.728$ lb

ΣF_y: $-240\sin 70° - 180 - 160 - 300\sin 40° = R_y$

OR $R_y = -758.36$ lb

THEN.. $R = \sqrt{(147.728)^2 + (758.36)^2}$ $\tan\theta = \dfrac{758.36}{147.728}$

$= 773$ lb

OR $\theta = 79.0°$

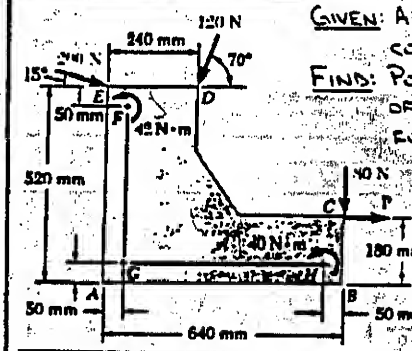
ALSO.. ΣM_A: $+$(4 ft)(240 lb) sin 70° − (6 ft)(240 lb) cos 70° − (8 ft)(180 lb) − (12 ft)(160 lb) − (20 ft)(300 lb) sin 40° + (6 ft)(300 lb) cos 40° = −d(758.36 lb)

OR $d = 9.54$ ft

$\therefore\ \mathbf{R} = 773$ lb $\measuredangle 79.0°$ ◀

AND THE LINE OF ACTION OF $\mathbf{R}$ INTERSECTS LINE AG 9.54 ft TO THE RIGHT OF A. ◀

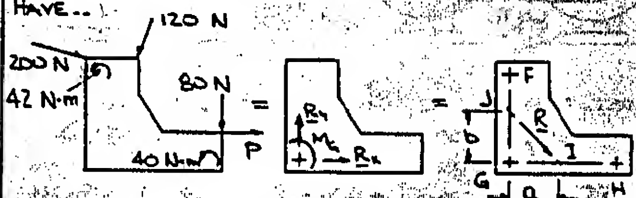
3.114 and 3.115



GIVEN: APPLIED FORCES AND COUPLES

FIND: POINT OF APPLICATION OF SINGLE EQUIVALENT FORCE FROM RIVET (a) ON LINE FG (b) ON LINE GH

HAVE..



FIRST REPLACE THE APPLIED FORCES AND COUPLES WITH AN EQUIVALENT FORCE-COUPLE SYSTEM AT G. THUS..

ΣF_x: $200\cos 15° - 120\cos 70° + P = R_x$

OR $R_x = (152.142 + P)$ N

ΣF_y: $-200\sin 15° - 120\sin 70° - 80 = R_y$

OR $R_y = -244.53$ N

ΣM_G: −(0.47 m)(200 N) cos 15° + (0.05 m)(200 N) sin 15° + (0.47 m)(120 N) cos 70° − (0.19 m)(120 N) sin 70° − (0.13 m)(P N) − (0.59 m)(80 N) + 42 N·m + 40 N·m = M_G

OR $M_G = -(55.544 + 0.13P)$ N·m

3.114   P = 0

NOW.. WITH $\mathbf{R}$ AT I.. ΣM_G: -55.544 N·m $= -a(244.53$ N$)$

OR $a = 0.227$ m

AND WITH $\mathbf{R}$ AT J.. ΣM_G: -55.544 N·m $= -b(152.142$ N$)$

OR $b = 0.365$ m

$\therefore$ (a) THE RIVET HOLE IS 0.365 m ABOVE G. ◀

(CONTINUED)

3.114 and 3.115 CONTINUED

(b) THE RIVET HOLE IS 0.227 m TO THE RIGHT OF G. ◀

3.115 $P = 60$ N

HAVE $R_x = (152.142 + 60) = 212.14$ N

$M_G = -[55.544 + 0.13(60)] = -63.344$ N·m

THEN.. WITH $\underline{R}$ AT I.. ΣM_G: -63.344 N·m $= -a(244.53$ N$)$

OR $a = 0.259$ m

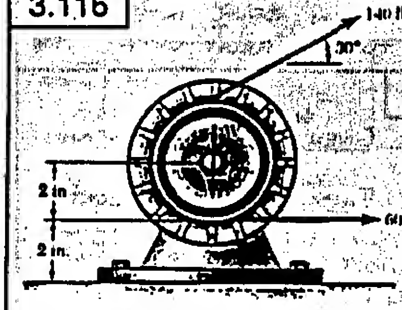
AND WITH $\underline{R}$ AT J.. ΣM_G: -63.344 N·m $= -b(212.14$ N$)$

OR $b = 0.299$ m

∴ (a) THE RIVET HOLE IS 0.299 m ABOVE G. ◀

(b) THE RIVET HOLE IS 0.259 m TO THE RIGHT OF G. ◀

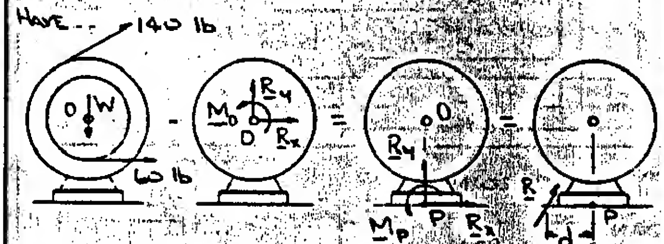
3.116



GIVEN: $W = 32$ lb

FIND: SINGLE EQUIVALENT FORCE $\underline{R}$ AND WHERE LINE OF ACTION OF $\underline{R}$ INTERSECTS THE FLOOR

HAVE..



FIRST REDUCE THE GIVEN FORCES TO AN EQUIVALENT FORCE-COUPLE SYSTEM AT O. THEN FOR EQUIVALENCE..

ΣF_x: $140 \cos 30° + 60 = R_x$ OR $R_x = 181.244$ lb

ΣF_y: $140 \sin 30° - 32 = R_y$ OR $R_y = 38$ lb

ΣM_O: $-(2$ in.$)(140$ lb$) + (2$ in.$)(60$ lb$) = M_O$

OR $M_O = -160$ lb·in

NEXT MOVE THE EQUIVALENT FORCE-COUPLE SYSTEM TO THE POINT P WHICH LIES ON THE FLOOR DIRECTLY BELOW O. THUS..

AT P.. $R_x = 181.244$ lb $R_y = 38$ lb

AND ΣM_P: -160 lb·in $- (4$ in.$)(181.244$ lb$) = M_P$

OR $M_P = -884.98$ lb·in.

FINALLY, REPLACE $(\underline{R}, \underline{M}_P)$ WITH THE SINGLE EQUIVALENT FORCE $\underline{R}$, WHERE..

$R = \sqrt{(181.244)^2 + (38)^2}$ $\qquad \tan\theta = \dfrac{38}{181.244}$

$= 185.2$ lb

OR $\theta = 11.84°$

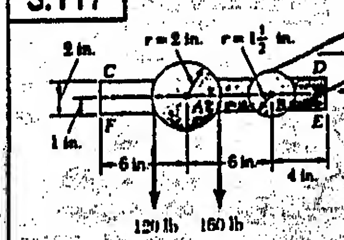
AND.. ΣM_P: -884.98 lb·in $= d(38$ lb$)$

OR $d = 23.3$ in.

∴ $\underline{R} = 185.2$ lb $\angle 11.84°$ ◀

AND THE LINE OF ACTION OF $\underline{R}$ INTERSECTS THE FLOOR AT A POINT 23.3 in. TO THE LEFT OF THE VERTICAL CENTER LINE OF THE MOTOR. ◀

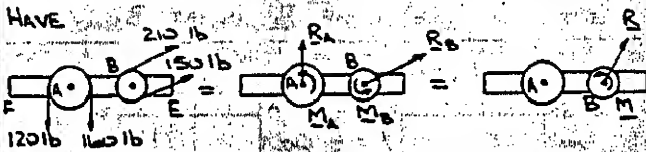
3.117



GIVEN: APPLIED FORCES

FIND: SINGLE EQUIVALENT FORCE $\underline{R}$ AND WHERE LINE OF ACTION OF $\underline{R}$ INTERSECTS EF

HAVE



FIRST REPLACE THE FORCES ACTING ON EACH PULLEY WITH AN EQUIVALENT FORCE-COUPLE SYSTEM ACTING AT THE CENTER OF EACH PULLEY.

PULLEY A: ΣF_y: $-120 - 160 = R_A$ OR $R_A = -280$ lb

ΣM_A: $(2$ in.$)(120$ lb$) - (2$ in.$)(160$ lb$) = M_A$

OR $M_A = -80$ lb·in.

PULLEY B: ΣF_y: $210 + 150 = R_B$

OR $R_B = 360$ lb $\angle 25°$

ΣM_B: $(1.5$ in.$)(150$ lb$) - (1.5$ in.$)(210$ lb$) = M_B$

OR $M_B = -90$ lb·in.

NEXT COMBINE $(\underline{R}_A, \underline{M}_A)$ AND $(\underline{R}_B, \underline{M}_B)$ INTO AN EQUIVALENT FORCE-COUPLE SYSTEM $(\underline{R}, \underline{M})$ AT B. HAVE..

ΣF_x: $360 \cos 25° = R_x$ OR $R_x = 326.27$ lb

ΣF_y: $-280 + 360 \sin 25° = R_y$ OR $R_y = -127.851$ lb

ΣM_B: -80 lb·in $+ (6$ in.$)(280$ lb$) - 90$ lb·in $= M$

OR $M = 1510$ lb·in

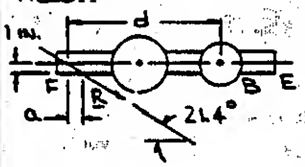
FINALLY, REPLACE $(\underline{R}, \underline{M})$ WITH THE SINGLE EQUIVALENT FORCE $\underline{R}$, WHERE..

$R = \sqrt{(326.27)^2 + (127.851)^2}$ $\qquad \tan\theta = \dfrac{127.851}{326.27}$

$= 350$ lb

OR $\theta = 21.4°$

ALSO..



ΣM_B: 1510 lb·in $= d(127.851$ lb$)$

OR $d = 11.810$ in.

AND $a = \dfrac{1 \text{ in.}}{\tan 21.4°}$

$= 2.552$ in.

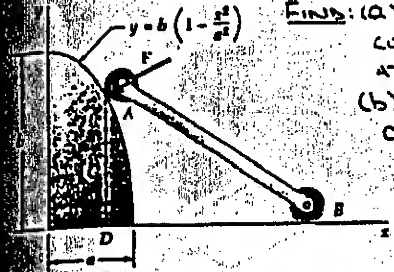
∴ $\underline{R} = 350$ lb $\angle 21.4°$ ◀

AND THE LINE OF ACTION OF $\underline{R}$ INTERSECTS THE LOWER EDGE OF THE BRACKET $(11.810 - 2.552)$ 9.26 in. TO THE LEFT OF THE CENTER OF PULLEY B AND $(12 - 9.26)$ 2.74 in. TO THE RIGHT OF CORNER F. ◀

3.118 GIVEN: $\underline{F}$ IS PERPENDICULAR TO THE SURFACE

FIND: (a) EQUIVALENT FORCE-COUPLE SYSTEM $(\underline{R}, \underline{M})$ AT D

(b) X FOR $\underline{M}_{MAX}$ IF $a = 1\,m$, $b = 2\,m$



(a) THE SLOPE AT ANY POINT ON THE SURFACE OF MEMBER C IS..

$$\frac{dy}{dx} = \frac{d}{dx}\left[b\left(1 - \frac{x^2}{a^2}\right)\right] = -\frac{2b}{a^2}x$$

SINCE $\underline{F}$ IS PERPENDICULAR TO THE SURFACE, IT FOLLOWS THAT

$$\tan\alpha = \frac{a^2}{2b}\frac{1}{x}$$

WHERE α IS THE ANGLE THAT $\underline{F}$ FORMS WITH THE HORIZONTAL. THEN FOR EQUIVALENCE..

$\Sigma\underline{F}$: $\underline{F} = \underline{R}$

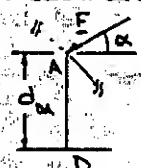
ΣM_D: $d_{DA} F\cos\alpha = M$

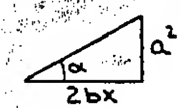
SINCE A IS A POINT ON THE SURFACE, HAVE

$d_{DA} = y$ AT A

ALSO,

$$\cos\alpha = \frac{2bx}{\sqrt{(a^2)^2 + (2bx)^2}}$$





THEN..

$$M = \left[b\left(1 - \frac{x^2}{a^2}\right)\right] \cdot F \cdot \frac{2bx}{\sqrt{a^4 + 4b^2x^2}} = \frac{2Fb^2\left(x - \frac{x^3}{a^2}\right)}{\sqrt{a^4 + 4b^2x^2}}$$

THE EQUIVALENT FORCE-COUPLE SYSTEM AT D IS.. $\underline{R} = F \measuredangle \tan^{-1}\left(\frac{a^2}{2bx}\right)$ ◀

$$\underline{M} = \frac{2Fb^2\left(x - \frac{x^3}{a^2}\right)}{\sqrt{a^4 + 4b^2x^2}} \circlearrowright$$ ◀

(b) SUBSTITUTING $a = 1\,m$, $b = 2\,m$ IN THE EXPRESSION FOR M YIELDS..

$$M = \frac{8F(x - x^3)}{\sqrt{1 + 16x^2}}$$

TO FIND THE VALUE OF X TO MAXIMIZE M..

$$\frac{dM}{dx} = 8F\,\frac{(1 - 3x^2)\sqrt{1 + 16x^2} - (x - x^3)\left[\frac{1}{2}(32x)(1 + 16x^2)^{-\frac{1}{2}}\right]}{(1 + 16x^2)} = 0$$

OR $(1 - 3x^2)(1 + 16x^2) - 16x(x - x^3) = 0$

OR $32x^4 + 3x^2 - 1 = 0$

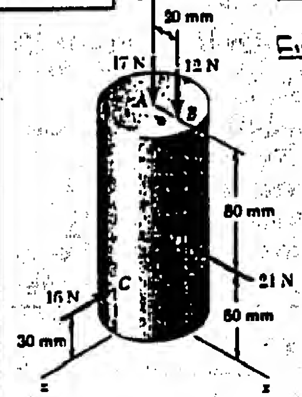
THEN $x^2 = \frac{-3 \pm \sqrt{(3)^2 - 4(32)(-1)}}{2(32)}$

TAKING THE POSITIVE ROOT SINCE $x^2 > 0$ YIELDS $x^2 = 0.136011\ m^2$

AND THEN FOR M_{MAX} $x = 0.369\ m$ ◀

3.119 GIVEN: DIAMETER = 60 mm, APPLIED FORCES

FIND: EQUIVALENT FORCE-COUPLE SYSTEM $(\underline{R}, \underline{M})$ AT C



FOR EQUIVALENCE..

$\Sigma\underline{F}$: $\underline{R} = \underline{F}_A + \underline{F}_B + \underline{F}_C + \underline{F}_D$
$= -17\underline{j} - 12\underline{j} - 16\underline{k} - 21\underline{i}$
$= -(21\ N)\underline{i} - (29\ N)\underline{j} - (16\ N)\underline{k}$

$\Sigma\underline{M}_C$: $\underline{M} = \underline{r}_{A/C} \times \underline{F}_A + \underline{r}_{B/C} \times \underline{F}_B + \underline{r}_{D/C} \times \underline{F}_D$

OR $\underline{M} = [(0.11\,m)\underline{j} - (0.03\,m)\underline{k}] \times [-(17\,N)]\underline{j}$
$+ [(0.02\,m)\underline{i} + (0.11\,m)\underline{j} - (0.03\,m)\underline{k}] \times [-(12\,N)]\underline{j}$
$+ [(0.03\,m)\underline{i} + (0.03\,m)\underline{j} - (0.03\,m)\underline{k}] \times [-(21\,N)]\underline{i}$
$= -(0.51\ N\cdot m)\underline{i} + [-(0.24\ N\cdot m)\underline{k} - (0.36\ N\cdot m)\underline{i}]$
$+ [(0.63\ N\cdot m)\underline{k} + (0.63\ N\cdot m)\underline{j}]$

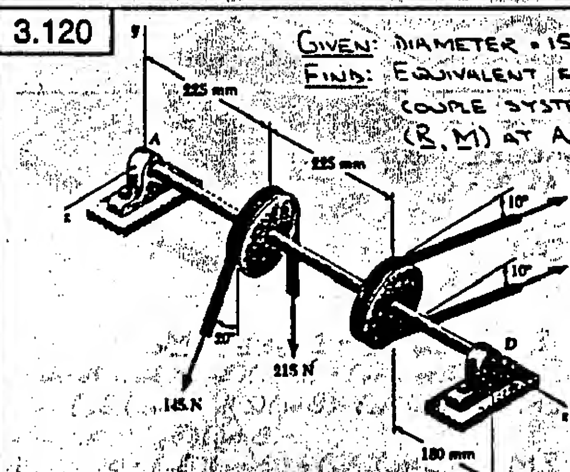
∴ THE EQUIVALENT FORCE-COUPLE SYSTEM AT C IS..

$\underline{R} = -(21\,N)\underline{i} - (29\,N)\underline{j} - (16\,N)\underline{k}$ ◀

$\underline{M} = -(0.87\ N\cdot m)\underline{i} + (0.63\ N\cdot m)\underline{j} + (0.39\ N\cdot m)\underline{k}$ ◀

3.120 GIVEN: DIAMETER = 150 mm

FIND: EQUIVALENT FORCE-COUPLE SYSTEM $(\underline{R}, \underline{M})$ AT A



FIRST REPLACE THE BELT FORCES ON EACH PULLEY WITH AN EQUIVALENT FORCE-COUPLE SYSTEM AT THE CENTER OF THE PULLEY; THIS ELIMINATES THE NEED TO DETERMINE WHERE THE BELTS CONTACT THE PULLEYS.

PULLEY B: $\Sigma\underline{F}$: $\underline{R}_B = -215\underline{j} + 145(-\cos 20°\underline{j} + \sin 20°\underline{k})$
$= -(351.26\ N)\underline{j} + (49.593\ N)\underline{k}$

$\Sigma\underline{M}_B$: $\underline{M}_B = [(0.075\,m)(145\,N) - (0.075\,m)(215\,N)]\underline{i}$
$= -(5.25\ N\cdot m)\underline{i}$

PULLEY C: $\Sigma\underline{F}$: $\underline{R}_C = (155 + 240)(-\sin 10°\,\underline{j} - \cos 10°\,\underline{k})$
$= -(68.591\ N)\underline{j} - (389.00\ N)\underline{k}$

$\Sigma\underline{M}_C$: $\underline{M}_C = [(0.075\,m)(240\,N) - (0.075\,m)(155\,N)]\underline{i}$
$= (6.375\ N\cdot m)\underline{i}$

THE EQUIVALENT FORCE-COUPLE SYSTEM AT A IS THEN

$\Sigma\underline{F}$: $\underline{R} = \underline{R}_B + \underline{R}_C$
$= (-351.26\underline{j} + 49.593\underline{k}) + (-68.591\underline{j} - 389.00\underline{k})$
$= -(420\ N)\underline{j} - (339\ N)\underline{k}$ ◀

(CONTINUED)

3.120 CONTINUED

AND ΣM_A: $\mathbf{M} = [(\mathbf{r}_{B/A} \times \mathbf{R}_B) + \mathbf{M}_B] + [(\mathbf{r}_{C/A} \times \mathbf{R}_C) + \mathbf{M}_C]$

$= \{(0.225\ \text{m})\mathbf{i} \times [-(351.26\ \text{N})\mathbf{j} + (49.593\ \text{N})\mathbf{k}] - (5.25\ \text{N·m})\mathbf{i}\}$

$+ \{(0.450\ \text{m})\mathbf{i} \times [-(168.591\ \text{N})\mathbf{j} - (389.00\ \text{N})\mathbf{k}] + (6.375\ \text{N·m})\mathbf{i}\}$

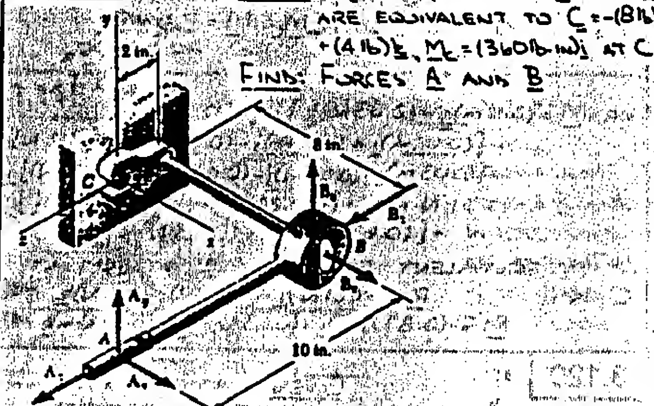
$= [(-5.25 + 6.375)\ \text{N·m}]\mathbf{i}$

$+ [(-11.1584 + 175.05)\ \text{N·m}]\mathbf{j}$

$- [(-79.034 - 30.866)\ \text{N·m}]\mathbf{k}$

$= (1.125\ \text{N·m})\mathbf{i} + (163.9\ \text{N·m})\mathbf{j} - (109.9\ \text{N·m})\mathbf{k}$ ◀

3.121



GIVEN: $A_z = 2$ lb, FORCES $\mathbf{A}$ AND $\mathbf{B}$ ARE EQUIVALENT TO $\mathbf{C} = -(8\ \text{lb})\mathbf{i} + (4\ \text{lb})\mathbf{k}$, $\mathbf{M}_C = (360\ \text{lb·in})\mathbf{j}$ AT C

FIND: FORCES $\mathbf{A}$ AND $\mathbf{B}$

FROM THE STATEMENT OF THE PROBLEM, EQUIVALENCE REQUIRES...

$\Sigma \mathbf{F}$: $\mathbf{A} + \mathbf{B} = \mathbf{C}$

OR ΣF_x: $A_x + B_x = -8$ lb (1)

ΣF_y: $A_y + B_y = 0$ (2)

ΣF_z: $2\ \text{lb} + B_z = 4$ lb (3)

AND $\Sigma \mathbf{M}_C$: $\mathbf{r}_{A/C} \times \mathbf{A} + \mathbf{r}_{B/C} \times \mathbf{B} = \mathbf{M}_C$

OR ΣM_x: $-(8\ \text{in.})(A_y) + (2\ \text{in.})(B_y) = 360$ lb·in. (4)

ΣM_y: $(8\ \text{in.})(A_x) - (8\ \text{in.})(2\ \text{lb}) - (2\ \text{in.})(B_x) - (8\ \text{in.})(B_z) = 0$ (5)

ΣM_z: $(8\ \text{in.})(A_y) + (8\ \text{in.})(B_y) = 0$ (6)

EQ. (2) OR (6) $\Rightarrow$ $A_y = -B_y$

SUBSTITUTING INTO EQ. (4)... $-8(-B_y) + 2B_y = 360$

OR $B_y = 36$ lb

AND $A_y = -36$ lb

EQ. (3) $\Rightarrow$ $B_z = 2$ lb

EQ. (1) $\Rightarrow$ $B_x = -(8 + A_x)$ lb

SUBSTITUTING INTO EQ. (5)...

$8A_x - 16 - 2[-(8 + A_x)] - 8(2) = 0$

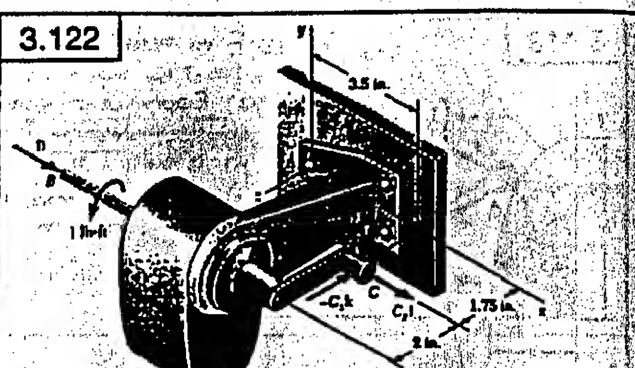
OR $A_x = 1.6$ lb

AND $B_x = -9.6$ lb

$\therefore$ $\mathbf{A} = (1.6\ \text{lb})\mathbf{i} - (36\ \text{lb})\mathbf{j} + (2\ \text{lb})\mathbf{k}$ ◀

$\mathbf{B} = -(9.6\ \text{lb})\mathbf{i} + (36\ \text{lb})\mathbf{j} + (2\ \text{lb})\mathbf{k}$ ◀

3.122



GIVEN: FORCES $\mathbf{B}$ AND $\mathbf{C}$ ARE EQUIVALENT TO $\mathbf{R} = (2.6\ \text{lb})\mathbf{i} - R_y\mathbf{j} - (0.7\ \text{lb})\mathbf{k}$, $\mathbf{M}_A^R = M_x\mathbf{i} + (1\ \text{lb·ft})\mathbf{j} - (0.72\ \text{lb·ft})\mathbf{k}$ AT A

FIND: (a) FORCES $\mathbf{B}$ AND $\mathbf{C}$

(b) R_y, M_x

(a) FROM THE STATEMENT OF THE PROBLEM, EQUIVALENCE REQUIRES...

$\Sigma \mathbf{F}$: $\mathbf{B} + \mathbf{C} = \mathbf{R}$

OR ΣF_x: $B_x + C_x = 2.6$ lb (1)

ΣF_y: $-C_y = R_y$ (2)

ΣF_z: $-C_z = -0.7$ lb OR $C_z = 0.7$ lb

AND $\Sigma \mathbf{M}_A$: $(\mathbf{r}_{B/A} \times \mathbf{B} + \mathbf{M}_B) + \mathbf{r}_{C/A} \times \mathbf{C} = \mathbf{M}_A^R$

OR ΣM_x: $(1\ \text{lb·ft}) + (\frac{1.75}{12}\ \text{ft})(C_y) = M_x$ (3)

ΣM_y: $(\frac{3.75}{12}\ \text{ft})(B_x) + (\frac{1.75}{12}\ \text{ft})(C_x) + (\frac{3.5}{12}\ \text{ft})(0.7\ \text{lb}) = 1\ \text{lb·ft}$

OR $3.75B_x + 1.75C_x = 9.55$

USING EQ. (1)... $3.75B_x + 1.75(2.6 - B_x) = 9.55$

OR $B_x = 2.5$ lb

AND $C_x = 0.1$ lb

ΣM_z: $-(\frac{3.5}{12}\ \text{ft})(C_y) = -0.72$ lb·ft

OR $C_y = 2.4686$ lb

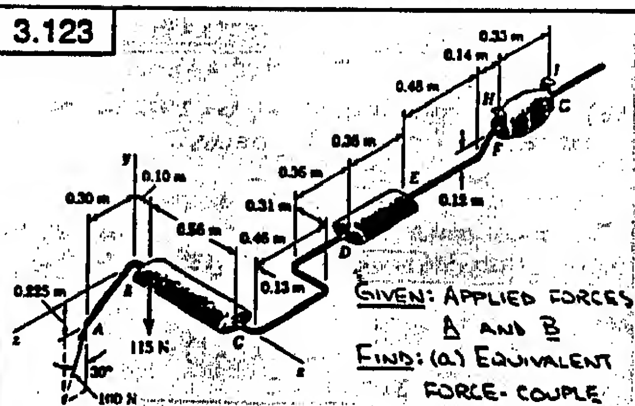
$\therefore$ $\mathbf{B} = (2.5\ \text{lb})\mathbf{i}$ $\mathbf{C} = (0.1\ \text{lb})\mathbf{i} - (2.47\ \text{lb})\mathbf{j} - (0.7\ \text{lb})\mathbf{k}$ ◀

(b) EQ. (2) $\Rightarrow$ $R_y = -2.47$ lb ◀

USING EQ. (3)... $1 + (\frac{1.75}{12})(2.4686) = M_x$

OR $M_x = 1.360$ lb·ft ◀

3.123



GIVEN: APPLIED FORCES $\mathbf{A}$ AND $\mathbf{B}$

FIND: (a) EQUIVALENT FORCE-COUPLE SYSTEM ($\mathbf{R}$, $\mathbf{M}_D$) AT D

(b) DIRECTION OF ROTATION OF PIPE CD RELATIVE TO MUFFLER DE

(CONTINUED)

3.123 CONTINUED

(a) EQUIVALENCE REQUIRES..

$\Sigma \underline{F}$: $\underline{R} = \underline{A} + \underline{B}$

$= (100\text{ N})(\cos 30^\circ \underline{j} - \sin 30^\circ \underline{k}) - (115\text{ N})\underline{j}$

$= -(28.4\text{ N})\underline{j} - (50\text{ N})\underline{k}$

AND $\Sigma \underline{M}_D$: $\underline{M}_D = \underline{r}_{A/D} \times \underline{F}_A + \underline{r}_{B/D} \times \underline{F}_B$

WHERE $\underline{r}_{A/D} = -(0.48\text{ m})\underline{i} - (0.225\text{ m})\underline{j} + (1.12\text{ m})\underline{k}$

$\underline{r}_{B/D} = -(0.38\text{ m})\underline{i} + (0.82\text{ m})\underline{k}$

THEN..

$$\underline{M}_D = 100\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -0.48 & -0.225 & 1.12 \\ 0 & \cos 30^\circ & -\sin 30^\circ \end{vmatrix} + 115\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -0.38 & 0 & 0.82 \\ 0 & -1 & 0 \end{vmatrix}$$

$= 100[(0.225 \sin 30^\circ - 1.12 \cos 30^\circ)\underline{i}$

$+ (-0.48 \sin 30^\circ)\underline{j} + (-0.48 \cos 30^\circ)\underline{k}]$

$+ 115[(0.82)\underline{i} + (0.38)\underline{k}]$

$= 8.56\underline{i} - 24.0\underline{j} + 2.13\underline{k}$

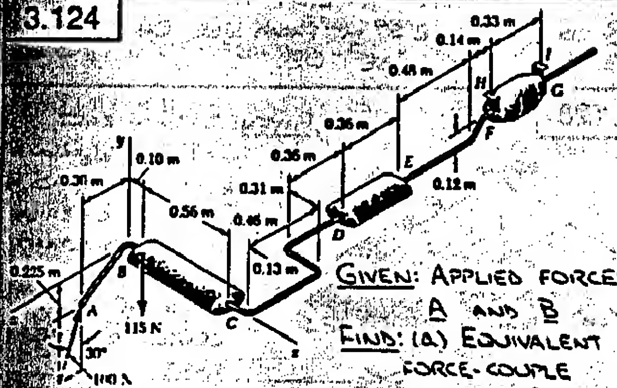
THE EQUIVALENT FORCE-COUPLE SYSTEM AT D IS..

$\underline{R} = -(28.4\text{ N})\underline{j} - (50\text{ N})\underline{k}$ ◀

$\underline{M}_D = (8.56\text{ N·m})\underline{i} - (24.0\text{ N·m})\underline{j} + (2.13\text{ N·m})\underline{k}$ ◀

(b) SINCE $(M_D)_z$ IS POSITIVE, PIPE CD WILL TEND TO ROTATE COUNTERCLOCKWISE RELATIVE TO MUFFLER DE. ◀

3.124



GIVEN: APPLIED FORCES A AND B

FIND: (a) EQUIVALENT FORCE-COUPLE SYSTEM $(\underline{R}, \underline{M}_F)$ AT F

(b) DIRECTION OF ROTATION OF PIPE EF RELATIVE TO THE MECHANIC

(a) EQUIVALENCE REQUIRES..

$\Sigma \underline{F}$: $\underline{R} = \underline{A} + \underline{B}$

$= (100\text{ N})(\cos 30^\circ \underline{j} - \sin 30^\circ \underline{k}) - (115\text{ N})\underline{j}$

$= -(28.4\text{ N})\underline{j} - (50\text{ N})\underline{k}$

AND $\Sigma \underline{M}_F$: $\underline{M}_F = \underline{r}_{A/F} \times \underline{A} + \underline{r}_{B/F} \times \underline{B}$

WHERE $\underline{r}_{A/F} = -(0.48\text{ m})\underline{i} - (0.345\text{ m})\underline{j} + (2.10\text{ m})\underline{k}$

$\underline{r}_{B/F} = -(0.38\text{ m})\underline{i} - (0.12\text{ m})\underline{j} + (1.80\text{ m})\underline{k}$

THEN..

$$\underline{M}_F = 100\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -0.48 & -0.345 & 2.10 \\ 0 & \cos 30^\circ & -\sin 30^\circ \end{vmatrix} + 115\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -0.38 & -0.12 & 1.80 \\ 0 & -1 & 0 \end{vmatrix}$$

(CONTINUED)

3.124 CONTINUED

$\underline{M}_F = 100[(0.345 \sin 30^\circ - 2.10 \cos 30^\circ)\underline{i}$

$+ (-0.48 \sin 30^\circ)\underline{j} + (-0.48 \cos 30^\circ)\underline{k}]$

$+ 115[(1.80)\underline{i} + (0.38)\underline{k}]$

$= 42.4\underline{i} - 24.0\underline{j} + 2.13\underline{k}$

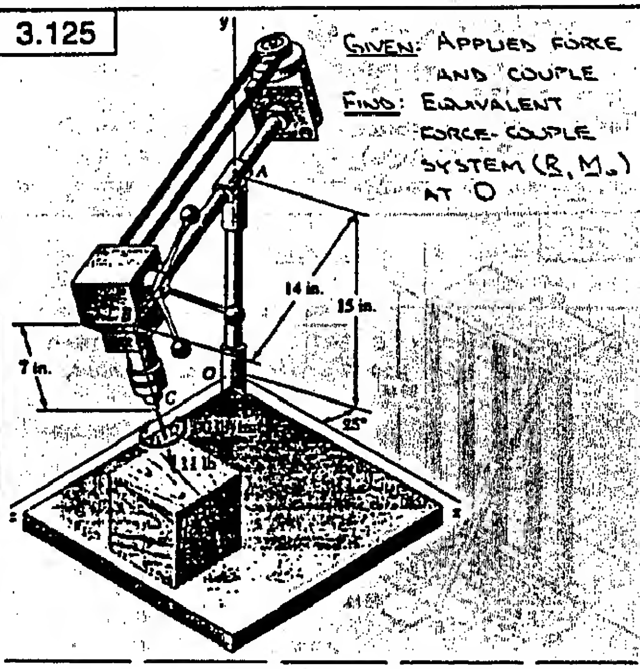
∴ THE EQUIVALENT FORCE-COUPLE SYSTEM AT F IS..

$\underline{R} = -(28.4\text{ N})\underline{j} - (50\text{ N})\underline{k}$ ◀

$\underline{M}_F = (42.4\text{ N·m})\underline{i} - (24.0\text{ N·m})\underline{j} + (2.13\text{ N·m})\underline{k}$ ◀

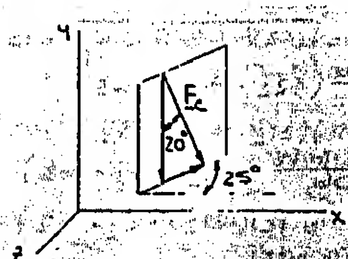
(b) SINCE $(M_F)_z$ IS POSITIVE, PIPE EF WILL TEND TO ROTATE COUNTERCLOCKWISE RELATIVE TO THE MECHANIC. ◀

3.125



GIVEN: APPLIED FORCE AND COUPLE

FIND: EQUIVALENT FORCE-COUPLE SYSTEM $(\underline{R}, \underline{M}_O)$ AT O



EQUIVALENCE REQUIRES..

$\Sigma \underline{F}$: $\underline{R} = \underline{F}_c$

$= (11\text{ lb})(\sin 20^\circ \cos 25^\circ \underline{i} - \cos 20^\circ \underline{j} - \sin 20^\circ \sin 25^\circ \underline{k})$

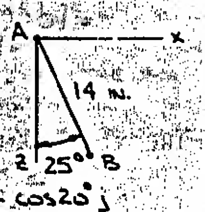
$= (3.41\text{ lb})\underline{i} - (10.34\text{ lb})\underline{j} - (1.590\text{ lb})\underline{k}$

TO SIMPLIFY THE COMPUTATION OF $\underline{M}_O$, SLIDE FORCE $\underline{F}_c$ ALONG ITS LINE OF ACTION TO B. THEN..

$\Sigma \underline{M}_O$: $\underline{M}_O = \underline{r}_{B/O} \times \underline{F}_c + \underline{M}_c$

WHERE

$\underline{r}_{B/O} = (14\text{ in.})\sin 25^\circ \underline{i} + (15\text{ in.})\underline{j} + (14\text{ in.})\cos 25^\circ \underline{k}$



$\underline{M}_c = (90\text{ lb·in.})(\sin 20^\circ \cos 25^\circ \underline{i} - \cos 20^\circ \underline{j} - \sin 20^\circ \sin 25^\circ \underline{k})$

(CONTINUED)

3.125 CONTINUED

THEN...

$$\underline{M}_O = 11\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 14\sin 25° & 15 & 14\cos 25° \\ \sin 20°\cos 25° & -\cos 20° & -\sin 20°\sin 25° \end{vmatrix} + 90(\sin 20°\cos 25°\,\underline{i} - \cos 20°\,\underline{j} - \sin 20°\sin 25°\,\underline{k})$$

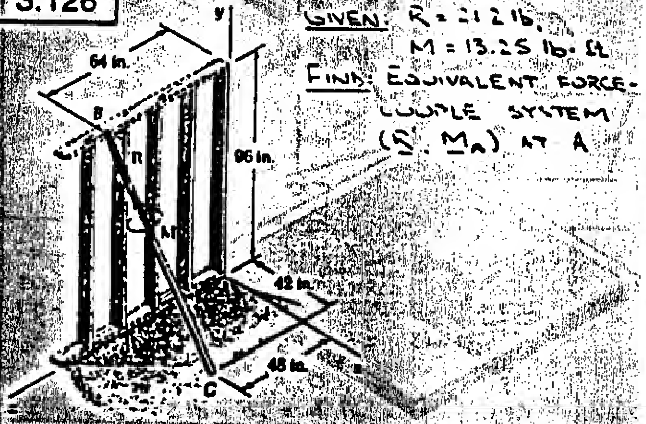
$$= [11(-15\sin 20°\sin 25° + 14\cos 25°\cos 20°) + 90(\sin 20°\cos 25°)]\,\underline{i}$$
$$+ [11(14\sin 20°\cos^2 25° + 14\sin 20°\sin^2 25°) + 90(-\cos 20°)]\,\underline{j}$$
$$+ [11(-14\sin 25°\cos 20° - 15\sin 20°\cos 25°) + 90(-\sin 20°\sin 25°)]\,\underline{k}$$
$$= (-23.849 + 131.154 + 27.898)\,\underline{i} + (43.263 + 9.407 - 84.572)\,\underline{j} + (-61.158 - 51.146 - 13.0095)\,\underline{k}$$
$$= 135.2\,\underline{i} - 31.9\,\underline{j} - 125.3\,\underline{k}$$

∴ THE EQUIVALENT FORCE-COUPLE SYSTEM AT O IS

$\underline{R} = (3.41\text{ lb})\underline{i} - (10.34\text{ lb})\underline{j} - (1.590\text{ lb})\underline{k}$

$\underline{M}_O = (135.2\text{ lb·in})\underline{i} - (31.9\text{ lb·in})\underline{j} - (125.3\text{ lb·in})\underline{k}$

3.126



GIVEN: $R = 21.2$ lb, $M = 13.25$ lb·ft

FIND: EQUIVALENT FORCE-COUPLE SYSTEM $(\underline{R}', \underline{M}_A)$ AT A

FIRST NOTE... $d_{BC} = \sqrt{(42)^2 + (-96)^2 + (-16)^2} = 106$ in.

THEN... $\underline{R} = \dfrac{21.2\text{ lb}}{106}(42\underline{i} - 96\underline{j} - 16\underline{k})$

$= (0.4\text{ lb})(21\underline{i} - 48\underline{j} - 8\underline{k})$

AND $\underline{M} = \dfrac{13.25\text{ lb·ft}}{106}(-42\underline{i} + 96\underline{j} + 16\underline{k})$

$= (0.25\text{ lb·ft})(-21\underline{i} + 48\underline{j} + 8\underline{k})$

EQUIVALENCE REQUIRES...

$\Sigma\underline{F}: \quad \underline{R}' = \underline{R}$

$\Sigma\underline{M}_A: \quad \underline{M}_A = \underline{r}_{C/A} \times \underline{R} + \underline{M}$

WHERE $\underline{r}_{C/A} = (42\text{ in.})\underline{i} + (48\text{ in.})\underline{k} = (3.5\text{ ft})\underline{i} + (4\text{ ft})\underline{k}$

THEN...

$$\underline{M}_A = 0.4\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3.5 & 0 & 4 \\ 21 & -48 & -8 \end{vmatrix} + (-5.25\underline{i} + 12\underline{j} + 2\underline{k})$$

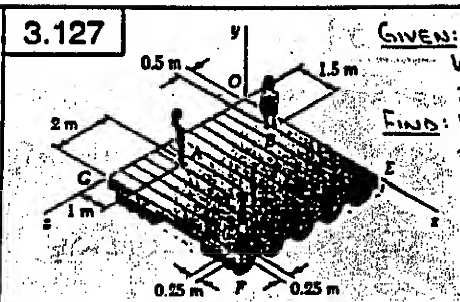
$$= [0.4(192) - 5.25]\,\underline{i} + [0.4(84 + 28) + 12]\,\underline{j} + [0.4(-168) + 2]\,\underline{k}$$
$$= 71.55\,\underline{i} + 56.8\,\underline{j} - 65.2\,\underline{k}$$

∴ THE EQUIVALENT FORCE-COUPLE SYSTEM AT A IS

$\underline{R}' = (8.4\text{ lb})\underline{i} - (19.2\text{ lb})\underline{j} - (3.2\text{ lb})\underline{k}$

$\underline{M}_A = (71.6\text{ lb·ft})\underline{i} + (56.8\text{ lb·ft})\underline{j} - (65.2\text{ lb·ft})\underline{k}$

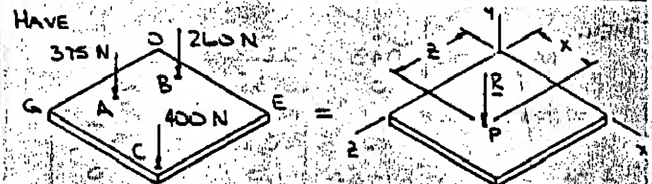
3.127



GIVEN: 5×5-m RAFT, $W_A = 375$ N, $W_B = 260$ N, $W_C = 400$ N

FIND: MAGNITUDE AND POINT OF APPLICATION OF THE RESULTANT $\underline{R}$

HAVE



EQUIVALENCE REQUIRES...

$\Sigma F_y: \; -375 - 260 - 400 = -R$

$R = 1035$ N

LET $\underline{R}$ BE APPLIED AT POINT P WHOSE COORDINATES ARE $(x, 0, z)$. THEN...

$\Sigma M_x: \; (3\text{ m})(375\text{ N}) + (0.5\text{ m})(260\text{ N}) + (4.75\text{ m})(400\text{ N}) = z(1035\text{ N})$

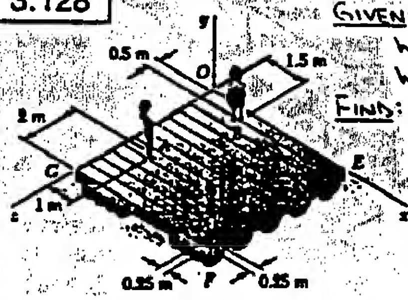
OR $z = 3.05$ m

$\Sigma M_z: \; -(1\text{ m})(375\text{ N}) - (1.5\text{ m})(260\text{ N}) - (4.75\text{ m})(400\text{ N}) = -x(1035\text{ N})$

OR $x = 2.57$ m

∴ $\underline{R}$ IS APPLIED 2.57 m FROM SIDE OG AND 3.05 m FROM SIDE OE.

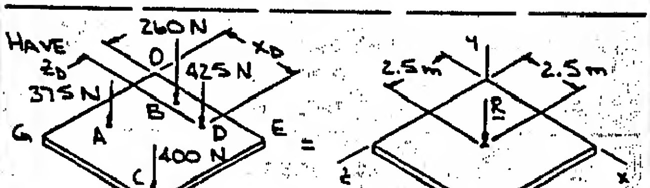
3.128



GIVEN: 5×5-m RAFT, $W_A = 375$ N, $W_B = 260$ N, $W_C = 400$ N, $W_D = 425$ N

FIND: POSITION OF CHILD D IF RESULTANT $\underline{R}$ PASSES THROUGH THE CENTER OF THE RAFT

HAVE



EQUIVALENCE REQUIRES...

$\Sigma F_y: \; -375 - 260 - 400 - 425 = -R \quad$ OR $R = 1460$ N

$\Sigma M_x: \; (3\text{ m})(375\text{ N}) + (0.5\text{ m})(260\text{ N}) + (4.75\text{ m})(400\text{ N}) + z_D(425\text{ N}) = (2.5\text{ m})(1460\text{ N})$

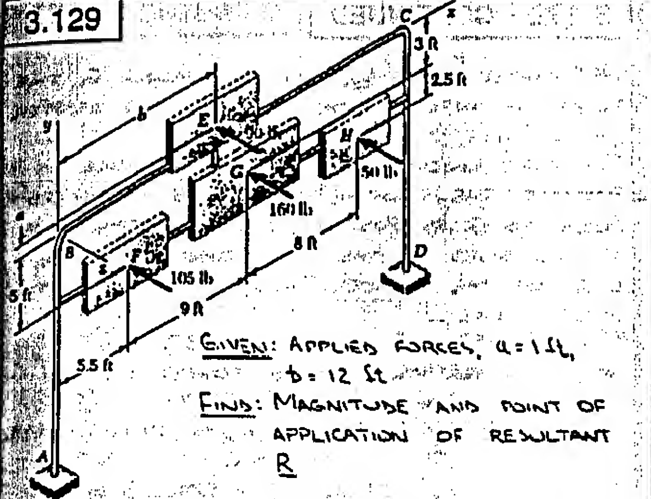
OR $z_D = 1.165$ m

$\Sigma M_z: \; -(1\text{ m})(375\text{ N}) - (1.5\text{ m})(260\text{ N}) - (4.75\text{ m})(400\text{ N}) - x_D(425\text{ N}) = -(2.5\text{ m})(1460\text{ N})$

OR $x_D = 2.32$ m

∴ THE CHILD SHOULD STAND 2.32 m FROM SIDE OG AND 1.165 m FROM SIDE OE

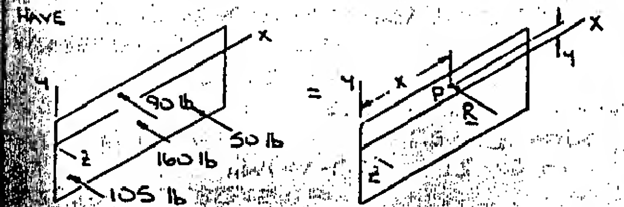
3.129



GIVEN: APPLIED FORCES, $a = 1$ ft, $b = 12$ ft

FIND: MAGNITUDE AND POINT OF APPLICATION OF RESULTANT $\underline{R}$

HAVE



ASSUME THAT THE RESULTANT $\underline{R}$ IS APPLIED AT POINT P WHOSE COORDINATES ARE $(x, y, 0)$.

EQUIVALENCE THEN REQUIRES...

$\Sigma F_z: -105 - 90 - 160 - 50 = -R$ OR $R = 405$ lb

$\Sigma M_x: (5\text{ ft})(105\text{ lb}) - (1\text{ ft})(90\text{ lb}) + (3\text{ ft})(160\text{ lb}) + (5.5\text{ ft})(50\text{ lb}) = -y(405\text{ lb})$

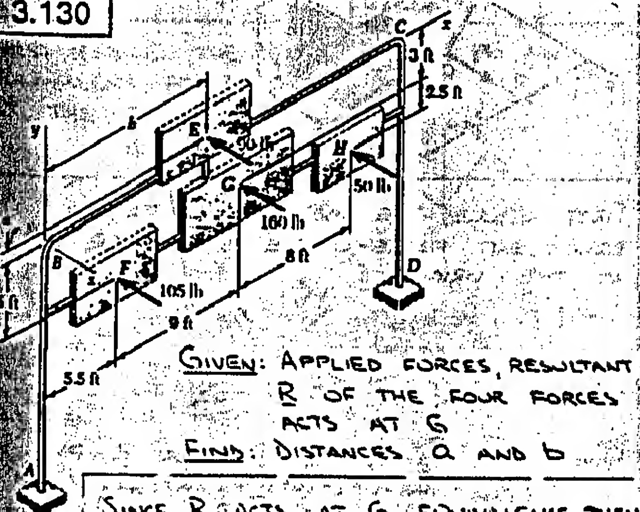
OR $y = -2.94$ ft

$\Sigma M_y: (5.5\text{ ft})(105\text{ lb}) + (12\text{ ft})(90\text{ lb}) + (14.5\text{ ft})(160\text{ lb}) + (22.5\text{ ft})(50\text{ lb}) - x(405\text{ lb})$

OR $x = 12.60$ ft

$\underline{R}$ ACTS 12.60 ft TO THE RIGHT OF MEMBER AB AND 2.94 ft BELOW MEMBER BC.

3.130



GIVEN: APPLIED FORCES, RESULTANT $\underline{R}$ OF THE FOUR FORCES ACTS AT G

FIND: DISTANCES a AND b

SINCE $\underline{R}$ ACTS AT G, EQUIVALENCE THEN REQUIRES THAT $\Sigma \underline{M}_G$ OF THE APPLIED

(CONTINUED)

3.130 CONTINUED

SYSTEM OF FORCES ALSO BE ZERO. THEN..

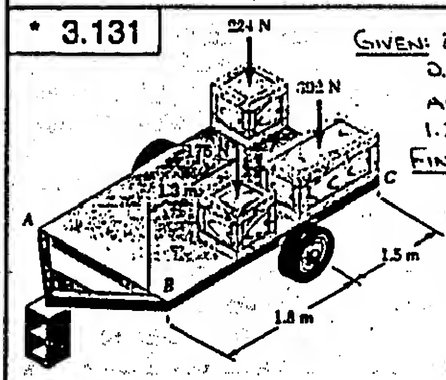
AT G: $\Sigma M_x: -(a+3)\text{ft}\cdot(90\text{ lb}) + (2\text{ ft})(105\text{ lb}) + (2.5\text{ ft})(50\text{ lb}) = 0$

OR $a = 0.722$ ft

$\Sigma M_y: -(9\text{ ft})(105\text{ ft}) - (14.5 - b)\text{ft}\cdot(90\text{ lb}) + (8\text{ ft})(50\text{ lb}) = 0$

OR $b = 20.6$ ft

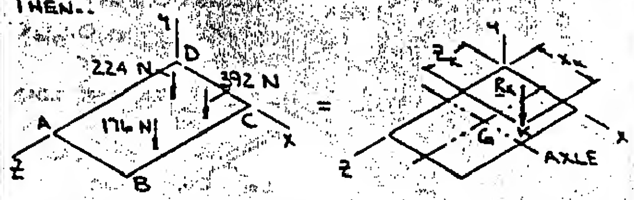
* 3.131



GIVEN: 2×3.3-m TRAILER, $0.66 \times 0.66 \times 0.66$-m AND $0.66 \times 0.66 \times 1.2$ m BOXES

FIND: SMALLEST LOAD AND LOCATION IF RESULTANT OF FOUR WEIGHTS PASSES THROUGH CENTER LINES OF AXLE AND TRAILER

FIRST REPLACE THE THREE KNOWN LOADS WITH A SINGLE EQUIVALENT FORCE $\underline{R}_K$ APPLIED AT POINT K WHOSE COORDINATES ARE $(x_K, 0, z_K)$. THEN..



EQUIVALENCE REQUIRES..

$\Sigma F_y: -224 - 392 - 176 = -R_K$ OR $\underline{R}_K = 792$ N $\downarrow$

$\Sigma M_x: (0.33\text{ m})(224\text{ N}) + (0.6\text{ m})(392\text{ N}) + (2\text{ m})(176\text{ N}) = z_K(792\text{ N})$

OR $z_K = 0.83475$ m

$\Sigma M_z: -(0.33\text{ m})(224\text{ N}) - (1.67\text{ m})(392\text{ N}) - (1.67\text{ m})(176\text{ N}) = -x_K(792\text{ N})$

OR $x_K = 1.29101$ m

FROM THE STATEMENT OF THE PROBLEM, IT IS KNOWN THAT THE RESULTANT OF $\underline{R}_K$ AND THE LIGHTEST LOAD $\underline{W}_L$ PASSES THROUGH G, THE POINT OF INTERSECTION OF THE TWO CENTER LINES. THUS, $\Sigma \underline{M}_G = 0$

FURTHER, SINCE W_L IS TO BE AS SMALL AS POSSIBLE, THE FOURTH BOX SHOULD BE PLACED AS FAR FROM G AS POSSIBLE. THESE TWO REQUIREMENTS IMPLY..

$0.33\text{ m} \le x_L \le 1.0\text{ m}$ AND $1.5\text{ m} \le z_L \le 2.97$ m

WHERE THE LOWER BOUND ON x AND THE UPPER BOUND ON z ARE IMPOSED SO THAT THE BOX DOES NOT OVERHANG THE TRAILER. SINCE THE BOX IS TO BE AS FAR FROM G AS POSSIBLE CONSIDER FIRST IF THESE BOUNDS ARE PHYSICALLY POSSIBLE.

(CONTINUED)

3.146 CONTINUED

WRENCH AND THE PRESCRIBED LINE OF ACTION (LINE AA') ARE KNOWN, IT FOLLOWS THAT THE DISTANCE a CAN BE DETERMINED. IN THE FOLLOWING SOLUTION, IT IS ASSUMED THAT a IS KNOWN. THEN, FOR EQUIVALENCE

$\Sigma F_x: \quad 0 = A\lambda_x + B_x \quad (1)$

$\Sigma F_y: \quad R = A\lambda_y + B_y \quad (2)$

$\Sigma F_z: \quad 0 = A\lambda_z + B_z \quad (3)$

$\Sigma M_x: \quad 0 = -zB_y \quad (4)$

$\Sigma M_y: \quad M = -aA\lambda_z + zB_x - xB_z \quad (5)$

$\Sigma M_z: \quad 0 = aA\lambda_y + xB_y \quad (6)$

THUS, THERE ARE SIX UNKNOWNS (A, B_x, B_y, B_z, x, z) AND SIX INDEPENDENT EQUATIONS. THEREFORE, IT WILL BE POSSIBLE TO OBTAIN A SOLUTION.

CASE 1: EQ. (4) $\Rightarrow z = 0$ ◁

NOW.. EQ. (2) $\Rightarrow A\lambda_y = R - B_y$

EQ. (3) $\Rightarrow B_z = -A\lambda_z$

EQ. (6) $\Rightarrow x = -\dfrac{aA\lambda_y}{B_y}$

$$= -\frac{a}{B_y}(R - B_y)$$

SUBSTITUTING INTO EQ. (5)..

$$M = -aA\lambda_z - \left[-\frac{a}{B_y}(R - B_y)\right](-A\lambda_z)$$

OR $\quad A = -\dfrac{1}{\lambda_z}\dfrac{M}{aR}B_y$

SUBSTITUTING INTO EQ (2)..

$$R = -\frac{1}{\lambda_z}\frac{M}{aR}B_y(\lambda_y) + B_y$$

OR $\quad B_y = \dfrac{\lambda_z aR^2}{\lambda_z aR - \lambda_y M}$

THEN.. $\quad A = -\dfrac{MR}{\lambda_z aR - \lambda_y M}$

$$B_x = \frac{\lambda_x MR}{\lambda_z aR - \lambda_y M}$$

$$B_z = \frac{\lambda_z MR}{\lambda_z aR - \lambda_y M}$$

IN SUMMARY..

$$\underline{A} = \frac{R}{\lambda_y - \frac{aR}{M}\lambda_z}\underline{\lambda} \quad ◁$$

$$\underline{B} = \frac{R}{\lambda_z aR - \lambda_y M}(\lambda_x M\underline{i} + \lambda_z aR\underline{j} + \lambda_z M\underline{k}) \quad ◁$$

ALSO.. $\quad x = a\left(1 - \dfrac{R}{B_y}\right) = a\left(1 - \dfrac{R}{\frac{\lambda_z aR^2}{\lambda_z aR - \lambda_y M}}\right)$

$$= \frac{\lambda_y}{\lambda_z}\frac{M}{R} \quad ◁$$

NOTE THAT FOR THIS CASE, THE LINES OF ACTION OF BOTH $\underline{A}$ AND $\underline{B}$ INTERSECT THE x AXIS.

CASE 2: EQ. (4) $\Rightarrow B_y = 0$

THEN EQ. (2) $\Rightarrow A = \dfrac{R}{\lambda_y}$

(CONTINUED)

3.146 CONTINUED

AND EQ. (1) $\Rightarrow B_x = -\dfrac{R}{\lambda_y}\lambda_x$

EQ. (3) $\Rightarrow B_z = -\dfrac{R}{\lambda_y}\lambda_z$

EQ. (6) $\Rightarrow aA\lambda_y = 0$ .. WHICH REQUIRES THAT $a = 0$

THEN, SUBSTITUTING INTO EQ. (5)..

$$M = z\left(-\frac{R}{\lambda_y}\lambda_x\right) - x\left(-\frac{R}{\lambda_y}\lambda_z\right)$$

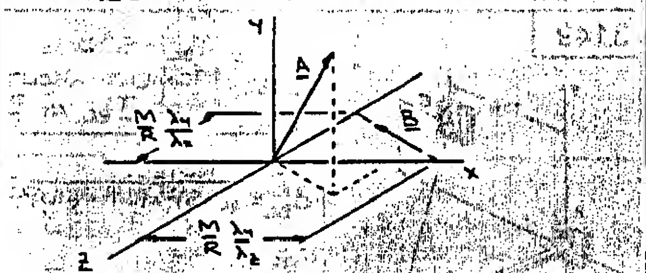
OR $\quad \lambda_z x - \lambda_x z = \dfrac{M}{R}\lambda_y$

THIS LAST EXPRESSION IS THE EQUATION OF THE LINE OF ACTION OF FORCE $\underline{B}$. IN SUMMARY..

$$\underline{A} = \frac{R}{\lambda_y}\underline{\lambda} \quad ◁$$

$$\underline{B} = \frac{R}{\lambda_y}(-\lambda_x\underline{i} - \lambda_z\underline{k}) \quad ◁$$

ASSUMING THAT $\lambda_x, \lambda_y, \lambda_z > 0$, THE EQUIVALENT FORCE SYSTEM IS..

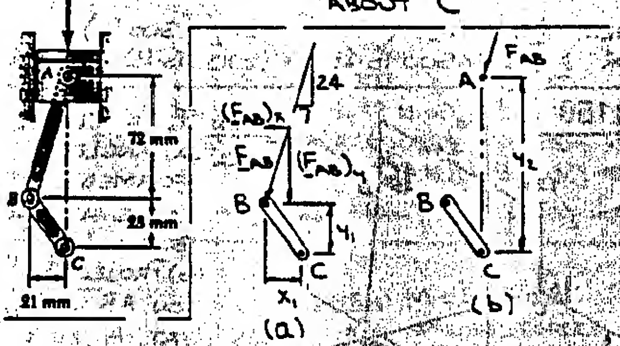


NOTE THAT THE COMPONENT OF $\underline{A}$ LYING IN THE xz PLANE IS PARALLEL TO $\underline{B}$.

3.147

GIVEN: $F_{AB} = 1.5$ kN

FIND: MOMENT OF $\underline{F}_{AB}$ ABOUT C



USING (a)

$$M_C = y_1(F_{AB})_x + x_1(F_{AB})_y$$

$$= (0.028\text{ m})\left(\tfrac{7}{25} \times 1500\text{ N}\right) + (0.021\text{ m})\left(\tfrac{24}{25} \times 1500\text{ N}\right)$$

$$= 42\text{ N}\cdot\text{m}$$

OR $\underline{M}_C = 42$ N·m ↺ ◀

USING (b)

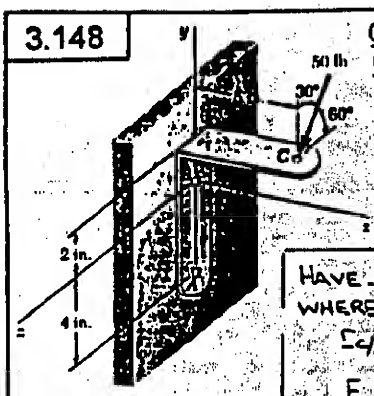
$$M_C = y_2(F_{AB})_x$$

$$= (0.1\text{ m})\left(\tfrac{7}{25} \times 1500\text{ N}\right)$$

$$= 42\text{ N}\cdot\text{m}$$

OR $\underline{M}_C = 42$ N·m ↺ ◀

3.148



GIVEN: FORCE $\underline{F}_C$

FIND: MOMENT OF $\underline{F}_C$ ABOUT A

HAVE.. $\underline{M}_A = \underline{r}_{C/A} \times \underline{F}_C$

WHERE

$\underline{r}_{C/A} = (5 \text{ in.})\underline{i} + (6 \text{ in.})\underline{j}$

$\underline{F}_C = -(50 \text{ lb})\cos 30°\,\underline{j} + (50 \text{ lb})\sin 30°\,\underline{k}$

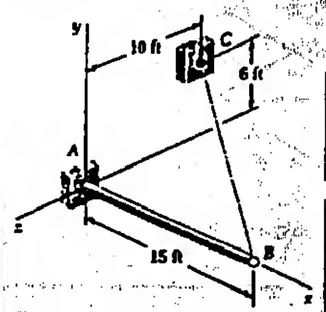
THEN..

$$\underline{M}_A = 50 \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 5 & 6 & 0 \\ 0 & -\cos 30° & \sin 30° \end{vmatrix}$$

$$= 50[(6 \sin 30°)\underline{i} - (5 \sin 30°)\underline{j} - (5 \cos 30°)\underline{k}]$$

OR $\underline{M}_A = (150 \text{ lb·in.})\underline{i} - (125 \text{ lb·in.})\underline{j} - (217 \text{ lb·in.})\underline{k}$ ◀

3.149



GIVEN: $T_{BC} = 570$ lb

FIND: MOMENT ABOUT A OF $\underline{T}_{BC}$ AT B

FIRST NOTE..

$d_{BC} = \sqrt{(-15)^2 + (6)^2 + (-10)^2} = 19$ ft

THEN..

$\underline{T}_{BC} = \frac{570 \text{ lb}}{19}(-15\underline{i} + 6\underline{j} - 10\underline{k})$

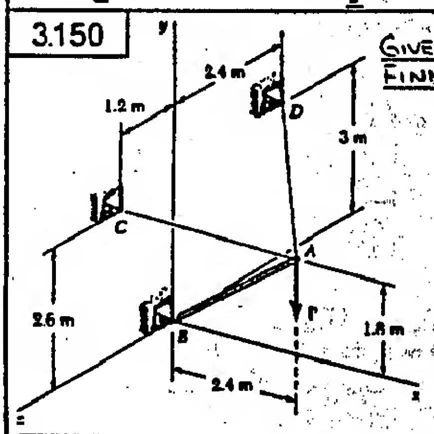
$= -(450 \text{ lb})\underline{i} + (180 \text{ lb})\underline{j} - (300 \text{ lb})\underline{k}$

HAVE.. $\underline{M}_A = \underline{r}_{B/A} \times \underline{T}_{BC}$ WHERE $\underline{r}_{B/A} = (15 \text{ ft})\underline{i}$

THEN.. $\underline{M}_A = 15\underline{i} \times (-450\underline{i} + 180\underline{j} - 300\underline{k})$

OR $\underline{M}_A = (4500 \text{ lb·ft})\underline{j} + (2700 \text{ lb·ft})\underline{k}$ ◀

3.150



GIVEN: $T_{AC} = 1260$ N

FIND: (a) ANGLE θ FORMED BY CABLE AC AND BOOM AB

(b) PROJECTION ON AB OF $\underline{T}_{AC}$ AT A

(a) FIRST NOTE.. $AC = \sqrt{(-2.4)^2 + (0.8)^2 + (1.2)^2}$

$= 2.8$ m

$AB = \sqrt{(-2.4)^2 + (-1.8)^2 + (0)^2}$

$= 3.0$ m

AND $\overrightarrow{AC} = -(2.4 \text{ m})\underline{i} + (0.8 \text{ m})\underline{j} + (1.2 \text{ m})\underline{k}$

$\overrightarrow{AB} = -(2.4 \text{ m})\underline{i} - (1.8 \text{ m})\underline{j}$

(CONTINUED)

3.150 CONTINUED

BY DEFINITION.. $\overrightarrow{AC} \cdot \overrightarrow{AB} = (AC)(AB)\cos\theta$

OR $(-2.4\underline{i} + 0.8\underline{j} + 1.2\underline{k}) \cdot (-2.4\underline{i} - 1.8\underline{j}) = (2.8)(3.0) \times \cos\theta$

OR $(-2.4)(-2.4) + (0.8)(-1.8) + (1.2)(0) = 8.4\cos\theta$

OR $\cos\theta = 0.51429$

OR $\theta = 59.0°$ ◀

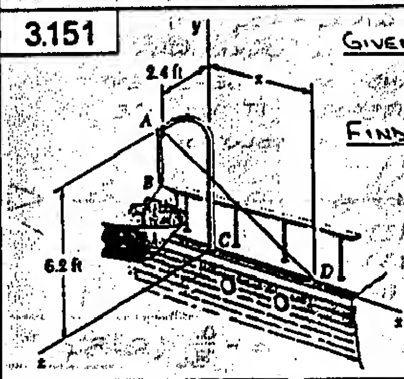
(b) HAVE.. $(T_{AC})_{AB} = \underline{T}_{AC} \cdot \underline{\lambda}_{AB}$

$= T_{AC}\cos\theta$

$= (1260 \text{ N})(0.51429)$

OR $(T_{AC})_{AB} = 648$ N ◀

3.151



GIVEN: $|M_z|$ OF $\underline{B}_A$ AT A ≤ 160 lb·ft, x = 4.8 ft

FIND: T_{MAX}

FIRST NOTE THAT $\underline{B}_A = 2\underline{T}_{AB} + \underline{T}_{AD}$ AND THEN OBSERVE THAT ONLY $\underline{T}_{AD}$ WILL CONTRIBUTE TO THE MOMENT ABOUT THE z AXIS. NOW..

$d_{AD} = \sqrt{(4.8)^2 + (-6.2)^2 + (-2.4)^2} = 8.2$ ft

THEN $\underline{T}_{AD} = \frac{T}{8.2}(4.8\underline{i} - 6.2\underline{j} - 2.4\underline{k})$

$= \frac{T}{41}(24\underline{i} - 31\underline{j} - 12\underline{k})$

NOW.. $M_z = \underline{k} \cdot (\underline{r}_{A/C} \times \underline{T}_{AD})$

WHERE $\underline{r}_{A/C} = (6.2 \text{ ft})\underline{j} + (2.4 \text{ ft})\underline{k}$

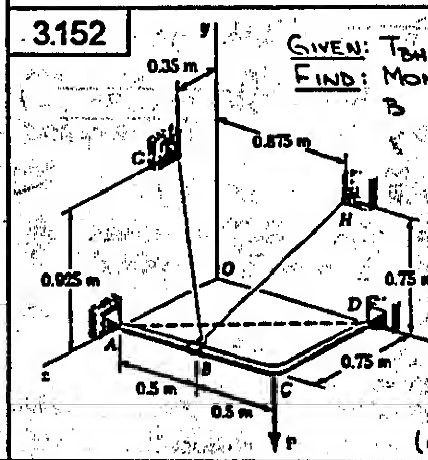
THEN FOR T_{MAX}..

$$160 = \left|\frac{T_{MAX}}{41}\begin{vmatrix} 0 & 0 & 1 \\ 0 & 6.2 & 2.4 \\ 24 & -31 & -12 \end{vmatrix}\right|$$

$$= \frac{T_{MAX}}{41}\left|-(1)(6.2)(24)\right|$$

OR $T_{MAX} = 44.1$ lb ◀

3.152



GIVEN: $T_{BH} = 450$ N

FIND: MOMENT OF $\underline{T}_{BH}$ AT B ABOUT DIAGONAL AD

(CONTINUED)

3.152 CONTINUED

$M_{AD} = \lambda_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BH})$

WHERE $\lambda_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$

$\mathbf{r}_{B/A} = (0.5\text{ m})\mathbf{i}$

AND $d_{BH} = \sqrt{(0.375)^2 + (0.75)^2 + (-0.75)^2} = 1.125\text{ m}$

THEN $\mathbf{T}_{BH} = \dfrac{450\text{ N}}{1.125}(0.375\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k})$

$= (150\text{ N})\mathbf{i} + (300\text{ N})\mathbf{j} - (300\text{ N})\mathbf{k}$

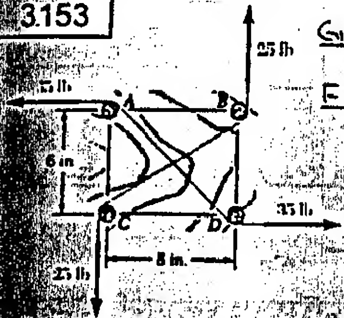
FINALLY..

$$M_{AD} = \frac{1}{5}\begin{vmatrix} 4 & 0 & -3 \\ 0.5 & 0 & 0 \\ 150 & 300 & -300 \end{vmatrix}$$

$= \frac{1}{5}[(-3)(0.5)(300)]$

OR $M_{AD} = -90\text{ N·m}$ ◀

3.153



GIVEN: APPLIED FORCES, $\mathbf{M} = 485$ lb·in. ↻

FIND: DIAMETER d OF THE PEGS

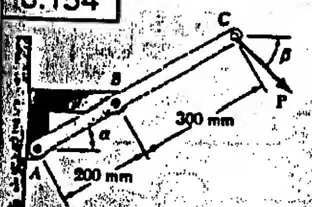
HAVE.. $\mathbf{M} = \mathbf{M}_{AD} + \mathbf{M}_{BC}$

OR $M = d_{AD}F_{AD} + d_{BC}F_{BC}$

OR $485\text{ lb·in.} = [(6+d)\text{ in.}](35\text{ lb}) + [(8+d)\text{ in.}](25\text{ lb})$

OR $d = 1.25\text{ in.}$ ◀

3.154



GIVEN: $P = 250\text{ N}$, $\alpha = 30°$, $\beta = 60°$

FIND: (a) EQUIVALENT FORCE-COUPLE SYSTEM $(\mathbf{F}, \mathbf{M})$ AT B

(b) EQUIVALENT SYSTEM $(\mathbf{F}_A, \mathbf{F}_B)$, WHERE $\mathbf{F}_A$ AND $\mathbf{F}_B$ ARE PARALLEL

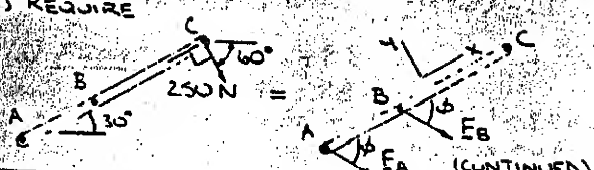
(a) EQUIVALENCE REQUIRES..

$\Sigma\mathbf{F}$: $\mathbf{F} = \mathbf{P}$ OR $\mathbf{F} = 250\text{ N} \measuredangle 60°$

ΣM_B: $M = -(0.3\text{ m})(250\text{ N}) = -75\text{ N·m}$

∴ THE EQUIVALENT FORCE-COUPLE SYSTEM AT B IS.. $\mathbf{F} = 250\text{ N} \measuredangle 60°$, $\mathbf{M} = 75\text{ N·m}$ ↻ ◀

(b) REQUIRE



(CONTINUED)

3.154 CONTINUED

EQUIVALENCE THEN REQUIRES..

ΣF_x: $0 = F_A\cos\phi + F_B\cos\phi$

∴ $F_A = -F_B$ OR $\cos\phi = 0$

ΣF_y: $-250 = -F_A\sin\phi - F_B\sin\phi$

NOW.. IF $F_A = -F_B \Rightarrow -250 = 0$ .. REJECT

∴ $\cos\phi = 0$

OR $\phi = 90°$

AND $F_A + F_B = 250$

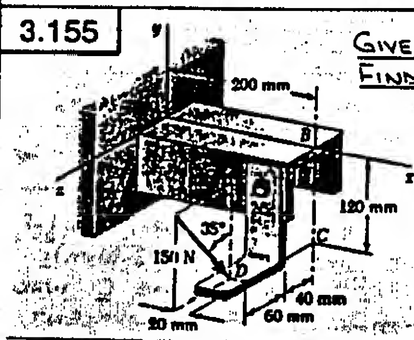
ALSO.. ΣM_B: $-(0.3\text{ m})(250\text{ N}) = (0.2\text{ m})F_A$

OR $F_A = -375\text{ N}$

AND $F_B = 625\text{ N}$

∴ $\mathbf{F}_A = 375\text{ N} \measuredangle 60°$, $\mathbf{F}_B = 625\text{ N} \measuredangle 60°$ ◀

3.155



GIVEN: 150-N FORCE

FIND: EQUIVALENT FORCE-COUPLE SYSTEM $(\mathbf{F}, \mathbf{M})$ AT A

EQUIVALENCE REQUIRES..

$\Sigma\mathbf{F}$: $\mathbf{F} = (150\text{ N})(-\cos 35°\,\mathbf{j} - \sin 35°\,\mathbf{k})$

$= -(122.873\text{ N})\mathbf{j} - (86.036\text{ N})\mathbf{k}$

$\Sigma\mathbf{M}_A$: $\mathbf{M} = \mathbf{r}_{D/A} \times \mathbf{F}$

WHERE $\mathbf{r}_{D/A} = (0.18\text{ m})\mathbf{i} - (0.12\text{ m})\mathbf{j} + (0.1\text{ m})\mathbf{k}$

THEN..

$$\mathbf{M} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.18 & -0.12 & 0.1 \\ 0 & -122.873 & -86.036 \end{vmatrix}$$

$= [(-0.12)(-86.036) - (0.1)(-122.873)]\mathbf{i} + [-(0.18)(-86.036)]\mathbf{j} + [(0.18)(-122.873)]\mathbf{k}$

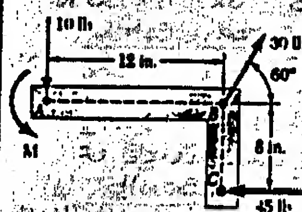
$= (22.6\text{ N·m})\mathbf{i} + (15.49\text{ N·m})\mathbf{j} - (22.1\text{ N·m})\mathbf{k}$

∴ THE EQUIVALENT FORCE-COUPLE SYSTEM AT A IS..

$\mathbf{F} = -(122.9\text{ N})\mathbf{j} - (86.0\text{ N})\mathbf{k}$ ◀

$\mathbf{M} = (22.6\text{ N·m})\mathbf{i} + (15.49\text{ N·m})\mathbf{j} - (22.1\text{ N·m})\mathbf{k}$ ◀

3.156



GIVEN: $M = 54$ lb·in., APPLIED FORCES

FIND: (a) RESULTANT $\mathbf{R}$

(b) POINTS WHERE LINE OF ACTION OF $\mathbf{R}$ INTERSECTS LINES AB AND BC

(a) HAVE.. $\Sigma\mathbf{F}$: $\mathbf{R} = (-10\mathbf{j}) + (30\cos 60°\,\mathbf{i} + 30\sin 60°\,\mathbf{j}) + (-45\mathbf{i})$

$= -(30\text{ lb})\mathbf{i} + (15.9808\text{ lb})\mathbf{j}$

OR $\mathbf{R} = 34.0\text{ lb} \measuredangle 28.0°$ ◀

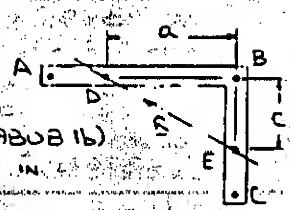
(CONTINUED)

3.156 CONTINUED

(b) FIRST REDUCE THE GIVEN FORCES AND COUPLE TO AN EQUIVALENT FORCE-COUPLE SYSTEM $(\underline{R}, \underline{M}_B)$ AT B. HAVE..

ΣM_B: $M_B = (54 \text{ lb}\cdot\text{in.}) + (12 \text{ in.})(10 \text{ lb}) - (8 \text{ in.})(45 \text{ lb})$

$= -186 \text{ lb}\cdot\text{in.}$



THEN WITH $\underline{R}$ AT D..

ΣM_B: $-186 \text{ lb}\cdot\text{in} = a(15.9808 \text{ lb})$

OR $a = 11.64$ IN.

AND WITH $\underline{R}$ AT E

ΣM_B: $-186 \text{ lb}\cdot\text{in} = C(30 \text{ lb})$

OR $C = 6.2$ IN

$\therefore$ THE LINE OF ACTION OF $\underline{R}$ INTERSECTS LINE AB 11.64 IN. TO THE LEFT OF B AND INTERSECTS LINE BC 6.2 IN. BELOW B.

3.157

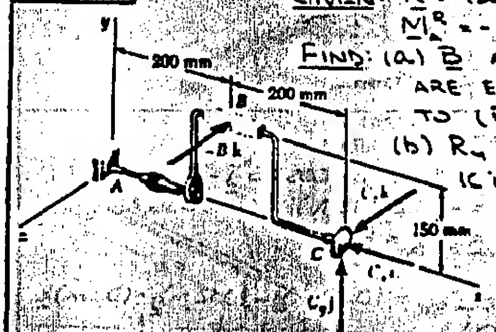
GIVEN: $\underline{R} = -(30\text{N})\underline{i} + R_y\underline{j} - R_z\underline{k}$

$\underline{M}_A^R = -(12 \text{ N}\cdot\text{m})\underline{i}$

FIND: (a) $\underline{B}$ AND $\underline{C}$ WHICH ARE EQUIVALENT TO $(\underline{R}, \underline{M}_A^R)$

(b) R_y AND R_z

(c) ORIENTATION OF SLOT TO MINIMIZE SLIPPING



(a) EQUIVALENCE REQUIRES..

$\Sigma \underline{F}$: $\underline{R} = \underline{B} + \underline{C}$

OR.. $-(30 \text{ N})\underline{i} + R_y\underline{j} - R_z\underline{k} = -B\underline{k} - (C_x\underline{i} + C_y\underline{j} + C_z\underline{k})$

EQUATING THE $\underline{i}$ COEFFICIENTS..

$\underline{i}$: $-30\text{N} = -C_x$ OR $C_x = 30$ N

ALSO.. $\Sigma \underline{M}_A$: $\underline{M}_A^R = \underline{r}_{B/A} \times \underline{B} + \underline{r}_{C/A} \times \underline{C}$

OR.. $-(12 \text{ N}\cdot\text{m})\underline{i} = [(0.2\text{m})\underline{i} + (0.15\text{m})\underline{j}] \times (-B)\underline{k} + (0.4\text{m})\underline{i} \times [-(40\text{N})\underline{i} + C_y\underline{j} + C_z\underline{k}]$

EQUATING COEFFICIENTS..

$\underline{i}$: $-12 \text{ N}\cdot\text{m} = -(0.15\text{m})B$ OR $B = 80$ N

$\underline{k}$: $0 = (0.4\text{m})C_y$ OR $C_y = 0$

$\underline{j}$: $0 = (0.2\text{m})(80\text{N}) - (0.4\text{m})C_z$ OR $C_z = 40\text{N}$

$\therefore$ $\underline{B} = -(80\text{N})\underline{k}$ $\underline{C} = -(30\text{N})\underline{i} - (40\text{N})\underline{k}$

(b) NOW HAVE FOR THE EQUIVALENCE OF FORCES.. $-(30\text{N})\underline{i} + R_y\underline{j} + R_z\underline{k} = -(80\text{N})\underline{k} + [-(30\text{N})\underline{i} - (40\text{N})\underline{k}]$

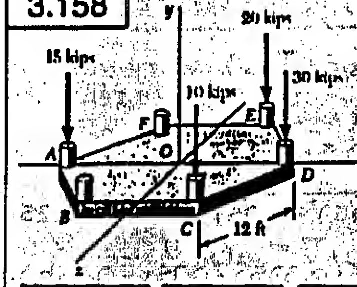
EQUATING COEFFICIENTS..

$\underline{j}$: $R_y = 0$ $R_y = 0$

$\underline{k}$: $R_z = -80 + 40$ OR $R_z = -40\text{N}$

(C) FIRST NOTE THAT $\underline{R} = -(30\text{N})\underline{i} - (40\text{N})\underline{k}$. THUS, THE SCREW IS BEST ABLE TO RESIST THE LATERAL FORCE R_z WHEN THE SLOT IN THE HEAD OF THE SCREW IS VERTICAL.

3.158



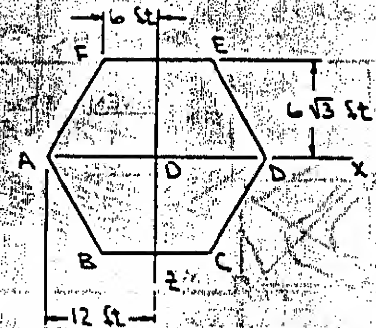
GIVEN: RESULTANT $\underline{R}$ PASSES THROUGH POINT O

FIND: VERTICAL FORCES F_B AND F_E

FROM THE STATEMENT OF THE PROBLEM IT CAN BE CONCLUDED THAT THE SIX APPLIED LOADS ARE EQUIVALENT TO THE RESULTANT $\underline{R}$ AT O. IT THEN FOLLOWS THAT

$\Sigma \underline{M}_O = 0$ OR $\Sigma M_x = 0$, $\Sigma M_z = 0$

FOR THE APPLIED LOADS.



THEN..

$\Sigma M_x = 0$: $(6\sqrt{3}\text{ft})F_B + (6\sqrt{3}\text{ft})(10 \text{ kips}) - (6\sqrt{3}\text{ft})(20 \text{ kips}) - (6\sqrt{3}\text{ft})F_F = 0$

OR $F_B - F_F = 10$ (1)

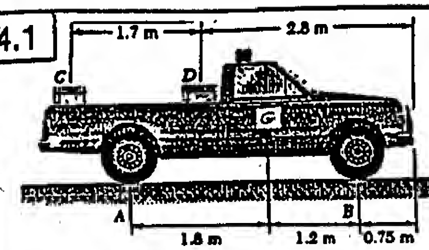
$\Sigma M_z = 0$: $(12\text{ft})(15 \text{ kips}) + (6\text{ft})F_B - (6\text{ft})(10 \text{ kips}) - (12\text{ft})(30 \text{ kips}) - (6\text{ft})(20 \text{ kips}) + (6\text{ft})F_F = 0$

OR $F_B + F_F = 60$ (2)

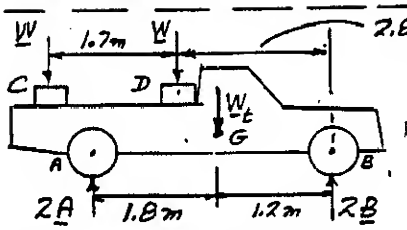
THEN (1) + (2) $\Rightarrow$ $\underline{F}_B = 35 \text{ kips} \downarrow$

AND $\underline{F}_F = 25 \text{ kips} \downarrow$

4.1



GIVEN: TRUCK, 1400 kg
CRATES, 350 kg (EACH)
FIND: REACTIONS AT EACH WHEEL



FREE BODY DIAGRAM

$W = (350\ kg)(9.81\ m/s^2) = 3.434\ kN$

$W_t = (1400\ kg)(9.81\ m/s^2) = 13.734\ kN$

(a) REAR WHEELS. $+\circlearrowleft \Sigma M_B = 0$

$W(1.7m + 2.05m) + W(2.05m) + W_t(1.2m) - 2A(3m) = 0$

$(3.434\ kN)(3.75m) + (3.434\ kN)(2.05m) + (13.734\ kN)(1.2m) - 2A(3m) = 0$

$A = +6.066\ kN$ $\qquad$ $A = 6.07\ kN \uparrow$ ◄

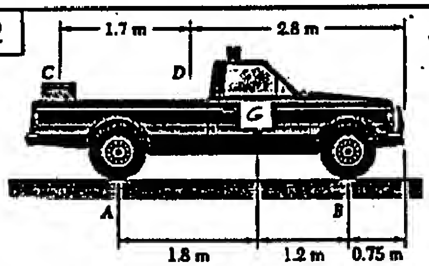
(b) FRONT WHEELS. $+\uparrow \Sigma F_y = 0$

$-W - W - W_t + 2A + 2B = 0$

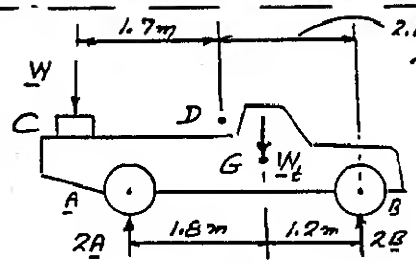
$-3.434\ kN - 3.434\ kN - 13.734\ kN + 2(6.066\ kN) + 2B = 0$

$B = +4.235\ kN$ $\qquad$ $B = 4.24\ kN \uparrow$ ◄

4.2



GIVEN: TRUCK: 1400 kg
CRATE C: 350 kg
(CRATE D HAS BEEN REMOVED)
FIND: REACTIONS AT EACH WHEEL



FREE-BODY DIAGRAM

$W = (350\ kg)(9.81\ m/s^2) = 3.434\ kN$

$W_t = (1400\ kg)(9.81\ m/s^2) = 13.734\ kN$

(a) REAR WHEELS $+\circlearrowleft \Sigma M_B = 0$

$W(1.7m + 2.05m) + W_t(1.2m) - 2A(3m) = 0$

$(3.434\ kN)(3.75\ m) + (13.734\ kN)(1.2m) - 2A(3m) = 0$

$A = +4.893\ kN$ $\qquad$ $A = 4.89\ kN \uparrow$ ◄

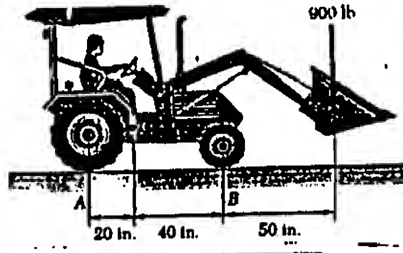
(b) FRONT WHEELS $+\uparrow \Sigma F_y = 0$

$-W - W_t + 2A + 2B = 0$

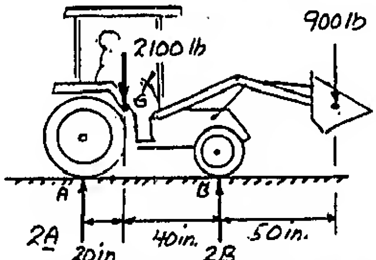
$-3.434\ kN - 13.734\ kN + 2(4.893\ kN) + 2B = 0$

$B = +3.691\ kN$ $\qquad$ $B = 3.69\ kN \uparrow$ ◄

4.3



GIVEN: 2100-lb TRACTOR
FIND: REACTIONS AT EACH WHEEL



a) REAR WHEELS $+\circlearrowleft \Sigma M_B = 0$

$+(2100\ lb)(40\ in.) - (900\ lb)(50\ in.) + 2A(60\ in.) = 0$

$A = +325\ lb$ $\qquad$ $A = 325\ lb \uparrow$ ◄

b) FRONT WHEELS $+\circlearrowleft \Sigma M_A$

$(2100\ lb)(20\ in.) - (900\ lb)(110\ in.) - 2B(60\ in.) = 0$

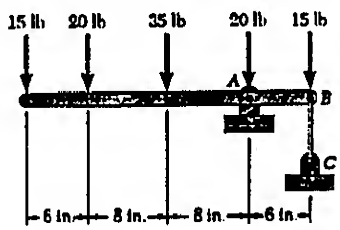
$B = +1175\ lb$ $\qquad$ $B = 1175\ lb \uparrow$ ◄

CHECK: $+\Sigma F_y = 0$: $2A + 2B - 2100\ lb - 900\ lb = 0$

$2(325\ lb) + 2(1175\ lb) - 2100\ lb - 900 = 0$

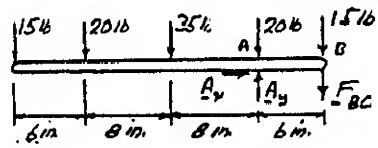
$0 = 0$ $\qquad$ (CHECKS)

4.4



FIND: (a) REACTION AT A
(b) TENSION IN CABLE BC

FREE-BODY DIAGRAM



(a) REACTION AT A: $\Sigma F_x = 0$ $\quad A_x = 0$

$+\circlearrowleft \Sigma M_B = 0$:

$(15\ lb)(28\ in.) + (20\ lb)(22\ in.) + (35\ lb)(14\ in.) + (20\ lb)(6\ in.) - A_y(6\ in.) = 0$

$A_y = +245\ lb$ $\qquad$ $A = 245\ lb \uparrow$ ◄

(b) TENSION IN BC

$+\circlearrowleft \Sigma M_A = 0$

$(15\ lb)(22\ in.) + (20\ lb)(16\ in.) + (35\ lb)(8\ in.) - (15\ lb)(6\ in.) - F_{BC}(6\ in.) = 0$

$F_{BC} = +140\ lb$ $\qquad$ $F_{BC} = 140\ lb$ ◄

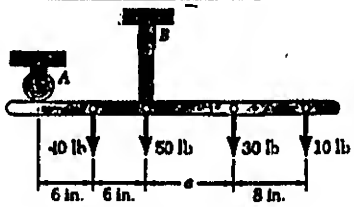
CHECK: $+\uparrow \Sigma F_y = 0$:

$-15\ lb - 20\ lb - 35\ lb - 15\ lb + A - F_{BC} = 0$

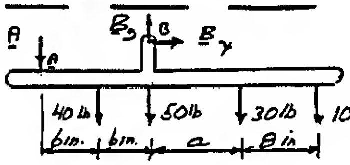
$-105\ lb + 245\ lb - 140\ lb = 0$

$0 = 0$ $\qquad$ (CHECKS)

4.5



FIND: REACTIONS AT A AND B



FREE-BODY DIAGRAM

$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$

$+\circlearrowleft \Sigma M_B = 0: (40 lb)(6 in.) - (30 lb)a - (10 lb)(a + 8 in.) + (12 in.)A = 0$

$A = (40a - 160)/12 \qquad (1)$

$+\circlearrowleft \Sigma M_A = 0:$

$-(40 lb)(6 in.) - (50 lb)(12 in.) - (30 lb)(a + 12 in.) - (10 lb)(a + 20 in.) + (12 in.)B_y = 0$

$B_y = (1400 + 40a)/12$

SINCE $B_x = 0$, $\quad B = (1400 + 40a)/12 \qquad (2)$

(a) FOR $a = 10$ in.

EQ (1): $A = (40 \times 10 - 160)/12 = +20$ lb $\qquad A = 20$ lb$\downarrow$ ◀

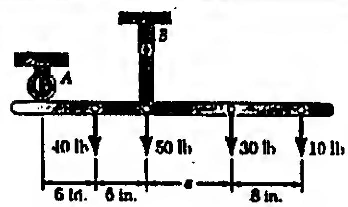
EQ. (2): $B = (1400 + 40 \times 10)/12 = +150$ lb $\qquad B = 150$ lb$\uparrow$ ◀

(b) FOR $a = 7$ in.

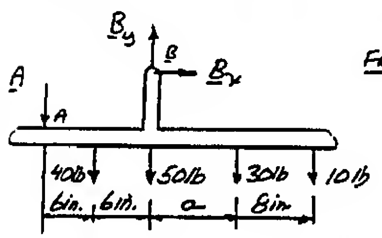
EQ. (1): $A = (40 \times 7 - 160)/12 = +10$ lb $\qquad A = 10$ lb$\downarrow$ ◀

EQ. (2): $B = (1400 + 40 \times 7)/12 = +140$ lb $\qquad B = 140$ lb$\uparrow$ ◀

4.6



FIND: SMALLEST DISTANCE a FOR NO MOTION



FREE-BODY DIAGRAM

FOR NO MOTION REACTION AT A MUST BE DOWNWARD OR ZERO SMALLEST DISTANCE a FOR NO MOTION CORRESPONDS TO $A = 0$

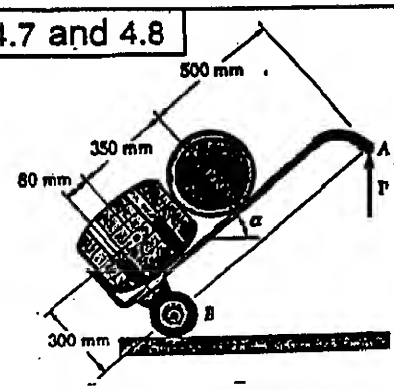
$+\circlearrowleft \Sigma M_B = 0$

$(40 lb)(6 in.) - (30 lb)a - (10 lb)(a + 8 in.) + (12 in.)A = 0$

$A = (40a - 160)/12$

$A = 0: \quad (40a - 160) = 0 \qquad a = 4$ in. ◀

4.7 and 4.8

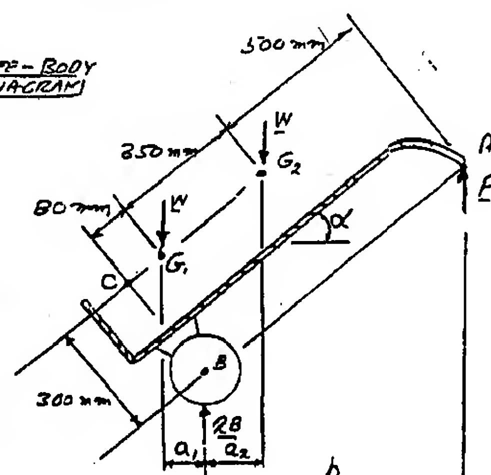


MASS OF EACH KEG IS 40 kg

FIND:
(a) P
(b) REACTION AT EACH WHEEL

FOR EACH KEG: $W = mg = (40 kg)(9.81 m/s^2) = 392.4$ N

FREE-BODY DIAGRAM



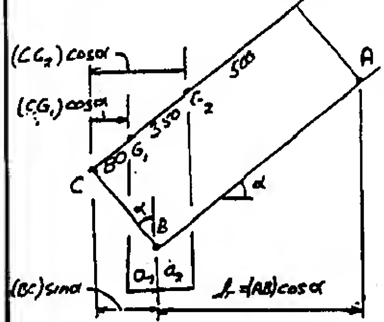
$+\circlearrowleft \Sigma M_B = 0: \; Wa_1 - Wa_2 + Pb = 0 \qquad P = W(a_2 - a_1)/b \qquad (1)$

$+\uparrow \Sigma F_y = 0: \; -W - W + P + 2B = 0 \qquad B = W - \frac{1}{2}P \qquad (2)$

GEOMETRY FIND a_1 and a_2 IN TERMS OF α

$BC = 300$ mm, $CG_1 = 80$ mm, $CG_2 = 80 + 350 = 430$ mm

$AB = 80 + 350 + 500 = 930$ mm



$a_1 = (BC)\sin\alpha - (CG_1)\cos\alpha$

$a_1 = 300\sin\alpha - 80\cos\alpha$

$a_2 = (CG_2)\cos\alpha - (BC)\sin\alpha$

$= 430\cos\alpha - 300\sin\alpha$

$b = (AB)\cos\alpha = 930\cos\alpha$

EQ(1): $P = W(a_2 - a_1)/b$

$P = W[(430\cos\alpha - 300\sin\alpha) - (300\sin\alpha - 80\cos\alpha)]/930\cos\alpha$

$= (392.4 N)(510\cos\alpha - 600\sin\alpha)/930\cos\alpha$

$P = (392.4)(0.5484 - 0.6452\tan\alpha)$

PROB. 4.7 $\quad \alpha = 35°$;

$P = 392.4(0.5484 - 0.6452\tan 35°) = +37.9$ N $\qquad P = 37.9$ N$\uparrow$ ◀

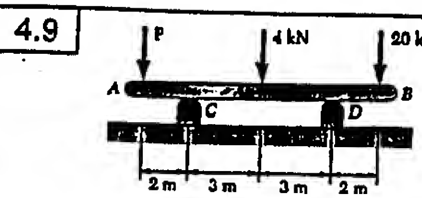
EQ(2) $B = W - \frac{1}{2}P = 392.4 N - \frac{1}{2}(37.9 N) = +374$ N $\qquad B = 374$ N$\uparrow$ ◀

PROB 4.8 $\quad \alpha = 40°$;

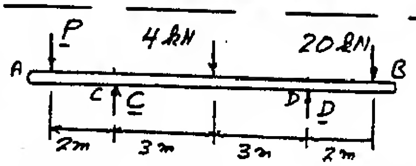
$P = 392.4(0.5484 - 0.6452\tan 40°) = +2.76$ N $\qquad P = 2.76$ N$\uparrow$ ◀

EQ(2): $B = W - \frac{1}{2}P = 392 N - \frac{1}{2}(2.76 N) = +391$ N $\qquad B = 391$ N$\uparrow$ ◀

4.9



FIND: RANGE OF VALUES OF P FOR EQUILIBRIUM



FREE-BODY DIAGRAM

$+\circlearrowleft \Sigma M_C = 0$: $P(2m) - (4kN)(3m) - (20kN)(8m) + D(6m) = 0$

$P = 86kN - 3D$ (1)

$+\circlearrowleft \Sigma M_D = 0$: $P(8m) + (4kN)(3m) - (20kN)(2m) - C(6m) = 0$

$P = 3.5kN + 0.75C$ (2)

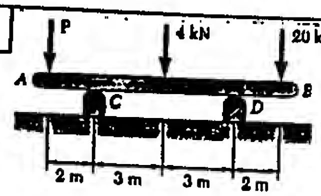
FOR NO MOTION $C \geq 0$ AND $D \geq 0$

FOR $C \geq 0$ FROM (2) $P \leq 3.5kN$

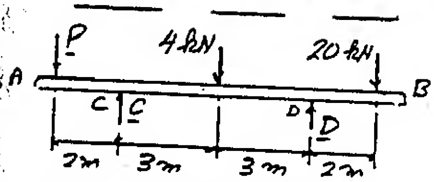
FOR $D \geq 0$ FROM (1) $P \leq 86kN$

RANGE OF P FOR NO MOTION: $3.5kN \leq P \leq 86kN$

4.10



FIND: RANGE OF VALUES OF P IF REACTIONS MUST BE $\leq 50kN$ AND BE DIRECTED UPWARD



FREE-BODY DIAGRAM

$+\circlearrowleft \Sigma M_C = 0$: $P(2m) - (4kN)(3m) - (20kN)(8m) + D(6m) = 0$

$P = 86kN - 3D$ (1)

$+\circlearrowleft \Sigma M_D = 0$: $P(8m) + (4kN)(3m) - (20kN)(2m) - C(6m) = 0$

$P = 3.5kN + 0.75C$ (2)

For $C \geq 0$, FROM (2): $P \geq 3.50 kN$

For $D \geq 0$, FROM (1): $P \leq 86 kN$

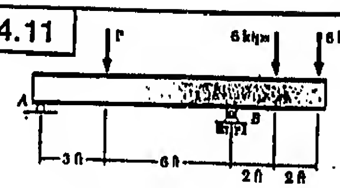
For $C \leq 50kN$, FROM (2):
$P \leq 3.5kN + 0.75(50kN)$
$P \leq 41kN$

For $D \leq 50kN$, FROM (1):
$P \geq 86kN - 3(50kN)$
$P \geq -64kN$

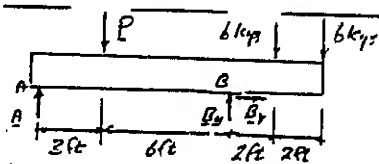
COMPARING THE FOUR CRITERIA, WE FIND

$3.50 kN \leq P \leq 41kN$

4.11



FIND: P_{MAX} FOR REACTIONS < 30 kips AND REACTION AT A UPWARD



$\Sigma F_x = 0$: $B_x = 0$
$\therefore B = B_y \uparrow$

$+\circlearrowleft \Sigma M_A = 0$: $-P(3ft) + B(9ft) - (6kips)(11ft) - (6kips)(13ft) = 0$

$P = 3B - 48kips$ (1)

$+\circlearrowleft \Sigma M_B = 0$: $-A(9ft) + P(6ft) - (6kips)(2ft) - (6kips)(4ft) = 0$

$P = 1.5A + 6kips$ (2)

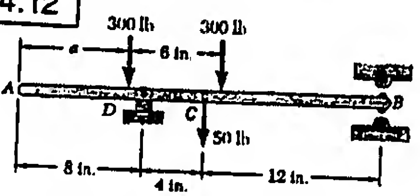
SINCE $B \leq 30kips$, EQ.(1) YIELDS
$P \leq (3)(30kips) - 48kips$ $\quad P \leq 42kips$

SINCE $0 \leq A \leq 30kips$, EQ.(2) YIELDS
$0 + 6kips \leq P \leq (1.5)(30kips) + 6kips$
$6kips \leq P \leq 51kips$

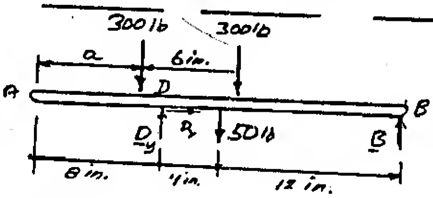
RANGE OF VALUES OF P FOR WHICH BEAM WILL BE SAFE:

$6kips \leq P \leq 42kips$

4.12

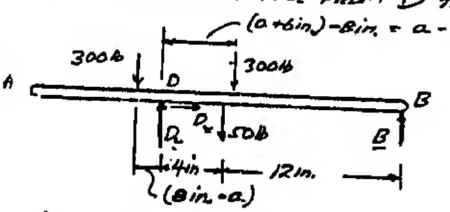


FIND: RANGE OF DISTANCE a FOR WHICH $B \leq 100lb \downarrow$ AND $B \leq 200lb \uparrow$



ASSUME B IS POSITIVE WHEN DIRECTED ↑

SKETCH SHOWING DISTANCE FROM D TO FORCES.



$+\circlearrowleft \Sigma M_D = 0$

$(300lb)(8in.-a) - (300lb)(a-2in.) - (50lb)(4in) - 16B = 0$

$-600a + 2800 + 16B = 0$

$a = (2800 + 16B)/600$ (1)

For $B = 100lb\downarrow = -100lb$, EQ(1) YIELDS:

$a \geq [2800 + 16(-100)]/600 = \frac{1200}{600} = 2in$

$a \geq 2in.$

For $B = 200\uparrow = +200lb$, EQ.(1) YIELDS:

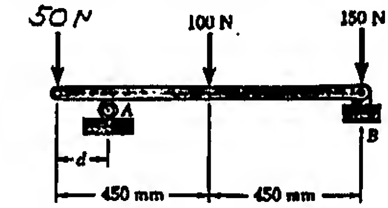
$a \leq [2800 + 16(200)]/600 = \frac{6000}{600} = 10in$

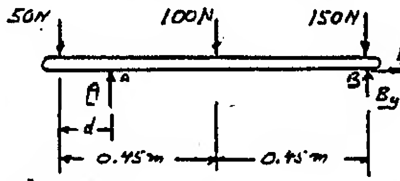
$a \leq 10in.$

REQUIRED RANGE: $2in. \leq a \leq 10in.$

4.13

FIND: RANGE OF DISTANCE d FOR WHICH THE REACTIONS ARE $\leq 180N$





$\Sigma F_x = 0$: $B_x = 0$
$\therefore B = B_y$

$+\circlearrowleft \Sigma M_A = 0$:

$(50N)d - (100N)(0.45m - d) - (150N)(0.9m - d) + B(0.9m - d) = 0$

$50d - 45 + 100d - 135 + 150d + 0.9B - Bd$

$$d = \frac{180 N\cdot m - (0.9m)B}{300N - B} \qquad (1)$$

$+\circlearrowleft \Sigma M_B = 0$:

$(50N)(0.9m) - A(0.9m - d) + (100N)(0.45m) = 0$

$45 - 0.9A + Ad + 45 = 0$

$$d = \frac{(0.9m)A - 90 N\cdot m}{A} \qquad (2)$$

SINCE $B \leq 180N$, EQ.(1) YIELDS.

$d \geq \frac{180 - (0.9)180}{300 - 180} = \frac{18}{120} = 0.15m \qquad d \geq 150mm$

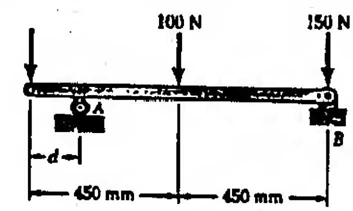
SINCE $A \leq 180N$, EQ.(2) YIELDS.

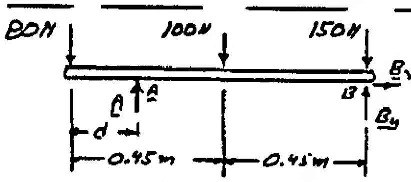
$d \leq \frac{(0.9)180 - 90}{180} = \frac{72}{180} = 0.40m \qquad d \leq 400mm$

RANGE: $150mm \leq d \leq 400mm$

4.14

FIND: RANGE OF DISTANCE d FOR WHICH REACTIONS ARE $\leq 180N$





$\Sigma F_x = 0$: $B_x = 0$
$\therefore B = B_y$

$+\circlearrowleft \Sigma M_A = 0$: $(80N)d - (100N)(0.45m - d) - (150N)(0.9m - d) + B(0.9m - d) = 0$

$80d - 45 + 100d - 135 + 150d + 0.9B - Bd = 0$

$$d = \frac{180 N\cdot m - 0.9B}{330N - B} \qquad (1)$$

$+\circlearrowleft \Sigma M_B = 0$: $(80N)(0.9m) - A(0.9m - d) + (100N)(0.45m) = 0$

$$d = \frac{0.9A - 117}{A} \qquad (2)$$

SINCE $B \leq 180N$, EQ.(1) YIELDS.

$d \geq (180 - 0.9 \times 180)/(330 - 180) = \frac{18}{150} = 0.12m \qquad d = 120mm$

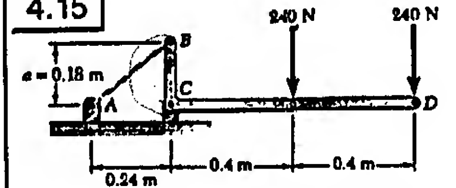
SINCE $A \leq 180N$, EQ.(2) YIELDS.

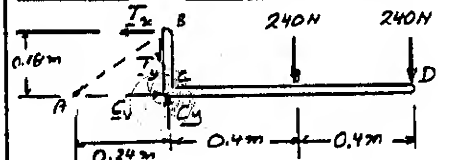
$d \leq (0.9 \times 180 - 117)/180 = \frac{45}{180} = 0.25m \qquad d = 250mm$

RANGE $120mm \leq d \leq 250mm$

4.15

FIND:
(a) TENSION IN AB
(b) REACTION AT C





AT B:

$$\frac{T_y}{T_x} = \frac{0.18m}{0.24m}$$

$$T_y = \frac{3}{4}T_x \qquad (1)$$

$+\circlearrowleft \Sigma M_C = 0$: $T_x(0.18m) - (240N)(0.4m) - (240N)(0.8m) = 0$

$T_x = +1600 N$

EQ.(1) $T_y = \frac{3}{4}(1600N) = 1200 N$

$T = \sqrt{T_x^2 + T_y^2} = \sqrt{1600^2 + 1200^2} = 2000 N$

$T = 2kN$

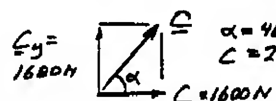
$\xrightarrow{+} \Sigma F_x = 0$: $C_x - T_x = 0$

$C_x - 1600N = 0 \qquad C_x = +1600N \qquad C_x = 1600 N \rightarrow$

$+\uparrow \Sigma F_y = 0$: $C_y - T_y - 240N - 240N = 0$

$C_y - 1200N - 480N = 0$

$C_y = +1680N \qquad C_y = 1680 N \uparrow$

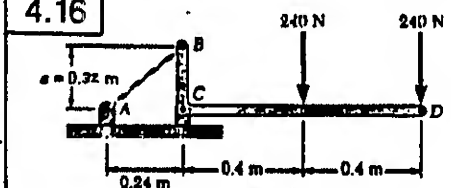


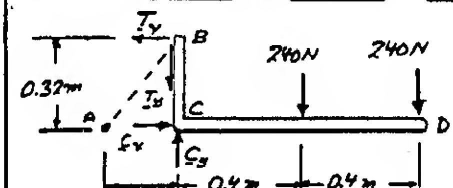
$\alpha = 46.4°$
$C = 2320 N$

$C = 2.32 N \angle 46.4°$

4.16

FIND:
(a) TENSION IN AB
(b) REACTION AT C





AT B:

$$\frac{T_y}{T_x} = \frac{0.32m}{0.24m}$$

$$T_y = \frac{4}{3}T_x$$

$+\circlearrowleft \Sigma M_C = 0$: $T_x(0.32m) - (240N)(0.4m) - (240N)(0.8m) = 0$

$T_x = 900 N$

EQ.(1) $T_y = \frac{4}{3}(900N) = 1200N$

$T = \sqrt{T_x^2 + T_y^2} = \sqrt{900^2 + 1200^2} = 1500N$

$T = 1.5kN$

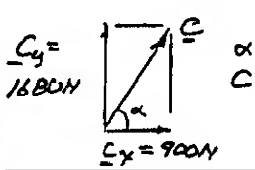
$\xrightarrow{+} \Sigma F_x = 0$: $C_x - T_x = 0$

$C_x - 900N = 0 \qquad C_x = +900N \qquad C_x = 900N \rightarrow$

$+\uparrow \Sigma F_y = 0$: $C_y - T_y - 240N - 240N = 0$

$C_y - 1200N - 480N = 0$

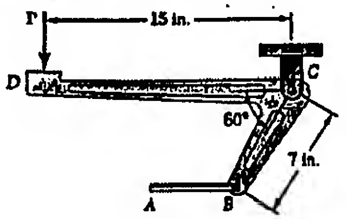
$C_y = +1680N \qquad C_y = 1680N \uparrow$



$\alpha = 61.8°$
$C = 1906N$

$C = 1.906 kN \angle 61.8°$

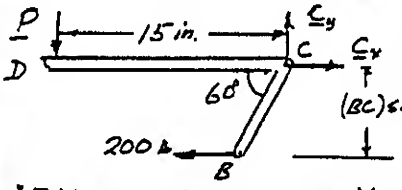
4.17



GIVEN: $T_{AB} = 200\,lb$

FIND: (a) FORCE $\underline{P}$ (b) REACTION AT C

FREE-BODY DIAGRAM



$BC = 7\,in.$

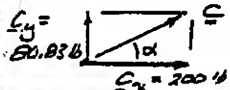
$(BC)\sin 60° = (7\,in)\sin 60° = 6.062\,in.$

$+\circlearrowleft \Sigma M_C = 0: \; P(15\,in.) - (200\,lb)(6.062\,in.) = 0$

$P = 80.83\,lb \qquad \underline{P} = 80.8\,lb\downarrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \; C_x - 200\,lb = 0 \qquad \underline{C}_x = 200\,lb \rightarrow$

$+\uparrow \Sigma F_y = 0: \; C_y - P = 0; \; C_y - 80.83\,lb = 0 \qquad \underline{C}_y = 80.83\,lb\uparrow$

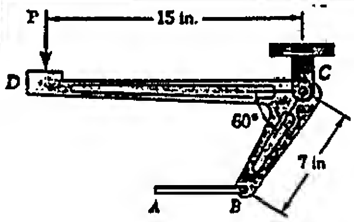


$\alpha = 22.0°$

$C = 215.7\,lb$

$\underline{C} = 216\,lb \angle 22.0°$ ◀

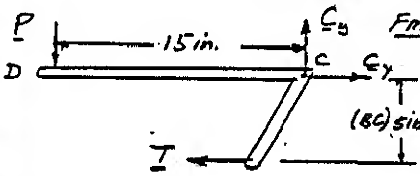
4.18



GIVEN: REACTION AT C = 250 lb

FIND: MAXIMUM ALLOWABLE TENSION IN AB

FREE-BODY DIAGRAM



$BC = 7\,in.$

$(BC)\sin 60° = (7\,in.)\sin 60° = 6.062\,in.$

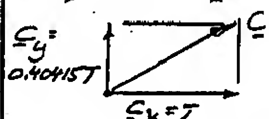
$+\circlearrowleft \Sigma M_C = 0: \; P(15\,in.) - T(6.062\,in.) = 0 \qquad P = 0.40415\,T$

$+\uparrow \Sigma F_y = 0: -P + C_y = 0; \; -0.40415\,P + C_y = 0$

$C_y = 0.40415\,T$

$\xrightarrow{+} \Sigma F_x = 0: \; -T + C_x = 0$

$C_x = T$



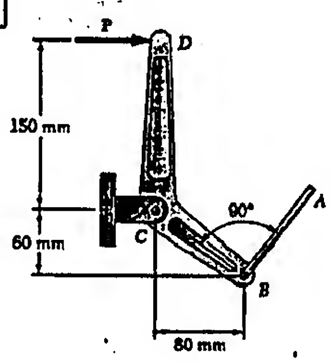
$C = \sqrt{C_x^2 + C_y^2} = \sqrt{T^2 + (0.40415T)^2}$

$C = 1.0786\,T$

FOR $C = 250\,lb$,

$250\,lb = 1.0786\,T \qquad T = 232\,lb$ ◀

4.19



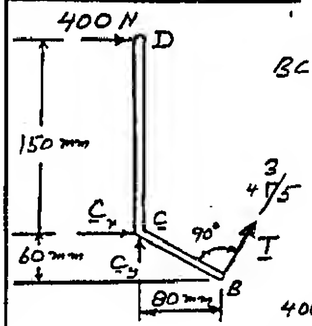
GIVEN: $P = 400\,N$

FIND: (a) TENSION IN ROD AB (b) REACTION AT C

(CONTINUED)

4.19 CONTINUED

FREE-BODY DIAGRAM



$BC = \sqrt{60^2 + 80^2} = 100\,mm$

$+\circlearrowleft \Sigma M_C = 0:$

$T(BC) - (400\,N)(CD) = 0$

$T(100\,mm) - (400\,N)(150\,mm) = 0$

$T = 600\,N$ ◀

$\xrightarrow{+} \Sigma F_x = 0:$

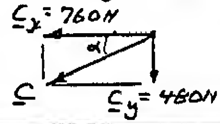
$400\,N + C_x + \frac{3}{5}T = 0$

$400\,N + C_x + \frac{3}{5}(600\,N) = 0$

$C_x = -760\,N \qquad \underline{C}_x = 760\,N \leftarrow$

$+\uparrow \Sigma F_y = 0: \; C_y + \frac{4}{5}T = 0$

$C_y + \frac{4}{5}(600\,N) = 0; \; C_y = -480\,N; \; \underline{C}_y = 480\,N\downarrow$

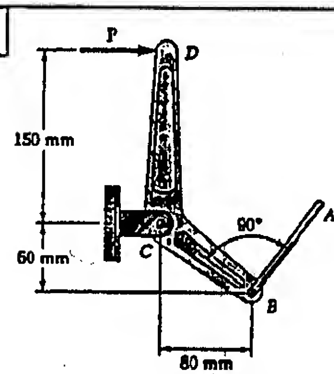


$\alpha = 32.28°$

$C = 898.9\,N$

$\underline{C} = 899\,N \measuredangle 32.3°$ ◀

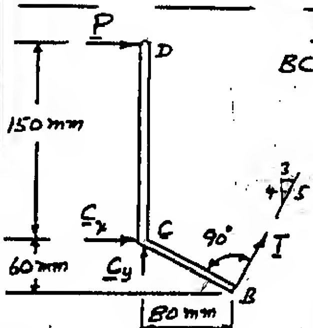
4.20



GIVEN: REACTION AT C = 1000 N

FIND: MAXIMUM ALLOWABLE FORCE $\underline{P}$

FREE-BODY DIAGRAM



$BC = \sqrt{60^2 + 80^2} = 100\,mm$

$+\circlearrowleft \Sigma M_C = 0:$

$T(BC) - P(CD) = 0$

$T(100\,mm) - P(150\,mm) = 0$

$T = 1.5P$

$\xrightarrow{+} \Sigma F_x = 0:$

$P + C_x + \frac{3}{5}T = 0$

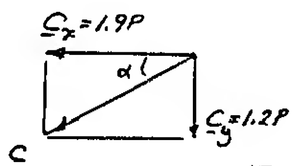
$P + C_x + \frac{3}{5}(1.5P) = 0$

$C_x = -1.9P \qquad \underline{C}_x = 1.9P \leftarrow$

$+\uparrow \Sigma F_y = 0: \; C_y + \frac{4}{5}T = 0$

$C_y + \frac{4}{5}(1.5P) = 0$

$C_y = -1.2P \qquad \underline{C}_y = 1.2P\downarrow$



$C = \sqrt{C_x^2 + C_y^2}$

$= \sqrt{(1.9P)^2 + (1.2P)^2}$

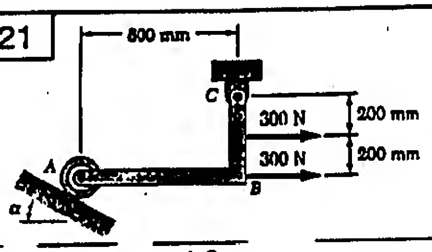
$C = 2.2472\,P$

FOR $C = 1000\,N$,

$1000\,N = 2.2472\,P$

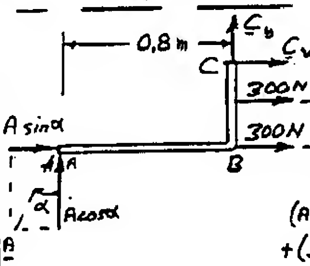
$P = 444.99\,N \qquad P = 445\,lb$ ◀

4.21



FIND: REACTIONS AT A AND C WHEN (a) $\alpha = 0$ (b) $\alpha = 30°$

FREE-BODY DIAGRAM



$+\circlearrowleft \Sigma M_C = 0:$

$(A\sin\alpha)(0.4m) - (A\cos\alpha)(0.8m) + (300N)(0.4m) + (300N)(0.2m) = 0$

$$A = \frac{180}{0.8\cos\alpha - 0.4\sin\alpha} \qquad (1)$$

$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + 300N + 300N + A\sin\alpha = 0$

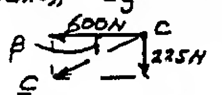
$C_x = -600 - A\sin\alpha \qquad (2)$

$+\uparrow \Sigma F_y = 0: \quad C_y + A\cos\alpha = 0 \qquad C_y = -A\cos\alpha \qquad (3)$

(a) WHEN $\alpha = 0$: EQ.(1), $A = \frac{180}{0.8} = 225N$ $\qquad A = 225N\uparrow$ ◀

EQ(2), $C_x = -600N$

EQ.(3), $C_y = -225N$



$\beta = 20.56°$

$C = 640.8N$

$C = 641N \angle 20.6°$ ◀

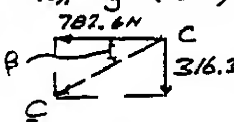
(b) WHEN $\alpha = 30°$:

EQ.(1), $A = \frac{180}{0.8\cos 30° - 0.4\sin 30°} = 365.2N$

$A = 365N \angle 60°$ ◀

EQ.(2), $C_x = -600 - (365.2)\sin 30° = -782.6N$

EQ.(3), $C_y = -(365.2)\cos 30° = -316.3N$

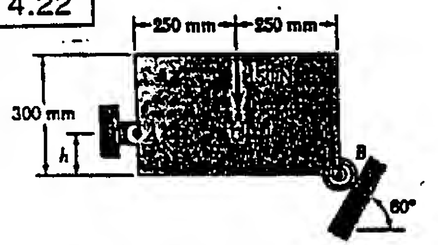


$\beta = 22.0°$

$C = 844.1N$

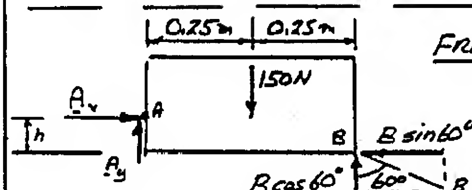
$C = 844N \angle 22.0°$ ◀

4.22



FIND: REACTIONS AT A AND B WHEN (a) $h = 0$ (b) $h = 200mm$

FREE-BODY DIAGRAM



$+\circlearrowleft \Sigma M_A = 0: \ (B\cos 60°)(0.5m) - (B\sin 60°)h - (150N)(0.25m) = 0$

$$B = \frac{37.5}{0.25 - 0.866\,h} \qquad (1)$$

(CONTINUED)

4.22 CONTINUED

(a) WHEN $h = 0$:

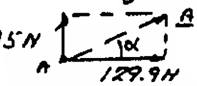
EQ.(1): $B = \frac{37.5}{0.25} = 150N \qquad B = 150N \angle 30°$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B\sin 60° = 0$

$A_x = (150)\sin 60° = 129.9N \qquad A_x = 129.9N \rightarrow$

$+\uparrow \Sigma F_y = 0: \quad A_y - 150 + B\cos 60° = 0$

$A_y = 150 - (150)\cos 60° = 75N \qquad A_y = 75N\uparrow$



$\alpha = 30°$

$A = 150N$

$A = 150N \angle 30°$ ◀

(b) WHEN $h = 200mm = 0.2m$

EQ(1): $B = \frac{37.5}{0.25 - 0.866(0.2)} = 488.3N$

$B = 488N \angle 30°$ ◀

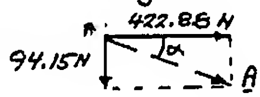
$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B\sin 60° = 0$

$A_x = (488.3)\sin 60° = 422.88N \qquad A_x = 422.8N \rightarrow$

$+\uparrow \Sigma F_y = 0: \quad A_y - 150 + B\cos 60° = 0$

$A_y = 150 - (488.3)\cos 60° = -94.15N$

$A_y = 94.15N\downarrow$



$\alpha = 12.55°$

$A = 433.2N$

$A = 433N \angle 12.6°$ ◀

4.23

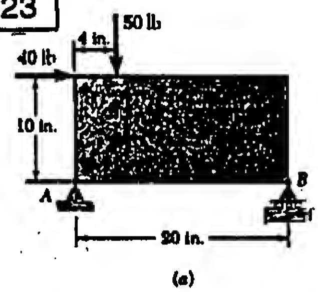
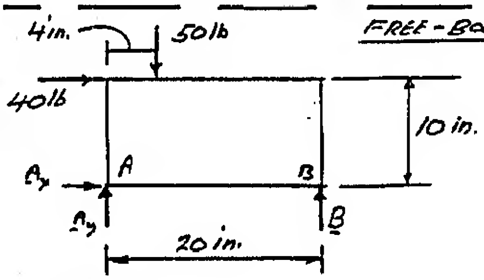


PLATE a:

FIND: REACTIONS AT A AND B

FREE-BODY DIAGRAM



$+\circlearrowleft \Sigma M_A = 0:$

$B(20in.) - (50lb)(4in.) - (40lb)(10in.) = 0$

$B = +30lb \qquad B = 30lb\uparrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 40lb = 0$

$A_x = -40lb \qquad A_x = 40lb \leftarrow$

$+\uparrow \Sigma F_y = 0: \quad A_y + B - 50lb = 0$

$A_y + 30lb - 50lb = 0$

$A_y = +20lb \qquad A_y = 20lb\uparrow$

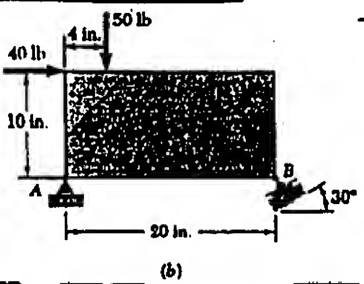
$\alpha = 26.56°$ $\qquad A = 44.72lb$

$A = 44.7lb \angle 26.6°$ ◀

(CONTINUED)

4.23 CONTINUED

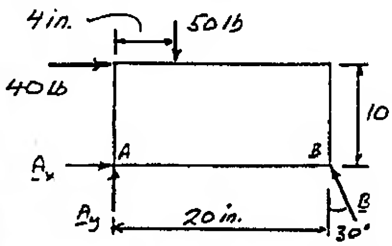
PLATE b:



(b)

FIND: REACTIONS AT A AND B

FREE-BODY DIAGRAM



$+\circlearrowleft \Sigma M_A = 0$: $(B\cos 30°)(20\text{ in.}) - (40\text{ lb})(10\text{ in.}) - (50\text{ lb})(4\text{ in.}) = 0$

$B = 34.64\text{ lb}$ $\qquad$ $\underline{B} = 34.6\text{ lb} \measuredangle 60°$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $A_x - B\sin 30° + 40\text{ lb}$

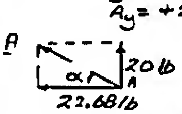
$A_x - (34.64\text{ lb})\sin 30° + 40\text{ lb} = 0$

$A_x = -22.68\text{ lb}$ $\qquad$ $A_x = 22.68\text{ lb} \leftarrow$

$+\uparrow \Sigma F_y = 0$: $A_y + B\cos 30° - 50\text{ lb} = 0$

$A_y + (34.64\text{ lb})\cos 30° - 50\text{ lb} = 0$

$A_y = +20\text{ lb}$ $\qquad$ $A_y = 20\text{ lb}\uparrow$

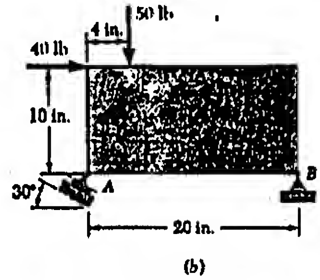


$\alpha = 41.4°$

$A = 30.24\text{ lb}$ $\qquad$ $\underline{A} = 30.2\text{ lb} \measuredangle 41.4°$ ◀

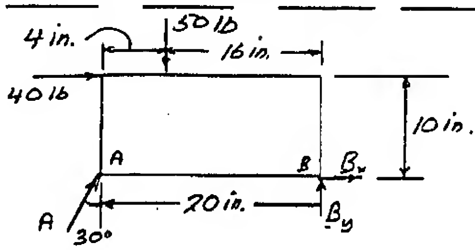
4.24 CONTINUED

PLATE b:



(b)

FIND: REACTIONS AT A AND B



$+\circlearrowleft \Sigma M_B = 0$: $-(A\cos 30°)(20\text{ in}) - (40\text{ lb})(10\text{ in.}) + (50\text{ lb})(16\text{ in.}) = 0$

$A = 23.09\text{ lb}$ $\qquad$ $\underline{A} = 23.11\text{ lb} \measuredangle 60°$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $A\sin 30° + 40\text{ lb} + B_x = 0$

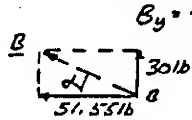
$(23.09\text{ lb})\sin 30° + 40\text{ lb} + B_x = 0$

$B_x = -51.55\text{ lb}$ $\qquad$ $B_x = 51.55\text{ lb} \leftarrow$

$+\uparrow \Sigma F_y = 0$: $A\cos 30° + B_y - 50\text{ lb} = 0$

$(23.09\text{ lb})\cos 30° + B_y - 50\text{ lb} = 0$

$B_y = +30\text{ lb}$ $\qquad$ $B_y = 30\text{ lb}\uparrow$

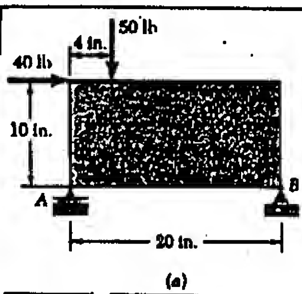


$\alpha = 30.2°$

$B = 57.64\text{ lb}$ $\qquad$ $\underline{B} = 57.6\text{ lb} \measuredangle 30.2°$ ◀

4.24

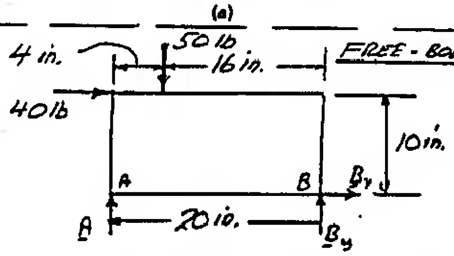
PLATE a:



(a)

FIND: REACTIONS AT A AND B

FREE-BODY DIAGRAM



$+\circlearrowleft \Sigma M_B = 0$: $-A(20\text{ in.}) + (50\text{ lb})(16\text{ in.}) - (40\text{ lb})(10\text{ in.}) = 0$

$A = +20\text{ lb}$ $\qquad$ $\underline{A} = 20\text{ lb}\uparrow$ ◀

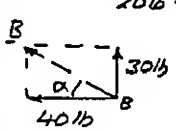
$\xrightarrow{+} \Sigma F_x = 0$: $40\text{ lb} + B_x = 0$

$B_x = -40\text{ lb}$ $\qquad$ $B_x = 40\text{ lb} \leftarrow$

$+\uparrow \Sigma F_y = 0$: $A + B_y - 50\text{ lb} = 0$

$20\text{ lb} + B_y - 50\text{ lb} = 0$

$B_y = +30\text{ lb}$ $\qquad$ $B_y = 30\text{ lb}\uparrow$



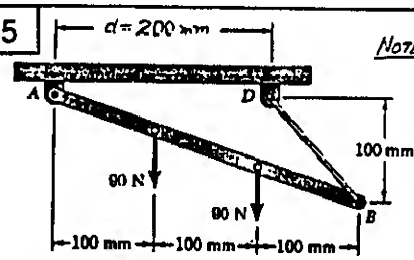
$\alpha = 36.87°$; $B = 50\text{ lb}$

$\underline{B} = 50\text{ lb} \measuredangle 36.9°$ ◀

(CONTINUED)

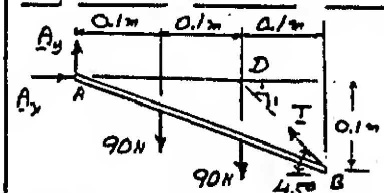
4.25

NOTE: $d = 200$ mm

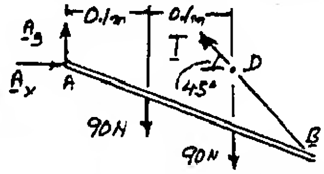


FIND: (a) TENSION IN CABLE BD (b) REACTION AT A

FREE-BODY DIAGRAM



MOVE T ALONG BD UNTIL IT ACTS AT POINT D.



$+\circlearrowleft \Sigma M_A = 0$:

$(T\sin 45°)(0.2\text{ m})$

$+(90\text{ N})(0.1\text{ m}) + (90\text{ N})(0.2\text{ m}) = 0$

$T = 190.92\text{ N}$

$T = 190.9\text{ N}$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $A_x - (190.92\text{ N})\cos 45° = 0$

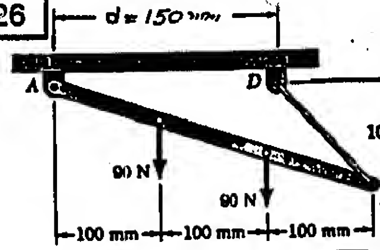
$A_x = +135\text{ N}$ $\qquad$ $A_x = 135\text{ N} \rightarrow$

$+\uparrow \Sigma F_y = 0$: $A_y - 90\text{ N} - 90\text{ N} + (190.92\text{ N})\sin 45° = 0$

$A_y = +45\text{ N}$ $\qquad$ $A_y = 45\text{ N}\uparrow$

$\underline{A} = 142.3\text{ N} \measuredangle 18.4°$ ◀

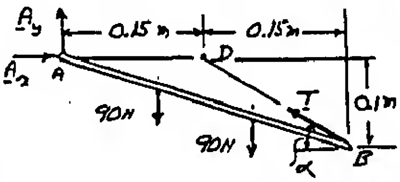
4.26

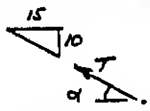


NOTE: $d = 150$ mm

FIND: (a) TENSION IN CABLE BD (b) REACTION AT A

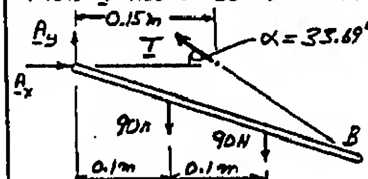
FREE-BODY DIAGRAM





$\tan\alpha = \frac{10}{15}$; $\alpha = 33.69°$

MOVE T ALONG BD UNTIL IT ACTS AT POINT D.



$+\circlearrowleft \Sigma M_A = 0:$

$(T \sin 33.69°)$

$-(90N)(0.1m) - (90N)(0.2m) = 0$

$T = 324.99$ N

$T = 324$ N ◀

$\xrightarrow{+} \Sigma F_x = 0: \; A_x - (324.99N)\cos 33.69° = 0$

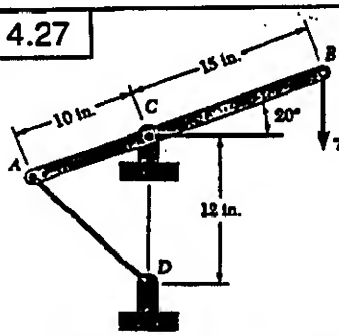
$A_x = +270N \qquad A_x = 270N \rightarrow$

$+\uparrow \Sigma F_y = 0: \; A_y - 90N - 90N + (324.99N)\sin 33.69° = 0$

$A_y = 0 \qquad A_y = 0$

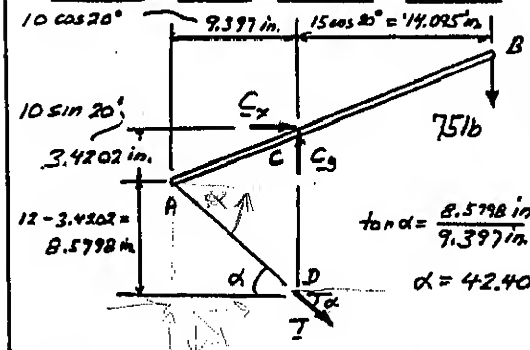
$A = 270N \rightarrow$ ◀

4.27



FIND: (a) TENSION IN CABLE AD (b) REACTION AT C

DRAW FREE-BODY DIAGRAM WITH TENSION IN AD ACTING AT D



AC = 10 in.
CB = 15 in.
CD = 12 in

$\tan\alpha = \frac{8.5798 \text{ in.}}{9.397 \text{ in.}}$

$\alpha = 42.40°$

$+\circlearrowleft \Sigma M_C = 0: \; (T\cos\alpha)(CD) - (75lb)(14.095in.) = 0$

$(T\cos 42.40°)(12in.) - (75lb)(14.095in) = 0$

$T = 119.29 lb \qquad T = 119.3$ lb ◀

$\xrightarrow{+} \Sigma F_x = 0: \; C_x + (119.29 lb)(\cos 42.40°) = 0$

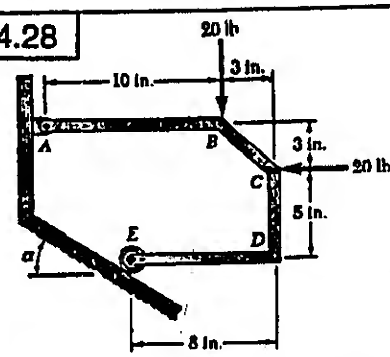
$C_x = -88.044$ lb $\qquad C_x = 88.044$ lb $\leftarrow$

$+\uparrow \Sigma F_y = 0: \; C_y - 75 lb - (119.29 lb)\sin 42.4°) = 0$

$C_y = +155.44 lb \qquad C_y = 155.44 lb \uparrow$

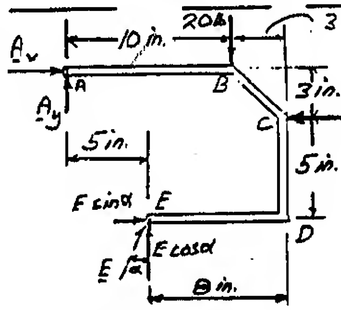
$C = 178.7$ lb ∠ 60.5° ◀

4.28



FIND: REACTIONS AT A AND E WHEN (a) $\alpha = 30°$ (b) $\alpha = 45°$

FREE-BODY DIAGRAM



$+\circlearrowleft \Sigma M_A = 0:$

$(E\sin\alpha)(8in.) + (E\cos\alpha)(5in) - (20 lb)(10in) - (20lb)(3in) = 0$

$E = \frac{260}{8\sin\alpha + 5\cos\alpha}$

(a) WHEN $\alpha = 30°$: $E = \frac{260}{8\sin 30° + 5\cos 30°} = 31.212$ lb

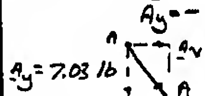
$E = 31.2$ lb ∠ 60° ◀

$\xrightarrow{+} \Sigma F_x = 0: \; A_x - 20 lb + (31.212 lb)\sin 30° = 0$

$A_x = +4.394$ lb $\qquad A_x = 4.394$ lb $\rightarrow$

$+\uparrow \Sigma F_y = 0: \; A_y - 20 + (31.212 lb)\cos 30° = 0$

$A_y = -7.03$ lb $\qquad A_y = 7.03$ lb $\downarrow$



$A = 8.29$ lb ∠ 58.0° ◀

(b) WHEN $\alpha = 45°$: $E = \frac{260}{8\sin 45° + 5\cos\alpha} = 28.28$ lb

$E = 28.3$ lb ∠ 45° ◀

$\xrightarrow{+} \Sigma F_x = 0: \; A_x - 20 lb + (28.28 lb)\sin 45° = 0$

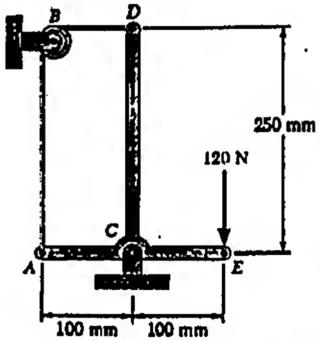
$A_x = 0 \qquad A_x = 0$

$+\uparrow \Sigma F_y = 0: \; A_y - 20 lb + (28.28 lb)\cos 45° = 0$

$A_y = 0 \qquad A_y = 0$

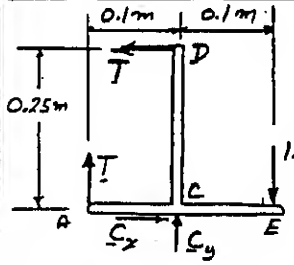
$A = 0$ ◀

4.29



FIND: TENSION IN CABLE ABD. REACTION AT C.

LET T EQUAL TENSION IN CABLE



FREE-BODY DIAGRAM

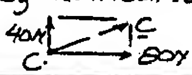
$+\circlearrowleft \Sigma M_C = 0$

$T(0.25\,m) - T(0.1\,m) - (120\,N)(0.1\,m) = 0$

$T = 80\,N$ ◁

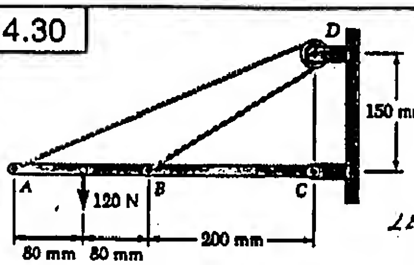
$\xrightarrow{+} \Sigma F_x = 0$: $C_x - 80N = 0$; $C_x = +80N$; $C_x = 80N \rightarrow$

$+\uparrow \Sigma F_y = 0$: $C_y - 120N + 80N = 0$; $C_y = +40N$; $C_y = 40N \uparrow$



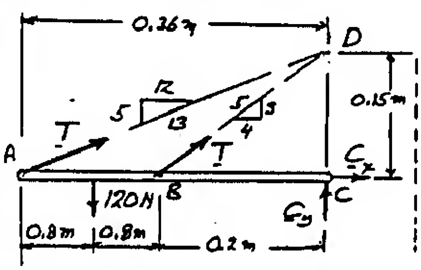
$C = 89.4\,N \measuredangle 26.6°$ ◁

4.30



FIND: (a) TENSION IN CABLE ADB. (b) REACTION AT C.

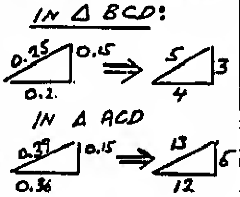
LET T EQUAL TENSION IN CABLE.



FREE-BODY DIAGRAM

IN Δ BCD:

IN Δ ACD



$+\circlearrowleft \Sigma M_C = 0$: $(120N)(0.28m) - (\frac{5}{13}T)(0.36m) - (\frac{3}{5}T)(0.2m) = 0$

$33.6 - T(0.13846 + 0.12) = 0$

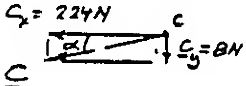
$T = 130.00\,N$ $\qquad T = 130\,N$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $C_x + \frac{12}{13}(130N) + \frac{4}{5}(130N) = 0$

$C_x = -224N$ $\qquad C_x = 224N \leftarrow$

$+\uparrow \Sigma F_y = 0$: $C_y - 120N + \frac{5}{13}(130N) + \frac{3}{5}(130N) = 0$

$C_y = -8.00\,N$ $\qquad C_y = 8N \downarrow$

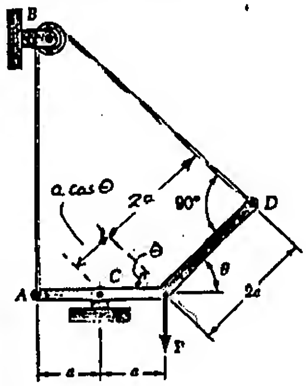


$C = 224.14\,N$

$\alpha = 2.045°$

$C = 224N \measuredangle 2.0°$ ◀

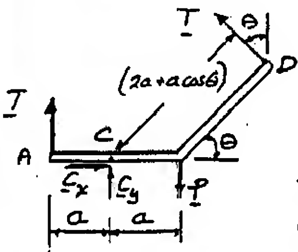
4.31 and 4.32



FIND: TENSION IN CABLE ABD REACTION AT C

WE NOTE THAT THE PERPENDICULAR DISTANCE FROM POINT C T PORTION BD OF CABLE IS $2a + a\cos\theta$

LET T EQUAL THE TENSION IN CABLE



$+\circlearrowleft \Sigma M_C = 0$:

$T(2a + a\cos\theta) - Ta - Pa = 0$

$T = \dfrac{P}{1+\cos\theta}$ $\qquad$ (1)

$\xrightarrow{+} \Sigma F_x = 0$: $C_x - T\sin\theta = 0$

$C_x = T\sin\theta = \dfrac{P\sin\theta}{1+\cos\theta}$

$+\uparrow \Sigma F_y = 0$: $C_y + T + T\cos\theta - P = 0$

$C_y = P - T(1+\cos\theta) = P - P\dfrac{1+\cos\theta}{1+\cos\theta}$; $\qquad C_y = 0$

SINCE $C_y = 0$, $C = C_x$ $\qquad C = P\dfrac{\sin\theta}{1+\cos\theta} \rightarrow$ (2)

FOR PROB 4.31 $\theta = 60°$:

EQ(1): $T = \dfrac{P}{1+\cos 60°} = \dfrac{P}{1+\frac{1}{2}}$ $\qquad T = \frac{2}{3}P$ ◀

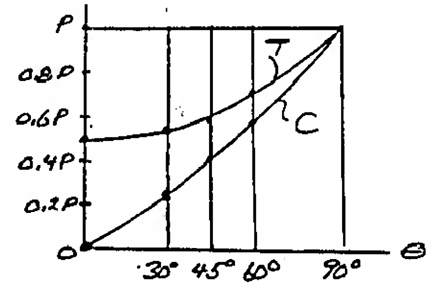
EQ(2): $C = P\dfrac{\sin 60°}{1+\cos 60°} = P\dfrac{0.866}{1+\frac{1}{2}}$ $\qquad C = 0.577P \rightarrow$ ◀

FOR PROB 4.32 $\theta = 45°$

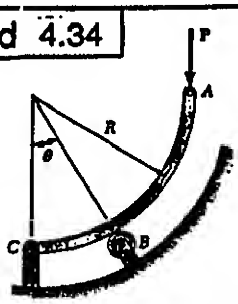
EQ(1): $T = \dfrac{P}{1+\cos 45°} = \dfrac{P}{1.7071}$ $\qquad T = 0.586P$ ◀

EQ(2): $C = P\dfrac{\sin 45°}{1+\cos 45°} = P\dfrac{0.7071}{1.7071}$ $\qquad C = 0.414P \rightarrow$ ◀

· THE FOLLOWING IS A PLOT OF T AND C FOR $0 \le \theta \le 90°$

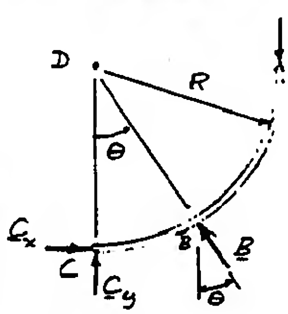


4.33 and 4.34



FIND: REACTION (a) AT B (b) AT C

FREE-BODY DIAGRAM



$+\circlearrowleft \Sigma M_D = 0$:

$C_x(R) - P(R) = 0$

$C_x = +P$

$\xrightarrow{+} \Sigma F_x = 0$: $C_x - B\sin\theta = 0$

$P - B\sin\theta = 0$

$B = P/\sin\theta$

$B = \frac{P}{\sin\theta} \measuredangle\theta$

$+\uparrow \Sigma F_y = 0$: $C_y + B\cos\theta - P = 0$

$C_y + (P/\sin\theta)\cos\theta - P = 0$

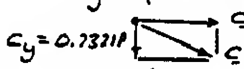
$C_y = P(1 - \frac{1}{\tan\theta})$

FOR PROB 4.33 $\theta = 30°$

(a) $B = P/\sin 30° = 2P$ $\quad B = 2P \measuredangle 60°$ ◀

(b) $C_x = +P$ $\quad C_x = P \rightarrow$

$C_y = P(1 - 1/\tan 30°) = -0.7321P$ $\quad C_y = 0.7321P\downarrow$



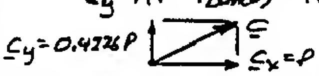
$C = 1.239P \measuredangle 36.2°$ ◀

FOR PROB. 4.34 $\theta = 60°$

(a) $B = P/\sin 60° = 1.1547P$ $\quad B = 1.155P \measuredangle 30°$ ◀

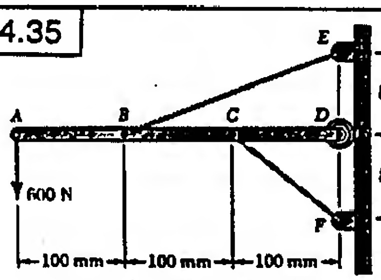
(b) $C_x = +P$ $\quad C_x = P \rightarrow$

$C_y = P(1 - 1/\tan 60°) = +0.4226P$ $\quad C_y = 0.4226P\downarrow$



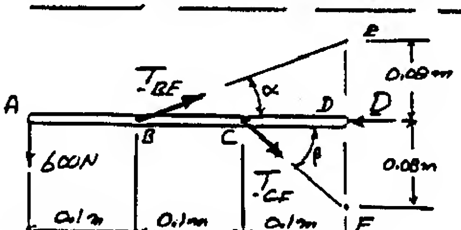
$C = 1.086P \measuredangle 22.9°$ ◀

4.35



FIND: TENSION IN EACH CABLE

REACTION AT D



$\tan\alpha = \frac{0.08\text{m}}{0.2\text{m}}$

$\alpha = 21.80°$

$\tan\beta = \frac{0.08\text{m}}{0.1\text{m}}$

$\beta = 38.66°$

(CONTINUED)

4.35 CONTINUED

$+\circlearrowleft \Sigma M_B = 0$: $(600N)(0.1m) - (T_{CF}\sin 38.66°)(0.1m) = 0$

$T_{CF} = 960.47N$ $\quad T_{CF} = 960N$ ◀

$+\circlearrowleft \Sigma M_C = 0$: $(600N)(0.2m) - (T_{BE}\sin 21.80°)(0.1m) = 0$

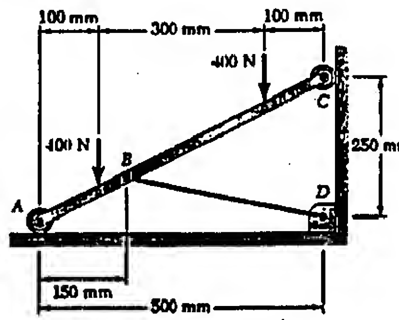
$T_{BE} = 3231.1N$ $\quad T_{BE} = 3230N$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $T_{BE}\cos\alpha + T_{CF}\cos\beta - D = 0$

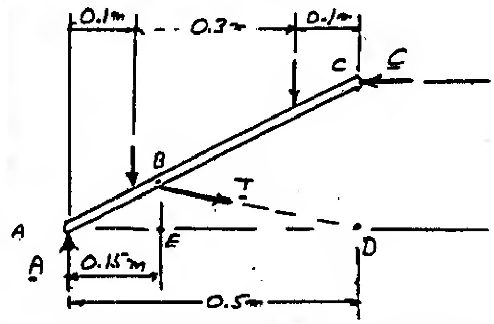
$(3231.1N)\cos 21.80° + (960.47N)\cos 38.66° - D = 0$

$D = 3750.03N$ $\quad D = 3750N \leftarrow$ ◀

4.36

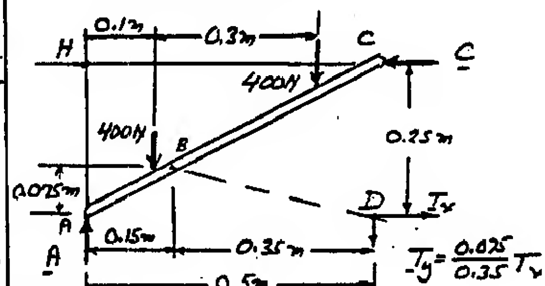


FIND: (a) TENSION IN CABLE BD, (b) REACTION AT A, (c) REACTION AT C.



SIMILAR TRIANGLES: ABE AND ACD

$\frac{AE}{AD} = \frac{BE}{CD}$; $\frac{0.15m}{0.5m} = \frac{BE}{0.25m}$; $BE = 0.075m$

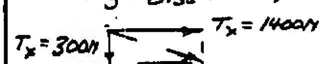


(a) $+\circlearrowleft \Sigma M_H = 0$:

$T_x(0.25m) - (\frac{0.075}{0.35}T_x)(0.5m) - (400N)(0.1m) - (400N)(0.4m) = 0$

$T_x = 1400N$

$T_y = \frac{0.075}{0.35}(1400N) = 300N$



$T = 1432 \text{ lb}$ ◀

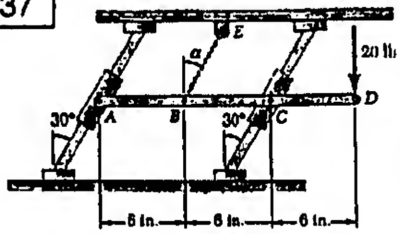
(b) $+\uparrow \Sigma F_y = 0$: $A - 300N - 400N - 400N = 0$

$A = +1100N$ $\quad A = 1100N\uparrow$ ◀

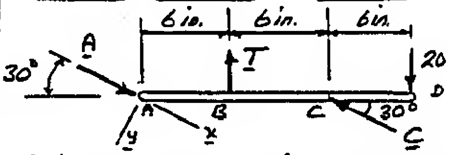
(c) $\xrightarrow{+} \Sigma F_x = 0$: $-C + 1400N = 0$

$C = +1400N$ $\quad C = 1400N \leftarrow$ ◀

4.37



IF CORD BE IS VERTICAL
$\alpha = 0$,
FIND:
TENSION IN BE
REACTIONS AT A AND C



FREE-BODY DIAGRAM

$+\swarrow \Sigma F_y = 0: \; -T\cos 30^\circ + (20\,lb)\cos 30^\circ = 0 \qquad T = 20\,lb$ ◀

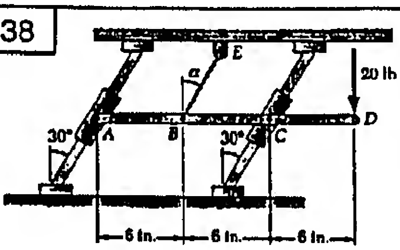
$+\circlearrowleft \Sigma M_C = 0: \; (A \sin 30^\circ)(12\,in.) - (20\,lb)(6\,in.) - (20\,lb)(6\,in.) = 0$

$A = +40\,lb \qquad \underline{A} = 40\,lb \measuredangle 30^\circ$ ◀

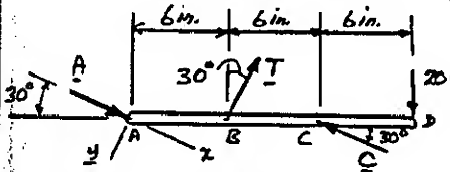
$+\circlearrowleft \Sigma M_A = 0: \; (20\,lb)(6\,in.) - (20\,lb)(18\,in.) + (C\sin 30^\circ)(12\,in.) = 0$

$C = +40\,lb \qquad \underline{C} = 40\,lb \measuredangle 30^\circ$ ◀

4.38



IF $\alpha = 30^\circ$,
FIND:
TENSION IN BE
REACTIONS AT A AND C



FREE-BODY DIAGRAM

$+\swarrow \Sigma F_y = 0: \; -T + (20\,lb)\cos 30^\circ = 0 \qquad T = 17.32\,lb$ ◀

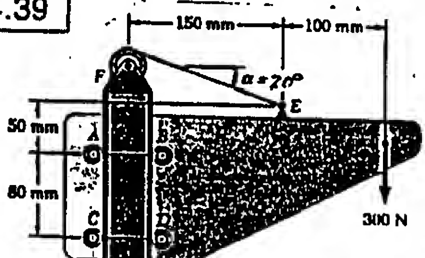
$+\circlearrowleft \Sigma M_C = 0: \; -(17.32\,lb)\cos 30^\circ(6\,in.) - (20\,lb)(6\,in.) - (A\sin 30^\circ)(12\,in.) = 0$

$A = +35\,lb \qquad \underline{A} = 35\,lb \measuredangle 30^\circ$ ◀

$+\circlearrowleft \Sigma M_A = 0: \; +(17.32\,lb)\cos 30^\circ(6\,in.) - (20\,lb)(18\,in.) + (C\sin 30^\circ)(12\,in.) = 0$

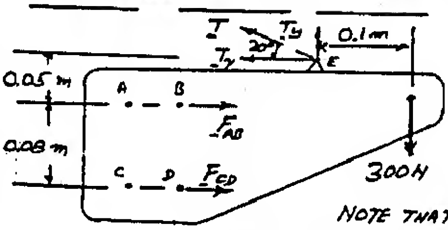
$C = +45\,lb \qquad \underline{C} = 45\,lb \measuredangle 30^\circ$ ◀

4.39



FIND: FORCE EXERTED ON POST BY EACH ROLLER.

DENOTE FORCE AT A + B BY F_{AB} AND AT C + D BY F_{CD}



FREE-BODY DIAGRAM

$T_x = T\cos 20^\circ$

$T_y = T\sin 20^\circ \quad (1)$

$+\uparrow \Sigma F_y = 0:$

$T_y = 300\,N$

NOTE THAT T_y AND 300 N↓ FORM A COUPLE: $(300\,N)(0.1\,m) = 30\,N\cdot m$ ↻

(CONTINUED)

4.39 CONTINUED

"COUPLE"

$+\circlearrowleft \Sigma M_C = 0: \; -F_{AB}(0.08\,m) + (T\cos 20^\circ)(0.130\,m) - 30\,N\cdot m = 0$

FROM EQ. (1) $\quad T = T_y/\sin 20^\circ = (300\,N)/\sin 20^\circ$

$-F_{AB}(0.08\,m) + (300\,N)\frac{\cos 20^\circ}{\sin 20^\circ}(0.130\,m) - 30\,N\cdot m = 0$

$F_{AB} = +964\,lb \qquad \underline{F}_{AB} = 964\,lb \rightarrow$

THUS F_{AB} ACTS AT B. ON BRACKET: $\underline{B} = 964\,lb \rightarrow$, $\underline{A} = 0$

ON POST: $\underline{B} = 964 \leftarrow$; $\underline{A} = 0$ ◀

$+\circlearrowleft \Sigma M_A = 0: \; +F_{CD}(0.08\,m) + (T\cos 20^\circ)(0.05\,m) - 30\,N\cdot m = 0$

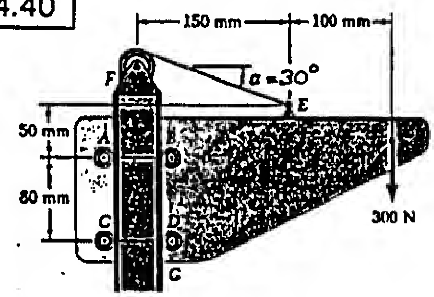
$F_{CD}(0.08\,m) + (300\,N)\frac{\cos 20^\circ}{\sin 20^\circ}(0.05\,m) - 30\,N\cdot m = 0$

$F_{CD} = -140.2\,N \qquad \underline{F}_{CD} = 140.2\,N \leftarrow$

THUS F_{CD} ACTS AT C. ON BRACKET: $\underline{C} = 140.2\,N \leftarrow$, $\underline{D} = 0$

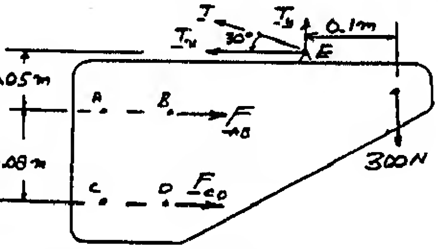
ON POST: $\underline{C} = 140.2\,N \rightarrow$; $D = 0$ ◀

4.40



FIND: FORCE EXERTED ON POST BY EACH ROLLER

DENOTE FORCE AT A AND B BY F_{AB} AND FORCE AT C AND D BY F_{CD}



FREE-BODY DIAGRAM

$+\uparrow \Sigma F_y = 0: \; T_y - 300\,N = 0 \qquad T_y = 300\,N \uparrow$

$T_y = T_x \tan 30^\circ; \; 300\,N = T_x \tan 30^\circ \qquad T_x = 519.62\,N \leftarrow$

NOTE THAT T_y AND 300-N LOAD FORM A COUPLE: $(300\,N)(0.1\,m) = 30\,N\cdot m$ ↻

$+\circlearrowleft \Sigma M_C = 0: \; -F_{AB}(0.08\,m) + T_x(0.13\,m) - 30\,N\cdot m = 0$

$-F_{AB}(0.08\,m) + (519.62\,N)(0.130\,m) - 30\,N\cdot m = 0$

$F_{AB} = +469.4\,lb \qquad \underline{F}_{AB} = 469.4\,N \rightarrow$

THUS F_{AB} ACTS AT B. ON BRACKET: $\underline{B} = 469\,N \rightarrow$, $\underline{A} = 0$

ON POST: $\underline{B} = 469\,N \leftarrow$; $\underline{A} = 0$ ◀

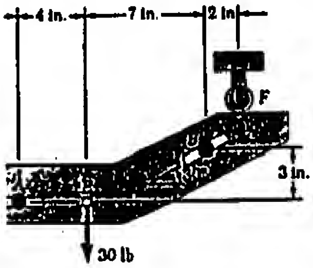
$+\circlearrowleft \Sigma M_A = 0: \; F_{CD}(0.08\,m) + T_x(0.05\,m) - 30\,N\cdot m = 0$

$F_{CD}(0.08\,m) + (519.62\,N)(0.05\,m) - 30\,N\cdot m = 0$

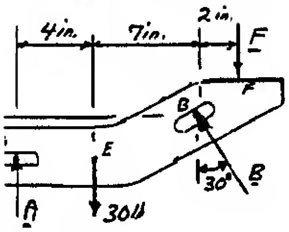
$F_{CD} = +50.2\,N \qquad \underline{F}_{CD} = 50.2\,N \rightarrow$

THUS F_{CD} ACTS AT D. ON BRACKET: $\underline{C} = 0$; $\underline{D} = 50.2\,N \rightarrow$

ON POST: $\underline{C} = 0$; $\underline{D} = 50.2\,N \leftarrow$ ◀



GIVEN:
P = 15 lb
FIND:
REACTIONS AT A AND B
REACTION AT F



FREE-BODY DIAGRAM

$- B \sin 30° = 0$ $\qquad B = 30\text{ lb} \measuredangle 60°$ ◀

$(4\text{ in.}) + B \sin 30°(3\text{ in.}) + B\cos 30°(11\text{ in.}) - F(13\text{ in.}) = 0$

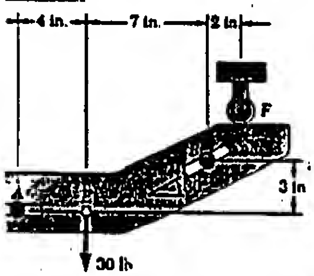
$+ (30\text{ lb})\sin 30°(3\text{ in.}) + (30\text{ lb})\cos 30°(11\text{ in.}) - F(13\text{ in.}) = 0$

$+ 16.2145\text{ lb}$ $\qquad F = 16.21\text{ lb}\downarrow$ ◀

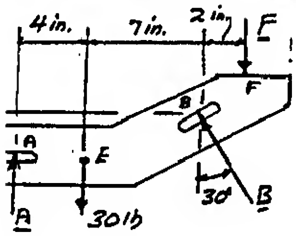
$0\text{ lb} + B\cos 30° - F = 0$

$\text{lb} + (30\text{ lb})\cos 30° - 16.2145\text{ lb} = 0$

20.23 lb $\qquad A = 20.2\text{ lb}\uparrow$ ◀



FIND:
RANGE OF P
FOR
$0 \le F \le 20\text{ lb}$



FREE-BODY DIAGRAM

$P - B \sin 30° = 0$ $\qquad B = 2P \measuredangle 60°$

$(4\text{ in.}) + B \sin 30°(3\text{ in.}) + B\cos 30°(11\text{ in.}) - F(13\text{ in.}) = 0$

$+ 2P \sin 30°(3\text{ in.}) + 2P\cos 30°(11\text{ in.}) - F(13\text{ in.}) = 0$

$+ 19.0525P - 13F = 0$

$\dfrac{13F + 120}{22.0525}$ $\qquad (1)$

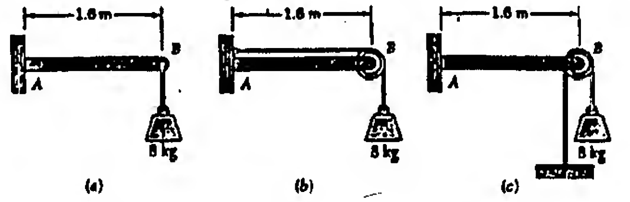
$: \quad P = \dfrac{13(0) + 120}{22.0525} = 5.442\text{ lb}$

$: \quad P = \dfrac{13(20) + 120}{22.0525} = 17.232\text{ lb}$

$20\text{ lb}:$ $\qquad 5.44\text{ lb} \le P \le 17.23\text{ lb}$ ◀

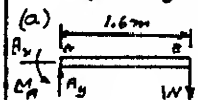
4.43

FIND: REACTION AT A IN EACH CASE.



$W = mg = (8\text{ kg})(9.81\text{ m/s}^2) = 78.48\text{ N}$

(a)



$\xrightarrow{+}\Sigma F_x = 0: \qquad A_x = 0$

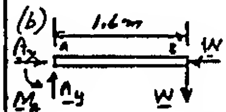
$+\uparrow\Sigma F_y = 0: \quad A_y - W = 0 \qquad A_y = 78.48\text{ N}\uparrow$

$+\circlearrowleft\Sigma M_A = 0: \quad M_A - W(1.6\text{ m}) = 0$

$M_A = +(78.48\text{ N})(1.6\text{ m}) \qquad M_A = 125.56\text{ N}\cdot\text{m}\circlearrowleft$

$A = 78.5\text{ N}\uparrow;\ M_A = 125.6\text{ N}\cdot\text{m}\circlearrowleft$ ◀

(b)



$\xrightarrow{+}\Sigma F_x = 0: \quad A_x - W = 0 \quad A_x = 78.48$

$+\uparrow\Sigma F_y = 0: \quad A_y - W = 0 \quad A_y = 78.48$

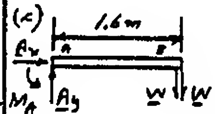
$A = (78.48\text{ N})\sqrt{2} = 110.99\text{ N} \measuredangle 45°$

$+\circlearrowleft\Sigma M_A = 0: \quad M_A - W(1.6\text{ m}) = 0$

$M_A = +(78.48\text{ N})(1.6\text{ m}) \qquad M_A = 125.57\text{ N}\cdot\text{m}\circlearrowleft$

$A = 111.0\text{ N} \measuredangle 45°;\ M_A = 125.6\text{ N}\cdot\text{m}\circlearrowleft$ ◀

(c)



$\Sigma F_x = 0: \qquad A_x = 0$

$+\uparrow\Sigma F_y = 0: \quad A_y - 2W = 0$

$A_y = 2W = 2(78.48\text{ N}) = 156.96\text{ N}\uparrow$

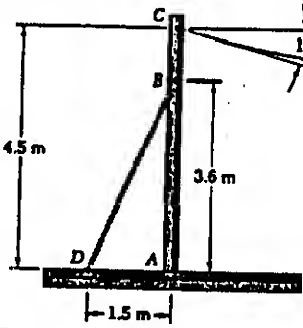
$+\circlearrowleft\Sigma M_A = 0: \quad M_A - 2W(1.6\text{ m}) = 0$

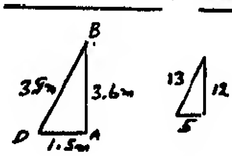
$M_A = +2(78.48\text{ N})(1.6\text{ m}) \qquad M_A = 251.1\text{ N}\cdot\text{m}\circlearrowleft$

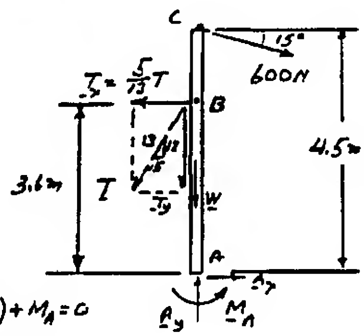
$A = 157.0\text{ N}\uparrow;\ M_A = 251\text{ N}\cdot\text{m}\circlearrowleft$ ◀

4.44

GIVEN: TENSION IN WIRE = 600 N
FIND: LARGEST AND SMALLEST TENSION IN CABLE BC FOR WHICH $|M_A|$ DOES NOT EXCEED 500 N·m







FREE-BODY DIAGRAM

$+\circlearrowleft\Sigma M_A = 0:$

$\frac{5}{13}T(3.6\text{ m}) - (600\text{ N})\cos 15°(4.5\text{ m}) + M_A = 0$

$(1.3846\text{ m})T - 2608\text{ N}\cdot\text{m} + M_A = 0$

$T = \dfrac{2608\text{ N}\cdot\text{m} + M_A}{1.3846\text{ m}}$

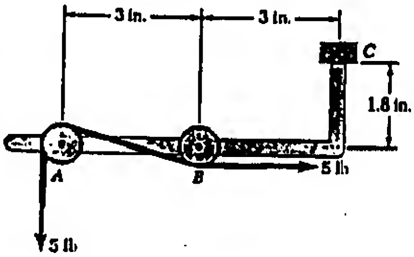
FOR $M_A = +500\text{ N}\cdot\text{m}$: $\quad T = \dfrac{2608 + 500}{1.3846} = 2244.7\text{ N}$

FOR $M_A = -500\text{ N}\cdot\text{m}$: $\quad T = \dfrac{2608 - 500}{1.3846} = 1522.4\text{ N}$

$T_{max} = 2240\text{ N};\ T_{min} = 1522\text{ N}$ ◀

4.45 and 4.46

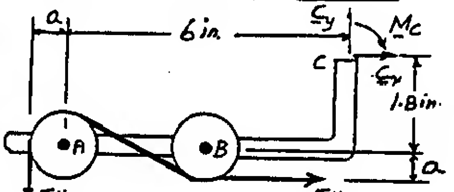
DENOTE: RADII OF PULLEYS BY a.



FIND: REACTION AT C

PROB. 4.45: $a = 0.4$ in.

PROB. 4.46: $a = 0.6$ in.



FREE-BODY DIAGRAM

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $C_x + 5\text{ lb} = 0$; $C_x = -5\text{ lb}$ $\quad C_x = 5\text{ lb} \leftarrow$

$+\uparrow\Sigma F_y = 0$: $C_y - 5\text{ lb} = 0$; $C_y = +5\text{ lb}$ $\quad C_y = 5\text{ lb}\uparrow$

$C_x = 5\text{ lb}$, $C_y = 5\text{ lb}$ $\qquad C = 7.07\text{ lb} \measuredangle 45°$

$+\circlearrowleft\Sigma M_C = 0$: $(5\text{ lb})(6\text{ in.} + a) + (5\text{ lb})(1.8\text{ in.} + a) - M_C = 0$

$M_C = 39\text{ lb}\cdot\text{in.} + (10\text{ lb})a \qquad (1)$

PROB. 4.45 WITH $a = 0.4$ in.

EQ(1): $M_C = 39\text{ lb}\cdot\text{in.} + (10\text{ lb})(0.4\text{ in.}) = +43.0\text{ lb}\cdot\text{in.}$

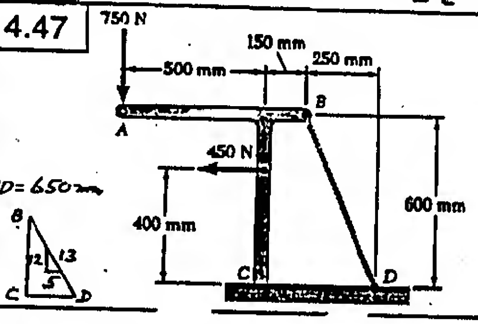
$C = 7.07\text{ lb} \measuredangle 45°$; $M_C = 43\text{ lb}\cdot\text{in.}\circlearrowleft$ ◀

PROB. 4.46 WITH $a = 0.6$ in.

EQ(1): $M_C = 39\text{ lb}\cdot\text{in.} + (10\text{ lb})(0.6\text{ in.}) = +45\text{ lb}\cdot\text{in.}$

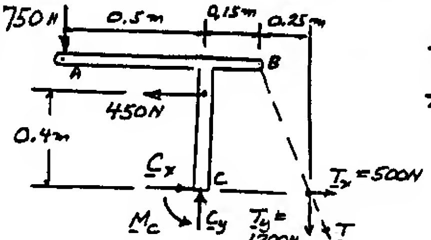
$C = 7.07\text{ lb} \measuredangle 45°$; $M_C = 45\text{ lb}\cdot\text{in.}\circlearrowleft$ ◀

4.47



GIVEN: $T_{BD} = 1300$ N

FIND: REACTION AT C



$T = 1300$ N

$T_x = \frac{5}{13}T = 500$ N

$T_y = \frac{12}{13}T = 1200$ N

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $C_x - 450\text{ N} + 500\text{ N} = 0$; $C_x = -50$ N; $C_x = 50\text{ N} \leftarrow$

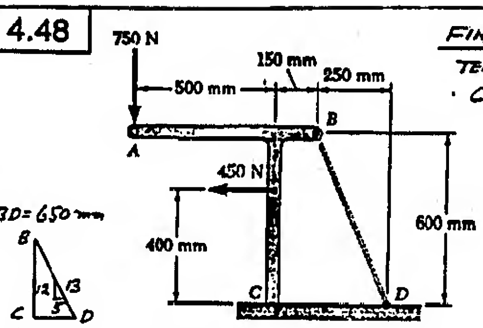
$+\uparrow\Sigma F_y = 0$: $C_y - 750\text{ N} - 1200\text{ N} = 0$; $C_y = +1950$ N; $C_y = 1950\text{ N}\uparrow$

$C_x = 50$ N, $C_y = 1950$ N $\qquad C = 1951\text{ N} \measuredangle 88.5°$ ◀

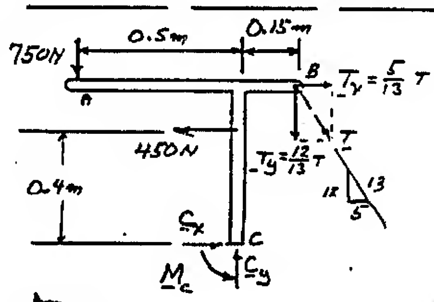
$+\circlearrowleft\Sigma M_C = 0$: $M_C + (750\text{ N})(0.5\text{ m}) + (450\text{ N})(0.4\text{ m}) - (1200\text{ N})(0.4\text{ m}) = 0$

$M_C = -75\text{ N}\cdot\text{m} \qquad M_C = 75\text{ N}\cdot\text{m}\circlearrowright$ ◀

4.48



FIND RANGE OF TENSION IN CABLE BD FOR WHICH $|M_C| \le 100$ N·m



$+\circlearrowleft\Sigma M_C = 0$:

$(750\text{ N})(0.5\text{ m}) + (450\text{ N})(0.4\text{ m}) - (\frac{5}{13}T)(0.6\text{ m}) - (\frac{12}{13}T)(0.15\text{ m}) + M_C = 0$

$375\text{ N}\cdot\text{m} + 180\text{ N}\cdot\text{m} - (\frac{4.8}{13}\text{ m})T + M_C = 0$

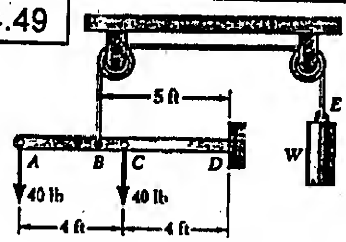
$T = \frac{13}{4.8}(555 - M_C)$

FOR $M_C = +100$ N·m: $T = \frac{13}{4.8}(555 - 100) = 1232$ N

FOR $M_C = -100$ N·m: $T = \frac{13}{4.8}(555 - (-100)) = 1774$ N

FOR $|M_C| \le 100$ N·m: $\qquad 1.232\text{ kN} \le T \le 1.774\text{ kN}$ ◀

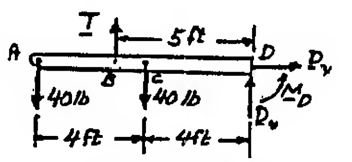
4.49



FIND: REACTION AT D

(a) WHEN $W = 100$ lb

(b) WHEN $W = 90$ lb



FREE-BODY DIAGRAM

TENSION IN CORD: $T = W$

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $D_x = 0$

$+\uparrow\Sigma F_y = 0$: $D_y - 40\text{ lb} - 40\text{ lb} + T = 0$

$D_y = 80\text{ lb} - T \qquad (1)$

$+\circlearrowleft\Sigma M_D = 0$: $M_D + (40\text{ lb})(8\text{ ft}) + (40\text{ lb})(4\text{ ft}) - T(5\text{ ft}) = 0$

$M_D = -480\text{ lb}\cdot\text{ft} + T(5\text{ ft}) \qquad (2)$

a. WHEN $W = 100$ lb: $T = 100$ lb, $D_x = 0$

EQ.(1): $D_y = 80\text{ lb} - 100\text{ lb} = -20\text{ lb}$ $\qquad D = 20\text{ lb}\downarrow$ ◀

EQ.(2): $M_D = -480\text{ lb}\cdot\text{ft} + (100\text{ lb})(5\text{ ft})$

$M_D = +20\text{ lb}\cdot\text{ft}$ $\qquad M_D = 20\text{ lb}\cdot\text{ft}\circlearrowleft$ ◀

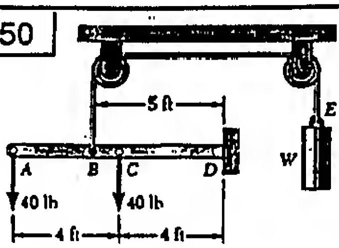
b. WHEN $W = 90$ lb: $T = 90$ lb, $D_x = 0$

EQ.(1): $D_y = 80\text{ lb} - 90\text{ lb} = -10\text{ lb}$ $\qquad D = 10\text{ lb}\downarrow$ ◀

EQ.(2): $M_D = -480\text{ lb}\cdot\text{ft} + (90\text{ lb})(5\text{ ft})$

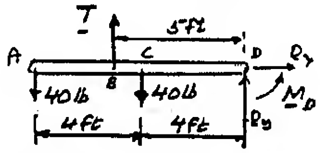
$M_D = -30\text{ lb}\cdot\text{ft}$ $\qquad M_D = 30\text{ lb}\cdot\text{ft}\circlearrowright$ ◀

4.50



FIND: RANGE OF W FOR WHICH $|M_D| \le 40\ lb\cdot ft$

FREE-BODY DIAGRAM



TENSION IN CORD: $T = W$

$+\circlearrowleft \Sigma M_D = 0: \quad M_D + (40\,lb)(8\,ft) + (40\,lb)(4\,ft) - T(5\,ft) = 0$

$T = \frac{1}{5\,ft}(480\ lb\cdot ft + M_D)$

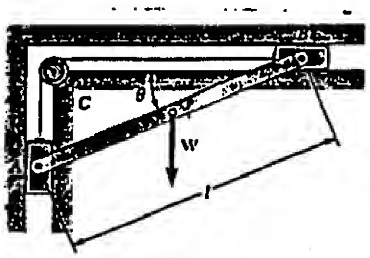
FOR $M_D = +40\ lb\cdot ft$: $T = \frac{1}{5}(480 + 40) = 104\ lb$

FOR $M_D = -40\ lb\cdot ft$: $T = \frac{1}{5}(480 - 40) = 88\ lb$

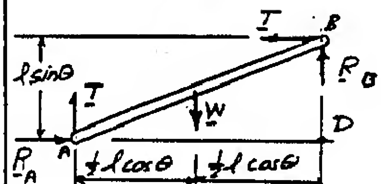
RECALL THAT WEIGHT W = TENSION T, WE HAVE

FOR $|M_D| \le 40\ lb\cdot ft$: $\quad 88\,lb \le W \le 104\,lb$ ◀

4.51



FIND:
(a) TENSION IN CORD IN TERMS OF W AND θ
(b) VALUE OF θ FOR $T = 3W$



(a) $+\circlearrowleft \Sigma M_D = 0$:

$T(l\sin\theta) - T(l\cos\theta) + W(\frac{1}{2}l\cos\theta) = 0$

$T = \frac{1}{2}W\frac{\cos\theta}{\cos\theta - \sin\theta}$

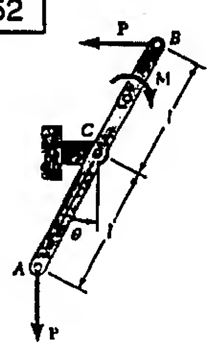
$T = \frac{1}{2}W/(1 - \tan\theta)$ ◀

(b) FOR $T = 3W$: $\quad 3W = \frac{1}{2}W/(1 - \tan\theta)$

$3 - 3\tan\theta = \frac{1}{2}W$

$\tan\theta = \frac{2.5}{3} = \frac{5}{6} \qquad \theta = 39.8^\circ$ ◀

4.52



FOR EQUILIBRIUM

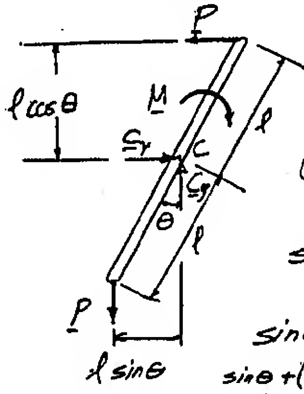
FIND:

(a) EQUATION IN θ, P, M, AND l

(b) VALUE OF θ, FOR $M = 150\ N\cdot m$, $P = 200\ N$, $l = 600\ mm$

(CONTINUED)

4.52 CONTINUED



FREE-BODY DIAGRAM

(a) $+\circlearrowleft \Sigma M_C = 0$:

$Pl\cos\theta + Pl\sin\theta - M = 0$

$\sin\theta + \cos\theta = \frac{M}{Pl}$ ◀

(b) FOR $M = 150\ N\cdot m$, $P = 200\ N$, AND $l = 600\ mm$

$\sin\theta + \cos\theta = \frac{150\ N\cdot m}{(200\,N)(0.6\,m)}$

$\sin\theta + \cos\theta = 1.25$

$\sin\theta + (1 - \sin^2\theta)^{1/2} = 1.25$

$(1 - \sin^2\theta)^{1/2} = 1.25 - \sin\theta$

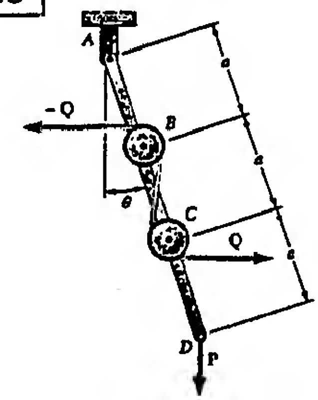
$1 - \sin^2\theta = 1.5625 - 2.5\sin\theta + \sin^2\theta$

$2\sin^2\theta - 2.5\sin\theta + 0.5625 = 0$

$\sin\theta = 0.2943$ and $\sin\theta = 0.9557$

$\theta = 17.1^\circ$ and $\theta = 72.9^\circ$ ◀

4.53

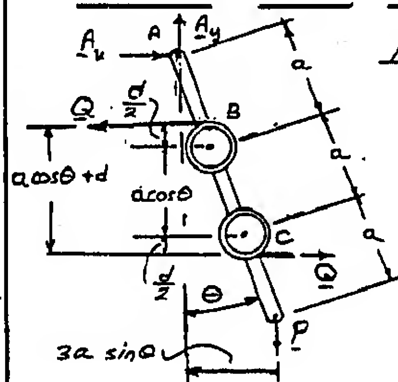


FOR EQUILIBRIUM

FIND:

(a) $P = f(Q, a, d, \theta)$

(b) MAGNITUDE OF P FOR $Q = 10\ lb$, $a = 5\ in.$, $d = 0.8\ in.$, AND $\theta = 30^\circ$.



FREE-BODY DIAGRAM

(a) $+\circlearrowleft \Sigma M_A = 0$

$Q(a\cos\theta + d) - P(3a\sin\theta) = 0$

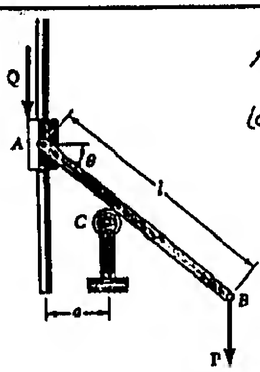
$P = \frac{Q}{3}\cdot\frac{a\cos\theta + d}{a\sin\theta}$ ◀

(b) FOR $Q = 10\ lb$, $a = 5\ in.$, $d = 0.8\ in$, $\theta = 30^\circ$

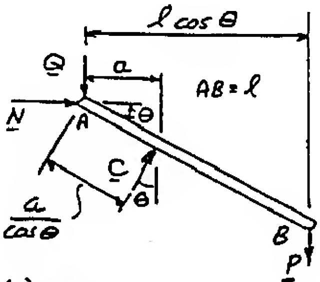
$P = \frac{10\ lb}{3}\cdot\frac{(5\ in.)\cos 30^\circ + 0.8\ in.}{(5\ in.)\sin 30^\circ}$

$P = 6.840\ lb \qquad\qquad P = 6.84\ lb$ ◀

4.54



FOR EQUILIBRIUM
FIND:
(a) EQUATION IN P, Q, a, l, AND θ.
(b) VALUE OF θ, FOR $P = 16$ lb, $Q = 12$ lb, $l = 20$ in., AND $a = 5$ in.



FREE-BODY DIAGRAM

$+\uparrow \Sigma F_y = 0$:

$C\cos\theta - P - Q = 0$

$C = \dfrac{P+Q}{\cos\theta}$

(a)

$+\circlearrowleft \Sigma M_A = 0$: $\quad C\dfrac{a}{\cos\theta} - Pl\cos\theta = 0$

$\dfrac{P+Q}{\cos\theta}\cdot\dfrac{a}{\cos\theta} - Pl\cos\theta = 0$

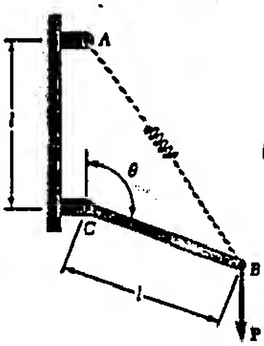
$\cos^3\theta = \dfrac{a(P+Q)}{Pl}$ ◀

(b) FOR $P = 16$ lb, $Q = 12$ lb, $l = 20$ in., AND $a = 5$ in:

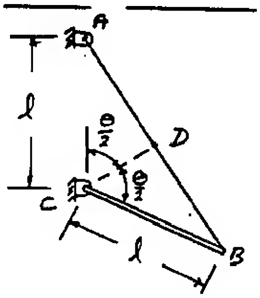
$\cos^3\theta = \dfrac{(5\text{ in.})(16\text{ lb} + 12\text{ lb})}{(16\text{ lb})(20\text{ in.})} = 0.4375$

$\cos\theta = 0.75915 \qquad \theta = 40.6^\circ$ ◀

4.55



FOR EQUILIBRIUM
FIND:
(a) $\theta = f(P, k, l)$.
(b) VALUE OF θ WHEN $P = \frac{1}{4}kl$.



GEOMETRY

$AB = 2l\sin\frac{\theta}{2}$

$CD = l\cos\frac{\theta}{2}$

LET ELONGATION OF SPRING $= s$

$s = (AB)_\theta - (AB)_{\theta = 90^\circ}$

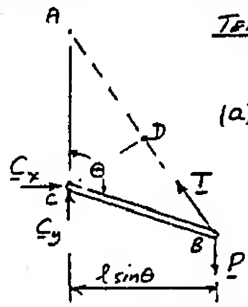
$s = 2l\sin\frac{\theta}{2} - 2l\sin 45^\circ$

$s = 2l(\sin\frac{\theta}{2} - \frac{1}{\sqrt{2}})$

(CONTINUED)

4.55 CONTINUED

FREE-BODY DIAGRAM



TENSION IN SPRING

$T = ks = 2kl(\sin\frac{\theta}{2} + \frac{1}{\sqrt{2}})$

(a) $\quad +\circlearrowleft \Sigma M_C = 0$:

$T(CD) - Pl\sin\theta = 0$

$2kl(\sin\frac{\theta}{2} + \frac{1}{\sqrt{2}})(l\cos\frac{\theta}{2})$

$- Pl\sin\theta = 0$

$2kl^2(\sin\frac{\theta}{2} + \frac{1}{\sqrt{2}})\cos\frac{\theta}{2} - Pl(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}) = 0$

$\cos\frac{\theta}{2}\left[2(kl - P)\sin\frac{\theta}{2} - \frac{2}{\sqrt{2}}kl\right] = 0$

$\cos\frac{\theta}{2} = 0$ OR $\sin\frac{\theta}{2} = \frac{1}{\sqrt{2}}\cdot\frac{kl}{kl - P}$

$\theta = 180^\circ$

(TRIVIAL)

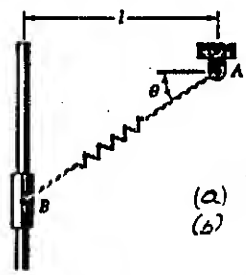
$\theta = 2\sin^{-1}\left[\frac{1}{\sqrt{2}}kl/(kl-P)\right]$ ◀

(b) FOR $P = \frac{1}{4}kl$:

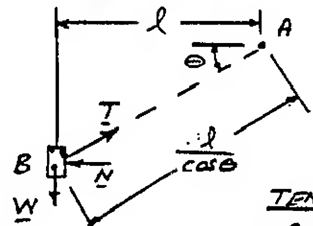
$\theta = 2\sin^{-1}\left[\frac{1}{\sqrt{2}}kl/(kl - \frac{1}{4}kl)\right] = 2\sin^{-1}(0.9428)$

$\theta = 2(70.529^\circ) = 141.06^\circ \qquad \theta = 141.1^\circ$ ◀

4.56



GIVEN:
W = WEIGHT OF COLLAR
SPRING IS UNDEFORMED FOR $\theta = 0$
FIND: FOR EQUILIBRIUM
(a) EQUATION IN θ, W, k, l
(b) VALUE OF θ WHEN $W = 300$ N, $l = 500$ mm, and $k = 800$ N/m.



FREE-BODY DIAGRAM

LET s = ELONGATION OF SPRING

$s = \dfrac{l}{\cos\theta} - l$

TENSION IN SPRING

$T = ks = kl(\frac{1}{\cos\theta} - 1)$

(a)

$+\uparrow \Sigma F_y = 0$: $\quad T\sin\theta - W = 0$

$kl(\frac{1}{\cos\theta} - 1)\sin\theta - W = 0$

$\dfrac{\sin\theta}{\cos\theta} - \sin\theta = \dfrac{W}{kl}$

$\tan\theta - \sin\theta = \dfrac{W}{kl}$ ◀

(b) $W = 300$ N, $l = 500$ mm, $k = 800$ N/m

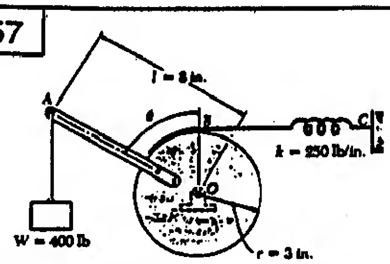
$\tan\theta - \sin\theta = \dfrac{300\text{ N}}{(800\text{ N/m})(0.5\text{ m})}$

$\tan\theta - \sin\theta = 0.75$

SOLVE BY TRIAL + ERROR: $\theta = 57.96^\circ$

$\theta = 58.0^\circ$ ◀

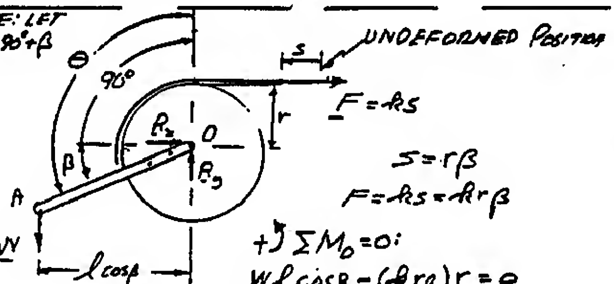
4.57



GIVEN: SPRING IS UNDEFORMED WHEN $\theta = 90^\circ$.

FIND: POSITION OF EQUILIBRIUM

NOTE: LET $\theta = 90^\circ + \beta$



$s = r\beta$

$F = ks = kr\beta$

$+\circlearrowleft \Sigma M_O = 0:$

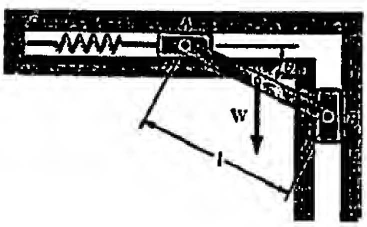
$W l \cos\beta - (kr\beta) r = 0$

$$\cos\beta = \frac{kr^2}{Wl} = \frac{(250\ \text{lb/in.})(3\ \text{in.})^2}{(400\ \text{lb})(8\ \text{in.})} = 0.703125\beta$$

SOLVE BY TRIAL AND ERROR: $\beta = 0.89245$ rad $\quad \beta = 51.1^\circ$

$\theta = 90^\circ + \beta = 90^\circ + 51.1^\circ \quad \theta = 141.1^\circ$ ◀

4.58



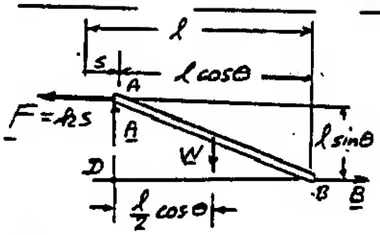
GIVEN: SPRING IS UNDEFORMED WHEN $\theta = 0$.

FIND: FOR EQUILIBRIUM,

(a) EQUATION IN W, l, k, θ

(b) VALUE OF θ WHEN $W = 75$ lb, $l = 30$ in., AND $k = 3$ lb/in.

FREE-BODY DIAGRAM



SPRING DEFORMATION

$s = l - l\cos\theta = l(1-\cos\theta)$

$F = ks = kl(1-\cos\theta)$

(a)

$+\circlearrowleft \Sigma M_D = 0: \quad F l \sin\theta - W\left(\frac{l}{2}\cos\theta\right) = 0$

$kl(1-\cos\theta)\sin\theta - \frac{1}{2} W l \cos\theta = 0$

$$(1-\cos\theta)\tan\theta = \frac{W}{2kl}$$ ◀

(b) WHEN $W = 75$ lb, $l = 30$ in., AND $k = 3$ lb/in.

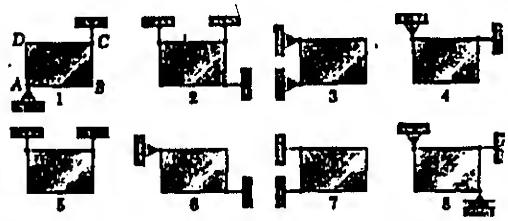
$$(1-\cos\theta)\tan\theta = \frac{75\ \text{lb}}{2(3\ \text{lb/in.})(30\ \text{in.})}$$

$(1-\cos\theta)\tan\theta = 0.41667$

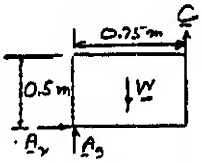
SOLVE BY TRIAL AND ERROR

$\theta = 49.71^\circ \qquad \theta = 49.7^\circ$ ◀

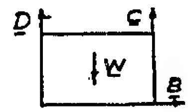
4.59



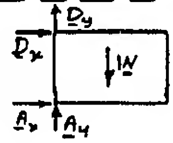
DETERMINE WHETHER (a) PLATE IS CONSTRAINED, (b) REACTIONS ARE DETERMINATE, (c) IF POSSIBLE, FIND REACTIONS. $m = 40$ kg; $W = (40\ \text{kg})9.81\ \text{m/s}^2 = 392.4$ N



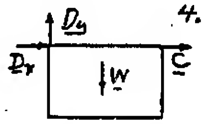
1. THREE NON-CONCURRENT, NON-PARALLEL REACTIONS:
   (a) PLATE: COMPLETELY CONSTRAINED
   (b) REACTIONS: DETERMINATE
   (c) EQUILIBRIUM MAINTAINED
   $A = C = 196.2$ N ↑



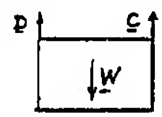
2. THREE NON-CONCURRENT, NON-PARALLEL REACTIONS
   (a) PLATE: COMPLETELY CONSTRAINED
   (b) REACTIONS: DETERMINATE
   (c) EQUILIBRIUM MAINTAINED
   $B = 0$, $C = D = 196.2$ N ↑



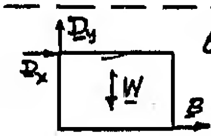
3. FOUR NON-CONCURRENT, NON-PARALLEL REACTIONS
   (a) PLATE: COMPLETELY CONSTRAINED
   (b) REACTIONS: INDETERMINATE
   (c) EQUILIBRIUM MAINTAINED
   $A_x = 294$ N →, $D_x = 294$ N ←
   $(A_y + D_y = 392$ N ↑)



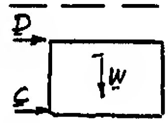
4. THREE CONCURRENT REACTIONS (THROUGH D)
   (a) PLATE: IMPROPERLY CONSTRAINED
   (b) REACTIONS: INDETERMINATE
   (c) NO EQUILIBRIUM $(\Sigma M_D \neq 0)$



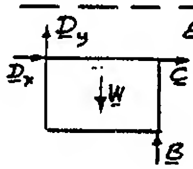
5. TWO REACTIONS
   (a) PLATE: IMPROPER CONSTRAINT
   (b) REACTIONS: DETERMINATE
   (c) EQUILIBRIUM MAINTAINED
   $C = D = 196.2$ N ↑



6. THREE NON-CONCURRENT, NON-PARALLEL REACTIONS
   (a) PLATE: COMPLETELY CONSTRAINED
   (b) REACTIONS: DETERMINATE
   (c) EQUILIBRIUM MAINTAINED
   $B = 294$ N →, $D = 491$ N $\measuredangle$ 53.1°



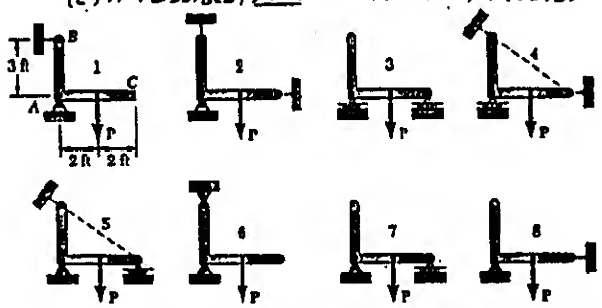
7. TWO REACTIONS
   (a) PLATE: IMPROPERLY CONSTRAINED
   (b) REACTIONS DETERMINED BY DYNAMICS
   (c) NO EQUILIBRIUM $(\Sigma F_y \neq 0)$



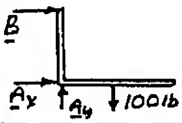
8. FOUR NON-CONCURRENT, NON-PARALLEL REACTIONS
   (a) PLATE: COMPLETELY CONSTRAINED
   (b) REACTIONS: INDETERMINATE
   (c) EQUILIBRIUM MAINTAINED
   $B = D_y = 196.2$ N ↑
   $(C + D_x = 0)$

4.60

DETERMINE WHETHER (a) BRACKET IS CONSTRAINED, (b) REACTIONS ARE DETERMINATE, (c) IF POSSIBLE, FIND REACTIONS. $P = 100\,lb$.



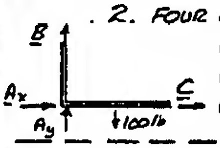
1. THREE NON-CONCURRENT, NON-PARALLEL REACTIONS



(a) BRACKET: COMPLETE CONSTRAINT
(b) REACTIONS: DETERMINATE
(c) EQUILIBRIUM MAINTAINED

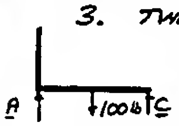
$\underline{A} = 120.2\,lb \angle 56.3^\circ$, $\underline{B} = 66.7\,lb \leftarrow$

2. FOUR CONCURRENT REACTIONS (THROUGH A)



(a) BRACKET: IMPROPER CONSTRAINT
(b) REACTIONS: INDETERMINATE
(c) NO EQUILIBRIUM $(\Sigma M_A \neq 0)$

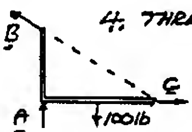
3. TWO REACTIONS



(a) BRACKET: PARTIAL CONSTRAINT
(b) REACTIONS: DETERMINATE
(c) EQUILIBRIUM MAINTAINED

$\underline{A} = 50\,lb \uparrow$, $\underline{C} = 50\,lb \uparrow$

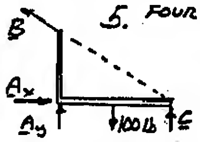
4. THREE NON-CONCURRENT, NON-PARALLEL REACTIONS



(a) BRACKET: COMPLETE CONSTRAINT
(b) REACTIONS: DETERMINATE
(c) EQUILIBRIUM MAINTAINED

$\underline{A} = 50\,lb\uparrow$, $\underline{B} = 83.3\,lb \measuredangle 36.9^\circ$, $\underline{C} = 66.7\,lb \rightarrow$

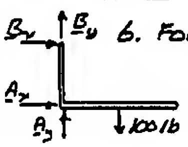
5. FOUR NON-CONCURRENT, NON-PARALLEL REACTIONS



(a) BRACKET: COMPLETE CONSTRAINT
(b) REACTIONS: INDETERMINATE
(c) EQUILIBRIUM MAINTAINED

$(\Sigma M_C = 0)$ $\underline{A}_y = 50\,lb\uparrow$

6. FOUR NON-CONCURRENT, NON-PARALLEL REACTIONS

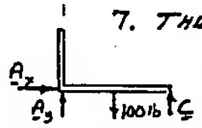


(a) BRACKET: COMPLETE CONSTRAINT
(b) REACTIONS: INDETERMINATE
(c) EQUILIBRIUM MAINTAINED

$\underline{A}_x = 66.7\,lb \rightarrow$, $\underline{B}_x = 66.7\,lb \leftarrow$

$(\underline{A}_y + \underline{B}_y = 100\,lb\uparrow)$

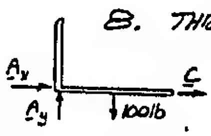
7. THREE NON-CONCURRENT, NON-PARALLEL REACTIONS



(a) BRACKET: COMPLETE CONSTRAINT
(b) REACTIONS: DETERMINATE
(c) EQUILIBRIUM MAINTAINED

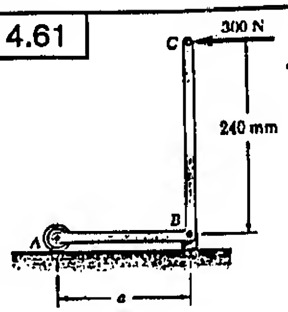
$\underline{A} = \underline{C} = 50\,lb\uparrow$

8. THREE CONCURRENT REACTIONS (THROUGH A)



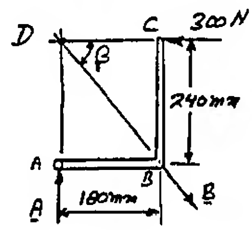
(a) BRACKET: IMPROPER CONSTRAINT
(b) REACTIONS: INDETERMINATE
(c) NO EQUILIBRIUM $(\Sigma M_A \neq 0)$

4.61



GIVEN: $a = 180\,mm$

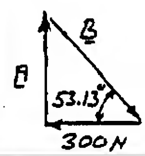
FIND: REACTIONS



FREE-BODY DIAGRAM (THREE-FORCE MEMBER)
REACTION AT B MUST PASS THROUGH D WHERE A AND 300-N LOAD INTERSECT.

ΔBCD: $\tan\beta = \frac{240}{180}$; $\beta = 53.13^\circ$

FORCE TRIANGLE



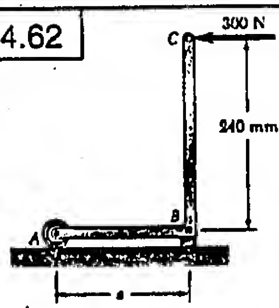
$A = (300\,N)\tan 53.13^\circ = 400\,N$

$\underline{A} = 400\,N \uparrow$ ◀

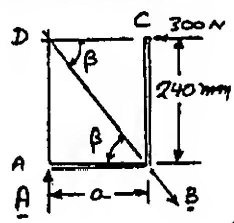
$B = \frac{300\,N}{\cos 53.13^\circ} = 500\,N$

$\underline{B} = 500\,N \measuredangle 53.1^\circ$ ◀

4.62

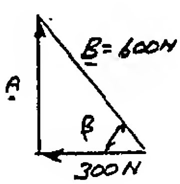


FIND: RANGE OF DISTANCE a FOR WHICH $B \le 600\,N$



FREE-BODY DIAGRAM (THREE-FORCE MEMBER)
REACTION AT B MUST PASS THROUGH D WHERE A AND 300-N LOAD INTERSECT.

$$a = \frac{240\,mm}{\sin\beta} \quad (1)$$



FORCE TRIANGLE (WITH $B = 600\,N$)

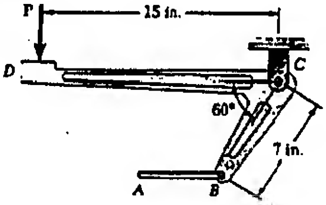
$$\cos\beta = \frac{300\,N}{600\,N} = 0.5$$

$$\beta = 60^\circ$$

EQ. (1) $a = \frac{240\,mm}{\tan 60^\circ} = 138.56\,mm$

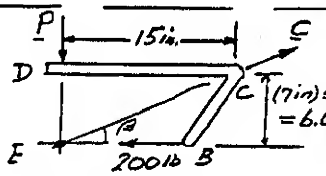
FOR $B \le 600\,N$; $a \ge 138.6\,mm$ ◀

4.63



GIVEN: TENSION IN AB = 200 lb.

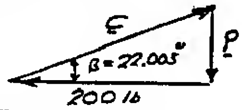
FIND:
(a) FORCE P.
(b) REACTION AT C.



FREE-BODY DIAGRAM (3-FORCE BODY)
REACTION AT C MUST PASS THROUGH E, WHERE D AND 200-lb. FORCE INTERSECT

$\tan\beta = \frac{6.062 \text{ in.}}{15 \text{ in.}}$; $\beta = 22.005°$

FORCE TRIANGLE



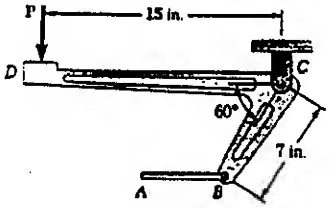
(a) $P = (200 \text{ lb}) \tan 22.005°$

$P = 80.831 \text{ lb}$ $\quad P = 80.8 \text{ lb} \downarrow$ ◄

(b) $C = \frac{200 \text{ lb}}{\cos 22.005°} = 215.7 \text{ lb}$

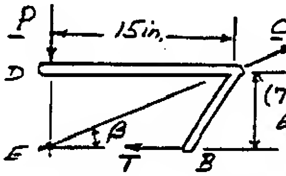
$C = 216 \text{ lb} \measuredangle 22.0°$ ◄

4.64



GIVEN: REACTION AT C = 250 lb.

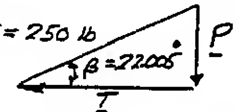
FIND: TENSION IN CABLE AB.



FREE-BODY DIAGRAM (3-FORCE BODY)
REACTION AT C MUST PASS THROUGH E, WHERE D AND THE FORCE T INTERSECT

$\tan\beta = \frac{6.062 \text{ in.}}{15 \text{ in.}}$; $\beta = 22.005°$

FORCE TRIANGLE

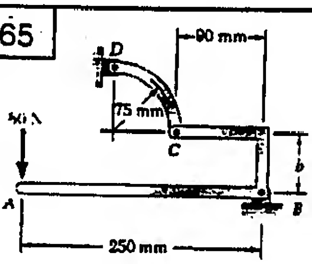


$T = (250 \text{ lb}) \cos 22.005°$

$T = 231.8 \text{ lb}$

$T = 232 \text{ lb}$ ◄

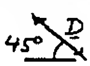
4.65



GIVEN: $b = 60$ mm

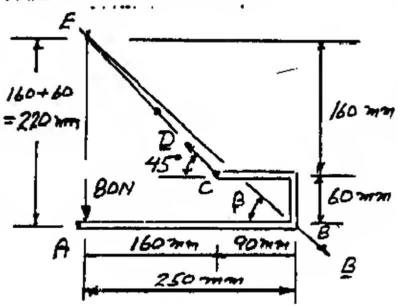
FIND: REACTIONS AT B AND D.

SINCE CD IS A TWO-FORCE MEMBER, THE LINE OF ACTION OF REACTION AT D MUST PASS THROUGH POINTS C AND D.



(CONTINUED)

4.65 CONTINUED

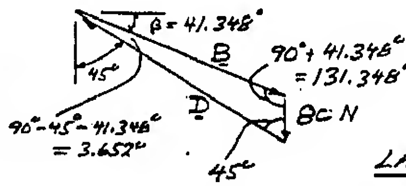


FREE-BODY DIAGRAM (3-FORCE BODY)
REACTION AT B MUST PASS THROUGH E, WHERE THE REACTION AT D AND 80-N FORCE INTERSECT.

$\tan\beta = \frac{220 \text{ mm}}{250 \text{ mm}}$

$\beta = 41.348°$

FORCE TRIANGLE



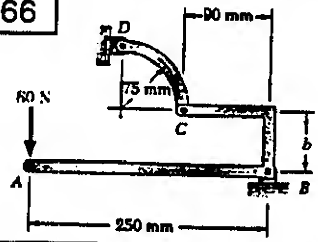
LAW OF SINES

$$\frac{80 \text{ N}}{\sin 3.652°} = \frac{B}{\sin 45°} = \frac{D}{\sin 131.348°}$$

$B = 888.0 \text{ N} \quad D = 942.8 \text{ N}$

$B = 888 \text{ N} \measuredangle 41.3°$, $D = 943 \text{ N} \measuredangle 45°$ ◄

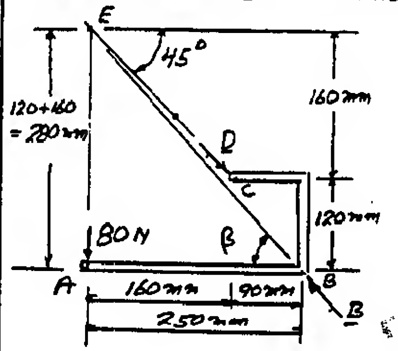
4.66



GIVEN: $b = 120$ mm

FIND: REACTIONS AT B AND D

SINCE CD IS A 2-FORCE MEMBER, LINE OF ACTION OF REACTION AT D MUST PASS THROUGH C & D $\measuredangle 45°$

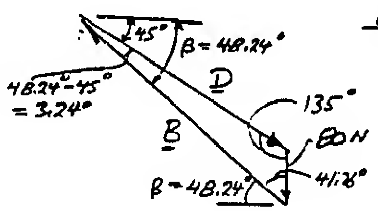


FREE-BODY DIAGRAM (3 FORCE BODY)
REACTION AT B MUST PASS THROUGH E, WHERE THE REACTION AT D AND 80-N FORCE INTERSECT

$\tan\beta = \frac{280 \text{ mm}}{250 \text{ mm}}$

$\beta = 48.24°$

FORCE TRIANGLE



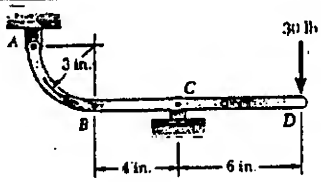
LAW OF SINES

$$\frac{80 \text{ N}}{\sin 3.24°} = \frac{B}{\sin 135°} = \frac{D}{\sin 41.76°}$$

$B = 1000.9 \text{ N} \quad D = 942.8 \text{ N}$

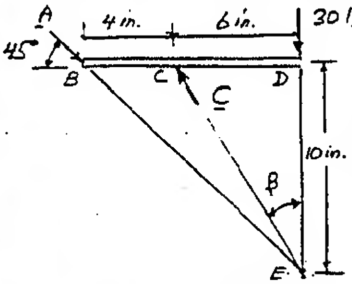
$B = 1001 \text{ N} \measuredangle 48.2°$, $D = 943 \text{ N} \measuredangle 45°$ ◄

4.67



FIND: REACTIONS AT A AND C.

SINCE AB IS A TWO-FORCE MEMBER, THE REACTION AT A MUST PASS THROUGH POINTS A AND B.

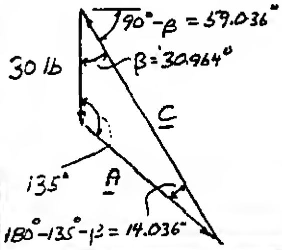


FREE-BODY DIAGRAM (3-FORCE BODY)

REACTION AT C MUST PASS THROUGH E WHERE REACTION AT A AND 30-lb FORCE INTERSECT.

ΔCDE:

$\tan\beta = \frac{6 \text{ in.}}{10 \text{ in.}}$; $\beta = 30.964°$



FORCE TRIANGLE

LAW OF SINES

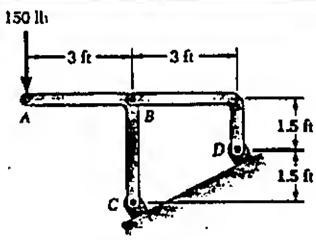
$$\frac{30 \text{ lb}}{\sin 14.036°} = \frac{A}{\sin 30.964°} = \frac{C}{\sin 135°}$$

$A = 63.64$ lb, $C = 87.46$ lb

$\underline{A} = 63.6$ lb ⦩ 45° ◀

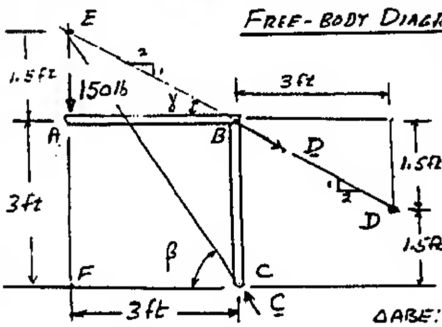
$\underline{C} = 87.5$ lb ⦪ 59.0° ◀

4.68



FIND: REACTIONS AT C AND D

SINCE BD IS A TWO-FORCE MEMBER, THE REACTION AT D MUST PASS THROUGH POINTS B AND D.



FREE-BODY DIAGRAM (3-FORCE BODY)

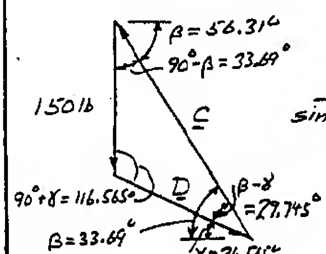
REACTION AT C MUST PASS THROUGH E WHERE REACTION AT D AND 150-lb LOAD INTERSECT

ΔCEF:

$\tan\beta = \frac{4.5 \text{ ft}}{3 \text{ ft}}$

$\beta = 56.31°$

ΔABE: $\tan\gamma = \frac{1}{2}$, $\gamma = 26.565°$



FORCE TRIANGLE LAW OF SINES

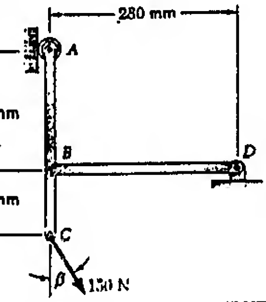
$$\frac{150 \text{ lb}}{\sin 29.745°} = \frac{C}{\sin 116.565°} = \frac{D}{\sin 33.69}$$

$C = 270.4$ lb, $D = 167.7$ lb

$\underline{C} = 270$ lb ⦪ 56.3° ◀

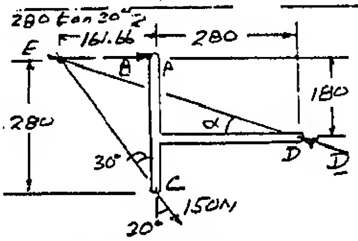
$\underline{D} = 167.7$ lb ⦩ 26.6° ◀

4.69



GIVEN: $\beta = 30°$

FIND: REACTIONS AT A AND D.

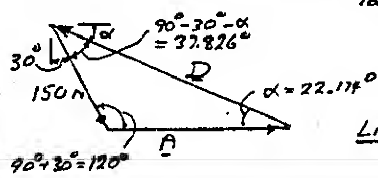


FREE-BODY DIAGRAM (3-FORCE BODY)

REACTION AT D MUST PASS THROUGH POINT E WHERE REACTION AT A AND 150-N LOAD INTERSECT

DIMENSIONS IN mm.

$\tan\alpha = \frac{180}{161.66 + 280}$: $\alpha = 22.174°$



FORCE TRIANGLE

LAW OF SINES

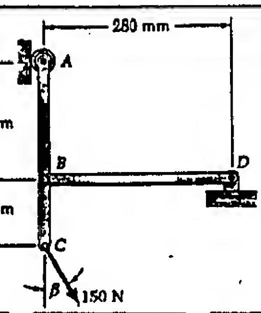
$$\frac{150 \text{ N}}{\sin 22.174°} = \frac{A}{\sin 37.826°} = \frac{D}{\sin 120°}$$

$A = 243.7$ N

$D = 344.2$ N

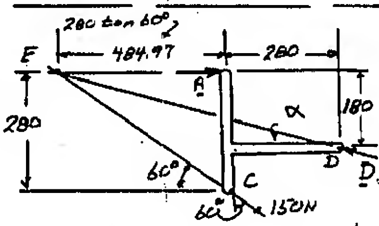
$\underline{A} = 244$ N ⟶ ; $\underline{D} = 344$ N ⦪ 22.2° ◀

4.70



GIVEN: $\beta = 60°$

FIND: REACTIONS AT A AND D

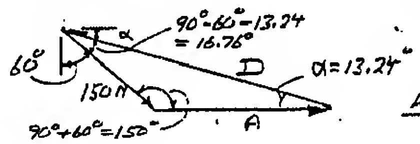


FREE-BODY DIAGRAM (3-FORCE BODY)

REACTION AT D MUST PASS THROUGH E WHERE REACTION AT A AND 150-N LOAD INTERSECT.

DIMENSIONS IN mm

$\tan\alpha = \frac{180}{484.97 + 280}$; $\alpha = 13.24°$



FORCE TRIANGLE

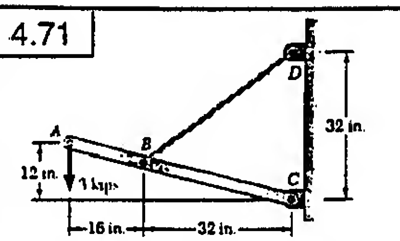
LAW OF SINES

$$\frac{150 \text{ N}}{\sin 13.24°} = \frac{A}{\sin 16.76°} = \frac{D}{\sin 150°}$$

$A = 188.8$ N $D = 327.4$ N

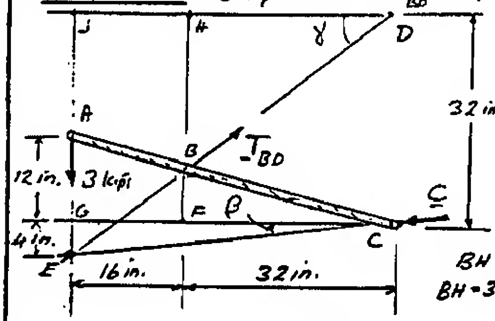
$\underline{A} = 188.4$ N ⟶ ; $\underline{D} = 327$ N ⦪ 13.2° ◀

4.71



FIND:
(a) TENSION IN CORD BD
(b) REACTION AT C

3-FORCE BODY: 3-kip LOAD AND T_{BD} INTERSECT AT E



GEOMETRY

$$\frac{BF}{AG} = \frac{CF}{CG}$$

$$\frac{BF}{12\,in.} = \frac{32\,in}{48\,in}$$

$BF = 8\,in$

$BH = 32\,in - BF$

$BH = 32 - 8 = 24\,in.$

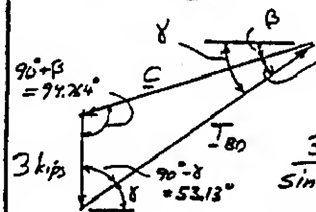
$$\frac{JE}{BH} = \frac{DJ}{DH}\,;\quad \frac{JE}{24\,in.} = \frac{48\,in.}{32\,in.}\,;\quad JE = 36\,in.$$

$EG = JE - JG = 36\,in. - 32\,in = 4\,in.$

IN Δ CEG: $\tan\beta = \frac{EG}{CG} = \frac{4\,in.}{48\,in.}$; $\beta = 4.764°$

IN Δ BDH: $\tan\gamma = \frac{BH}{DH} = \frac{24\,in.}{32\,in.}$; $\gamma = 36.87°$

FORCE TRIANGLE FOR 3 FORCES INTERSECTING AT E



$\gamma - \beta = 36.87° - 4.764° = 32.106°$

LAW OF SINES

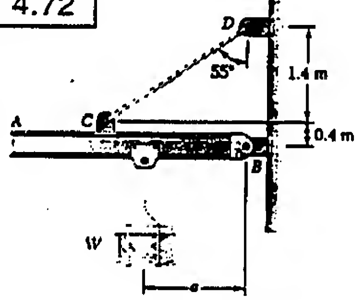
$$\frac{3\,kips}{\sin 32.106°} = \frac{T_{BD}}{\sin 94.764°} = \frac{C}{\sin 53.13°}$$

$T_{BD} = 5.625\,kips$; $C = 4.516\,kips$

(a) $T_{BD} = 5.63\,kips$ ◀

(b) $C = 4.52\,kips$ ⦨ 4.8° ◀

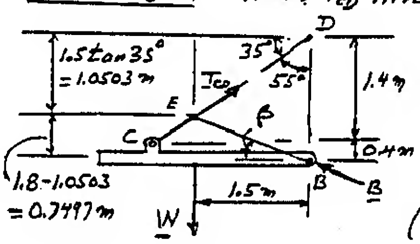
4.72



GIVEN:
$a = 1.5\,m$
$W = 50\,kg$

FIND:
(a) TENSION IN CABLE CD
(b) REACTION AT B

3-FORCE BODY: W AND T_{CD} INTERSECT AT E



FREE-BODY DIAGRAM

$$\tan\beta = \frac{0.7497\,m}{1.5\,m}$$

$\beta = 26.56°$

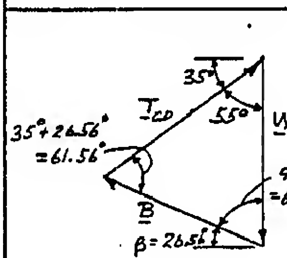
(CONTINUED)

4.72 CONTINUED

FORCE TRIANGLE

3 FORCES INTERSECT AT E

$W = (50\,kg)9.81\,m/s^2 = 490.5\,N$



LAW OF SINES

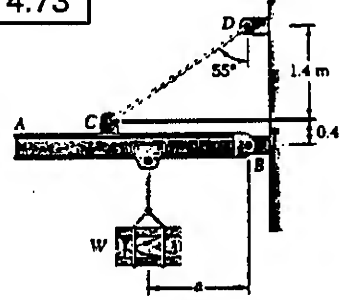
$$\frac{490.5\,N}{\sin 61.56°} = \frac{T_{CD}}{\sin 63.44°} = \frac{B}{\sin 55°}$$

$T_{CD} = 498.9\,N$; $B = 456.9\,N$

(a) $T_{CD} = 499\,N$ ◀

(b) $B = 457\,N$ ⦪ 26.6° ◀

4.73

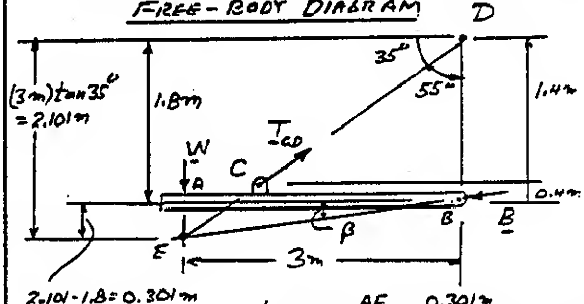


GIVEN:
$a = 3\,m$
$W = 50\,kg$

FIND:
(a) TENSION IN CABLE CD
(b) REACTION AT B

3-FORCE BODY W AND T_{CD} INTERSECT AT E

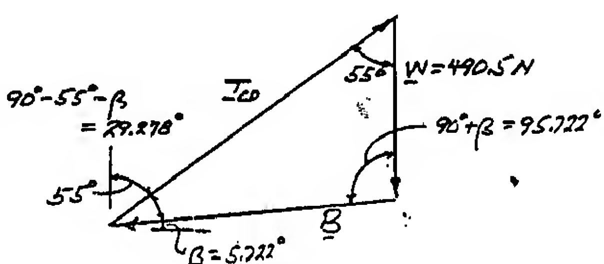
FREE-BODY DIAGRAM



$$\tan\beta = \frac{AE}{AB} = \frac{0.301\,m}{3\,m}\,;\quad \beta = 5.722°$$

FORCE TRIANGLE (3 FORCES INTERSECT AT E)

$W = (50\,kg)9.81\,m/s^2 = 490.5\,N$



LAW OF SINES

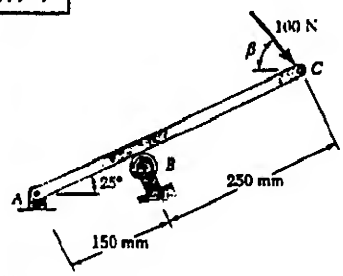
$$\frac{490.5\,N}{\sin 29.278°} = \frac{T_{CD}}{\sin 95.722°} = \frac{B}{\sin 55°}$$

$T_{CD} = 997.99\,N$; $B = 821.59\,N$

(a) $T_{CD} = 998\,N$ ◀

(b) $B = 822\,N$ ⦨ 5.7° ◀

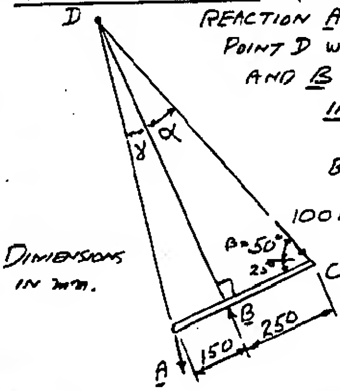
4.74



GIVEN: $\beta = 50°$

FIND: REACTIONS AT A AND B.

FREE-BODY DIAGRAM (3-FORCE BODY)



REACTION $\underline{A}$ MUST PASS THROUGH POINT D WHERE 100-N FORCE AND $\underline{B}$ INTERSECT

IN RIGHT △ BCD:

$\alpha = 90° - 75° = 15°$

$BD = 250 \tan 75° = 933.0$ mm

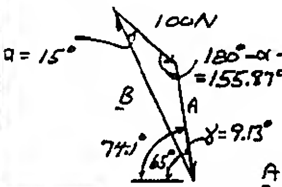
IN RIGHT △ ABD:

$\tan\gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933 \text{ mm}}$

$\gamma = 9.13°$

DIMENSIONS IN mm.

FORCE TRIANGLE



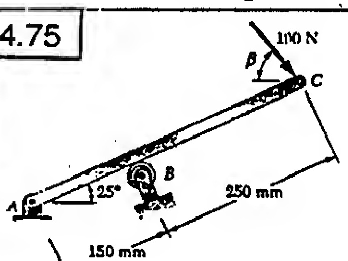
LAW OF SINES

$$\frac{100N}{\sin 9.13°} = \frac{A}{\sin 15°} = \frac{B}{\sin 155.87°}$$

$A = 163.1$ N; $B = 257.6$ N

$\underline{A} = 163.1$ N ∡ 74.1°; $\underline{B} = 258$ N ∡ 65° ◀

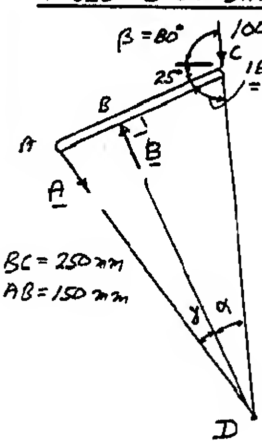
4.75



GIVEN: $\beta = 80°$

FIND: REACTIONS AT A AND B

FREE-BODY DIAGRAM (3-FORCE BODY)



REACTION A MUST PASS THROUGH POINT D WHERE 100N FORCE AND B INTERSECT

IN RIGHT △ BCD:

$\alpha = 90° - 75° = 15°$

$BD = BC \tan 75° = 250 \tan 75$

$BD = 933.0$ mm

IN RIGHT △ ABD:

$\tan\gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933 \text{ mm}}$

$\gamma = 9.13°$

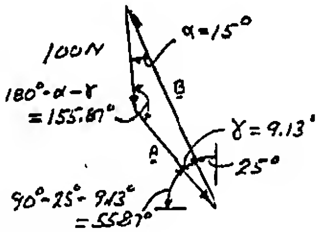
BC = 250 mm

AB = 150 mm

(CONTINUED)

4.75 CONTINUED

FORCE TRIANGLE



LAW OF SINES

$$\frac{100N}{\sin 9.13°} = \frac{A}{\sin 15°} = \frac{B}{\sin 155.87°}$$

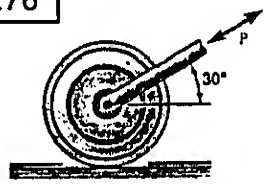
$A = 163.1$ N

$B = 257.6$ N

$A = 163.1$ N ∡ 55.9° ◀

$B = 258$ N ∡ 65° ◀

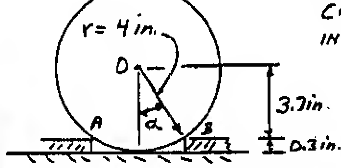
4.76



GIVEN: 40-lb ROLLER OF DIAMETER 8 in. THICKNESS OF TILE IS 0.3 in.

FIND: FORCE $\underline{P}$ TO MOVE ROLLER ONTO TILES IF ROLLER IS (a) PUSHED ←, (b) PULLED →

GEOMETRY FOR EACH CASE AS ROLLER COMES INTO CONTACT WITH TILE

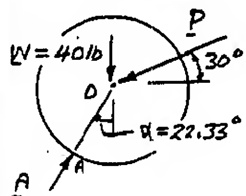


$\alpha = \cos^{-1}\frac{2.7 \text{ in.}}{3 \text{ in.}}$

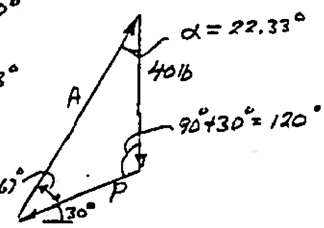
$\alpha = 22.33°$

(a) ROLLER PUSHED TO LEFT (3-FORCE BODY)

FORCES MUST PASS THROUGH O.



FORCE TRIANGLE



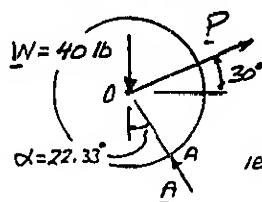
LAW OF SINES

$$\frac{40 \text{ lb}}{\sin 37.67°} = \frac{P}{\sin 22.33}; \quad P = 24.86 \text{ lb}$$

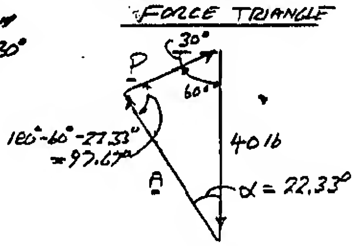
$\underline{P} = 24.9$ lb ∡ 30° ◀

(b) ROLLER PULLED TO RIGHT (3-FORCE BODY)

FORCES MUST PASS THROUGH O



FORCE TRIANGLE

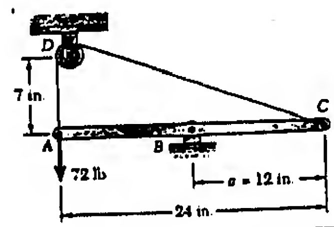


LAW OF SINES

$$\frac{40 \text{ lb}}{\sin 97.67°} = \frac{P}{\sin 22.33°}; \quad P = 15.334 \text{ lb}$$

$\underline{P} = 15.33$ lb ∡ 30° ◀

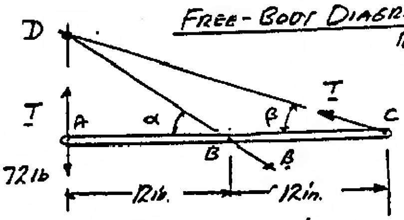
4.77



FIND: TENSION IN CORD, REACTION AT B

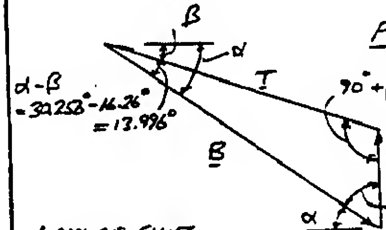
FREE-BODY DIAGRAM

REACTION AT B MUST PASS THROUGH D.



$\tan\alpha = \frac{7\text{ in.}}{12\text{ in.}}$; $\alpha = 30.256°$ $\qquad \tan\beta = \frac{7\text{ in.}}{24\text{ in.}}$; $\beta = 16.26°$

FORCE TRIANGLE



LAW OF SINES

$$\frac{T}{\sin 59.744°} = \frac{T - 72\text{ lb}}{\sin 13.996°} = \frac{B}{\sin 106.26}$$

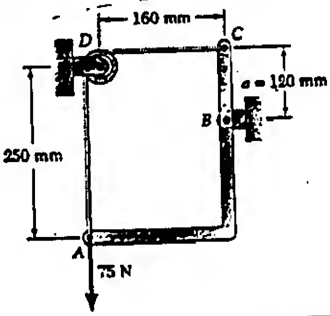
$T(\sin 13.996°) = (T - 72\text{ lb})(\sin 59.744°)$

$T(0.24185) = (T - 72)(0.86378)$

$T = 100.00\text{ lb}$ $\qquad T = 100\text{ lb}$

$B = (100\text{ lb})\frac{\sin 106.26°}{\sin 59.744} = 111.14\text{ lb}$ $\qquad B = 111.1\text{ lb} \measuredangle 30.3°$

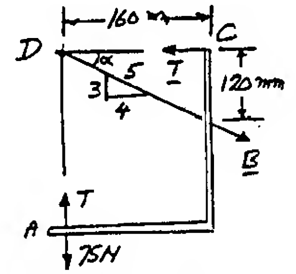
4.78



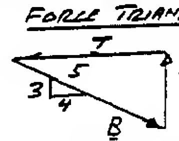
FIND: TENSION IN CORD, REACTION AT B

FREE-BODY DIAGRAM

REACTION AT B MUST PASS THROUGH D



FORCE TRIANGLE



$\tan\alpha = \frac{120}{160}$; $\alpha = 36.9°$

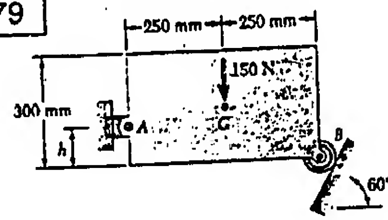
$$\frac{T}{4} = \frac{T - 75\text{ lb}}{3} = \frac{B}{5}$$

$3T = 4T - 300$; $\quad T = 300\text{ lb}$

$B = \frac{5}{4}T = \frac{5}{4}(300\text{ lb}) = 375\text{ lb}$

$B = 375\text{ lb} \measuredangle 36.9°$

4.79

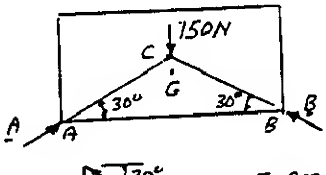


FIND: REACTIONS AT A AND B WHEN (a) $h = 0$ (b) $h = 200$ mm

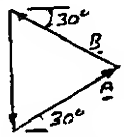
(a) $h = 0$

FREE-BODY DIAGRAM

REACTION A MUST PASS THROUGH C WHERE 150-N WEIGHT AND B INTERSECT



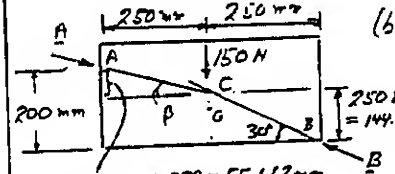
FORCE TRIANGLE IS EQUILATERAL



$A = 150\text{ N} \measuredangle 30°$

$B = 150\text{ N} \measuredangle 30°$

(b) $h = 200$ mm

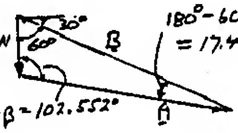


$200 - 144.338 = 55.662$ mm

$\tan\beta = \frac{55.662}{250}$; $\beta = 12.552°$

FORCE TRIANGLE

LAW OF SINES



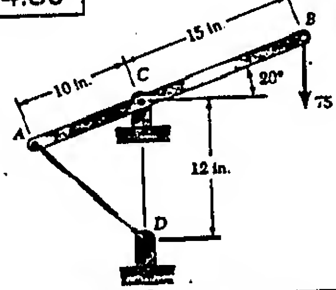
$$\frac{150\text{ N}}{\sin 17.448°} = \frac{A}{\sin 60°} = \frac{B}{\sin 102.552°}$$

$A = 433.247\text{ N}$; $B = 488.31\text{ N}$

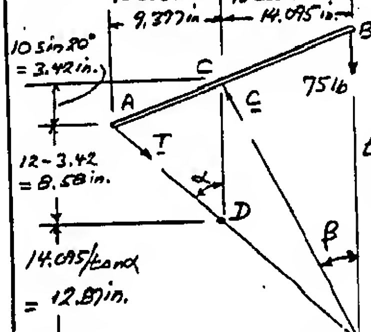
$A = 433\text{ N} \measuredangle 12.6°$

$B = 488\text{ N} \measuredangle 30°$

4.80



FIND: (a) TENSION IN CABLE AD (b) REACTION AT C



FREE-BODY DIAGRAM

REACTION C MUST PASS THROUGH E WHERE 75-lb FORCE AND T INTERSECT

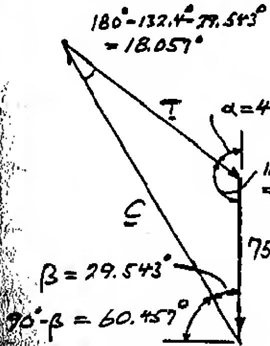
$\tan\alpha = \frac{9.397\text{ in.}}{8.58\text{ in.}}$; $\alpha = 47.6°$

$\tan\beta = \frac{14.095\text{ in.}}{CD + 12.87\text{ in.}} = \frac{14.095}{24.87}$

$\beta = 29.543°$

(CONTINUED)

4.80 CONTINUED



FORCE TRIANGLE
LAW OF SINES

$$\frac{75 \text{ lb}}{\sin 18.057°} = \frac{T}{\sin 29.543°} = \frac{C}{\sin 132.4°}$$

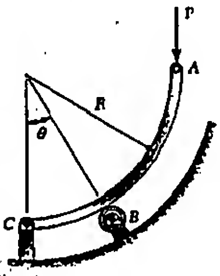
$T = 119.31$ lb

$C = 178.68$ lb

(a) $T = 119.3$ lb ◀

(b) $\underline{C} = 178.7$ lb $\measuredangle 60.5°$ ◀

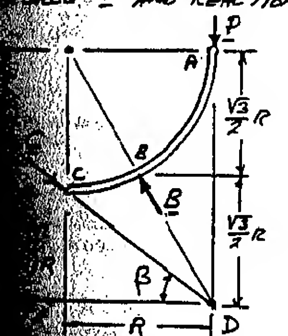
4.81



GIVEN: $\theta = 30°$

FIND: REACTION
(a) AT B
(b) AT C

FREE-BODY DIAGRAM (3-FORCE BODY)
REACTION AT C MUST PASS THROUGH D WHERE FORCE $\underline{P}$ AND REACTION AT B INTERSECT

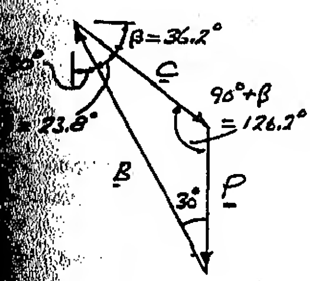


IN Δ CDE:

$$\tan\beta = \frac{(\sqrt{3}-1)R}{R} = \sqrt{3}-1$$

$\beta = 36.2°$

FORCE TRIANGLE



LAW OF SINES

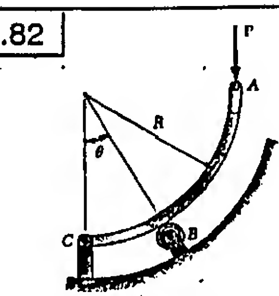
$$\frac{P}{\sin 23.8°} = \frac{B}{\sin 126.2°} = \frac{C}{\sin 30°}$$

$B = 2.00P$; $C = 1.239P$

(a) $\underline{B} = 2P \measuredangle 60°$ ◀

(b) $\underline{C} = 1.239P \measuredangle 36.2°$ ◀

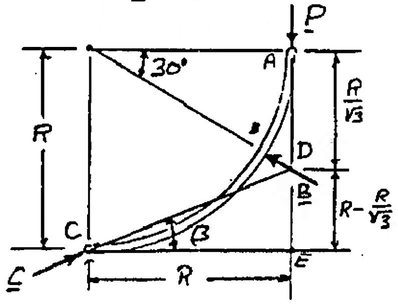
4.82



GIVEN: $\theta = 60°$

FIND: REACTION
(a) AT B
(b) AT C

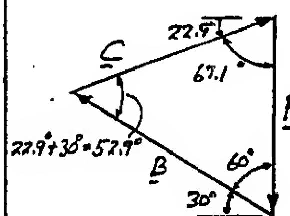
FREE-BODY DIAGRAM (3-FORCE BODY)
REACTION AT C MUST PASS THROUGH D WHERE FORCE $\underline{P}$ AND REACTION AT B INTERSECT.



IN Δ CDE:

$$\tan\beta = \frac{R - R/\sqrt{3}}{R} = 1 - \frac{1}{\sqrt{3}}$$

$\beta = 22.9°$



FORCE TRIANGLE
LAW OF SINES

$$\frac{P}{\sin 52.9°} = \frac{B}{\sin 67.1°} = \frac{C}{\sin 60°}$$

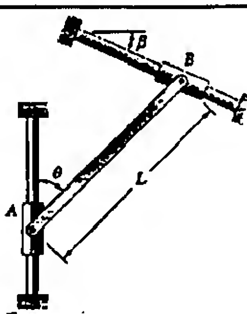
$B = 1.155P$; $C = 1.086P$

(a) $\underline{B} = 1.155P \measuredangle 30°$ ◀

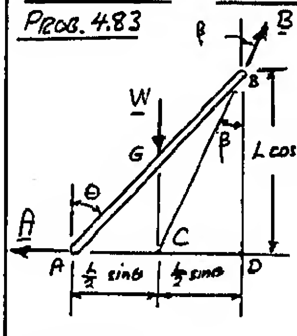
(b) $\underline{C} = 1.086P \measuredangle 22.9°$ ◀

4.83 and 4.84

FOR EQUILIBRIUM,
PROB. 4.83:
FIND: $\theta = f(\beta)$.
PROB. 4.84:
GIVEN: $m = 8$ kg, $\beta = 30°$
FIND: (a) ANGLE θ.
(b) REACTIONS AT A AND B.



PROB. 4.83

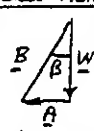


FREE-BODY DIAGRAM (3-FORCE BODY) FORCES INTERSECT AT C.
IN Δ BCD

$$\tan\beta = \frac{\frac{1}{2}L\sin\theta}{L\cos\theta} = \frac{1}{2}\frac{\sin\theta}{\cos\theta}$$

$\tan\theta = 2\tan\beta$ ◀

FORCE TRIANGLE



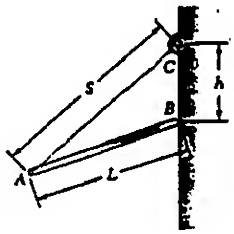
$A = W\tan\theta$
$B = W/\cos\theta$

PROB. 4.84
GIVEN: $m = 8$ kg; $W = (8 \text{ kg})9.81 \text{ m/s}^2 = 78.48$ N, $\beta = 30°$

(a) $\tan\theta = 2\tan 30° = 1.1547$ $\theta = 49.1°$ ◀

(b) $A = W\tan\beta = (78.48\text{N})\tan 30°$ $\underline{A} = 45.3\text{N} \leftarrow$ ◀
$B = W/\cos\beta = (78.48\text{N})/\cos 30°$ $\underline{B} = 90.6\text{N} \measuredangle 60°$ ◀

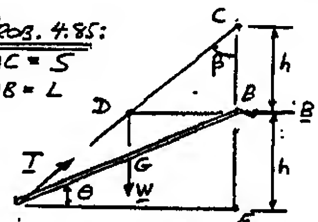
4.85 and 4.86



PROB. 4.85:
FIND: EXPRESSION FOR h IN TERMS OF S AND L

PROB 4.86:
GIVEN: $L = 20$ in., $S = 30$ in., AND $W = 10$ lb
FIND: (a) DISTANCE h
(b) TENSION IN AC
(c) REACTION AT B

PROB. 4.85:
$AC = S$
$AB = L$

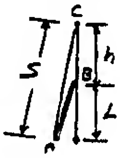


FREE-BODY DIAGRAM (3-FORCE BODY)
THE FORCES W AND B MUST INTERSECT AT D ON LINE OF ACTION OF T.

IN Δ ACE: $(2h)^2 + (AE)^2 = S^2$ (1)
IN Δ ABE: $h^2 + (AE)^2 = L^2$ (2)
EQ(1)−EQ(2): $3h^2 = S^2 - L^2$ (3)

$h = \sqrt{(S^2 - L^2)/3}$

AS LENGTH S INCREASES RELATIVE TO L, ANGLE θ INCREASES UNTIL ROD AB IS VERTICAL AND



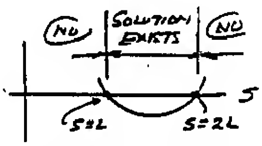
$h \geq S - L$

$\sqrt{(S^2-L^2)/3} \geq S - L$

$S^2 - L^2 \geq 3(S^2 - 2SL + L^2)$

$0 \geq 2S^2 - 6SL + 4L^2$

$0 \geq 2(S-L)(S-2L)$



$\therefore$ NO SOLUTION FOR $S > 2L$

PROB 4.86 $L = 20$ in., $S = 30$ in., $W = 10$ lb

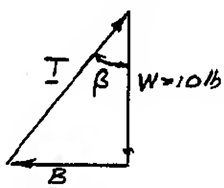
$h = \sqrt{(S^2 - L^2)/3} = \sqrt{(30^2 - 20^2)/3} = \sqrt{500/3}$

(a) $h = 12.91$ in.

IN Δ ACE: $\cos\beta = \dfrac{2h}{S} = \dfrac{2(12.91\text{ in.})}{30\text{ in.}} = 0.8607$

$\beta = 30.609°$

FORCE TRIANGLE



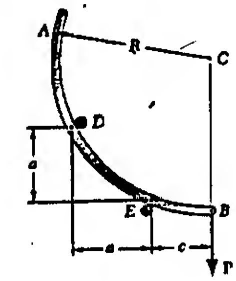
$T = \dfrac{W}{\cos\beta} = \dfrac{10\text{ lb}}{\cos 30.609°}$

(b) $T = 11.62$ lb ◀

$B = W\tan\beta = (10\text{ lb})\tan 30.609°$

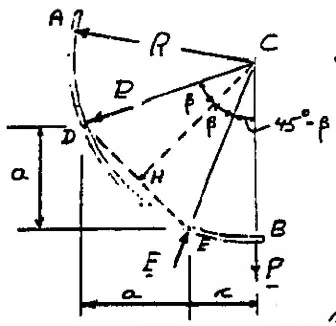
(c) $B = 5.92$ lb ← ◀

4.87



GIVEN:
$a = 20$ mm
$R = 100$ mm

FIND: DISTANCE c CORRESPONDING TO EQUILIBRIUM



SLOPE OF DE IS 45°
$\therefore$ SLOPE OF CH IS 45°

$DE = \sqrt{2}\,a$

$DH = HE = \frac{1}{2}DE = \frac{\sqrt{2}}{2}a$

IN Δ DHC AND IN Δ CEH:

$\sin\beta = \dfrac{\frac{\sqrt{2}}{2}a}{R} = \dfrac{a}{\sqrt{2}R}$

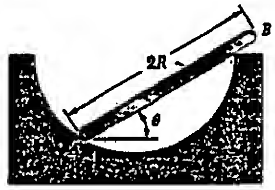
$c = R\sin(45° - \beta)$

FOR $a = 20$ mm, $R = 100$ mm

$\sin\beta = \dfrac{20\text{ mm}}{\sqrt{2}(100\text{ mm})}$; $\beta = 8.13°$

$c = (100\text{ mm})\sin(45° - 8.13°)$ $\quad c = 60.0$ mm ◀

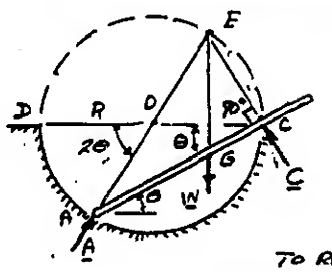
4.88



GIVEN: RADIUS OF BOWL IS R.

FIND: ANGLE θ FOR EQUILIBRIUM

FREE-BODY DIAGRAM (3-FORCE BODY)



POINT E IS POINT OF INTERSECTION OF A AND C.
SINCE A PASSES THROUGH O AND SINCE C IS PERPENDICULAR TO ROD, TRIANGLE ACE IS A RIGHT TRIANGLE INSCRIBED IN THE CIRCLE. THUS E IS A POINT ON THE CIRCLE.

NOTE THAT $\angle$ DOA IS THE CENTRAL ANGLE CORRESPONDING TO THE INSCRIBED ANGLE DCA.
THUS $\angle DOA = 2\theta$

HORIZONTAL PROJECTIONS OF AE AND AG ARE EQUAL.

$(AE)\cos 2\theta = (AG)\cos\theta$

$(2R)\cos 2\theta = (R)\cos\theta$

SET: $\cos 2\theta = 2\cos^2\theta - 1$

$4\cos^2\theta - 2 = \cos\theta$

$4\cos^2\theta - \cos\theta - 2 = 0$

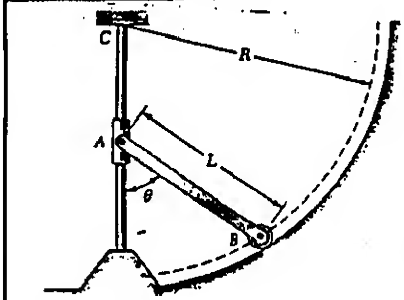
$\cos\theta = 0.84307$ $\quad \theta = 32.5°$ ◀

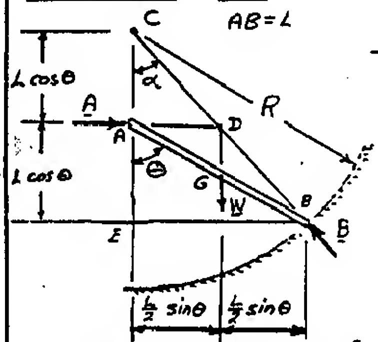
$\cos\theta = -0.59307$ $\quad \theta = 126.4°$ (DISCARD)

4.89 and 4.90

PROB. 4.89: DERIVE EQUATION IN θ, L, AND R FOR POSITION OF EQUILIBRIUM

PROB. 4.90: GIVEN: L = 15 in., R = 20 in. AND W = 10 lb. FIND: ANGLE θ FOR EQUILIBRIUM





AB = L

FREE-BODY DIAGRAM (3-FORCE BODY)

REACTION $\underline{B}$ MUST PASS THROUGH D WHERE $\underline{B}$ AND $\underline{W}$ INTERSECT.

NOTE THAT ΔABC AND ΔBGD ARE SIMILAR.

$\therefore AC = AE = L\cos\theta$

PROB. 4.89

IN ΔABC: $(CE)^2 + (BE)^2 = (BC)^2$

$(2L\cos\theta)^2 + (L\sin\theta)^2 = R^2$

$(R/L)^2 = 4\cos^2\theta + \sin^2\theta$

$(R/L)^2 = 4\cos^2\theta + 1 - \cos^2\theta$

$(R/L)^2 = 3\cos^2\theta + 1$

$$\cos^2\theta = \frac{1}{3}\left[\left(\frac{R}{L}\right)^2 - 1\right]$$

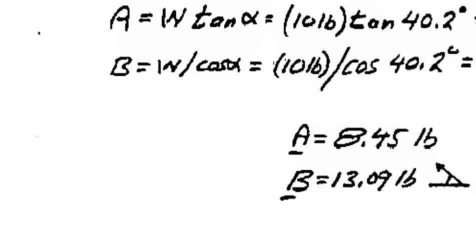
PROB. 4.90. FOR L = 15 in., R = 20 in., AND W = 10 lb.

$\cos^2\theta = \frac{1}{3}\left[\left(\frac{20\text{ in.}}{15\text{ in.}}\right)^2 - 1\right]$; $\cos\theta = 57.39°$; $\theta = 59.4°$

IN ΔABC: $\tan\alpha = \frac{BE}{CE} = \frac{L\sin\theta}{2L\cos\theta} = \frac{1}{2}\tan\theta$

$\tan\alpha = \frac{1}{2}\tan 57.39° = 0.8452$; $\alpha = 40.2°$

FORCE TRIANGLE



$A = W\tan\alpha = (10\text{ lb})\tan 40.2° = 8.45\text{ lb}$

$B = W/\cos\alpha = (10\text{ lb})/\cos 40.2° = 13.09\text{ lb}$

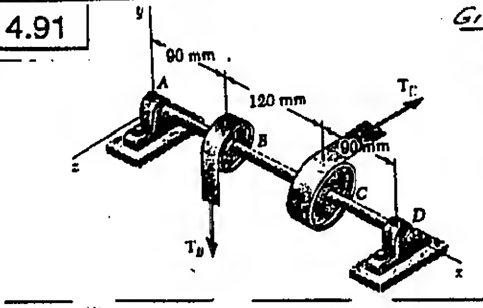
$\underline{A} = 8.45\text{ lb}$

$\underline{B} = 13.09\text{ lb} \measuredangle 47.8°$

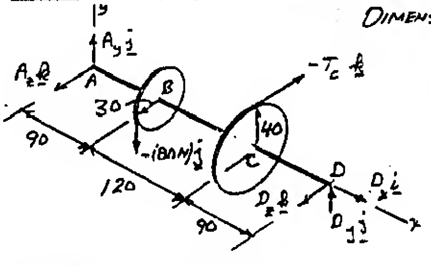
4.91

GIVEN: $T_B = 80$ N, $r_B = 30$ mm, $r_C = 40$ mm

FIND: REACTIONS AT A AND D.



DIMENSIONS IN mm



WE HAVE 6 UNKNOWNS AND 6 EQS. OF EQUILIBRIUM.

$\Sigma M_A = 0$: $(90\mathbf{i} + 30\mathbf{k}) \times (-80\mathbf{j}) + (210\mathbf{i} + 40\mathbf{j}) \times (-T_C\mathbf{k}) + (300\mathbf{i}) \times (D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}) = 0$

$-7200\mathbf{k} + 2400\mathbf{i} + 210T_C\mathbf{j} - 40T_C\mathbf{i} + 300D_y\mathbf{k} - 300D_z\mathbf{j} = 0$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO:

($\mathbf{i}$) $2400 - 40T_C = 0$; $T_C = 60$ N

($\mathbf{j}$) $210T_C - 300D_z = 0$; $(210)(60) - 300D_z = 0$; $D_z = 42$ N

($\mathbf{k}$) $-7200 + 300D_y = 0$; $D_y = 24$ N

$\Sigma F_x = 0$: $D_x = 0$

$\Sigma F_y = 0$: $A_y + D_y - 80\text{ N} = 0$; $A_y = 80 - 24 = 56$ N

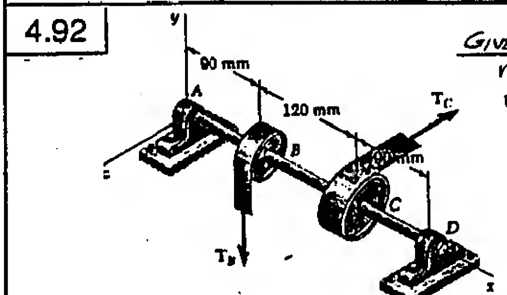
$\Sigma F_z = 0$: $A_z + D_z - 60\text{ N} = 0$; $A_z = 60 - 42 = 18$ N

$\underline{A} = (56\text{ N})\mathbf{j} + (18\text{ N})\mathbf{k}$; $\underline{D} = (24\text{ N})\mathbf{j} + (42\text{ N})\mathbf{k}$

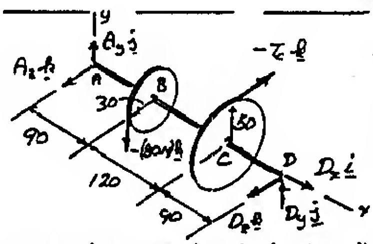
4.92

GIVEN: $T_B = 80$ N, $r_B = 30$ mm, $r_C = 50$ mm

FIND: REACTIONS AT A AND D



DIMENSIONS IN mm



WE HAVE 6 UNKNOWNS AND 6 EQS. OF EQUILIBRIUM

$\Sigma M_A = 0$: $(90\mathbf{i} + 30\mathbf{k}) \times (-80\mathbf{j}) + (210\mathbf{i} + 50\mathbf{j}) \times (-T_C\mathbf{k}) + (300\mathbf{i}) \times (D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}) = 0$

$-7200\mathbf{k} + 2400\mathbf{i} + 210T_C\mathbf{j} - 50T_C\mathbf{i} + 300D_y\mathbf{k} - 300D_z\mathbf{j} = 0$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO:

($\mathbf{i}$) $2400 - 50T_C = 0$; $T_C = 48$ N

($\mathbf{j}$) $210T_C - 300D_z = 0$; $(210)(48) - 300D_z = 0$; $D_z = 33.6$ N

($\mathbf{k}$) $-7200 + 300D_y = 0$; $D_y = 24$ N

$\Sigma F_x = 0$: $D_x = 0$

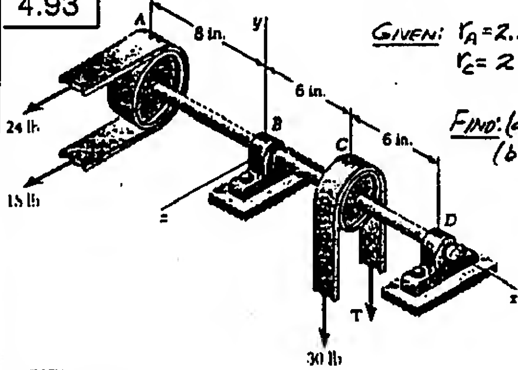
$\Sigma F_y = 0$: $A_y + D_y - 80\text{ N} = 0$; $A_y = 80 - 24 = 56$ N

$\Sigma F_z = 0$: $A_z + D_z - 48 = 0$; $A_z = 48 - 33.6 = 14.4$ N

$\underline{A} = (56\text{ N})\mathbf{j} + (14.4\text{ N})\mathbf{k}$

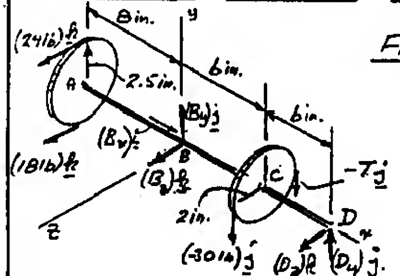
$\underline{D} = (24\text{ N})\mathbf{j} + (33.6\text{ N})\mathbf{k}$

4.93



GIVEN: $r_A = 2.5$ in. $r_C = 2$ in.

FIND: (a) TENSION T (b) REACTIONS AT A AND D



FREE-BODY DIAGRAM

WE HAVE 6 UNKNOWNS AND 6 EQUATIONS OF EQUILIBRIUM

$$\Sigma M_B = 0: (-8\mathbf{i}+2.5\mathbf{j})\times(24\mathbf{k}) + (-8\mathbf{i}-2.5\mathbf{j})\times(18\mathbf{k}) + (6\mathbf{i}+2\mathbf{k})\times(-30\mathbf{j}) + (6\mathbf{i}-2\mathbf{k})\times(-T\mathbf{j}) + (12\mathbf{i})\times(D_y\mathbf{j}+D_z\mathbf{k}) = 0$$

$$192\mathbf{j}+60\mathbf{i}+144\mathbf{j}-45\mathbf{i}-180\mathbf{k}+60\mathbf{i}-6T\mathbf{k}-2T\mathbf{i}+12D_y\mathbf{k}-12D_z\mathbf{j}=0$$

EQUATING TO ZERO THE COEFFICIENTS OF UNIT VECTORS:

$\mathbf{i}$: $60-45+60-2T=0$ $\quad T = 37.5$ lb

$\mathbf{j}$: $192+144-12D_z=0$ $\quad D_z = 28$ lb

$\mathbf{k}$: $-180-6(37.5)+12D_y=0$ $\quad D_y = 33.75$ lb

$\Sigma F_x = 0$: $B_x = 0$ $\quad B_x = 0$

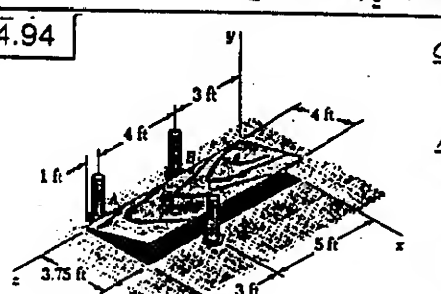
$\Sigma F_y = 0$: $B_y - 30 - 37.5 + 33.75 = 0$ $\quad B_y = 33.75$ lb

$\Sigma F_z = 0$: $B_z + 24 + 18 + 28 = 0$ $\quad B_z = -70$ lb

$\mathbf{B} = (33.75\text{ lb})\mathbf{j} - (70\text{ lb})\mathbf{k}$

$\mathbf{D} = (33.75\text{ lb})\mathbf{j} + (28\text{ lb})\mathbf{k}$

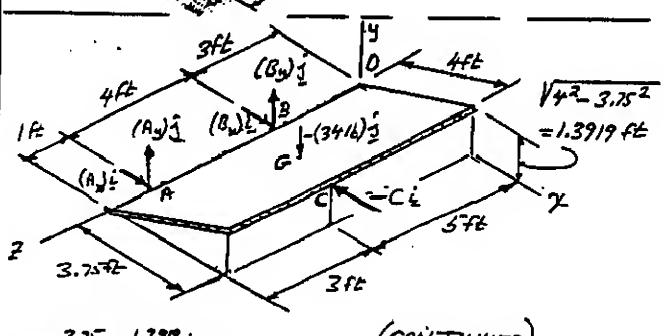
4.94



GIVEN: WEIGHT = 34 lb

FIND: REACTIONS AT A, B, AND C

NEGLECT THE EFFECT OF FRICTION



$\sqrt{4^2-3.75^2} = 1.3919$ ft

$\mathbf{r}_{G/B} = \frac{3.75}{2}\mathbf{i} + \frac{1.3919}{2}\mathbf{j} + \mathbf{k}$

(CONTINUED)

4.94 CONTINUED

WE HAVE 5 UNKNOWNS AND 6 EQS. OF EQUILIBRIUM.

PLYWOOD SHEET IS FREE TO MOVE IN Z DIRECTION, BUT EQUILIBRIUM IS MAINTAINED ($\Sigma F_z = 0$)

$$\Sigma M_B = 0: \mathbf{r}_{A/B}\times(A_x\mathbf{i}+A_y\mathbf{j}) + \mathbf{r}_{C/B}\times(-C\mathbf{i}) + \mathbf{r}_{G/B}\times(-W\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ A_x & A_y & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.75 & 1.3919 & 2 \\ -C & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.875 & 0.696 & 1 \\ 0 & -34 & 0 \end{vmatrix} = 0$$

$$-4A_y\mathbf{i} + 4A_x\mathbf{j} - 2C\mathbf{j} + 1.3919C\mathbf{k} + 34\mathbf{i} - 63.75\mathbf{k} = 0$$

EQUATING COEFFICIENTS OF UNIT VECTORS TO ZERO:

$\mathbf{i}$: $-4A_y + 34 = 0$ $\quad A_y = 8.5$ lb

$\mathbf{j}$: $-2C + 4A_x = 0$; $\quad A_x = \frac{1}{2}C = \frac{1}{2}(45.80) = 22.9$ lb

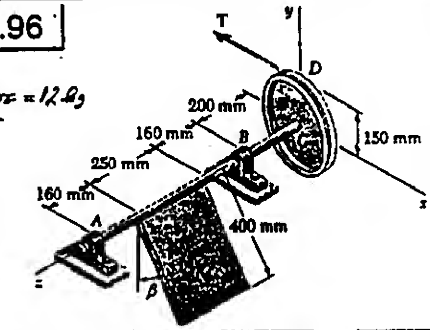
$\mathbf{k}$: $1.3919C - 63.75 = 0$; $\quad C = 45.80$ lb $\quad C = 45.8$ lb

$\Sigma F_x = 0$: $A_x + B_x - C = 0$: $\quad B_x = 45.8 - 22.9 = 22.9$ lb

$\Sigma F_y = 0$: $A_y + B_y - W = 0$: $\quad B_y = 34 - 8.5 = 25.5$ lb

$\mathbf{A} = (22.9\text{ lb})\mathbf{i} + (8.5\text{ lb})\mathbf{j}$; $\mathbf{B} = (22.9\text{ lb})\mathbf{i} + (25.5\text{ lb})\mathbf{j}$; $\mathbf{C} = -(45.8\text{ lb})\mathbf{i}$

4.95 and 4.96



GIVEN: MASS OF PLATE = 12 kg

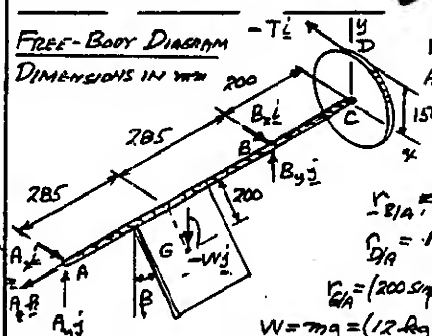
FIND: (a) TENSION T (b) REACTIONS AT A AND B

PROB. 4.95: $\beta = 30°$

PROB. 4.96: $\beta = 60°$

FREE-BODY DIAGRAM

DIMENSIONS IN mm



WE HAVE 6 UNKNOWNS AND 6 EQS. OF EQUIL.

$\mathbf{r}_{B/A} = -570\mathbf{k}$

$\mathbf{r}_{D/A} = 150\mathbf{j} - 770\mathbf{k}$

$\mathbf{r}_{G/A} = (200\sin\beta)\mathbf{i} - (200\cos\beta)\mathbf{j} - 285\mathbf{k}$

$W = mg = (12\text{ kg})9.81\text{ m/s}^2 = 117.72$ N

$$\Sigma M_A = 0: \mathbf{r}_{G/A}\times(-W\mathbf{j}) + \mathbf{r}_{D/A}\times(-T\mathbf{i}) + \mathbf{r}_{B/A}\times(B_x\mathbf{i} + B_y\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 200\sin\beta & -200\cos\beta & -285 \\ 0 & -W & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 150 & -770 \\ -T & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -570 \\ B_x & B_y & 0 \end{vmatrix} = 0$$

$$-285W\mathbf{i} - (200\sin\beta)W\mathbf{k} + 770T\mathbf{j} + 150T\mathbf{k} + 570B_y\mathbf{i} - 570B_x\mathbf{j} = 0$$

EQUATING COEFFICIENTS OF UNIT VECTORS

$\mathbf{i}$: $-285W + 570B_y = 0$; $\quad B_y = (285/570)W$ $\quad$ (1)

$\mathbf{j}$: $770T - 570B_x = 0$; $\quad B_x = (770/570)T$ $\quad$ (2)

$\mathbf{k}$: $-(200\sin\beta)W + 150T = 0$; $\quad T = (200/150)\sin\beta\, W$ $\quad$ (3)

$\Sigma F_x = 0$: $A_x + B_x - T = 0$ $\quad A_x = T - B_x$ $\quad$ (4)

$\Sigma F_y = 0$: $A_y + B_y - W = 0$ $\quad A_y = W - B_y$ $\quad$ (5)

$\Sigma F_z = 0$: $A_z = 0$ $\quad A_z = 0$ $\quad$ (6)

(CONTINUED)

4.95 and 4.96 CONTINUED

PROB 4.95: $\beta = 30°$; $W = 117.72\ N$

EQ.(1): $B_y = (285/570)117.72N = 58.86N$

EQ.(3): $T = (200/150)(\sin 30°)117.72N = 78.48N$

EQ.(2): $B_z = (770/570)78.48N = 106.02N$

EQ.(4): $A_x = 78.48N - 106.02N = -27.54N$

EQ.(5): $A_y = 117.72N - 58.86N = 58.86N$

EQ.(6): $A_z = 0$

(a) $T = 78.5N$ ◀

(b) $\underline{A} = -(27.5N)\underline{i} + (58.9N)\underline{j}$; $\underline{B} = (106.0N)\underline{i} + (58.9N)\underline{j}$ ◀

PROB. 4.96: $\beta = 60°$, $W = 117.72N$

EQ.(1): $B_y = (285/570)117.72N = 58.86N$

EQ.(3): $T = (200/150)(\sin 60°)117.72N = 135.93N$

EQ.(2): $B_x = (770/570)135.93N = 183.63N$

EQ.(4): $A_x = 135.93N - 183.63N = -47.70N$

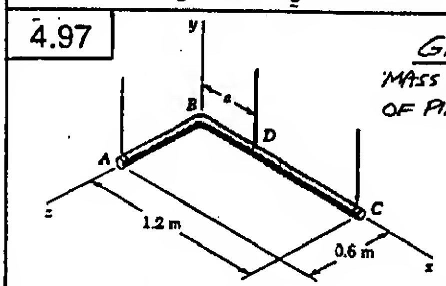
EQ.(5): $A_y = 117.72N - 58.86N = 58.86N$

EQ.(6): $A_z = 0$

(a) $T = 135.9N$ ◀

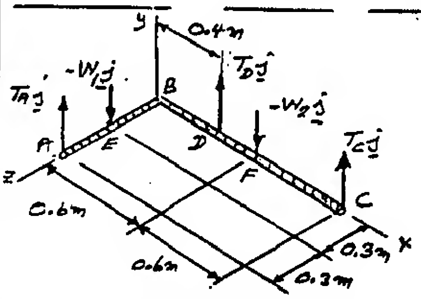
(b) $\underline{A} = -(47.7N)\underline{i} + (58.9N)\underline{j}$; $\underline{B} = (183.6N)\underline{i} + (58.9N)\underline{j}$ ◀

4.97



GIVEN: $a = 0.4m$, MASS PER UNIT LENGTH OF PIPES: $m' = 8\ kg/m$

FIND: TENSION IN WIRES AT A, B, AND C.



$W_1 = 0.6m'g$

$W_2 = 1.2m'g$

$\Sigma M_D = 0$: $\underline{r}_{A/D} \times T_A \underline{j} + \underline{r}_{E/D} \times (-W_1 \underline{j}) + \underline{r}_{F/D} \times (-W_2 \underline{j}) + \underline{r}_{C/D} \times T_C \underline{j} = 0$

$(-0.4\underline{i} + 0.6\underline{k}) \times T_A \underline{j} + (-0.4\underline{i} + 0.3\underline{k}) \times (-W_1 \underline{j}) + 0.2\underline{i} \times (-W_2 \underline{j})$
$+ 0.8\underline{i} \times T_C \underline{j} = 0$

$-0.4 T_A \underline{k} - 0.6 T_A \underline{i} + 0.4 W_1 \underline{k} + 0.3 W_1 \underline{i} - 0.2 W_2 \underline{k} + 0.8 T_C \underline{k} = 0$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO.

(i) $-0.6 T_A + 0.3 W_1 = 0$; $T_A = \frac{1}{2} W_1 = \frac{1}{2} 0.6m'g = 0.3m'g$

(k) $-0.4 T_A + 0.4 W_1 - 0.2 W_2 + 0.8 T_C = 0$

$-0.4(0.3m'g) + 0.4(0.6m'g) - 0.2(1.2m'g) + 0.8 T_C = 0$

$T_C = (0.12 - 0.24 - 0.24)m'g/0.8 = 0.15m'g$

$\Sigma F_y = 0$: $T_A + T_C + T_D - W_1 - W_2 = 0$

$0.3m'g + 0.15m'g + T_D - 0.6m'g - 1.2m'g = 0$

$T_D = 1.35m'g$

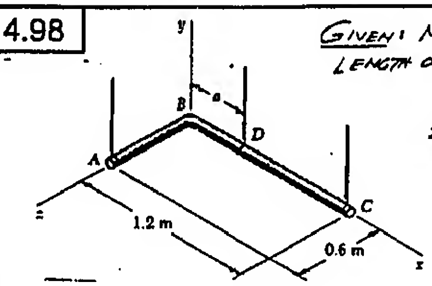
$m'g = (8\ kg/m)(9.81\ m/s^2) = 78.48\ N/m$

$T_A = 0.3m'g = 0.3 \times 78.45$ $T_A = 23.5N$ ◀

$T_B = 0.15m'g = 0.15 \times 78.45$ $T_B = 11.77N$ ◀

$T_C = 1.35m'g = 1.35 \times 78.45$ $T_C = 105.9N$ ◀

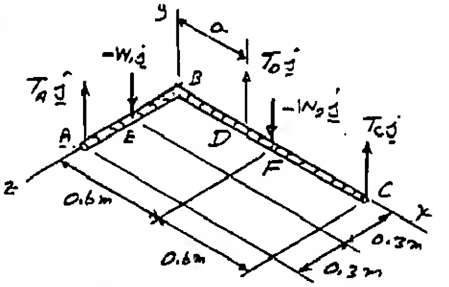
4.98



GIVEN: MASS PER UNIT LENGTH OF PIPES: $m' = 8\ kg/m$

FIND: (a) LARGEST VALUE OF a FOR NO TIPPING

(b) CORRESPONDING TENSION IN EACH WIRE



$W_1 = 0.6m'g$

$W_2 = 1.2m'g$

$\Sigma M_D = 0$: $\underline{r}_{A/D} \times T_A \underline{j} + \underline{r}_{E/D} \times (-W_1 \underline{j}) + \underline{r}_{F/D} \times (-W_2 \underline{j}) + \underline{r}_{C/D} \times T_C \underline{j} = 0$

$(-a\underline{i} + 0.6\underline{k}) \times T_A \underline{j} + (-a\underline{i} + 0.3\underline{k}) \times (-W_1 \underline{j})$
$+ (0.6 - a)\underline{i} \times (-W_2 \underline{j}) + (1.2 - a)\underline{i} \times T_C \underline{j} = 0$

$-T_A a \underline{k} - 0.6 T_A \underline{i} + W_1 a \underline{k} + 0.3 W_1 \underline{i} - W_2(0.6 - a)\underline{k} + T_C(1.2 - a)\underline{k} = 0$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO

(i) $-0.6 T_A + 0.3 W_1 = 0$; $T_A = \frac{1}{2} W_1 = \frac{1}{2} 0.6m'g = 0.3m'g$

(k) $-T_A a + W_1 a - W_2(0.6 - a) + T_C(1.2 - a) = 0$

$-0.3m'g\,a + 0.6m'g\,a - 1.2m'g(0.6 - a) + T_C(1.2 - a) = 0$

$T_C = \dfrac{0.3a - 0.6a + 1.2(0.6 - a)}{1.2 - a}$ FOR MAX a AND NO TIPPING, $T_C = 0$

(a) $-0.3a + 1.2(0.6 - a) = 0$

$-0.3a + 0.72 - 1.2a = 0$

$1.5a = 0.72$ $a = 0.48m$ ◀

(b) REACTIONS: $m'g = (8\ kg/m)9.81\ m/s^2 = 78.45\ N/m$

$T_A = 0.3m'g = 0.3 \times 78.45 = 23.535N$

$T_A = 23.5N$ ◀

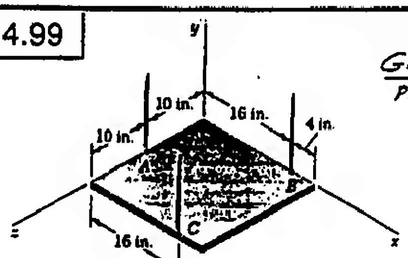
$\Sigma F_y = 0$: $T_A + T_C + T_D - W_1 - W_2 = 0$

$T_A + 0 + T_D - 0.6m'g - 1.2m'g = 0$

$T_D = 1.8m'g - T_A = 1.8 \times 78.45 - 23.535 = 117.67N$

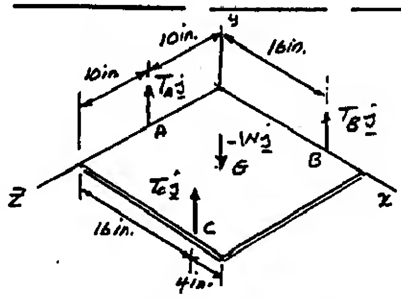
$T_D = 117.7N$ ◀

4.99



GIVEN: WEIGHT OF PLATE $W = 56\ lb$

FIND: TENSION IN EACH WIRE



$\Sigma M_A = 0$: $r_{B/A} \times T_B j + r_{C/A} \times T_C j + r_{G/A} \times (-Wj) = 0$

$(16i - 10k) \times T_B j + (16i + 10k) \times T_C j + 10i \times (-Wj) = 0$

$16T_B k + 10T_B i + 16T_C k - 10T_C i - 10Wk = 0$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO

i: $10T_B - 10T_C = 0$; $T_B = T_C$

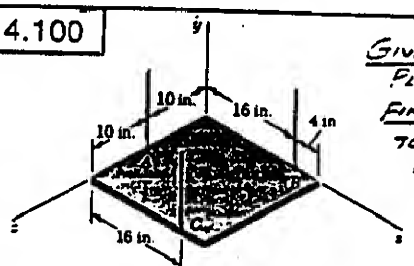
k: $16T_B + 16T_C - 10W = 0$

$16T_B + 16T_B - 10(56\ lb) = 0$; $T_B = T_C = 17.5\ lb$

$\Sigma F_y = 0$: $T_A + T_B + T_C - W = 0$

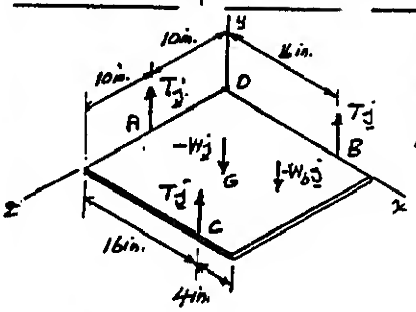
$T_A + 17.5 + 17.5 - 56 = 0$ $\quad T_A = 21.0\ lb$

4.100



GIVEN WEIGHT OF PLATE $W = 56\ lb$.

FIND: LIGHTEST BLOCK TO PLACE ON PLATE FOR WHICH TENSIONS IN THE 3 WIRES ARE EQUAL



LET $-W_b j$ BE THE WEIGHT OF BLOCK AND x AND z THE BLOCK'S COORDINATES.

SINCE TENSIONS IN WIRES ARE EQUAL, LET, $T_A = T_B = T_C = T$

$\Sigma M_D = 0$: $r_A \times Tj + r_B \times Tj + r_C \times Tj + r_G \times (-Wj) + (xi + zk) \times (-W_b j) = 0$

$(10k) \times Tj + (16i) \times Tj + (16i + 20k) \times Tj$

$+ (10i + 10k) \times (-Wj) + (xi + zk) \times (-W_b j) = 0$

$-10Ti + 16Tk + 16Tk - 20Ti - 10Wk + 10Wi - W_b x k + W_b z i = 0$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO

i: $-30T + 10W + W_b z = 0$ (1)

k: $32T - 10W - W_b x = 0$ (2)

ALSO

$\Sigma F_y = 0$: $3T - W - W_b = 0$ (3)

(CONTINUED)

4.100 CONTINUED

NOW, ELIMINATE T:

(EQ.1) + 10(EQ.3): $(z - 10)W_b = 0$ (4)

3(EQ.2) − 32(EQ.3): $2W + (-3x + 32)W_b = 0$ (5)

NOTE THAT $x \le 20\ in.$ AND $z \le 20\ in.$

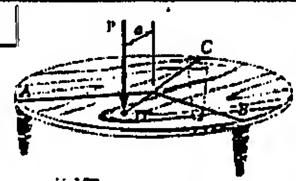
FROM EQ.(4): $z = 10\ in.$ OR

FROM EQ.(5): $\frac{W_b}{W} = \frac{2}{3x - 32} \ge \frac{2}{3(20) - 32} = \frac{2}{28} = \frac{1}{14}$

$\therefore W_B = \frac{1}{14} W = \frac{1}{14}(56\ lb) = 4\ lb$

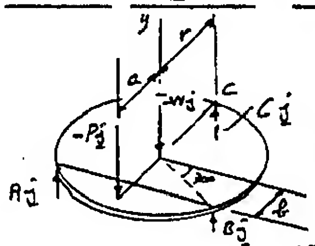
$W = 4\ lb$ AT $x = 20\ in.$, $z = 10\ in.$

4.101



GIVEN: $P = 100\ lb$, TABLE, $W = 30\ lb$, $r = 2\ ft$.

FIND: MINIMUM a FOR NO TIPPING



$r = 2\ ft$ $\quad b = r \sin 30° = 1\ ft$

WE SHALL SUM MOMENTS ABOUT AB

$(b + r)C + (a - b)P - bW = 0$

$(1 + 2)C + (a - 1)100 - (1)30 = 0$

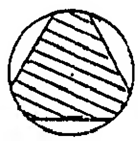
$C = \frac{1}{3}[30 - (a - 1)100]$

IF TABLE IS NOT TO TIP, $C \ge 0$

$[30 - (a - 1)100] \ge 0$

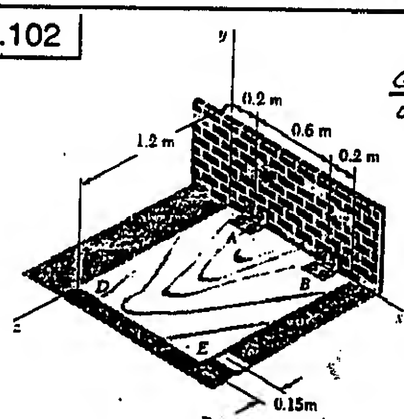
$30 \ge (a - 1)100$

$a - 1 \le 0.1$ $\quad a \le 1.3\ ft$ $\quad a = 1.300\ ft$



ONLY ⊥ DISTANCE FROM P TO AB MATTERS. SAME CONDITION MUST BE SATISFIED FOR EACH LEG. ∴ P MUST BE LOCATED IN SHADED AREA FOR NO TIPPING

4.102



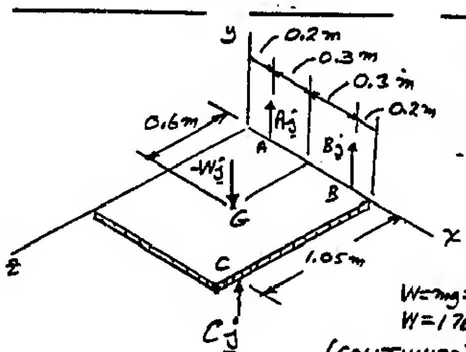
GIVEN: MASS OF PLYWOOD SHEET, $m = 18\ kg$

FIND: REACTIONS

(a) AT A.

(b) AT B.

(c) AT C.



$r_{B/A} = 0.6i$

$r_{C/A} = 0.8i + 1.05k$

$r_{G/A} = 0.3i + 0.6k$

$W = mg = (18\ kg)9.81$

$W = 176.58\ N$

(CONTINUED)

4.102 CONTINUED

$\Sigma M_A = 0$: $\mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$

$(0.6\mathbf{i}) \times B\mathbf{j} + (0.8\mathbf{i} + 1.05\mathbf{k}) \times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k}) \times (-W\mathbf{j}) = 0$

$0.6B\mathbf{k} + 0.8C\mathbf{k} - 1.05C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO.

(i) $-1.05C + 0.6W = 0$; $C = (0.6/1.05)176.58\text{N} = 100.90\text{ N}$

(k) $0.6B + 0.8C - 0.3W = 0$

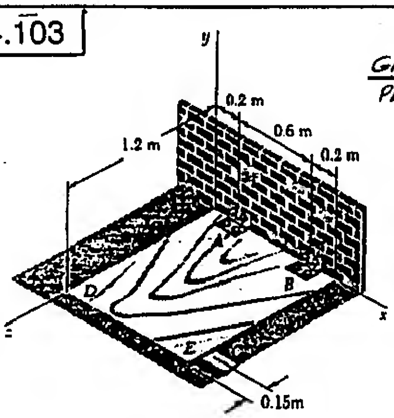
$0.6B + 0.8(100.90\text{N}) - 0.3(176.58\text{N}) = 0$, $B = -46.24\text{N}$

$\Sigma F_y = 0$: $A + B + C - W = 0$

$A - 46.24\text{N} + 100.90\text{N} + 176.58\text{N} = 0$, $A = 121.92\text{N}$

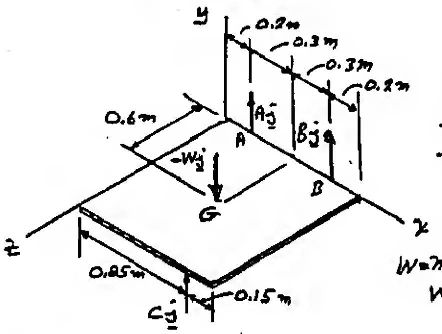
(a) $A = 121.9\text{N}$ (b) $B = -46.2\text{N}$. (c) $C = 100.9\text{N}$ ◀

4.103



GIVEN: MASS OF PLYWOOD SHEET $m = 10\text{ kg}$

FIND: REACTIONS
(a) AT A.
(b) AT B.
(c) AT C.



$\mathbf{r}_{B/A} = 0.6\mathbf{i}$
$\mathbf{r}_{C/A} = 0.65\mathbf{i} + 1.2\mathbf{k}$
$\mathbf{r}_{G/A} = 0.3\mathbf{i} + 0.6\mathbf{k}$

$W = mg = (18\text{ kg})\,9.81\text{ m/s}^2$
$W = 176.58\text{ N}$

$\Sigma M_A = 0$: $\mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$

$0.6\mathbf{i} \times B\mathbf{j} + (0.65\mathbf{i} + 1.2\mathbf{k}) \times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k}) \times (-W\mathbf{j}) = 0$

$0.6B\mathbf{k} + 0.65C\mathbf{k} - 1.2C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO.

(i) $-1.2C + 0.6W = 0$; $C = (0.6/1.2)176.58\text{N} = 88.29\text{ N}$

(k) $0.6B + 0.65C - 0.3W = 0$

$0.6B + 0.65(88.29\text{N}) - 0.3(176.58\text{N}) = 0$

$B = -7.36\text{ N}$

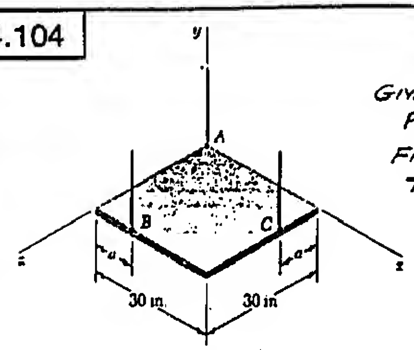
$\Sigma F_y = 0$: $A + B + C - W = 0$

$A - 7.36\text{N} + 88.29\text{N} - 176.58\text{N} = 0$

$A = 95.648\text{N}$

(a) $A = 95.6\text{N}$. (b) -7.36N. (c) 88.3N ◀

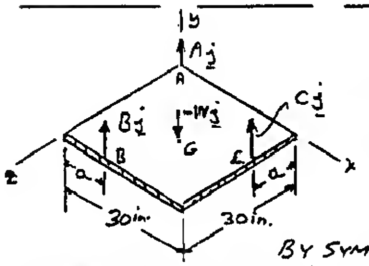
4.104



GIVEN: WEIGHT OF PLATE $W = 24\text{ lb}$

FIND: (a) WIRE TENSIONS WHEN $a = 10\text{ in.}$

(b) VALUE OF a FOR WHICH TENSION IN EACH WIRE IS 8 lb



$\mathbf{r}_{B/A} = a\mathbf{i} + 30\mathbf{k}$
$\mathbf{r}_{C/A} = 30\mathbf{i} + a\mathbf{k}$
$\mathbf{r}_{G/A} = 15\mathbf{i} + 15\mathbf{k}$

BY SYMMETRY: $B = C$

$\Sigma M_A = 0$: $\mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_C \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$

$(a\mathbf{i} + 30\mathbf{k}) \times B\mathbf{j} + (30\mathbf{i} + a\mathbf{k}) \times B\mathbf{j} + (15\mathbf{i} + 15\mathbf{k}) \times (-W\mathbf{j}) = 0$

$Ba\mathbf{k} - 30B\mathbf{i} + 30B\mathbf{k} - Ba\mathbf{i} - 15W\mathbf{k} + 15W\mathbf{i} = 0$

EQUATE COEFFICIENT OF UNIT VECTOR $\mathbf{i}$ TO ZERO

(i) $-30B - Ba + 15W = 0$

$$B = \frac{15W}{30 + a} \qquad C = B = \frac{15W}{30 + a} \qquad (1)$$

$\Sigma F_y = 0$: $A + B + C - W = 0$

$$A + 2\left[\frac{15W}{30 + a}\right] - W = 0; \quad A = \frac{aW}{30 + a} \qquad (2)$$

(a) FOR $a = 10\text{ in.}$

EQ.(1) $C = B = \dfrac{15(24\text{ lb})}{30 + 10} = 9\text{ lb}$

EQ.(2) $A = \dfrac{10(24\text{ lb})}{30 + 10} = 6\text{ lb}$

$A = 6\text{ lb}$; $B = C = 9\text{ lb}$ ◀

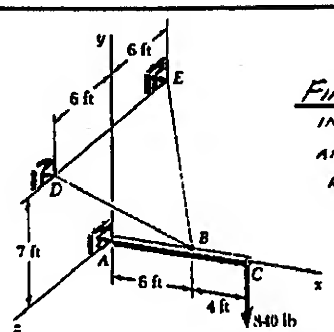
(b) FOR TENSION IN EACH WIRE = 8 lb

EQ.(1) $8\text{ lb} = \dfrac{15(24\text{ lb})}{30 + a}$

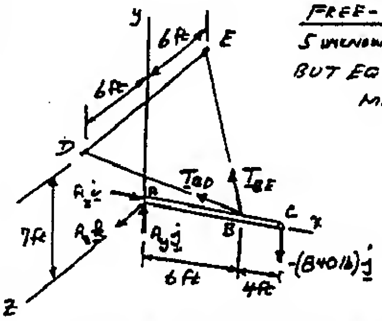
$30\text{ in.} + a = 45$

$a = 15\text{ in.}$ ◀

4.105



FIND: TENSION IN EACH CABLE AND REACTION AT A.



FREE-BODY DIAGRAM WE HAVE 5 UNKNOWNS AND 6 EQS. OF EQUIL., BUT EQUILIBRIUM IS MAINTAINED ($\Sigma M_x = 0$)

$\overrightarrow{BD} = (-6\text{ ft})\mathbf{i} + (7\text{ ft})\mathbf{j} + (6\text{ ft})\mathbf{k}$ $\quad BD = 11$ ft

$\overrightarrow{BE} = (-6\text{ ft})\mathbf{i} + (7\text{ ft})\mathbf{j} - (6\text{ ft})\mathbf{k}$ $\quad BE = 11$ ft

$$\mathbf{T}_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = \frac{T_{BD}}{11}(-6\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$$

$$\mathbf{T}_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = \frac{T_{BE}}{11}(-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k})$$

$$\Sigma M_A = 0: \quad \mathbf{r}_B \times \mathbf{T}_{BD} + \mathbf{r}_B \times \mathbf{T}_{BE} + \mathbf{r}_C \times (-840\mathbf{j}) = 0$$

$$6\mathbf{i} \times \frac{T_{BD}}{11}(-6\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}) + 6\mathbf{i} \times \frac{T_{BE}}{11}(-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k}) + 10\mathbf{i} \times (-840\mathbf{j}) = 0$$

$$\frac{42}{11}T_{BD}\mathbf{k} - \frac{36}{11}T_{BD}\mathbf{j} + \frac{42}{11}T_{BE}\mathbf{k} + \frac{36}{11}T_{BE}\mathbf{j} - 8400\mathbf{k}$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO.

($\mathbf{j}$) $-\frac{36}{11}T_{BD} + \frac{36}{11}T_{BE} = 0$; $\quad T_{BE} = T_{BD}$

($\mathbf{k}$) $\frac{42}{11}T_{BD} + \frac{42}{11}T_{BE} - 8400 = 0$

$2\left(\frac{42}{11}T_{BD}\right) = 8400$; $\quad T_{BD} = 1100$ lb ◀

$T_{BE} = 1100$ lb ◀

$\Sigma F_x = 0$: $A_x - \frac{6}{11}(1100 \text{ lb}) - \frac{6}{11}(1100 \text{ lb}) = 0$

$A_x = 1200$ lb

$\Sigma F_y = 0$: $A_y + \frac{7}{11}(1100 \text{ lb}) + \frac{7}{11}(1100 \text{ lb}) - 840 \text{ lb} = 0$

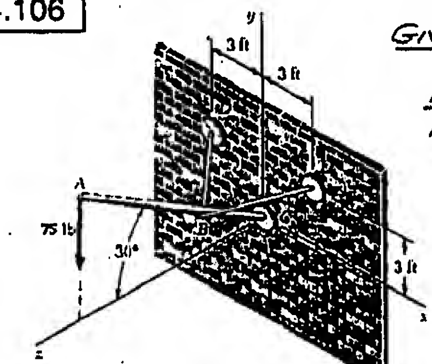
$A_y = -560$ lb

$\Sigma F_z = 0$: $A_z + \frac{6}{11}(1100 \text{ lb}) - \frac{6}{11}(1100 \text{ lb}) = 0$

$A_z = 0$

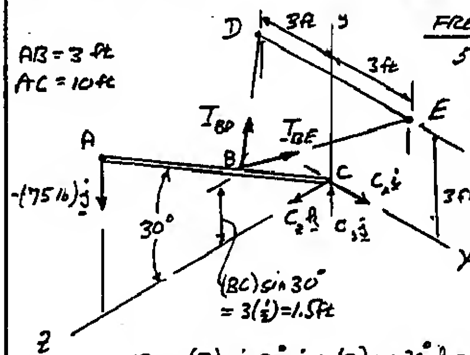
$\mathbf{A} = (1200 \text{ lb})\mathbf{i} - (560 \text{ lb})\mathbf{j}$ ◀

4.106



GIVEN: $BC = 3$ ft, $AC = 10$ ft

FIND: TENSION IN EACH BRACE AND REACTION AT C.



$AB = 3$ ft

$AC = 10$ ft

FREE-BODY DIAGRAM 5 UNKNOWNS AND 6 EQS. OF EQUIL. BUT EQUIL. IS MAINTAINED ($\Sigma M_x = 0$)

$\mathbf{r}_B = (3)\sin 30°\,\mathbf{j} + (3)\cos 30°\,\mathbf{k} = 1.5\mathbf{j} + 2.598\mathbf{k}$

$\mathbf{r}_A = (10)\sin 30°\,\mathbf{j} + (10)\cos 30°\,\mathbf{k} = 5\mathbf{j} + 8.66\mathbf{k}$

$\mathbf{r}_D = -3\mathbf{i} + 3\mathbf{j}$

$\mathbf{r}_E = 3\mathbf{i} + 3\mathbf{j}$

$\overrightarrow{BD} = \mathbf{r}_D - \mathbf{r}_B = -3\mathbf{i} + 3\mathbf{j} - 1.5\mathbf{j} - 2.598\mathbf{k}$

$\overrightarrow{BD} = -3\mathbf{i} + 1.5\mathbf{j} - 2.598\mathbf{k}$ $\quad BD = 4.243$ ft

$\overrightarrow{BE} = 3\mathbf{i} + 1.5\mathbf{j} - 2.598\mathbf{k}$ $\quad BE = 4.243$ ft

$$\mathbf{T}_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = T_{BD}(-0.707\mathbf{i} + 0.3535\mathbf{j} - 0.6123\mathbf{k})$$

$$\mathbf{T}_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = T_{BE}(0.707\mathbf{i} + 0.3535\mathbf{j} - 0.6123\mathbf{k})$$

$$\Sigma M_C = 0: \quad \mathbf{r}_B \times \mathbf{T}_{BD} + \mathbf{r}_B \times \mathbf{T}_{BE} + (5\mathbf{j} + 8.66\mathbf{k}) \times (-75 \text{ lb})\mathbf{j}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.5 & 2.598 \\ -0.707 & +0.3535 & -0.6123 \end{vmatrix} T_{BD} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.5 & 2.598 \\ 0.707 & 0.3535 & -0.6123 \end{vmatrix} T_{BE} + 649.5\mathbf{i} = 0$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO

($\mathbf{j}$) $-1.837\,T_{BD} + 1.837\,T_{BE} = 0$; $\quad T_{BE} = T_{BD}$

($\mathbf{i}$) $-1.837\,T_{BD} - 1.837\,T_{BE} + 649.5 = 0$; $\quad T_{BD} = 176.8$ lb ◀

$T_{BE} = 176.8$ lb ◀

$\Sigma F_x = 0$: $C_x + (176.8)(-0.707) + (176.8)(0.707) = 0$

$C_x = 0$

$\Sigma F_y = 0$: $C_y + (176.8)(0.3535) + (176.8)(0.3535) - 75 \text{ lb} = 0$

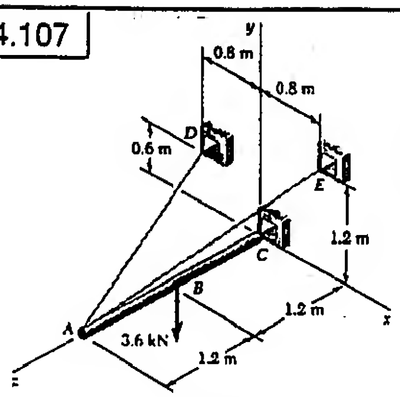
$C_y = -50$

$\Sigma F_z = 0$: $C_z + (176.8)(-0.6123) + (176.8)(-0.6123) = 0$

$C_z = 216$ lb

$\mathbf{C} = -(50 \text{ lb})\mathbf{j} + (216 \text{ lb})\mathbf{k}$ ◀

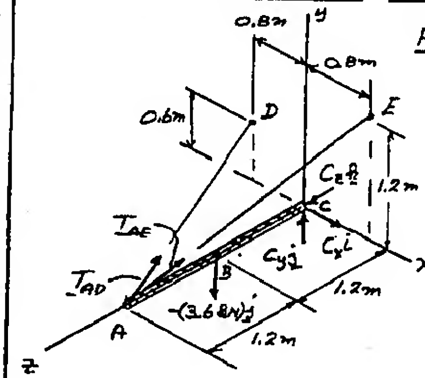
4.107



FIND: TENSION IN EACH CABLE AND REACTION AT C.

FREE-BODY DIAGRAM

5 UNKNOWNS AND 6 EQS. OF EQUIL. EQUILIBRIUM MAINTAINED ($\Sigma M_{AC} = 0$)



$r_B = 1.2k$

$r_A = 2.4k$

$\overrightarrow{AD} = -0.8i + 0.6j - 2.4k \qquad AD = 2.6\ m$

$\overrightarrow{AE} = 0.8i + 1.2j - 2.4k \qquad AE = 2.8\ m$

$$T_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8i + 0.6j - 2.4k)$$

$$T_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8i + 1.2j - 2.4k)$$

$\Sigma M_C = 0$: $r_A \times T_{AD} + r_A \times T_{AE} + r_B \times (-3.6\ kN)j = 0$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & 2.4 \\ -0.8 & 0.6 & -2.4 \end{vmatrix}\frac{T_{AD}}{2.6} + \begin{vmatrix} i & j & k \\ 0 & 0 & 2.4 \\ 0.8 & 1.2 & -2.4 \end{vmatrix}\frac{T_{AE}}{2.8} + 1.2k \times (-3.6\ kN)j = 0$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO

i: $-0.55385\, T_{AD} - 1.02857\, T_{AE} + 4.32 = 0$ (1)

j: $-0.73846\, T_{AD} + 0.68571\, T_{AE} = 0$

$T_{AD} = 0.92857\, T_{AE}$ (2)

EQ.(1): $-0.55385(0.92857)T_{AE} - 1.02857\, T_{AE} + 4.32 = 0$

$1.54286\, T_{AE} = 4.32$

$T_{AE} = 2.800\ kN \qquad T_{AE} = 2.80\ kN$ ◀

EQ.(2): $T_{AD} = 0.92857(2.80) = 2.600\ kN \qquad T_{AD} = 2.60\ kN$ ◀

$\Sigma F_x = 0$: $C_x - \frac{0.8}{2.6}(2.6\ kN) + \frac{0.8}{2.8}(2.8\ kN) = 0$; $C_x = 0$

$\Sigma F_y = 0$: $C_y + \frac{0.6}{2.6}(2.6\ kN) + \frac{1.2}{2.8}(2.8\ kN) - (3.6\ kN) = 0$

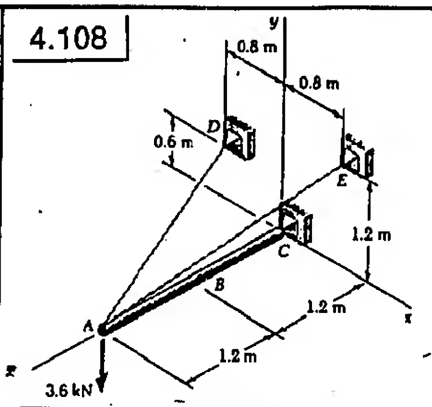
$C_y = 1.800\ kN$

$\Sigma F_z = 0$: $C_z - \frac{2.4}{2.6}(2.6\ kN) - \frac{2.4}{2.8}(2.8\ kN) = 0$

$C_z = 4.80\ kN$

$C = (1.800\ kN)j + (4.80\ kN)k$ ◀

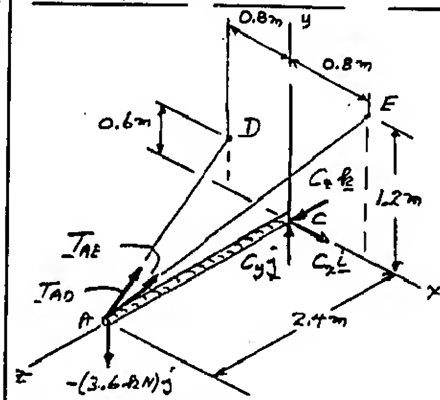
4.108



FIND: TENSION IN EACH CABLE AND REACTION AT C

FREE-BODY DIAGRAM

5 UNKNOWNS AND 6 EQS. OF EQUI- EQUILIBRIUM MAINTAINED ($\Sigma M_{AC} = 0$)



$\overrightarrow{AD} = -0.8i + 0.6j - 2.4k \qquad AD = 2.6\ m$

$\overrightarrow{AE} = 0.8i + 1.2j - 2.4k \qquad AE = 2.8\ m$

$$T_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8i + 0.6j - 2.4k)$$

$$T_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8i + 1.2j - 2.4k)$$

$\Sigma M_C = 0$: $r_A \times T_{AD} + r_A \times T_{AE} + r_A \times (-3.6\ kN)j$

FACTOR r_A: $r_A \times (T_{AD} + T_{AE} - (3.6\ kN)j)$

OR: $T_{AD} + T_{AE} - (3\ kN)j = 0$ (FORCES CONCURRENT AT A)

COEFF. OF i: $-\frac{T_{AD}}{2.6}(0.8) + \frac{T_{AE}}{2.8}(0.8) = 0$

$T_{AD} = \frac{2.6}{2.8}T_{AE}$ (1)

COEFF OF j: $\frac{T_{AD}}{2.6}(0.6) + \frac{T_{AE}}{2.8}(1.2) - 3.6\ kN = 0$

$\frac{2.6}{2.8}T_{AE}\left(\frac{0.6}{2.6}\right) + \frac{1.2}{2.8}T_{AE} - 3.6\ kN = 0$

$T_{AE}\left(\frac{0.6 + 1.2}{2.8}\right) = 3.6\ kN$

$T_{AE} = 5.600\ kN \qquad T_{AE} = 5.60\ kN$ ◀

EQ(1): $T_{AD} = \frac{2.6}{2.8}(5.6) = 5.200\ kN \qquad T_{AD} = 5.20\ kN$ ◀

$\Sigma F_x = 0$: $C_x - \frac{0.8}{2.6}(5.2\ kN) + \frac{0.8}{2.8}(5.6\ kN) = 0$; $C_x = 0$

$\Sigma F_y = 0$: $C_y + \frac{0.6}{2.6}(5.2\ kN) + \frac{1.2}{2.8}(5.6\ kN) - 3.6\ kN = 0$

$C_y = 0$

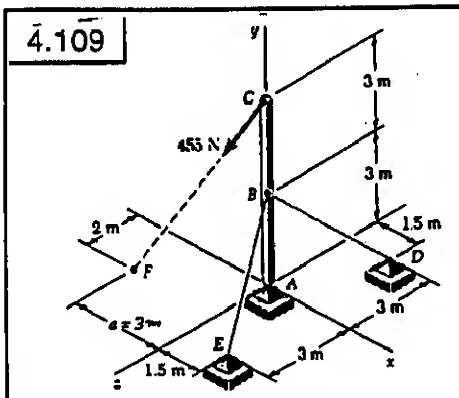
$\Sigma F_z = 0$: $C_z - \frac{2.4}{2.6}(5.2\ kN) - \frac{2.4}{2.8}(5.6\ kN) = 0$

$C_z = 9.6\ kN$

$C = (9.6\ kN)k$ ◀

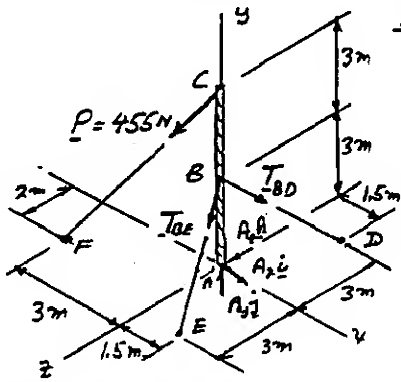
NOTE: SINCE FORCES + REACTION ARE CONCURRENT AT A, WE COULD HAVE USED THE METHODS OF CHAPTER 2.

4.109



GIVEN: $a = 3$ m

FIND: TENSION IN EACH CABLE AND REACTION AT A.



FREE-BODY DIAGRAM

5 UNKNOWNS AND 6 EQS. OF EQUIL. BUT, EQUILIBRIUM MAINTAINED ($\Sigma M_{AC}=0$)

$\underline{r}_B = 3\underline{j}$

$\underline{r}_C = 6\underline{j}$

$\overrightarrow{CF} = -3\underline{i} - 6\underline{j} + 2\underline{k}$ $\qquad CF = 7$ m

$\overrightarrow{BD} = 1.5\underline{i} - 3\underline{j} - 3\underline{k}$ $\qquad BD = 4.5$ m

$\overrightarrow{BE} = 1.5\underline{i} - 3\underline{j} + 3\underline{k}$ $\qquad BE = 4.5$ m

$$\underline{P} = P\frac{\overrightarrow{CF}}{CF} = \frac{P}{7}(-3\underline{i} - 6\underline{j} + 2\underline{k})$$

$$\underline{T}_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = \frac{T_{BD}}{4.5}(1.5\underline{i} - 3\underline{j} - 3\underline{k}) = \frac{T_{BD}}{3}(\underline{i} - 2\underline{j} - 2\underline{k})$$

$$\underline{T}_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = \frac{T_{BD}}{3}(\underline{i} - 2\underline{j} + 2\underline{k})$$

$\Sigma \underline{M}_A = 0$: $\underline{r}_B \times \underline{T}_{BD} + \underline{r}_B \times \underline{T}_{BE} + \underline{r}_C \times \underline{P} = 0$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 3 & 0 \\ 1 & -2 & -2 \end{vmatrix}\frac{T_{BD}}{3} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 3 & 0 \\ 1 & -2 & 2 \end{vmatrix}\frac{T_{BE}}{3} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 6 & 0 \\ -3 & -6 & 2 \end{vmatrix}\frac{P}{7} = 0$$

COEFF. OF $\underline{i}$: $-2T_{BD} + 2T_{BE} + \frac{12}{7}P = 0$ $\qquad$ (1)

COEFF. OF $\underline{k}$: $-T_{BD} - T_{BE} + \frac{18}{7}P = 0$ $\qquad$ (2)

EQ(1)+2EQ(2): $-4T_{BD} + \frac{48}{7}P = 0$ $\qquad T_{BD} = \frac{12}{7}P$

EQ(2): $-\frac{12}{7}P - T_{BE} + \frac{18}{7}P = 0$ $\qquad T_{BE} = \frac{6}{7}P$

SINCE $P = 445$ N, $T_{BD} = \frac{12}{7}(455)$ $\qquad T_{BD} = 780$ N ◀

$T_{BE} = \frac{6}{7}(455)$ $\qquad T_{BE} = 390$ N ◀

$\Sigma \underline{F} = 0$: $\underline{T}_{BD} + \underline{T}_{BE} + \underline{P} + \underline{A} = 0$

COEFF. OF $\underline{i}$: $\frac{780}{3} + \frac{390}{3} - \frac{455}{7}(3) + A_x = 0$

$260 + 130 - 195 + A_x = 0;$ $\qquad A_x = -195$ N

COEFF. OF $\underline{j}$: $-\frac{780}{3}(2) - \frac{390}{3}(2) - \frac{455}{7}(6) + A_y = 0$

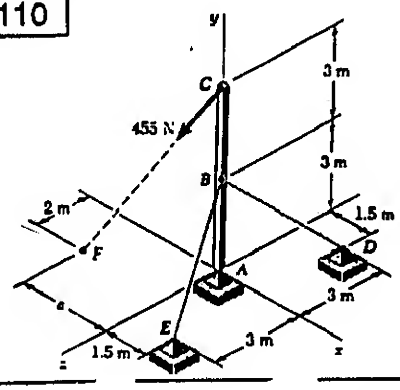
$-520 - 260 - 390 + A_y = 0;$ $\qquad A_y = 1170$ N

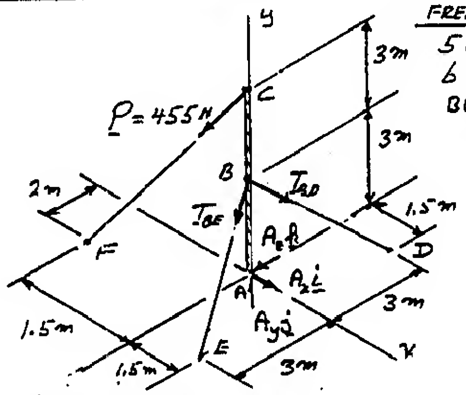
COEFF. OF $\underline{k}$: $-\frac{780}{3}(2) + \frac{390}{3}(2) + \frac{455}{7}(2) + A_z = 0$

$-520 + 260 + 130 + A_z = 0;$ $\qquad A_z = +130$ N

$$\underline{A} = -(195\text{ N})\underline{i} + (1170\text{ N})\underline{j} + (130\text{ N})\underline{k}$$ ◀

4.110





FREE-BODY DIAGRAM

5 UNKNOWNS AND 6 EQS. OF EQUIL. BUT, EQUILIBRIUM MAINTAINED ($\Sigma M_{AC}=0$)

$\underline{r}_B = 3\underline{j}$

$\underline{r}_C = 6\underline{j}$

$\overrightarrow{CF} = -1.5\underline{i} - 6\underline{j} + 2\underline{k}$ $\qquad CF = 6.5$ m

$\overrightarrow{BD} = 1.5\underline{i} - 3\underline{j} - 3\underline{k}$ $\qquad BD = 4.5$ m

$\overrightarrow{BE} = 1.5\underline{i} - 3\underline{j} + 3\underline{k}$ $\qquad BE = 4.5$ m

$$\underline{P} = P\frac{\overrightarrow{CF}}{CF} = \frac{P}{6.5}(-1.5\underline{i} - 6\underline{j} + 2\underline{k}) = \frac{P}{13}(-3\underline{i} - 12\underline{j} + 4\underline{k})$$

$$\underline{T}_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = \frac{T_{BD}}{4.5}(1.5\underline{i} - 3\underline{j} - 3\underline{k}) = \frac{T_{BD}}{3}(\underline{i} - 2\underline{j} - 2\underline{k})$$

$$\underline{T}_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = \frac{T_{BE}}{3}(\underline{i} - 2\underline{j} + 2\underline{k})$$

$\Sigma \underline{M}_A = 0$: $\underline{r}_B \times \underline{T}_{BD} + \underline{r}_B \times \underline{T}_{BE} + \underline{r}_C \times \underline{P} = 0$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 3 & 0 \\ 1 & -2 & -2 \end{vmatrix}\frac{T_{BD}}{3} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 3 & 0 \\ 1 & -2 & 2 \end{vmatrix}\frac{T_{BE}}{3} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 6 & 0 \\ -3 & -12 & +4 \end{vmatrix}\frac{P}{13} = 0$$

COEFF. OF $\underline{i}$: $-2T_{BD} + 2T_{BE} + \frac{24}{13}P = 0$ $\qquad$ (1)

COEFF. OF $\underline{k}$: $-T_{BD} - T_{BE} + \frac{18}{13}P = 0$ $\qquad$ (2)

EQ.(1)+2EQ.(2): $-4T_{BD} + \frac{60}{13}P = 0$ $\qquad T_{BD} = \frac{15}{13}P$

EQ(2): $-\frac{15}{13}P - T_{BE} + \frac{18}{13}P = 0$ $\qquad T_{BE} = \frac{3}{13}P$

SINCE $P = 455$ N, $T_{BD} = \frac{15}{13}(455)$ $\qquad T_{BD} = 525$ N ◀

$T_{BE} = \frac{3}{13}(455)$ $\qquad T_{BE} = 105$ N ◀

$\Sigma \underline{F} = 0$: $\underline{T}_{BD} + \underline{T}_{BE} + \underline{P} + \underline{A} = 0$

COEFF. OF $\underline{i}$: $\frac{525}{3} + \frac{105}{3} - \frac{455}{13}(3) + A_x = 0$

$175 + 35 - 105 + A_x = 0;$ $\qquad A_x = -105$ N

COEFF. OF $\underline{j}$: $-\frac{525}{3}(2) - \frac{105}{3}(2) - \frac{455}{13}(12) + A_y = 0$

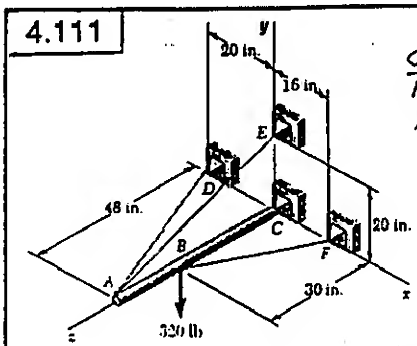
$-350 - 70 - 420 + A_y = 0;$ $\qquad A_y = 840$ N

COEFF. OF $\underline{k}$: $-\frac{525}{3}(2) + \frac{105}{3}(2) + \frac{455}{13}(4) + A_z = 0$

$-350 + 70 + 140 + A_z = 0;$ $\qquad A_z = 140$ N

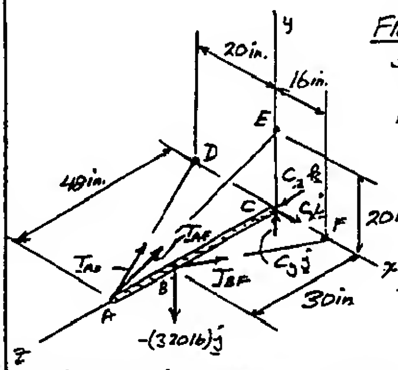
$$\underline{A} = -(105\text{ N})\underline{i} + (840\text{ N})\underline{j} + (140\text{ N})\underline{k}$$ ◀

4.111



GIVEN: CABLE DAE PASSES OVER A PULLEY AT A

FIND: TENSION IN EACH CABLE AND REACTION AT C.



FREE-BODY DIAGRAM
5 UNKNOWNS AND 6 EQS. OF EQUIL.
BUT EQUILIBRIUM MAINTAINED ($\Sigma M_{AC} = 0$)

T = TENSION IN BOTH PARTS OF CABLE DAE.

$r_B = 30k$
$r_A = 48k$

$\overrightarrow{AD} = -20i - 48k \qquad AD = 52 in.$
$\overrightarrow{AE} = 20j - 48k \qquad AE = 52 in.$
$\overrightarrow{BF} = 16i - 30k \qquad BF = 34 in.$

$$T_{AD} = T\frac{\overrightarrow{AD}}{AD} = \frac{T}{52}(-20i - 48k) = \frac{T}{13}(-5i - 12k)$$

$$T_{AE} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{52}(20j - 48k) = \frac{T}{13}(5j - 12k)$$

$$T_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = \frac{T_{BF}}{34}(16i - 30k) = \frac{T_{BF}}{17}(8i - 15k)$$

$$\Sigma M_C = 0: \; r_A \times T_{AD} + r_A \times T_{AE} + r_B \times T_{BF} + r_B \times (-320\,lb)j = 0$$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & 48 \\ -5 & 0 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} i & j & k \\ 0 & 0 & 48 \\ 0 & 5 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} i & j & k \\ 0 & 0 & 30 \\ 8 & 0 & -15 \end{vmatrix}\frac{T_{BF}}{17} + (30k)\times(-320j) = 0$$

COEFF. OF i: $-\frac{240}{13}T + 9600 = 0 \qquad T = 520\,lb$

COEFF. OF j: $-\frac{240}{13}T + \frac{240}{17}T_{BD} = 0$

$$T_{BD} = \frac{17}{13}T = \frac{17}{13}(520);\; T_{BD} = 680\,lb$$

$\Sigma F = 0: \; T_{AD} + T_{AE} + T_{BF} - 320j + C = 0$

COEFF. OF i: $-\frac{20}{52}(520) + \frac{8}{17}(680) + C_x = 0$
$-200 + 320 + C_x = 0 \qquad C_x = -120\,lb$

COEFF. OF j: $\frac{20}{52}(520) - 320 + C_y = 0$
$200 - 320 + C_y = 0 \qquad C_y = 120\,lb$

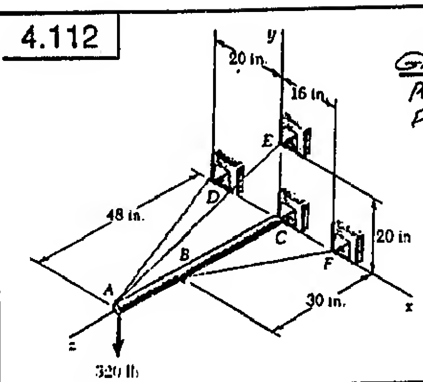
COEFF OF k: $-\frac{48}{52}(520) - \frac{48}{52}(520) - \frac{30}{34}(680) + C_z = 0$
$-480 - 480 - 600 + C_z = 0$
$C_z = 1560\,lb$

ANSWERS: $T_{DAE} = T$

$T_{DAE} = 520\,lb$ ◀
$T_{BD} = 680\,lb$ ◀

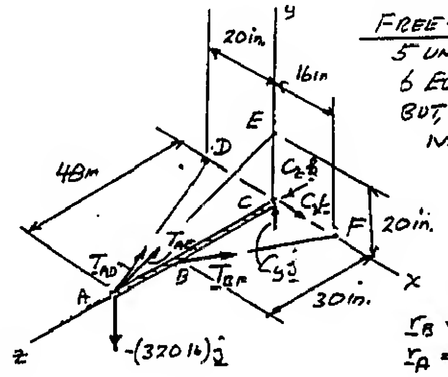
$C = -(120\,lb)i + (120\,lb)j + (1560\,lb)k$ ◀

4.112



GIVEN: CABLE DAE PASSES OVER A PULLEY AT A

FIND: TENSION IN EACH CABLE AND REACTION AT C.



FREE-BODY DIAGRAM
5 UNKNOWNS AND 6 EQS. OF EQUIL.
BUT EQUILIBRIUM MAINTAINED ($\Sigma M_{AC} = 0$)

T = TENSION IN BOTH PARTS OF CABLE DAE.

$r_B = 30k$
$r_A = 48k$

$\overrightarrow{AD} = -20i - 48k \qquad AD = 52 in.$
$\overrightarrow{AE} = 20j - 48k \qquad AE = 52 in.$
$\overrightarrow{BF} = 16i - 30k \qquad BF = 34 in.$

$$T_{AD} = T\frac{\overrightarrow{AD}}{AD} = \frac{T}{52}(-20i - 48k) = \frac{T}{13}(-5i - 12k)$$

$$T_{AE} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{52}(20j - 48k) = \frac{T}{13}(5j - 12k)$$

$$T_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = \frac{T_{BF}}{34}(16i - 30k) = \frac{T_{BF}}{17}(8i - 15k)$$

$$\Sigma M_C = 0: \; r_A \times T_{AD} + r_A \times T_{AE} + r_B \times T_{BF} + r_A \times (-320\,lb)j = 0$$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & 48 \\ -5 & 0 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} i & j & k \\ 0 & 0 & 48 \\ 0 & 5 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} i & j & k \\ 0 & 0 & 30 \\ 8 & 0 & 15 \end{vmatrix}\frac{T_{BF}}{17} + 48k\times(-320j) = 0$$

COEFF OF i: $-\frac{240}{13}T + 15360 = 0 \qquad T = 832\,lb$

COEFF OF j: $-\frac{240}{13}T + \frac{240}{17}T_{BD} = 0$

$$T_{BD} = \frac{17}{13}T = \frac{17}{13}(832) \qquad T_{BD} = 1088\,lb$$

$\Sigma F = 0: \; T_{AD} + T_{AE} + T_{BF} - 320j + C = 0$

COEFF. OF i: $-\frac{20}{52}(832) + \frac{8}{17}(1088) + C_x = 0$
$-320 + 512 + C_x = 0 \qquad C_x = -192\,lb$

COEFF. OF j: $\frac{20}{52}(832) - 320 + C_y = 0$
$320 - 320 + C_y = 0 \qquad C_y = 0$

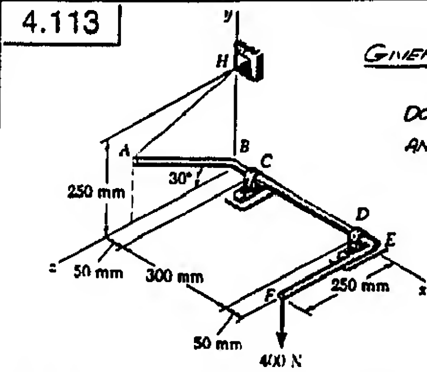
COEFF. OF k: $-\frac{48}{52}(832) - \frac{48}{52}(852) - \frac{30}{34}(1088) + C_z = 0$
$-768 - 768 - 960 + C_z = 0 \qquad C_z = 2496\,lb$

ANSWERS: $T_{DAE} = T$

$T_{DAE} = 832\,lb$ ◀
$T_{BD} = 1088\,lb$ ◀

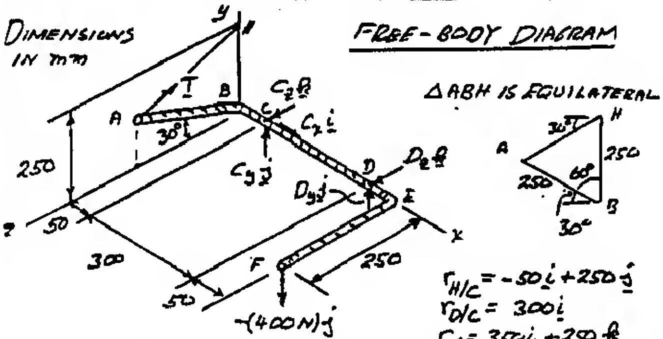
$C = -(192\,lb)i + (2496\,lb)k$ ◀

4.113



GIVEN: Rod AB = 250 mm
BEARING AT D
DOES NOT EXERT
ANY AXIAL THRUST.

FIND:
(a) TENSION IN WIRE AH.
(b) REACTIONS AT C AND D.



$r_{H/C} = -50i + 250j$
$r_{D/C} = 300i$
$r_{F/C} = 350i + 250k$

$T = T(\sin 30°)j - T(\cos 30°)k = T(0.5j - 0.866k)$

$\Sigma M_C = 0: \; r_{H/C} \times T + r_D \times D + r_{F/C} \times (-400j) = 0$

$$\begin{vmatrix} i & j & k \\ -50 & 250 & 0 \\ 0 & 0.5 & -0.866 \end{vmatrix} T + \begin{vmatrix} i & j & k \\ 300 & 0 & 0 \\ 0 & D_y & D_z \end{vmatrix} + \begin{vmatrix} i & j & k \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} = 0$$

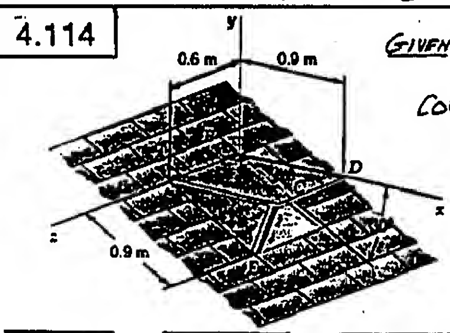
COEFF. OF i: $-216.5T + 100 \times 10^3 = 0$
$T = 461.9 N$ $\qquad T = 462 N$ ◄

COEFF. OF j: $-43.3T - 300D_z = 0$
$-43.3(461.9) - 300D_z = 0; \qquad D_z = -66.67 N$

COEFF. OF k: $-25T + 300D_y - 140 \times 10^3 = 0$
$-25(461.9) + 300D_y - 140 \times 10^3 = 0 \qquad D_y = 505.1 N$
$D = (505 N)j - (66.7 N)k$ ◄

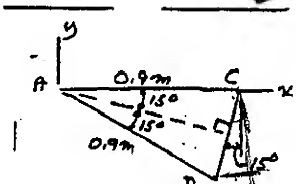
$\Sigma F = 0: \; C + D + T - 400j = 0$
COEFF. OF i: $C_x = 0$ $\qquad C_x = 0$
COEFF. OF j: $C_y + (461.9)0.5 + 505.1 - 400 = 0; \quad C_y = -336 N$
COEFF. OF k: $C_z - (461.9)0.866 - 66.67 = 0 \qquad C_z = 467 N$
$C = -(336 N)j + (467 N)k$ ◄

4.114



GIVEN: $m_{cover} = 20 \text{ kg}$
$A_z = 0$
COVER IS HORIZONTAL

FIND: (a) FORCE EXERTED BY CE
(b) REACTIONS AT A AND B

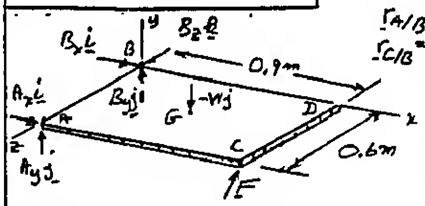


FORCE EXERTED BY CD

$F = F(\sin 75°)i + F(\cos 75°)j$
$F = F(0.2588i + 0.9659j)$

(CONTINUED)

4.114 CONTINUED



$W = mg = 20 \text{ kg}(9.81 \text{ m/s}^2) = 196.2 N$
$r_{A/B} = 0.6k$
$r_{C/B} = 0.9i + 0.6k$
$r_{G/B} = 0.45i + 0.3k$

$F = F(0.2588i + 0.9659j)$

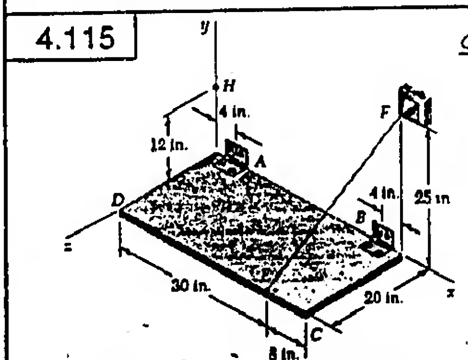
$\Sigma M_B = 0: \; r_{G/B} \times (-196.2j) + r_{C/B} \times F + r_{A/B} \times A = 0$

$$\begin{vmatrix} i & j & k \\ 0.45 & 0 & 0.3 \\ 0 & -196.2 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0.9 & 0 & 0.6 \\ 0.2588 & 0.9659 & 0 \end{vmatrix} F + \begin{vmatrix} i & j & k \\ 0 & 0 & 0.6 \\ A_x & A_y & 0 \end{vmatrix} = 0$$

COEFF. OF i: $+58.86 - 0.5796F - 0.6A_y = 0$ (1)
COEFF. OF j: $-0.1553F + 0.6A_x = 0$ (2)
COEFF. OF k: $-88.29 + 0.8693F = 0: \quad F = 101.56 N$
EQ.(1): $+58.86 - 0.5796(101.56) - 0.6A_y = 0; \quad A_y = 0$
EQ.(2): $-0.1553(101.56) + 0.6A_x = 0; \qquad A_x = 26.29$
$F = 101.6 N; \quad A = (26.3 N)i$ ◄

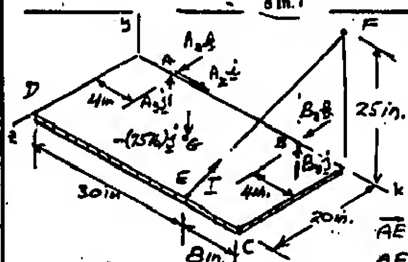
$\Sigma F: \; A + B + F - Wj = 0$
COEFF. OF i: $26.29 + B_x + 0.2588(101.56) = 0; \quad B_x = 0$
COEFF. OF j: $B_y + 0.9659(101.56) - 196.2 = 0; \quad B_y = 98.1$
COEFF. OF k: $B_z = 0$ $\qquad B = 98.1 N$ ◄

4.115



GIVEN: W = ...
B_x = ...

FIND
(a) TENSION IN CABLE ...
(b) REACTIONS AT ...



$r_{B/A} = (38-8)i = 30i$
$r_{E/A} = (30-4)i + 20k$
$= 26i + 20k$
$r_{G/A} = \frac{38}{2}i + 10k$
$= 19i + 10k$

$\overrightarrow{AE} = 8i + 25j - 20k$
$AE = 33 \text{ in.}$

$T = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{33}(8i + 25j - 20k)$

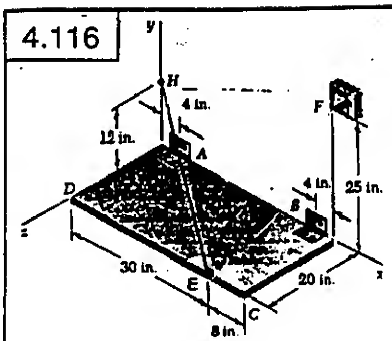
$\Sigma M_A = 0: \; r_{E/A} \times T + r_{G/A} \times (-75j) + r_{B/A} \times B = 0$

$$\begin{vmatrix} i & j & k \\ 26 & 0 & 20 \\ 8 & 25 & -20 \end{vmatrix} \frac{T}{33} + \begin{vmatrix} i & j & k \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 30 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0.$$

COEFF. OF i: $-(25)(20)\frac{T}{33} + 750 = 0:$ $\qquad T = 49.5 \text{ lb}$ ◄
COEFF. OF j: $(160 + 520)\frac{49.5}{33} - 30B_z = 0:$ $\qquad B_z = 34 \text{ lb}$
COEFF. OF k: $(26)(25)\frac{49.5}{33} - 1425 + 30B_y = 0:$ $\qquad B_y = 15 \text{ lb}$
$B = (15 \text{ lb})j + (34 \text{ lb})k$ ◄

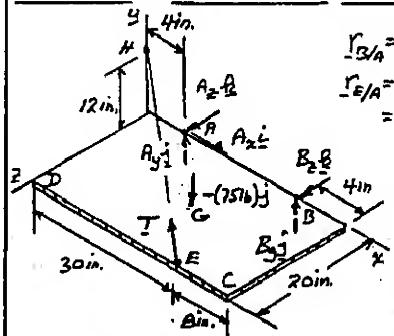
$\Sigma F = 0: \; A + B + T - (75 \text{ lb})j = 0$
COEFF. OF i: $A_x + \frac{8}{33}(49.5) = 0$ $\qquad A_x = -12 \text{ lb}$
COEFF. OF j: $A_y + 15 + \frac{25}{33}(49.5) - 75 = 0$ $\qquad A_y = 22.5 \text{ lb}$
COEFF. OF k: $A_z + 34 - \frac{20}{33}(49.5) = 0$ $\qquad A_z = -4 \text{ lb}$
$A = -(12 \text{ lb})i + (22.5 \text{ lb})j - (4 \text{ lb})k$ ◄

4.116



GIVEN: $W = 75\ lb$, $B_x = 0$

FIND: (a) TENSION IN CABLE EH. (b) REACTIONS AT A AND B.



$r_{B/A} = (38-8)\mathbf{i} = 30\mathbf{i}$

$r_{E/A} = (30-4)\mathbf{i} + 20\mathbf{k} = 26\mathbf{i} + 20\mathbf{k}$

$r_{G/A} = \frac{38}{2}\mathbf{i} + 10\mathbf{k} = 19\mathbf{i} + 10\mathbf{k}$

$\vec{EH} = -30\mathbf{i} + 12\mathbf{j} - 20\mathbf{k}$

$EH = 38\ in.$

$$\mathbf{T} = T\frac{\vec{EH}}{EH} = \frac{T}{38}(-30\mathbf{i} + 12\mathbf{j} - 20\mathbf{k})$$

$\Sigma M_A = 0$: $r_{E/A} \times \mathbf{T} + r_{G/A} \times (-75\mathbf{j}) + r_{B/A} \times \mathbf{B} = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ -30 & 12 & -20 \end{vmatrix}\frac{T}{38} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 30 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

COEFF. OF $\mathbf{i}$: $-(12)(20)\frac{T}{38} + 750 = 0$; $T = 118.75$; $T = 118.8\ lb$ ◀

COEFF. OF $\mathbf{j}$: $(-600 + 520)\frac{118.75}{38} - 30B_z = 0$; $B_z = -8.33\ lb$

COEFF. OF $\mathbf{k}$: $(26)(12)\frac{118.75}{38} - 1425 + 30B_y = 0$; $B_y = 15.00\ lb$

$\mathbf{B} = (15\ lb)\mathbf{j} - (8.33\ lb)\mathbf{k}$ ◀

$\Sigma F = 0$: $\mathbf{A} + \mathbf{B} + \mathbf{T} - (75\ lb)\mathbf{j} = 0$

COEFF. OF $\mathbf{i}$: $A_x - \frac{30}{38}(118.75) = 0$ $\quad A_x = 93.75\ lb$

COEFF. OF $\mathbf{j}$: $A_y + 15 + \frac{12}{38}(118.75) - 75 = 0$ $\quad A_y = 22.5\ lb$

COEFF. OF $\mathbf{k}$: $A_z - 8.33 - \frac{20}{38}(118.75) = 0$ $\quad A_z = 70.83\ lb$

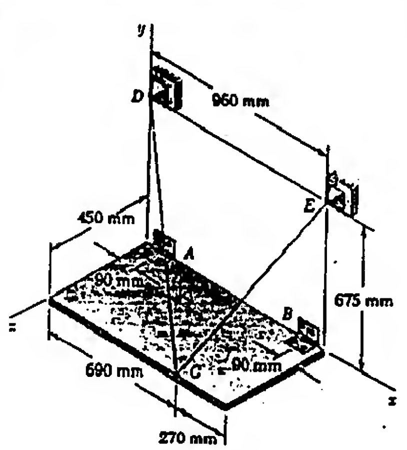
$\mathbf{A} = (93.8\ lb)\mathbf{i} + (22.5\ lb)\mathbf{j} + (70.8\ lb)\mathbf{k}$ ◀

4.117 and 4.118

GIVEN: $m_{plate} = 100\ kg$, $B_x = 0$

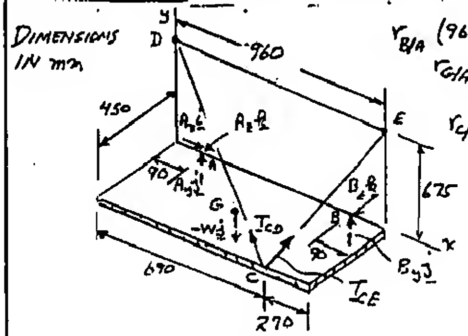
CABLE DCE PASSES OVER PULLEY AT C.

FIND: (a) TENSION IN CABLE DCE. (b) REACTIONS AT A AND B.



4.117 and 4.118 CONTINUED

DIMENSIONS IN mm



$r_{B/A} = (960-180)\mathbf{i} = 780\mathbf{i}$

$r_{G/A} = (\frac{960}{2} - 90)\mathbf{i} + \frac{450}{2}\mathbf{k} = 390\mathbf{i} + 225\mathbf{k}$

$r_{C/A} = 600\mathbf{i} + 450\mathbf{k}$

T = TENSION IN CABLE DCE

$\vec{CD} = -690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k}$ $\quad CD = 1065\ mm$

$\vec{CE} = 270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k}$ $\quad CE = 855\ mm$

$\mathbf{T}_{CD} = \frac{T}{1065}(-690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$

$\mathbf{T}_{CE} = \frac{T}{855}(270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$

$\mathbf{W} = -mg\mathbf{j} = -(100\ kg)(9.81\ m/s^2)\mathbf{j} = -(981\ N)\mathbf{j}$

PROB. 4.117

$\Sigma M_A = 0$: $r_{C/A} \times \mathbf{T}_{CD} + r_{C/A} \times \mathbf{T}_{CE} + r_{G/A} \times (-W\mathbf{j}) + r_{B/A} \times \mathbf{B} = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ -690 & 675 & -450 \end{vmatrix}\frac{T}{1065} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix}\frac{T}{855} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 390 & 0 & 225 \\ 0 & -981 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 780 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

COEFF. OF $\mathbf{i}$: $-(450)(675)\frac{T}{1065} - (450)(675)\frac{T}{855} + 220.725 \times 10^3 = 0$

$T = 344.6\ N$ $\quad T = 345\ N$ ◀

COEFF. OF $\mathbf{j}$: $(-690 \times 450 + 600 \times 450)\frac{344.6}{1065} + (270 \times 450 + 600 \times 450)\frac{344.6}{855} - 780B_z = 0$

$B_z = 185.49\ N$

COEFF. OF $\mathbf{k}$: $(600)(675)\frac{344.6}{1065} + (600)(675)\frac{344.6}{855} - 382.59 \times 10^3 + 780B_y$; $B_y = 113.2\ N$

$\mathbf{B} = (113.2\ N)\mathbf{j} + (185.5\ N)\mathbf{k}$ ◀

$\Sigma F = 0$: $\mathbf{A} + \mathbf{B} + \mathbf{T}_{CD} + \mathbf{T}_{CE} + \mathbf{W} = 0$

COEFF. OF $\mathbf{i}$: $A_x - \frac{690}{1065}(344.6) + \frac{270}{855}(344.6) = 0$; $\quad A_x = 114.4\ N$

COEFF. OF $\mathbf{j}$: $A_y + 113.2 + \frac{675}{1065}(344.6) + \frac{675}{855}(344.6) - 981 = 0$; $\quad A_y = 377\ N$

COEFF. OF $\mathbf{k}$: $A_z + 185.5 - \frac{450}{1065}(344.6) - \frac{450}{855}(344.6) = 0$; $\quad A_z = 141.5\ N$

$\mathbf{A} = (114.4\ N)\mathbf{i} + (377\ N)\mathbf{j} + (141.5\ N)\mathbf{k}$ ◀

PROB. 4.118

$\Sigma M_A = 0$: $r_{C/A} \times \mathbf{T}_{CE} + r_{G/A} \times (-W\mathbf{j}) + r_{B/A} \times \mathbf{B} = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix}\frac{T}{855} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 390 & 0 & 225 \\ 0 & -981 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 780 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

COEFF. OF $\mathbf{i}$: $-(450)(675)\frac{T}{855} + 220.725 \times 10^3 = 0$

$T = 621.3\ N$ $\quad T = 621\ N$ ◀

COEFF. OF $\mathbf{j}$: $(270 \times 450 + 600 \times 450)\frac{621.3}{855} - 780B_z = 0$; $\quad B_z = 364.7\ N$

COEFF. OF $\mathbf{k}$: $(600)(675)\frac{621.3}{855} - 382.59 \times 10^3 + 780B_y = 0$; $\quad B_y = 113.2\ N$

$\mathbf{B} = (113.2\ N)\mathbf{j} + (365\ N)\mathbf{k}$ ◀

$\Sigma F = 0$: $\mathbf{A} + \mathbf{B} + \mathbf{T}_{CE} + \mathbf{W} = 0$

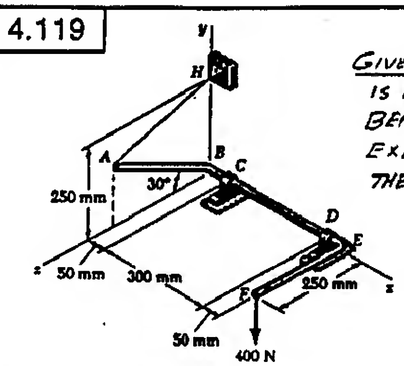
COEFF. OF $\mathbf{i}$: $A_x + \frac{270}{855}(621.3) = 0$ $\quad A_x = -196.2\ N$

COEFF. OF $\mathbf{j}$: $A_y + 113.2 + \frac{675}{855}(621.3) - 981 = 0$; $\quad A_y = 377.3\ N$

COEFF. OF $\mathbf{k}$: $A_z + 364.7 - \frac{450}{855}(621.3) = 0$ $\quad A_z = -37.7\ N$

$\mathbf{A} = -(196.2\ N)\mathbf{i} + (377\ N)\mathbf{j} - (37.7\ N)\mathbf{k}$ ◀

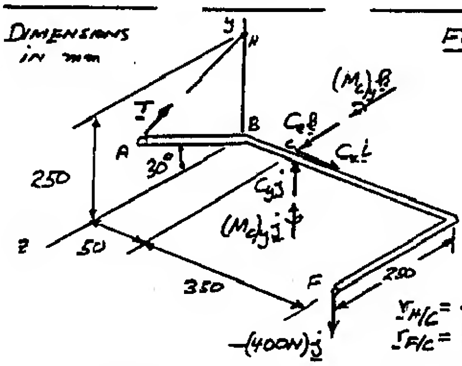
4.119



GIVEN: BEARING AT D IS REMOVED AND BEARING AT C CAN EXERT COUPLES ABOUT THE y AND z AXES.

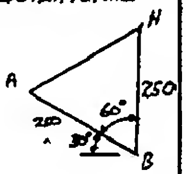
FIND: TENSION IN WIRE AH AND REACTION AT C.

DIMENSIONS IN mm



FREE-BODY DIAGRAM

ΔABH IS EQUILATERAL



$\mathbf{r}_{H/C} = -50\mathbf{i} + 250\mathbf{j}$

$\mathbf{r}_{F/C} = 350\mathbf{i} + 250\mathbf{k}$

$\mathbf{T} = T(\sin 30°)\mathbf{j} - T(\cos 30°)\mathbf{k} = T(0.5\mathbf{j} - 0.866\mathbf{k})$

$\Sigma \mathbf{M}_C = 0: \ \mathbf{r}_{F/C} \times (-400\mathbf{j}) + \mathbf{r}_{H/C} \times \mathbf{T} + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50 & 250 & 0 \\ 0 & 0.5 & -0.866 \end{vmatrix} T + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$$

COEFF. OF $\mathbf{i}$: $+100\times10^3 - 216.5\,T = 0$; $T = 461.9N$; $T = 462N$ ◄

COEFF. OF $\mathbf{j}$: $-43.3(461.9) + (M_C)_y = 0$

$(M_C)_y = 20\times10^3 \ N\cdot mm$; $(M_C)_y = 20 N\cdot m$

COEFF. OF $\mathbf{k}$: $-140\times10^3 - 25(461.9) + (M_C)_z = 0$

$(M_C)_z = 151.54\times10^3 N\cdot mm$; $(M_C)_z = 151.5 N\cdot m$

$\Sigma \mathbf{F} = 0: \mathbf{C} + \mathbf{T} - 400\mathbf{j} = 0$ $\quad \mathbf{M}_C = (20N\cdot m)\mathbf{j} + (151.5N\cdot m)\mathbf{k}$ ◄

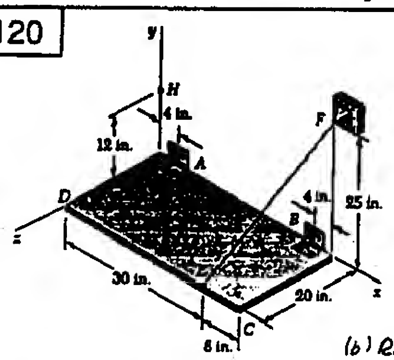
COEFF. OF $\mathbf{i}$: $C_x = 0$ $\quad C_x = 0$

COEFF. OF $\mathbf{j}$: $C_y + 0.5(461.9) - 400 = 0$ $\quad C_y = 169.1N$

COEFF. OF $\mathbf{k}$: $C_z - 0.866(461.9) = 0$ $\quad C_z = 400N$

$\mathbf{C} = (169.1N)\mathbf{j} + (400N)\mathbf{k}$ ◄

4.120



GIVEN: $W = 75$ lb

HINGE AT B IS REMOVED

HINGE AT A CAN EXERT COUPLES PARALLEL TO y AND z AXES

FIND:

(a) TENSION IN CABLE EF

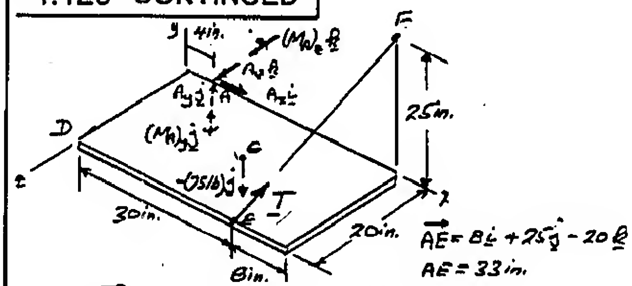
(b) REACTION AT A

$\mathbf{r}_{E/A} = (30-4)\mathbf{i} + 20\mathbf{k} = 26\mathbf{i} + 20\mathbf{k}$

$\mathbf{r}_{G/A} = (0.5\times38)\mathbf{i} + 10\mathbf{k} = 19\mathbf{i} + 10\mathbf{k}$

(CONTINUED)

4.120 CONTINUED



$\overrightarrow{AE} = 8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k}$

$AE = 33$ in.

$\mathbf{T} = T\dfrac{\overrightarrow{AE}}{AE} = \dfrac{T}{33}(8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k})$

$\Sigma \mathbf{M}_A = 0: \ \mathbf{r}_{E/A} \times \mathbf{T} + \mathbf{r}_{G/A} \times (-75\mathbf{j}) + (M_A)_y\mathbf{j} + (M_A)_z\mathbf{k} = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ 8 & 25 & -10 \end{vmatrix} \frac{T}{33} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + (M_A)_y\mathbf{j} + (M_A)_z\mathbf{k} = 0$$

COEFF. OF $\mathbf{i}$: $-(20)(25)\frac{T}{33} + 750 = 0$ $\quad T = 49.5$ lb ◄

COEFF. OF $\mathbf{j}$: $(160 + 520)\frac{49.5}{33} + (M_A)_y = 0$; $\quad (M_A)_y = -1020$ lb·in.

COEFF. OF $\mathbf{k}$: $(26)(25)\frac{49.5}{33} - 1425 + (M_A)_z = 0$; $\quad (M_A)_z = 450$ lb·in.

$\Sigma \mathbf{F} = 0: \ \mathbf{A} + \mathbf{T} - 75\mathbf{j} = 0$ $\quad \mathbf{M}_A = -(1020 \text{ lb·in})\mathbf{j} + (450 \text{ lb·in})\mathbf{k}$ ◄

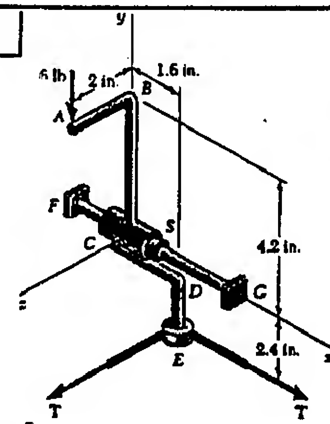
COEFF. OF $\mathbf{i}$: $A_x + \frac{8}{33}(49.5) = 0$ $\quad A_x = -12$ lb

COEFF. OF $\mathbf{j}$: $A_y + \frac{25}{33}(49.5) - 75 = 0$ $\quad A_y = 37.5$ lb

COEFF. OF $\mathbf{k}$: $A_z - \frac{20}{33}(49.5)$ $\quad A_z = 30$ lb

$\mathbf{A} = -(12\text{lb})\mathbf{i} + (37.5\text{lb})\mathbf{j} + (30\text{lb})\mathbf{k}$ ◄

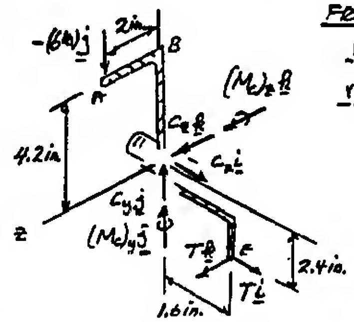
4.121



FIND:

(a) TENSION IN TAPE

(b) REACTION AT C



FREE-BODY DIAGRAM

$\mathbf{r}_{A/C} = 4.2\mathbf{j} + 2\mathbf{k}$

$\mathbf{r}_{E/C} = 1.6\mathbf{i} - 2.4\mathbf{j}$

$\Sigma \mathbf{M}_C = 0$

$\mathbf{r}_{A/C} \times (-6\mathbf{j}) + \mathbf{r}_{E/C} \times T(\mathbf{i} + \mathbf{k}) + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$

$(4.2\mathbf{j} + 2\mathbf{k}) \times (-6\mathbf{j}) + (1.6\mathbf{i} - 2.4\mathbf{j}) \times T(\mathbf{i} + \mathbf{k}) + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$ ◄

COEFF. OF $\mathbf{i}$: $12 - 2.4T = 0$; $\quad T = 5$ lb

COEFF. OF $\mathbf{j}$: $-1.6(5) + (M_C)_y = 0$ $\quad (M_C)_y = 8$ lb·in.

COEFF. OF $\mathbf{k}$: $2.4(5) + (M_C)_z = 0$ $\quad (M_C)_z = -12$ lb·in.

$\mathbf{M}_C = (8 \text{ lb·in.})\mathbf{j} - (12 \text{ lb·in.})\mathbf{k}$ ◄

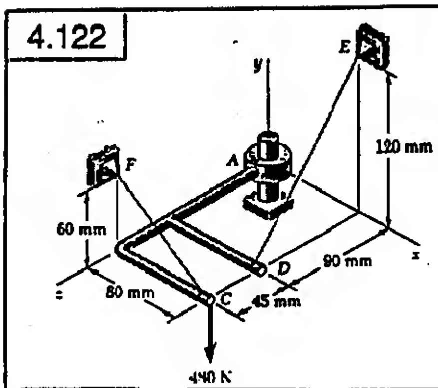
$\Sigma \mathbf{F} = 0: \ C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k} - (6lb)\mathbf{j} + (5lb)\mathbf{i} + (5lb)\mathbf{k} = 0$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO

$C_x = -5$ lb $\quad C_y = 6$ lb $\quad C_z = -5$ lb

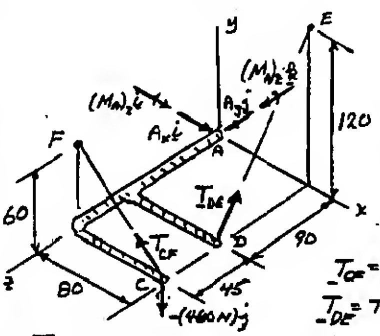
$\mathbf{C} = -(5lb)\mathbf{i} + (6lb)\mathbf{j} - (5lb)\mathbf{k}$ ◄

4.122



FIND: TENSION IN EACH CABLE, REACTION AT A.

FREE-BODY DIAGRAM



$\mathbf{r}_{C/A} = 80\mathbf{i} + 135\mathbf{k}$

$\mathbf{r}_{D/A} = 80\mathbf{i} + 90\mathbf{k}$

$\overrightarrow{CF} = -80\mathbf{i} + 60\mathbf{j}$; $CF = 100$

$\overrightarrow{DE} = 120\mathbf{j} - 90\mathbf{k}$; $DE = 150$

$\mathbf{T}_{CF} = T\frac{\overrightarrow{CF}}{CF} = T_{CF}(-0.8\mathbf{i} + 0.6\mathbf{j})$

$\mathbf{T}_{DE} = T\frac{\overrightarrow{DE}}{DE} = T_{DE}(0.8\mathbf{j} - 0.6\mathbf{k})$

$\Sigma M_A = 0: \mathbf{r}_{C/A}\times\mathbf{T}_{CF} + \mathbf{r}_{D/A}\times\mathbf{T}_{DE} + \mathbf{r}_{C/A}\times(-480\mathbf{j}) + (M_A)_x\mathbf{i} + (M_A)_z\mathbf{k} = 0$

$$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 80 & 0 & 135\\ 0.8 & 0.6 & 0\end{vmatrix}T_{CF} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 80 & 0 & 90\\ 0 & 0.8 & -0.6\end{vmatrix}T_{DE} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 80 & 0 & 135\\ 0 & -480 & 0\end{vmatrix} + (M_A)_x\mathbf{i} + (M_A)_z\mathbf{k} = 0$$

COEFF. OF $\mathbf{i}$: $-81T_{CF} - 72T_{DE} + 64.8\times10^3 + (M_A)_x\mathbf{i} = 0$ (1)

COEFF. OF $\mathbf{j}$: $108T_{CF} + 48T_{DE} = 0$; $T_{CF} = \frac{4}{9}T_{DE}$ (2)

COEFF. OF $\mathbf{k}$: $48T_{CF} + 64T_{DE} - 38.4\times10^3 + (M_A)_z\mathbf{k} = 0$ (3)

$\Sigma F = 0$: $\mathbf{T}_{CF} + \mathbf{T}_{DE} - 480\mathbf{j} + \mathbf{A}$

COEFF. OF $\mathbf{j}$: $0.6T_{CF} + 0.8T_{DE} - 480 = 0$

USE EQ.(2) $0.6\left(\frac{4}{9}\right)T_{DE} + 0.8T_{DE} = 480$; $T_{DE} = 450\text{ N}$ ◀

$T_{CF} = \frac{4}{9}T_{DE} = \frac{4}{9}(450)$ $T_{CF} = 200\text{ N}$ ◀

COEFF. OF $\mathbf{i}$: $A_x - 0.8(200) = 0$; $A_x = 160\text{ N}$

COEFF. OF $\mathbf{k}$: $A_z - 0.6(450) = 0$; $A_z = 270\text{ N}$

$\mathbf{A} = (160\text{ N})\mathbf{i} + (270\text{ N})\mathbf{k}$ ◀

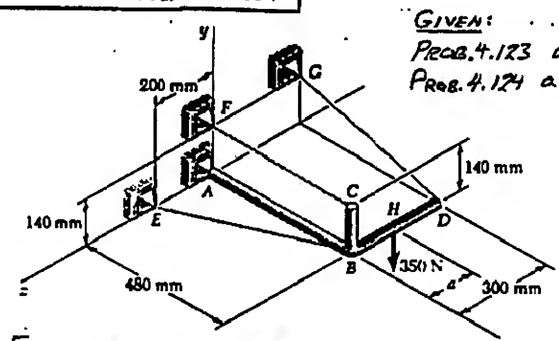
EQ(1): $-81(200) - 72(450) + 64.8\times10^3 + (M_A)_x = 0$

$(M_A)_x = -16.2\times10^3\text{ N}\cdot\text{mm}$; $(M_A)_x = -16.2\text{ N}\cdot\text{m}$

EQ(3): $48(200) + 64(450) - 38.4\times10^3 + (M_A)_z = 0$; $(M_A)_z = 0$

$\mathbf{M}_A = -(16.2\text{ N}\cdot\text{m})\mathbf{i}$ ◀

4.123 and 4.124



GIVEN:

PROB. 4.123 $a = 150$ mm

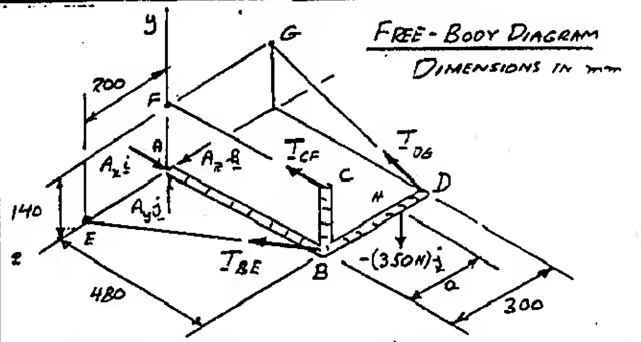
PROB. 4.124 $a = 300$ mm

FIND: TENSION IN CABLES. REACTION AT A.

(CONTINUED)

4.123 and 4.124 CONTINUED

FREE-BODY DIAGRAM
DIMENSIONS IN mm



$\overrightarrow{BE} = -480\mathbf{i} + 200\mathbf{k}$ $BE = 520\text{ mm}$; $\boldsymbol{\lambda}_{BE} = \frac{1}{13}(-12\mathbf{i} + 5\mathbf{k})$

$\overrightarrow{DG} = -480\mathbf{i} + 140\mathbf{j}$ $DG = 500\text{ mm}$; $\boldsymbol{\lambda}_{DG} = \frac{1}{25}(-24\mathbf{i} + 7\mathbf{j})$

$\mathbf{T}_{BE} = T_{BE}\boldsymbol{\lambda}_{BE} = \frac{T_{BE}}{13}(-12\mathbf{i} + 5\mathbf{k})$; $\mathbf{T}_{CF} = -T_{CF}\mathbf{i}$

$\mathbf{T}_{DG} = T_{DG}\boldsymbol{\lambda}_{DG} = \frac{T_{DG}}{25}(-24\mathbf{i} + 7\mathbf{j})$

PROB. 4.123

$a = 150\text{ mm}$ $\mathbf{r}_{H/A} = 480\mathbf{i} - 150\mathbf{k}$

$\Sigma M_A = 0: \mathbf{r}_{B/A}\times\mathbf{T}_{BE} + \mathbf{r}_{F/A}\times\mathbf{T}_{CF} + \mathbf{r}_{G/A}\times\mathbf{T}_{DG} + \mathbf{r}_{H/A}\times(-350\mathbf{j}) = 0$

$$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 480 & 0 & 0\\ -12 & 0 & 5\end{vmatrix}\frac{T_{BE}}{13} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 140 & 0\\ -1 & 0 & 0\end{vmatrix}T_{CF} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 140 & -300\\ -24 & 7 & 0\end{vmatrix}\frac{T_{DG}}{25} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 480 & 0 & -150\\ 0 & -350 & 0\end{vmatrix} = 0$$

COEFF. OF $\mathbf{i}$: $2100\frac{T_{DG}}{25} - (150\times350) = 0$ $T_{DG} = 625\text{ N}$ ◀

COEFF. OF $\mathbf{j}$: $-2400\frac{T_{BE}}{13} + 7200\frac{625}{25} = 0$ $T_{BE} = 975\text{ N}$ ◀

COEFF. OF $\mathbf{k}$: $140T_{CF} + (24\times140)\frac{625}{25} - 168\times10^3 = 0$. $T_{CF} = 600\text{ N}$ ◀

$\Sigma F = 0$: $\mathbf{A} + \mathbf{T}_{BE} + \mathbf{T}_{DG} + \mathbf{T}_{CF} - 350\mathbf{j} = 0$

COEFF. OF $\mathbf{i}$: $A_x - \frac{12}{13}975 - \frac{24}{25}625 - 600 = 0$ $A_x = 2100\text{ N}$

COEFF. OF $\mathbf{j}$: $A_y + \frac{7}{25}625 - 350 = 0$ $A_y = 175\text{ N}$

COEFF. OF $\mathbf{k}$: $A_z + \frac{5}{13}975 = 0$ $A_z = -375\text{ N}$

$\mathbf{A} = (2100\text{ N})\mathbf{i} + (175\text{ N})\mathbf{j} - (375\text{ N})\mathbf{k}$ ◀

PROB. 4.124

$a = 300\text{ mm}$

$\mathbf{r}_{H/A} = 480\mathbf{i} - 300\mathbf{k}$

$\Sigma M_A = 0: \mathbf{r}_{B/A}\times\mathbf{T}_{BE} + \mathbf{r}_{F/A}\times\mathbf{T}_{CF} + \mathbf{r}_{G/A}\times\mathbf{T}_{DG} + \mathbf{r}_{H/A}\times(-350\mathbf{j}) = 0$

$$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 480 & 0 & 0\\ -12 & 0 & 5\end{vmatrix}\frac{T_{BE}}{13} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 140 & 0\\ -1 & 0 & 0\end{vmatrix}T_{CF} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 140 & -300\\ -24 & 7 & 0\end{vmatrix}\frac{T_{DG}}{25} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 480 & 0 & -300\\ 0 & -350 & 0\end{vmatrix} = 0$$

COEFF. OF $\mathbf{i}$: $2100\frac{T_{DG}}{25} - (300\times350) = 0$ $T_{DG} = 1250\text{ N}$ ◀

COEFF. OF $\mathbf{j}$: $-2400\frac{T_{BE}}{13} + 7200\frac{1250}{25} = 0$ $T_{BE} = 1950\text{ N}$ ◀

COEFF. OF $\mathbf{k}$: $140T_{CF} + (24\times140)\frac{1250}{25} - 168\times10^3 = 0$; $T_{CF} = 0$ ◀

$\Sigma F = 0$: $\mathbf{A} + \mathbf{T}_{BE} + \mathbf{T}_{DG} + \mathbf{T}_{CF} - 350\mathbf{j} = 0$

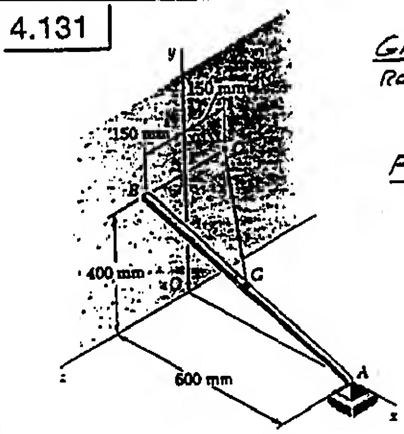
COEFF. OF $\mathbf{i}$: $A_x - \frac{12}{13}1950 - \frac{24}{25}1250 + 0 = 0$ $A_x = 3000\text{ N}$

COEFF. OF $\mathbf{j}$: $A_y + \frac{7}{25}1250 - 350 = 0$ $A_y = 0$

COEFF. OF $\mathbf{k}$: $A_z + \frac{5}{13}1950 = 0$ $A_z = -750\text{ N}$

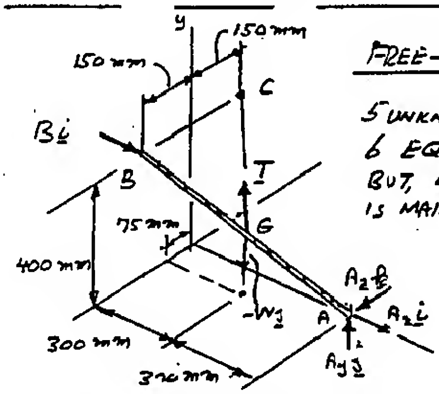
$\mathbf{A} = (3000\text{ N})\mathbf{i} - (750\text{ N})\mathbf{k}$ ◀

4.131



GIVEN: MASS OF ROD AB: $m = 10\ kg$

FIND: (a) TENSION IN CORD CG, (b) REACTIONS AT A AND B.



FREE-BODY DIAGRAM

5 UNKNOWNS AND 6 EQS. OF EQUIL. BUT, EQUILIBRIUM IS MAINTAINED $(\Sigma M_{AB} = 0)$

$W = mg = (10\,kg)9.81\ m/s^2$

$W = 98.1\ N$

$\overrightarrow{GC} = -300\underline{i} + 200\underline{j} - 225\underline{k} \qquad GC = 425\ mm$

$\underline{T} = T\dfrac{\overrightarrow{GC}}{GC} = \dfrac{T}{425}(-300\underline{i} + 200\underline{j} - 225\underline{k})$

$\underline{r}_{B/A} = -600\underline{i} + 400\underline{j} + 150\ mm$

$r_{G/A} = -300\underline{i} + 200\underline{j} + 75\ mm$

$\Sigma \underline{M}_A = 0: \ \underline{r}_{B/A}\times\underline{B} + \underline{r}_{G/A}\times\underline{T} + r_{G/A}\times(-W\underline{j}) = 0$

$$\begin{vmatrix} i & j & k \\ -600 & 400 & 150 \\ B & 0 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ -300 & 200 & 75 \\ -300 & 200 & -225 \end{vmatrix}\frac{T}{425} + \begin{vmatrix} i & j & k \\ -300 & 200 & 75 \\ 0 & -98.1 & 0 \end{vmatrix}$$

COEFF. OF $\underline{i}$: $(-105.88 - 35.29)T + 7357.5 = 0$

$T = 52.12\ N \qquad T = 52.1\ N$ ◀

COEFF. OF $\underline{j}$: $150B - (300\times75 + 300\times225)\dfrac{52.12}{425} = 0$

$B = 73.58\ N \qquad \underline{B} = (73.6\ N)\underline{i}$

$\Sigma \underline{F} = 0: \ \underline{A} + \underline{B} + \underline{T} - W\underline{j} = 0$

COEFF. OF $\underline{i}$: $A_x + 73.58 - 52.15\dfrac{300}{425} = 0; \qquad A_x = 37.8\ N$ ◀

COEFF. OF $\underline{j}$: $A_y + 52.15\dfrac{200}{425} - 98.1 = 0 \qquad A_y = 73.6\ N$ ◀

COEFF. OF $\underline{k}$: $A_z - 52.15\dfrac{225}{425} = 0 \qquad A_z = 27.6\ N$ ◀

ALTERNATE COMPUTATION OF $\underline{B}$:

$\overrightarrow{AC} = -600\underline{i} + 400\underline{j} - 150\underline{k}; \quad \lambda_{AC} = \dfrac{\overrightarrow{AC}}{AC}$

$\Sigma M_{AC} = 0: \ \lambda_{AC}\cdot \underline{M}_A = \lambda_{AC}\cdot(\underline{r}_{B/A}\times\underline{B}) + \lambda_{AC}\cdot(\underline{r}_{G/A}\times -W\underline{j}) = 0$

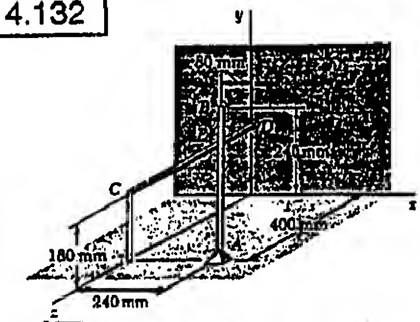
$$\begin{vmatrix} -600 & +400 & -150 \\ -600 & +400 & +150 \\ B & 0 & 0 \end{vmatrix}\frac{1}{AC} + \begin{vmatrix} -600 & +400 & -150 \\ -300 & +200 & +75 \\ & -W & \end{vmatrix}\frac{1}{AC} = 0$$

$B(400\times150 + 400\times150) - W(300\times150 + 600\times75) = 0$

$120\times10^3 B - 90\times10^3 W = 0$

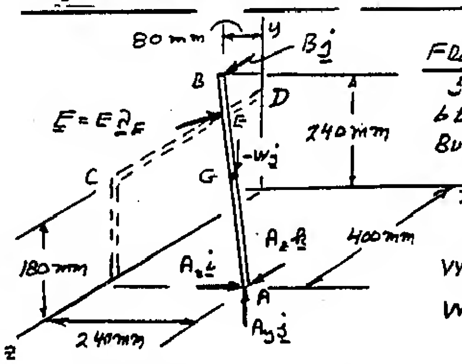
$B = \frac{3}{4}W = \frac{3}{4}(98.1) = 73.6\ N; \quad B = 73.6\ N$ ◁

4.132



GIVEN: MASS OF ROD AB $m = 5\ kg$

FIND: (a) FORCE CD EXERTS ON AB (b) REACTIONS AT A AND B



FREE-BODY DIAGRAM

5 UNKNOWNS AND 6 EQS. OF EQUIL. BUT, EQUILIBRIUM MAINTAINED $(\Sigma M_{AC} = 0)$

$W = mg = (5\,kg)9.8\ \frac{m}{s^2}$

$W = 49.05\ N$

λ_E = UNIT VECTOR ⊥ TO AB AND CD

$\overrightarrow{AB} = -320\underline{i} + 240\underline{j} - 400\underline{k}$

UNIT VECTOR ALONG ROD CD IS $\underline{k}$

$\lambda_E = \dfrac{\overrightarrow{AB}\times\underline{k}}{|\overrightarrow{AB}\times\underline{k}|} = \dfrac{(-320\underline{i} + 240\underline{j} - 400\underline{k})\times\underline{k}}{|\overrightarrow{AB}\times\underline{k}|}$

$\lambda_E = \dfrac{320\underline{j} + 240\underline{i}}{\sqrt{320^2 + 240^2}} = \dfrac{320\underline{j} + 240\underline{i}}{400}; \quad \lambda_E = 0.8\underline{j} + 0.6\underline{i}$

$\underline{E} = E\lambda_E; \quad \underline{E} = E(0.6\underline{i} + 0.8\underline{j}) \quad (1)$

$\underline{r}_{B/A} = \overrightarrow{AB} = -320\underline{i} + 240\underline{j} - 400\underline{k}$

$r_{G/A} = \frac{1}{2}\overrightarrow{AB} = -160\underline{i} + 120\underline{j} - 200\underline{k}$

$r_{E/A} = \frac{180}{240}\overrightarrow{AB} = -240\underline{i} + 180\underline{j} - 300\underline{k}$

$\Sigma \underline{M}_A = 0: \ \underline{r}_{B/A}\times\underline{B} + \underline{r}_{G/A}\times(-W\underline{j}) + \underline{r}_{E/A}\times\underline{E} = 0$

$$\begin{vmatrix} i & j & k \\ -320 & +240 & -400 \\ 0 & 0 & B \end{vmatrix} + \begin{vmatrix} i & j & k \\ -160 & 120 & -200 \\ 0 & -W & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ -240 & 180 & -300 \\ 0.6 & 0.8 & 0 \end{vmatrix}E = 0$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO

COEFF. OF $\underline{k}$: $+160W + (-240\times0.8 - 180\times0.6)E = 0$

$160(49.05) - 300E = 0 \qquad E = 26.16\ N$

COEFF. OF $\underline{i}$: $240B - 200W + 300\times0.8E = 0$

$240B - 200(49.05) + 240(26.16) = 0; \quad B = 14.715\ N$

THUS! $\underline{E} = E(0.6\underline{i} + 0.8\underline{j}) = 26.16(0.6\underline{i} + 0.8\underline{j})$

$\underline{E} = (15.70\ N)\underline{i} + (20.9\ N)\underline{j}$ ◀

$\underline{B} = B\underline{k} \qquad \underline{B} = (14.72\ N)\underline{k}$ ◀

$\Sigma \underline{F} = 0: \ \underline{A} + \underline{B} + \underline{E} - W\underline{j} = 0$

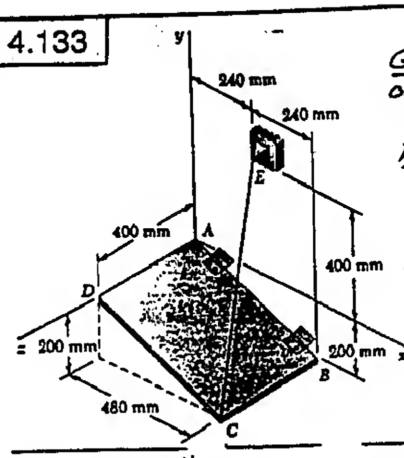
(i) $A_x + 15.70 = 0 \qquad A_x = -15.70\ N$

(j) $A_y + 20.9 - 49.05 = 0 \qquad A_y = 28.1\ N$

(k) $A_z + 14.72 = 0 \qquad A_z = -14.72\ N$

$\underline{A} = -(15.70\ N)\underline{i} + (28.1\ N)\underline{j} - (14.72\ N)\underline{k}$ ◀

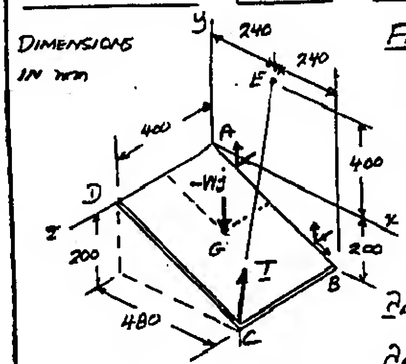
4.133



GIVEN: MASS OF PLATE $m = 50\,kg$

FIND: TENSION IN WIRE CE

DIMENSIONS IN mm



FREE-BODY DIAGRAM

$W = mg = (50\,kg)\,9.81\,m/s^2$

$W = 490.5\,N$

$\vec{CE} = -240\mathbf{i} + 600\mathbf{j} - 400\mathbf{k}$

$CE = 760\,mm$

$\mathbf{T} = T\frac{\vec{CE}}{CE}$

$\mathbf{T} = \frac{T}{760}(-240\mathbf{i} + 600\mathbf{j} - 400\mathbf{k})$

$\lambda_{AB} = \frac{\vec{AB}}{AB} = \frac{480\mathbf{i} - 200\mathbf{j}}{520}$

$\lambda_{AB} = \frac{1}{13}(12\mathbf{i} - 5\mathbf{j})$

$\Sigma M_{AB} = 0:\ \lambda_{AB}\cdot(\mathbf{r}_{E/A}\times\mathbf{T}) + \lambda_{AB}\cdot(\mathbf{r}_{G/A}\times -W\mathbf{j}) = 0$

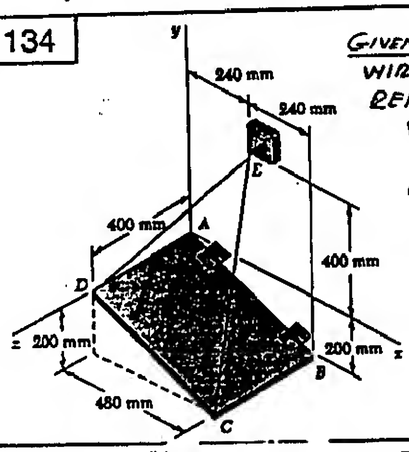
$\mathbf{r}_{E/A} = 240\mathbf{i} + 400\mathbf{j}\ ;\ \mathbf{r}_{G/A} = 240\mathbf{i} - 100\mathbf{j} + 200\mathbf{k}$

$$\begin{vmatrix} 12 & -5 & 0 \\ 240 & +400 & 0 \\ -240 & 600 & -400 \end{vmatrix}\frac{T}{13\times760} + \begin{vmatrix} 12 & 5 & 0 \\ 240 & -100 & 200 \\ 0 & -W & 0 \end{vmatrix}\frac{1}{13} = 0$$

$(-12\times400\times400 - 5\times240\times400)\,T/760 + 12\times200\,W = 0$

$T = 0.76\,W = 0.76(490.5\,N);\qquad T = 373\,N$ ◀

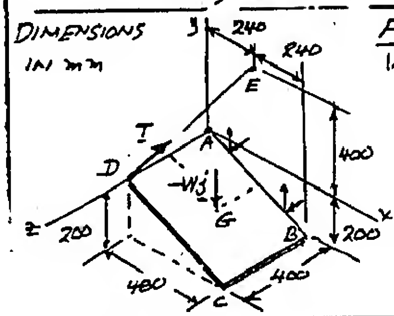
4.134



GIVEN: REMOVE WIRE CE AND REPLACE BY WIRE DE. MASS OF PLATE: $m = 50\,kg$

FIND: TENSION IN WIRE DE

DIMENSIONS IN mm



FREE-BODY DIAGRAM

$W = mg = (50\,kg)\,9.81\,m/s^2$

$W = 490.5\,N$

$\vec{DE} = 240\mathbf{i} + 400\mathbf{j} - 400\mathbf{k}$

$DE = 614.5\,mm$

$\mathbf{T} = T\frac{\vec{DE}}{DE}$

$\mathbf{T} = \frac{T}{614.5}(240\mathbf{i} + 400\mathbf{j} - 400\mathbf{k})$

(CONTINUED)

4.134 CONTINUED

$\lambda_{AB} = \frac{\vec{AB}}{AB} = \frac{480\mathbf{i} - 200\mathbf{j}}{520}\ ;\quad \lambda_{AB} = \frac{1}{13}(12\mathbf{i} - 5\mathbf{j})$

$\mathbf{r}_{E/A} = 240\mathbf{i} + 400\mathbf{j}\ ;\quad \mathbf{r}_{G/A} = 240\mathbf{i} - 100\mathbf{j} + 200\mathbf{k}$

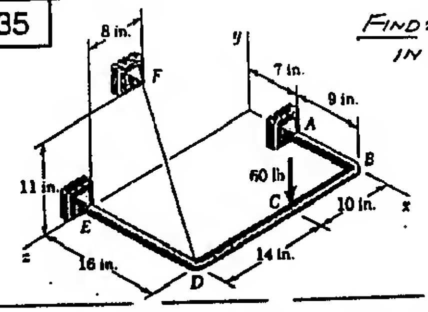
$\Sigma M_{AB} = 0:\ \lambda_{AB}\cdot(\mathbf{r}_{E/A}\times\mathbf{T}) + \lambda_{AB}\cdot(\mathbf{r}_{G/A}\times -W\mathbf{j}) = 0$

$$\begin{vmatrix} 12 & -5 & 0 \\ 240 & 400 & 0 \\ 240 & 400 & -400 \end{vmatrix}\frac{T}{13\times614.5} + \begin{vmatrix} 12 & -5 & 0 \\ 240 & -100 & 200 \\ 0 & -W & 0 \end{vmatrix}\frac{1}{13} = 0$$

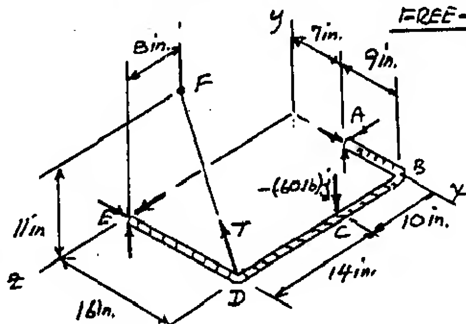
$(-12\times400\times400 - 5\times240\times400)\,T/614.5 + 12\times200\times W = 0$

$T = 0.6145\,W = 0.6145(490.5\,N);\qquad T = 301\,N$ ◀

4.135



FIND: TENSION IN CABLE DF



FREE-BODY DIAGRAM

$\vec{DF} = -16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k}\qquad DF = 21\,in.$

$\mathbf{T} = T\frac{\vec{DF}}{DF} = \frac{T}{21}(-16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k})$

$\mathbf{r}_{D/E} = 16\mathbf{i}$

$\mathbf{r}_{C/E} = 16\mathbf{i} - 14\mathbf{k}$

$\lambda_{EA} = \frac{\vec{EA}}{EA} = \frac{7\mathbf{i} - 24\mathbf{k}}{25}$

$\Sigma M_{EA} = 0:\ \lambda_{EA}\cdot(\mathbf{r}_{D/E}\times\mathbf{T}) + \lambda_{EA}\cdot(\mathbf{r}_{C/E}\times(-60\mathbf{j})) = 0$

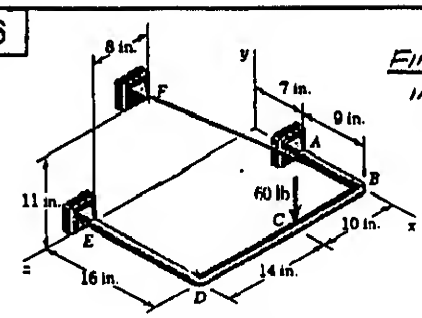
$$\begin{vmatrix} 7 & 0 & -24 \\ 16 & 0 & 0 \\ -16 & 11 & -8 \end{vmatrix}\frac{T}{21\times25} + \begin{vmatrix} 7 & 0 & -24 \\ 16 & 0 & -14 \\ 0 & -60 & 0 \end{vmatrix}\frac{1}{25} = 0$$

$-\frac{24\times16\times11}{21\times25}T + \frac{-7\times14\times60 + 24\times16\times60}{25} = 0$

$201.14\,T + 17{,}160 = 0$

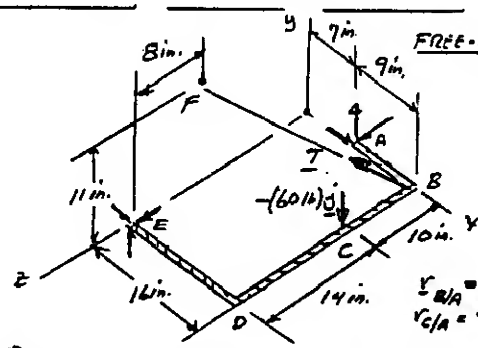
$T = 85.31\,lb\qquad T = 85.3\,lb$ ◀

4.136



FIND: TENSION IN CABLE BF

FREE-BODY DIAGRAM



$r_{B/A} = 9i$

$r_{C/A} = 9i + 10k$

$\vec{BF} = -16i + 11j + 16k \qquad BF = 25.16\ in.$

$T = T\frac{\vec{BF}}{BF} = \frac{T}{25.16}(-16i + 11j + 16k)$

$\lambda_{AE} = \frac{\vec{AE}}{AE} = \frac{7i - 24k}{25}$

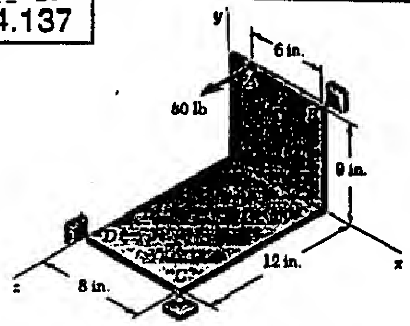
$\Sigma M_{AE} = 0: \quad \lambda_{AE}\cdot(r_{B/A}\times T) + \lambda_{AE}\cdot(r_{C/A}\times(-60j)) = 0$

$$\begin{vmatrix} 7 & 0 & -24 \\ 9 & 0 & 0 \\ -16 & 11 & 16 \end{vmatrix}\frac{T}{25\times25.16} + \begin{vmatrix} 7 & 0 & -24 \\ 9 & 0 & 10 \\ 0 & -60 & 0 \end{vmatrix}\frac{1}{25} = 0$$

$$-\frac{24\times9\times11}{25\times25.16}T + \frac{24\times9\times60 + 7\times10\times60}{25}$$

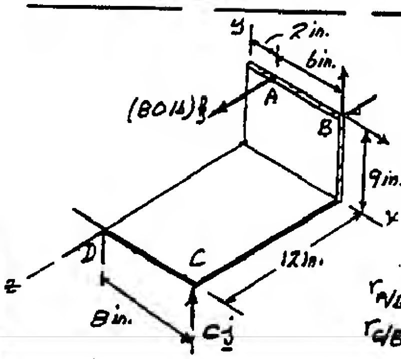
$94.426T - 17{,}160 = 0 \qquad T = 181.7\ lb$

4.137



FIND: REACTION AT C.

FREE-BODY DIAGRAM



$\vec{BD} = -8i - 9j + 12k$

$BD = 17\ in$

$\lambda_{BD} = \frac{\vec{BD}}{BD}$

$\lambda_{BD} = \frac{-8i - 9j + 12k}{17}$

$r_{A/B} = -6i$

$r_{C/B} = -9j + 12k$

(CONTINUED)

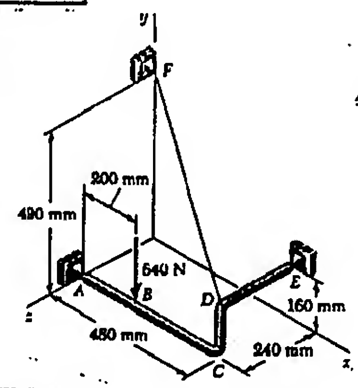
4.137 CONTINUED

$\Sigma M_{BD} = 0: \quad \lambda_{BD}\cdot(r_{C/B}\times C) + \lambda_{BD}\cdot(r_{A/B}\times(80\ lb)k) = 0$

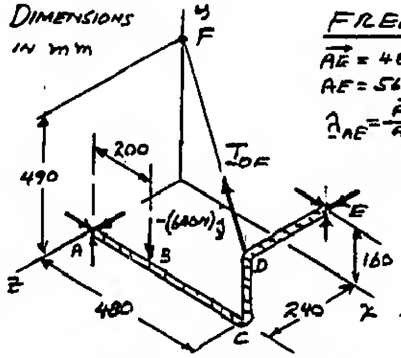
$$\begin{vmatrix} -8 & -9 & 12 \\ 0 & -9 & 12 \\ 0 & C & 0 \end{vmatrix}\frac{1}{17} + \begin{vmatrix} -8 & -9 & 12 \\ -6 & 0 & 0 \\ 0 & 0 & 80 \end{vmatrix}\frac{1}{17} = 0$$

$\frac{8\times12\times C}{17} - \frac{9\times6\times80}{17} = 0; \quad C = 45\ lb \qquad C = (45\ lb)j$

4.138



FIND: TENSION IN WIRE DF



FREE-BODY DIAGRAM

$\vec{AE} = 480i + 160j - 240k$

$AE = 560\ mm$

$\lambda_{AE} = \frac{\vec{AE}}{AE} = \frac{480i + 160j - 240k}{560}$

$\lambda_{AE} = \frac{6i + 2j - 3k}{7}$

$r_{B/A} = 200i$

$r_{D/A} = 480i + 160j$

$\vec{DF} = -480i + 330j - 240k; \qquad DF = 630\ mm$

$T_{DF} = T_{DF}\frac{\vec{DF}}{DF} = T_{DF}\frac{-480i + 330j - 240k}{630} = T_{DF}\frac{-16i + 11j - 8k}{21}$

$\Sigma M_{AE} = \lambda_{AE}\cdot(r_{D/A}\times T_{DF}) + \lambda_{AE}\cdot(r_{B/A}\times(-600j)) = 0$

$$\begin{vmatrix} 6 & 2 & -3 \\ 480 & 160 & 0 \\ -16 & 11 & -8 \end{vmatrix}\frac{T_{DF}}{21\times7} + \begin{vmatrix} 6 & 2 & -3 \\ 200 & 0 & 0 \\ 0 & -640 & 0 \end{vmatrix}\frac{1}{7} = 0$$

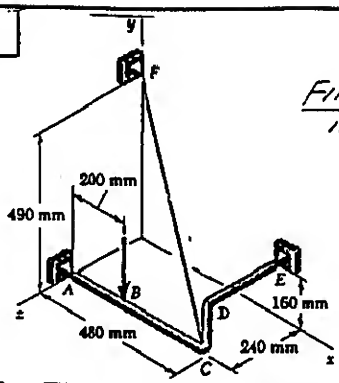
$$\frac{-6\times160\times8 + 2\times480\times8 - 3\times480\times11 - 3\times160\times16}{21\times7}T_{DF} + \frac{3\times200\times640}{7} = 0$$

$-1120\,T_{DF} + 384\times10^3 = 0$

$T_{DF} = 342.9\ N$

$T_{DF} = 343\ N$

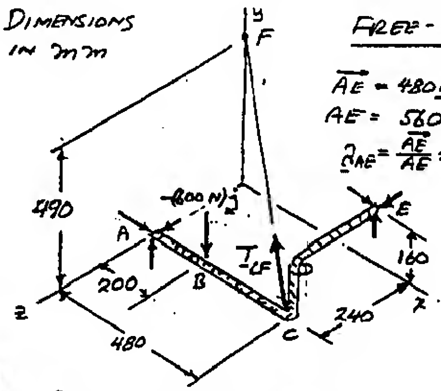
4.139



FIND: TENSION IN WIRE CF

DIMENSIONS IN mm

FREE-BODY DIAGRAM



$\vec{AE} = 480\underline{i} + 160\underline{j} - 240\underline{k}$

$AE = 560$ mm

$\underline{\lambda}_{AE} = \frac{\vec{AE}}{AE} = \frac{480\underline{i} + 160\underline{j} - 240\underline{k}}{560}$

$\underline{\lambda}_{AE} = \frac{6\underline{i} + 2\underline{j} - 3\underline{k}}{7}$

$\underline{r}_{B/A} = 200\underline{i}$

$\underline{r}_{C/A} = 480\underline{i}$

$\vec{CF} = -480\underline{i} + 490\underline{j} - 240\underline{k} \qquad CF = 726.7$ mm

$\underline{T}_{CF} = T_{CF}\frac{\vec{CF}}{CF} = \frac{-480\underline{i} + 490\underline{j} - 240\underline{k}}{726.7}$

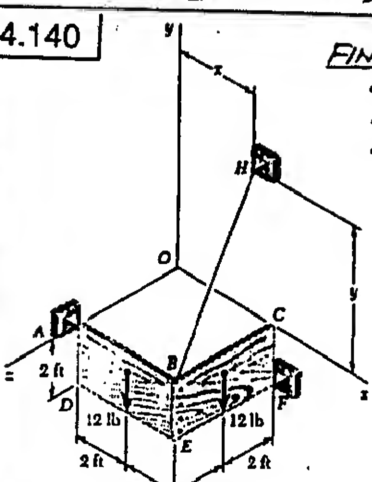
$\Sigma M_{AE} = 0: \quad \underline{\lambda}_{AE} \cdot (\underline{r}_{C/A} \times \underline{T}_{CF}) + \underline{\lambda}_{AE} \cdot (\underline{r}_{B/A} \times (-600\underline{j})) = 0$

$$\begin{vmatrix} 6 & 2 & -3 \\ 480 & 0 & 0 \\ -480 & +490 & -240 \end{vmatrix} \frac{T_{CF}}{726.7 \times 7} + \begin{vmatrix} 6 & 2 & -3 \\ 200 & 0 & 0 \\ 0 & -640 & 0 \end{vmatrix} \frac{1}{7} = 0$$

$$\frac{2 \times 480 \times 240 - 3 \times 480 \times 490}{726.7 \times \cancel{7}} T_{CF} + \frac{3 \times 200 \times 640}{\cancel{7}} = 0$$

$-653.91\, T_{CF} + 384 \times 10^2 = 0; \qquad T_{CF} = 587\text{ N}$ ◀

4.140



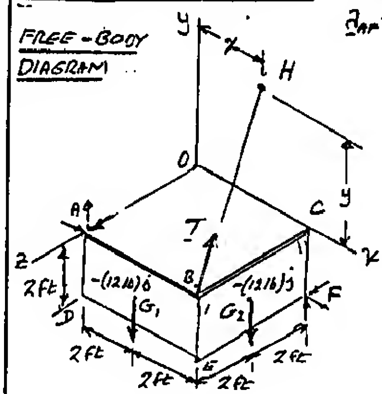
FIND: (a) LOCATION OF H IN xy PLANE FOR WHICH TENSION IN WIRE BH IS MINIMUM

(b) CORRESPONDING MINIMUM TENSION

(CONTINUED)

4.140 CONTINUED

FREE-BODY DIAGRAM



$\vec{AF} = 4\underline{i} - 2\underline{j} - 4\underline{k} \qquad AF = 3$ ft

$\underline{\lambda}_{AF} = \frac{1}{3}(2\underline{i} - \underline{j} - 2\underline{k})$

$\underline{r}_{G_1/A} = 2\underline{i} - \underline{j}$

$\underline{r}_{G_2/A} = 4\underline{i} - \underline{j} - 2\underline{k}$

$\underline{r}_{B/A} = 4\underline{i}$

$\Sigma M_{AF} = 0: \underline{\lambda}_{AF} \cdot (\underline{r}_{G_1/A} \times (-12\underline{j})) + \underline{\lambda}_{AF} \cdot (\underline{r}_{G_2/A} \times (-12\underline{j})) + \underline{\lambda}_{AF} \cdot (\underline{r}_{B/A} \times \underline{T}) = 0$

$$\begin{vmatrix} 2 & -1 & -2 \\ 2 & -1 & 0 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \begin{vmatrix} 2 & -1 & -2 \\ 4 & -1 & -2 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \underline{\lambda}_{AF} \cdot (\underline{r}_{B/A} \times \underline{T}) = 0$$

$$(2 \times 2 \times 12)\frac{1}{3} + (-2 \times 2 \times 12 + 2 \times 4 \times 12)\frac{1}{3} + \underline{\lambda}_{AF} \cdot (\underline{r}_{B/A} \times \underline{T}) = 0$$

$\underline{\lambda}_{AF} \cdot (\underline{r}_{B/A} \times \underline{T}) = -32$ OR; $\underline{T} \cdot (\underline{\lambda}_{AF} \times \underline{r}_{B/A}) = -32 \qquad (1)$

PROJECTION OF $\underline{T}$ ON $(\underline{\lambda}_{AF} \times \underline{r}_{B/A})$ IS CONSTANT. THUS, T_{min} IS PARALLEL TO

$$\underline{\lambda}_{AF} \times \underline{r}_{B/A} = \frac{1}{3}(2\underline{i} - \underline{j} - 2\underline{k}) \times 4\underline{i} = \frac{1}{3}(-8\underline{j} + 4\underline{k})$$

CORRESPONDING UNIT VECTOR IS $\frac{1}{\sqrt{5}}(-2\underline{j} + \underline{k})$

$T_{min} = T(-2\underline{j} + \underline{k})\frac{1}{\sqrt{5}} \qquad (2)$

EQ (1): $\frac{T}{\sqrt{5}}(-2\underline{j} + \underline{k}) \cdot \left[\frac{1}{3}(2\underline{i} - \underline{j} - 2\underline{k}) \times 4\underline{i}\right] = -32$

$\frac{T}{\sqrt{5}}(-2\underline{j} + \underline{k}) \cdot \frac{1}{3}(-8\underline{j} + 4\underline{k}) = -32$

$\frac{T}{3\sqrt{5}}(16 + 4) = -32; \qquad T = -\frac{3\sqrt{5}(32)}{20} = 4.8\sqrt{5}$

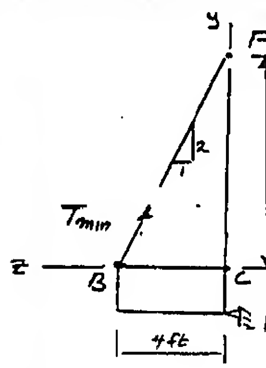
$T = 10.733$ lb

EQ. (2) $\quad T_{min} = T(-2\underline{j} + \underline{k})\frac{1}{\sqrt{5}}$

$= 4.8\sqrt{5}(-2\underline{j} + \underline{k})\frac{1}{\sqrt{5}}$

$\underline{T}_{min} = -(9.6\text{ lb})\underline{j} + (4.8\text{ lb})\underline{k}$

SINCE T_{min} HAS NO $\underline{i}$ COMPONENT, WIRE BH IS PARALLEL TO THE yz PLANE, AND $x = 4$ ft



$y = (4\text{ ft})(2) = 8$ ft

ANSWERS:

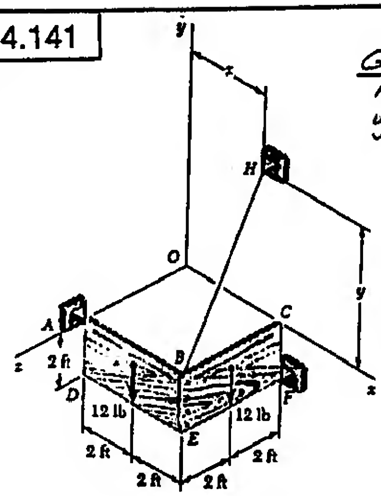
(a) $x = 4$ ft, $y = 8$ ft ◀

(b) $T_{min} = 10.73$ lb ◀

PLATE: $m = 50$ kg

$\underline{\lambda}_{EA} = \frac{EA}{EA} = \frac{76 - \ldots}{25}$

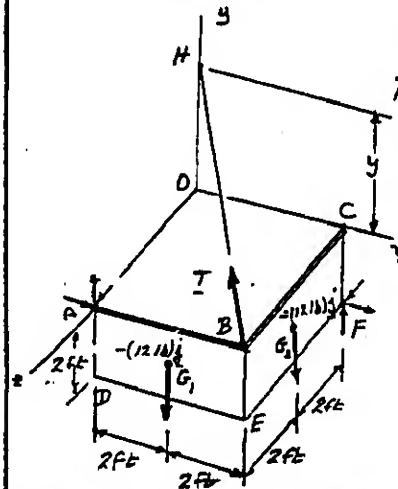
4.141



GIVEN: SUPPORT H MUST BE ON y AXIS (i.e., x = 0)

FIND:
(a) DISTANCE y FOR WHICH TENSION IN WIRE BH IS MINIMUM.
(b) CORRESPONDING MINIMUM TENSION.

FREE-BODY DIAGRAM



$\vec{AF} = 4\underline{i} - 2\underline{j} - 4\underline{k}$

$\lambda_{AF} = \frac{1}{3}(2\underline{i} - \underline{j} - 2\underline{k})$

$r_{G_1/A} = 2\underline{i} - \underline{j}$

$r_{G_2/A} = 4\underline{i} - \underline{j} - 2\underline{k}$

$r_{B/A} = 4\underline{i}$

$\Sigma M_{AF} = 0: \lambda_{AF}\cdot(r_{G_1/A} \times (-12\underline{j})) + \lambda_{AF}\cdot(r_{G_2/A} \times (-12\underline{j})) + \lambda_{AF}\cdot(r_{B/A} \times \underline{T}) = 0$

$$\begin{vmatrix} 2 & -1 & 2 \\ 2 & -1 & 0 \\ 0 & -12 & 0 \end{vmatrix}\frac{1}{3} + \begin{vmatrix} 2 & -1 & 2 \\ 4 & -1 & -2 \\ 0 & -12 & 0 \end{vmatrix}\frac{1}{3} + \lambda_{AF}\cdot(r_{B/A} \times \underline{T}) = 0$$

$(2\times2\times12)\frac{1}{3} + (-2\times2\times12 + 2\times4\times12)\frac{1}{3} + \lambda_{AF}\cdot(r_{B/A} \times \underline{T}) = 0$

$$\lambda_{AF}\cdot(r_{B/A} \times \underline{T}) = -32 \qquad (1)$$

$\vec{BH} = -4\underline{i} + y\underline{j} - 4\underline{k} \qquad BH = (32 + y^2)^{1/2}$

$\underline{T} = T\frac{\vec{BH}}{BH} = T\frac{-4\underline{i} + y\underline{j} - 4\underline{k}}{(32+y^2)^{1/2}}$

EQ. (1):

$$\lambda_{AF}\cdot(r_{B/A} \times \underline{T}) = \begin{vmatrix} 2 & -1 & -2 \\ 4 & 0 & 0 \\ -4 & y & -4 \end{vmatrix}\frac{T}{3(32+y^2)^{1/2}} = -32$$

$(-16 - 8y)T = -3\times32(32+y^2)^{1/2}; \quad T = 96\frac{(32+y^2)^{1/2}}{8y+16} \qquad (2)$

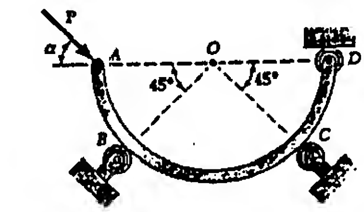
$\frac{dT}{dy} = 0: \; 96\frac{(8y+16)\frac{1}{2}(32+y^2)^{-1/2}(2y) - (32+y^2)^{1/2}(8)}{(8y+16)^2}$

NUMERATOR = 0: $(8y+16)y = (32+y^2)8$

$\cancel{8y^2} + 16y = 32\times8 + \cancel{8y^2}$ $\qquad y = 16\text{ ft}$ ◀

EQ. (2): $T = 96\frac{(32+16^2)^{1/2}}{8\times16+16} = 11.313\text{ lb} \qquad T_{min} = 11.31\text{ lb}$ ◀

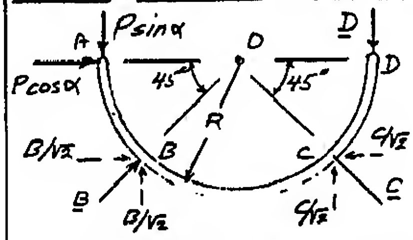
4.142 and 4.143



PROB. 4.141: FOR $\alpha = 45°$, FIND REACTIONS AT B, C, AND D.

PROB. 4.142: FIND RANGE OF α FOR EQUILIBRIUM.

FREE-BODY DIAGRAM



$+\circlearrowleft\Sigma M_O = 0: (P\sin\alpha)R - D(R) = 0 \qquad D = P\sin\alpha \qquad (1)$

$\xrightarrow{+}\Sigma F_x = 0: \; P\cos\alpha + B/\sqrt{2} - C/\sqrt{2} = 0 \qquad (2)$

$+\uparrow\Sigma F_y = 0: \; -P\sin\alpha + B/\sqrt{2} + C/\sqrt{2} - P\sin\alpha = 0$

$-2P\sin\alpha + B/\sqrt{2} + C/\sqrt{2} = 0 \qquad (3)$

(2)+(3): $P(\cos\alpha - 2\sin\alpha) + 2B/\sqrt{2} = 0$

$B = \frac{\sqrt{2}}{2}(2\sin\alpha - \cos\alpha)P \qquad (4)$

(2)−(3): $P(\cos\alpha + 2\sin\alpha) - 2C/\sqrt{2} = 0$

$C = \frac{\sqrt{2}}{2}(2\sin\alpha + \cos\alpha)P \qquad (5)$

PROB 4.142 FOR $\alpha = 45°$; $\sin\alpha = \cos\alpha = \frac{1}{\sqrt{2}}$

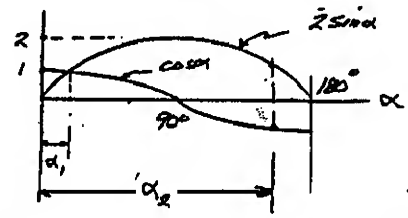
EQ(4): $B = \frac{\sqrt{2}}{2}\left(\frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)P = \frac{1}{2}P$; $\quad \underline{B} = \frac{1}{2}P \angle 45°$ ◀

EQ(5): $C = \frac{\sqrt{2}}{2}\left(\frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)P = \frac{3}{2}P$; $\quad \underline{C} = \frac{3}{2}P \measuredangle 45°$ ◀

EQ.(1): $D = P/\sqrt{2} \qquad \underline{D} = P/\sqrt{2}\downarrow$ ◀

PROB. 4.143 RANGE OF α FOR EQUILIBRIUM

FOR $B \geq 0$:
FROM EQ.(4): $2\sin\alpha - \cos\alpha \geq 0$
FOR $C \geq 0$:
FROM EQ(5): $2\sin\alpha + \cos\alpha \geq 0$



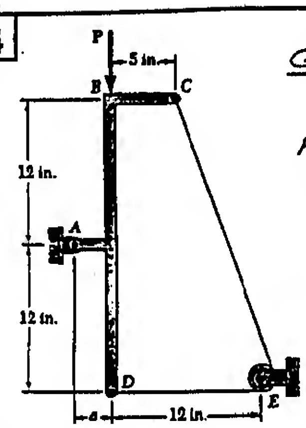
$2\sin\alpha_1 \geq \cos\alpha_1$
$\tan\alpha_1 \geq 0.5$
$\alpha_1 \geq 26.6°$

$2\sin\alpha_2 \geq -\cos\alpha_2$
$\tan\alpha_2 \geq -0.5$
$\alpha_2 \leq 153.4°$

$26.6° \leq \alpha \leq 153.4°$ ◀

FOR THIS RANGE $\sin\alpha \geq 0$, THUS EQ(1) YIELDS $D \geq 0$, OK

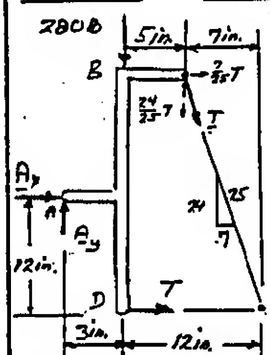
4.144



GIVEN: $a = 3\text{in.}$, $P = 280\text{ lb.}$

FIND:
(a) TENSION IN CABLE DEC.
(b) REACTION AT A.

FREE-BODY DIAGRAM



$+\circlearrowleft \Sigma M_A = 0$: $-(280\text{ lb})(8\text{ in.}) - T(12\text{ in.}) - \frac{7}{25}T(12\text{ in.}) - \frac{24}{25}T(8\text{ in.}) = 0$

$(12 - 11.04)T = 840$

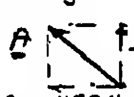
$T = 875\text{ lb}$ ◀

$\overset{+}{\rightarrow} \Sigma F_x = 0$: $\frac{7}{25}(875\text{ lb}) + 875\text{ lb} + A_x = 0$

$A_x = -1120$ $\quad A_x = 1120\text{ lb} \leftarrow$

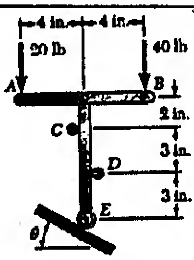
$+\uparrow \Sigma F_y = 0$: $A_y - 280\text{ lb} - \frac{24}{25}(875\text{ lb}) = 0$

$A_y = +1120$ $\quad A_y = 1120\text{ lb}\uparrow$



$A = 1584\text{ lb} \searrow 45°$ ◀

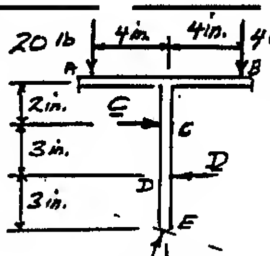
4.145



GIVEN: $\theta = 30°$.

FIND: REACTIONS AT C, D, AND E.

FREE-BODY DIAGRAM



$+\uparrow \Sigma F_y = 0$: $E\cos 30° - 20 - 40 = 0$

$E = \frac{60\text{ lb}}{\cos 30°} = 69.28\text{ lb}$

$E = 69.3\text{ lb} \measuredangle 60°$ ◀

$+\circlearrowleft \Sigma M_D = 0$:

$(20\text{ lb})(4\text{ in.}) - (40\text{ lb})(4\text{ in.}) - C(3\text{ in.}) + E\sin 30°(3\text{ in.}) = 0$

$-80 - 3C + 69.28(0.5)(3) = 0$

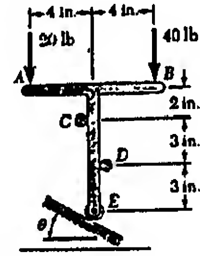
$C = 7.974\text{ lb}$ $\quad C = 7.97\text{ lb} \rightarrow$ ◀

$\overset{+}{\rightarrow} \Sigma F_x = 0$: $E\sin 30° + C - D = 0$

$(69.28\text{ lb})(0.5) + 7.974\text{ lb} - D = 0$

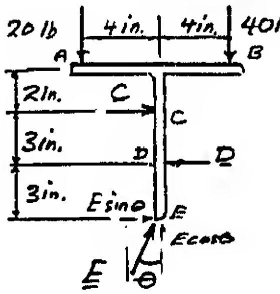
$D = 42.61\text{ lb}$ $\quad D = 42.6\text{ lb} \leftarrow$ ◀

4.146



FIND:
(a) SMALLEST θ FOR EQUILIBRIUM.
(b) CORRESPONDING REACTIONS AT C, D, AND E

FREE-BODY DIAGRAM



$+\uparrow \Sigma F_y = 0$: $E\cos\theta - 20 - 40 = 0$

$E = \frac{60}{\cos\theta}$

$+\circlearrowleft \Sigma M_D = 0$:

$(20\text{ lb})(4\text{ in.}) - (40\text{ lb})(4\text{ in.}) - C(3\text{ in.}) + \left(\frac{60}{\cos\theta}\sin\theta\right)3\text{ in.} = 0$

$C = \frac{1}{3}(180\tan\theta - 80)$

(a) For $C = 0$, $\quad 180\tan\theta = 80$

$\tan\theta = 4/9$; $\quad \theta = 23.96°$ $\quad \theta = 24.0°$ ◀

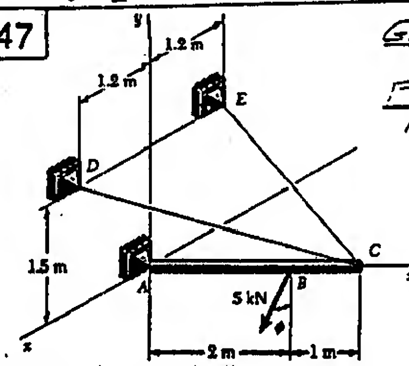
EQ.(1) $E = 60/\cos 23.96° = 65.66\text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0$: $-D + C + E\sin\theta = 0$

$D = (65.66)\sin 23.96 = 26.67\text{ lb}$

(b) $C = 0$; $\quad D = 26.7\text{ lb} \leftarrow$; $E = 65.7\text{ lb} \measuredangle 70.0°$ ◀

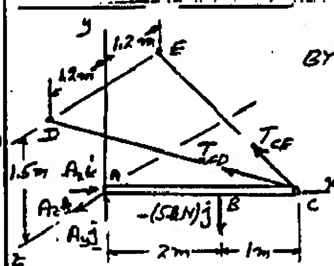
4.147



GIVEN: $\phi = 0$

FIND: (a) TENSION IN CD AND CE
(b) REACTION AT A.

FREE-BODY DIAGRAM BY SYMMETRY WITH xy PLANE



$T_{CD} = T_{CE} = T$

$\overline{CD} = -3i + 1.5j + 1.2k$

$CD = 3.562\text{ m}$

$T_{CD} = T\frac{\overline{CD}}{CD} = T\frac{-3i + 1.5j + 1.2k}{3.562}$

$T_{CE} = T\frac{-3i + 1.5j - 1.2k}{3.562}$

$r_{B/A} = 2i \quad r_{C/A} = 3i$

$\Sigma M_A = 0$: $r_{C/A} \times T_{CD} + r_{C/A} \times T_{CE} + r_{B/A} \times (-5\text{ kN})j = 0$

$$\begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ -3 & 1.5 & 1.2 \end{vmatrix}\frac{T}{3.562} + \begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ -3 & 1.5 & -1.2 \end{vmatrix}\frac{T}{3.562} + \begin{vmatrix} i & j & k \\ 2 & 0 & 0 \\ 0 & -5 & 0 \end{vmatrix} = 0$$

COEFF. OF k: $2\left[3 \times 1.5 \times \frac{T}{3.562}\right] - 10 = 0$; $\quad T = 3.958\text{ kN}$

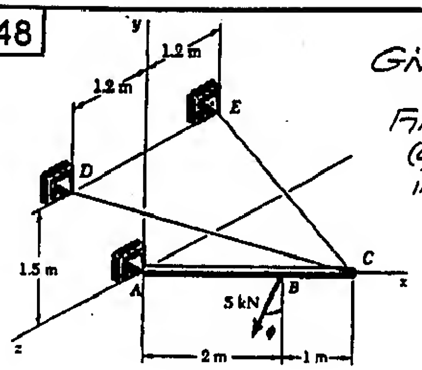
$\Sigma F = 0$: $A + T_{CD} + T_{CE} - 5j = 0$; COEFF. of k: $A_z = 0$

COEFF. OF i: $A_x - 2[3.958 \times 3/3.562] = 0$ $\quad A_x = 6.67\text{ kN}$

COEFF. OF j: $A_y + 2[3.958 \times 1.5/3.562] - 5 = 0$ $\quad A_y = 1.667\text{ kN}$

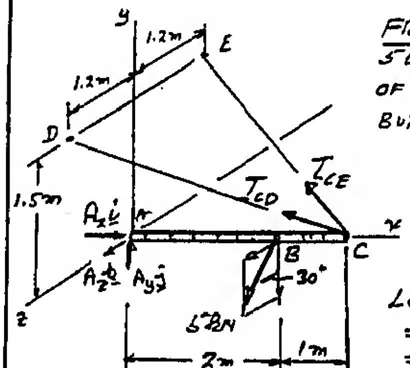
(a) $T_{CD} = T_{CE} = 3.96\text{ kN}$. (b) $A = (6.67\text{ kN})i + (1.667\text{ kN})j$ ◀

4.148



GIVEN: $\phi = 30°$.

FIND:
(a) TENSION IN CD AND CE.
(b) REACTION AT A.



FREE-BODY DIAGRAM
5 UNKNOWNS AND 6 EQS. OF EQUILIBRIUM
BUT, EQUIL. MAINTAINED ($\Sigma M_{AC} = 0$)

$\mathbf{r}_{B/A} = 2\mathbf{i}$

$\mathbf{r}_{C/A} = 3\mathbf{i}$

LOAD AT B,
$= -(5\cos 30°)\mathbf{j} + (5\sin 30°)\mathbf{k}$
$= -4.33\mathbf{j} + 2.5\mathbf{k}$

$\overrightarrow{CD} = -3\mathbf{i} + 1.5\mathbf{j} + 1.2\mathbf{k} \qquad CD = 3.562\,\text{m}$

$\mathbf{T}_{CD} = T_{CD}\dfrac{\overrightarrow{CD}}{CD} = \dfrac{T}{3.562}(-3\mathbf{i} + 1.5\mathbf{j} + 1.2\mathbf{k})$

SIMILARLY, $\mathbf{T}_{CE} = \dfrac{T}{3.562}(-3\mathbf{i} + 1.5\mathbf{j} - 1.2\mathbf{k})$

$\Sigma M_A = 0$: $\mathbf{r}_{C/A} \times \mathbf{T}_{CD} + \mathbf{r}_{C/A} \times \mathbf{T}_{CE} + \mathbf{r}_{B/A} \times (-4.33\mathbf{j} + 2.5\mathbf{k}) = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -3 & 1.5 & 1.2 \end{vmatrix}\frac{T_{CD}}{3.562} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -3 & 1.5 & -1.2 \end{vmatrix}\frac{T_{CE}}{3.562} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 0 \\ 0 & -4.33 & 2.5 \end{vmatrix} = 0$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO

(j) $-3.6\dfrac{T_{CD}}{3.562} + 3.6\dfrac{T_{CE}}{3.562} - 5 = 0$

$-3.6T_{CD} + 3.6T_{CE} - 17.810 = 0 \qquad (1)$

(k) $4.5\dfrac{T_{CD}}{3.562} + 4.5\dfrac{T_{CE}}{3.562} - 8.66 = 0$

$4.5T_{CD} + 4.5T_{CE} = 30.846 \qquad (2)$

(2)+1.25(1): $9T_{CE} - 53.11 = 0; \quad T_{CE} = 5.901\,\text{kN}$

EQ (1): $-3.6T_{CD} + 3.6(5.901) - 17.810 = 0$

$T_{CD} = 0.954\,\text{kN}$

$\Sigma F = 0$: $\mathbf{A} + \mathbf{T}_{CD} + \mathbf{T}_{CE} - 4.33\mathbf{j} + 2.5\mathbf{k} = 0$

(i) $A_x + \dfrac{0.954}{3.562}(-3) + \dfrac{5.901}{3.562}(-3) = 0; \quad A_x = 5.77\,\text{kN}$

(j) $A_y + \dfrac{0.954}{3.562}(1.5) + \dfrac{5.901}{3.562}(1.5) - 4.33 = 0$

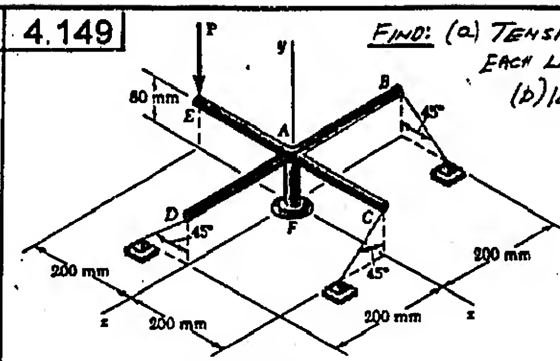
$A_y = 1.443\,\text{kN}$

(k) $A_z + \dfrac{0.954}{3.562}(1.2) + \dfrac{5.901}{3.562}(-1.2) + 2.5 = 0$

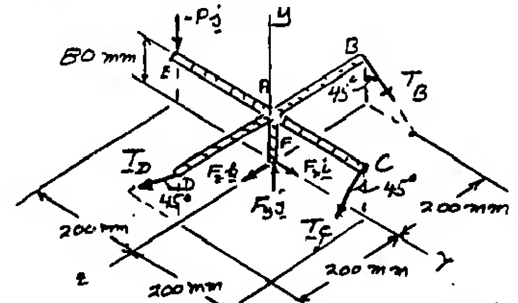
$A_z = -0.833\,\text{kN}$.

ANSWERS:
(a) $T_{CD} = 0.954\,\text{kN}; \quad T_{CE} = 5.90\,\text{kN}$ ◀
(b) $\mathbf{A} = (5.77\,\text{kN})\mathbf{i} + (1.443\,\text{kN})\mathbf{j} - (0.833\,\text{kN})\mathbf{k}$ ◀

4.149



FIND: (a) TENSION IN EACH LINK.
(b) REACTION AT F



$\mathbf{T}_B = T_B(\mathbf{i} - \mathbf{j})/\sqrt{2}$

$\mathbf{T}_C = T_C(-\mathbf{j} + \mathbf{k})/\sqrt{2}$

$\mathbf{T}_D = T_D(-\mathbf{i} - \mathbf{j})/\sqrt{2}$

$\mathbf{r}_{E/F} = -200\mathbf{i} + 80\mathbf{j}$

$\mathbf{r}_{B/F} = 80\mathbf{j} - 200\mathbf{k}$

$\mathbf{r}_{C/F} = 200\mathbf{i} + 80\mathbf{j}$

$\mathbf{r}_{D/F} = 80\mathbf{j} + 200\mathbf{k}$

$\Sigma M_F = 0$: $\mathbf{r}_{B/F} \times \mathbf{T}_B + \mathbf{r}_{C/F} \times \mathbf{T}_C + \mathbf{r}_{D/F} \times \mathbf{T}_D + \mathbf{r}_{E/F} \times (-P\mathbf{j}) = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & -200 \\ 1 & -1 & 0 \end{vmatrix}\frac{T_B}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 200 & 80 & 0 \\ 0 & -1 & 1 \end{vmatrix}\frac{T_C}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & 200 \\ -1 & -1 & 0 \end{vmatrix}\frac{T_D}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -200 & 80 & 0 \\ 0 & -P & 0 \end{vmatrix} = 0$$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO AND MULTIPLY EACH EQUATION BY $\sqrt{2}$.

(i) $-200T_B + 80T_C + 200T_D = 0 \qquad (1)$

(j) $-200T_B - 200T_C - 200T_D = 0 \qquad (2)$

(k) $-80T_B - 200T_C + 80T_D + 200\sqrt{2}P = 0 \qquad (3)$

$\frac{80}{200}(2)$: $-80T_B - 80T_C - 80T_D = 0 \qquad (4)$

(3)+(4): $-160T_B - 280T_C + 200\sqrt{2}P = 0 \qquad (5)$

(1)+(2): $-400T_B - 120T_C = 0$

$T_B = -\frac{120}{400}T_C = -0.3T_C \qquad (6)$

(6)→(5): $-160(-0.3T_C) - 280T_C + 200\sqrt{2}P = 0$

$-232T_C + 200\sqrt{2}P = 0$

$T_C = 1.2191P \qquad T_C = 1.219P$ ◀

(6): $T_B = -0.3(1.2191P) = -0.36574P \qquad T_B = -0.366P$ ◀

(2): $-200(-0.36574P) - 200(1.2191P) - 200T_D = 0$

$T_D = -0.8534P \qquad T_D = -0.853P$ ◀

$\Sigma F = 0$: $\mathbf{F} + \mathbf{T}_B + \mathbf{T}_C + \mathbf{T}_D - P\mathbf{j} = 0$

(i) $F_x + (-0.36574P)/\sqrt{2} - (-0.8534P)/\sqrt{2} = 0$

$F_x = -0.3448P \qquad F_x = -0.345P$

(j) $F_y - (-0.36574P)/\sqrt{2} - (1.2191P)/\sqrt{2} - (-0.8534P)/\sqrt{2} - 200 = 0$

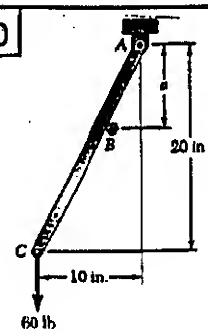
$F_y = P \qquad F_y = P$

(k) $F_z + (1.2191P)/\sqrt{2} = 0$

$F_z = -0.8620P \qquad F_z = -0.862P$

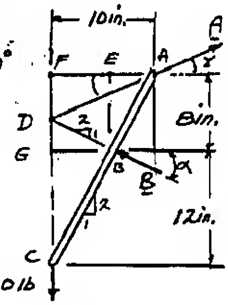
$\mathbf{F} = -0.345P\mathbf{i} + P\mathbf{j} - 0.862P\mathbf{k}$ ◀

4.150



FIND:
(a) REACTIONS AT A AND B WHEN $a = 8$ in.
(b) DISTANCE a FOR WHICH REACTION AT A IS HORIZONTAL AND CORRESPONDING REACTIONS AT A AND B.

(a) $\tan\alpha = \frac{1}{2}$; $\alpha = 26.57°$



3-FORCE BODY

REACTION AT A PASSES THROUGH D WHERE B AND 60-lb LOAD INTERSECT

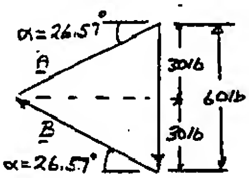
$AE = \frac{1}{2}EB = \frac{1}{2}(8) = 4$ in.

$EF = BG = 10 - 4 = 6$ in.

$DG = \frac{1}{2}BG = \frac{1}{2}(6) = 3$ in.

$FD = FG - DG = 8 - 3 = 5$ in.

$\tan\gamma = \frac{FD}{AF} = \frac{5}{10}$; $\gamma = 26.57°$



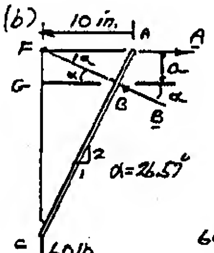
FORCE TRIANGLE

$A = B = \frac{30 \text{ lb}}{\sin 26.57°} = 67.08$ lb

$A = 67.1$ lb $\angle 26.6°$

$B = 67.1$ lb $\searrow 26.6°$

(b)



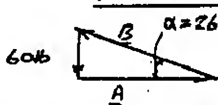
FOR A HORIZONTAL,

$\triangle ABF$: $BF = AF\cos\alpha$

$\triangle BFG$: $FG = BF\sin\alpha$

$a = FG = AF\cos\alpha\sin\alpha$

$a = (10 \text{ in.})\cos 26.57° \sin 26.57°$; $a = 4$ in.



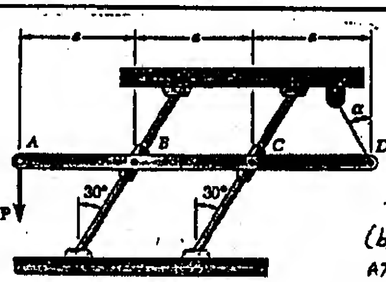
FORCE TRIANGLE

$A = 60/\tan\alpha = 120$ lb

$A = 120$ lb $\rightarrow$

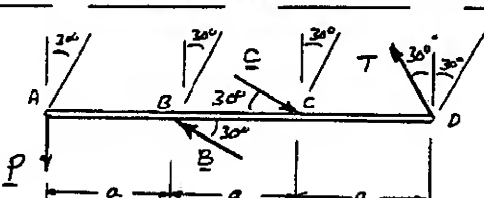
$B = (60 \text{ lb})/\sin\alpha = 134.16$ lb $B = 134.1$ lb $\searrow 26.6°$

4.151



GIVEN: $\alpha = 30°$

FIND:
(a) TENSION IN WIRE.
(b) REACTIONS AT B AND C.



(CONTINUED)

4.151 CONTINUED

$30° \nearrow \Sigma F = 0$: $-P\cos 30° + T\cos 60° = 0$

$T = P\frac{\cos 30°}{\cos 60°} = P\frac{\sqrt{3}/2}{1/2}$ $T = \sqrt{3}P$

$+\circlearrowleft \Sigma M_B = 0$: $Pa - (C\sin 30°)a + T\cos 30°(2a) = 0$

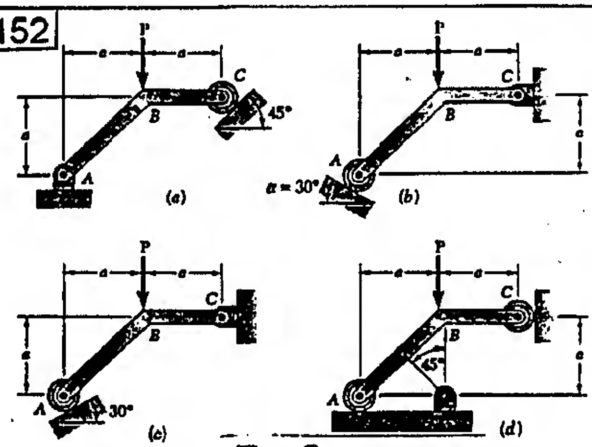
$Pa - (\frac{1}{2}C)a + \sqrt{3}P(\sqrt{3}/2)2a = 0$

$-\frac{1}{2}C + (1+3)P = 0$; $C = 8P$: $C = 8P \measuredangle 30°$

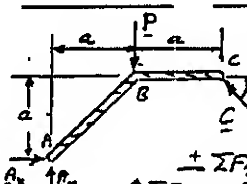
$+\Sigma F = 0$: $-B\cos 30° + C\cos 30° - T\sin 30° = 0$

$-B\frac{\sqrt{3}}{2} + 8P\frac{\sqrt{3}}{2} - \sqrt{3}P(\frac{1}{2}) = 0$; $B = 7P$; $B = 7P \searrow 30°$

4.152



FIND: REACTIONS



(a) $+\circlearrowleft \Sigma M_A = 0$: $-Pa + (C\sin 45°)2a + (C\cos 45°)a = 0$

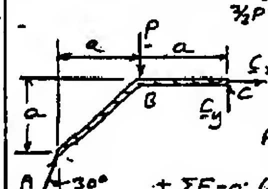
$3\frac{C}{\sqrt{2}} = P$; $C = \frac{\sqrt{2}}{3}P$

$C = 0.471P \searrow 45°$

$\pm \Sigma F_x = 0$: $A_x - (\frac{\sqrt{2}}{3}P)\frac{1}{\sqrt{2}}$; $A_x = P/3 \rightarrow$

$+\uparrow \Sigma F_y = 0$: $A_y - P + (\frac{\sqrt{2}}{3}P)\frac{1}{\sqrt{2}}$; $A_y = \frac{2P}{3}\uparrow$

$\frac{2}{3}P$, $P/3$ $A = 0.745P \angle 63.4°$



(b) $+\circlearrowleft \Sigma M_C = 0$:

$+Pa - (A\cos 30°)2a + (A\sin 30°)a = 0$

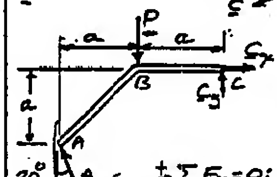
$A(1.732 - 0.5) = P$; $A = 1.232P$

$A = 1.232P \angle 60°$

$\pm \Sigma F_x = 0$: $(1.232P)\sin 30° + C_x = 0$; $C_x = 0.616P \leftarrow$

$+\uparrow \Sigma F_y = 0$: $(1.232P)\cos 30° - P + C_y = 0$; $C_y = 0.067P \downarrow$

$0.616P$, $0.067P$ $C = 0.620P \measuredangle 6.2°$



(c) $+\circlearrowleft \Sigma M_C = 0$

$+Pa - (A\cos 30°)2a + (A\sin 30°)a = 0$

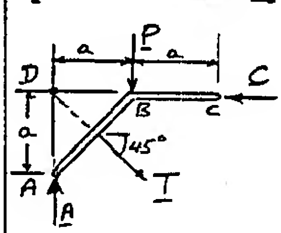
$A(1.732 + 0.5) = P$ $A = 0.448P$

$A = 0.448P \searrow 60°$

$\pm \Sigma F_x = 0$: $-(0.448P)\sin 30° + C_x = 0$; $C_x = 0.224P \rightarrow$

$+\uparrow \Sigma F_y = 0$: $(0.448P)\cos 30° - P + C_y = 0$; $C_y = 0.612P\uparrow$

$0.612P$, $0.224P$ $C = 0.652P \angle 69.9°$



(d)

FORCE T EXERTED BY WIRE AND REACTIONS A AND C ALL INTERSECT AT POINT D.

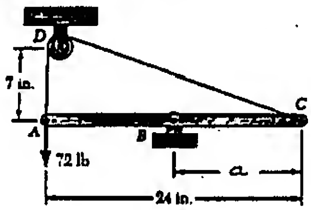
$+\circlearrowleft \Sigma M_D = 0$: $Pa = 0$

EQUILIBRIUM NOT MAINTAINED

ROD IS IMPROPERLY CONSTRAINED

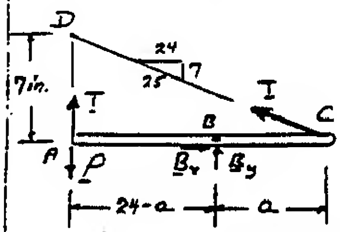
*4.153

FOR THE RIGID BODIES OF THE FOLLOWING PROBLEMS, FIND THE VALUE OF a OR α WHICH RESULTS IN IMPROPER CONSTRAINTS.



(a) PROB. 4.77

FREE-BODY DIAGRAM



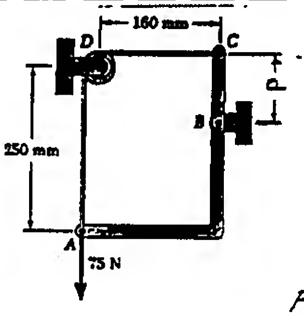
$+\circlearrowleft \Sigma M_B = 0$:

$$P(24-a) - T(24-a) + \frac{7}{25}Ta = 0$$

$$T = \frac{P(24-a)}{24-a-\frac{7}{25}a}$$

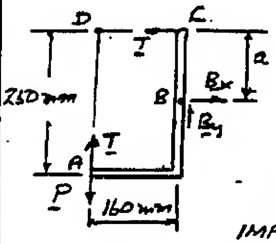
T BECOMES ∞ WHEN $24-a-\frac{7}{25}a=0$

IMPROPER CONSTRAINT: $a = 18.75$ in.



(b) PROB. 4.78

FREE-BODY DIAGRAM



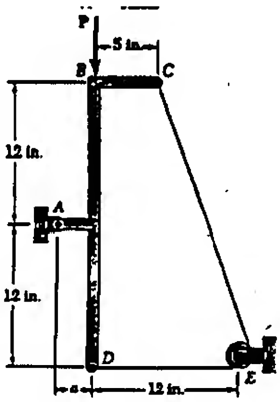
$+\circlearrowleft \Sigma M_B = 0$:

$$P(160) - T(160) + T(a) = 0$$

$$T = \frac{160P}{160-a}$$

T BECOMES INFINITE WHEN $160 - a = 0$

IMPROPER CONSTRAINT: $a = 160$ mm

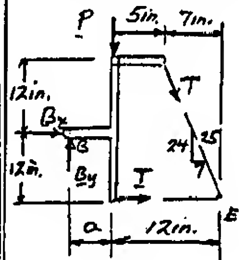


(k) PROB. 4.144

(CONTINUED)

*4.153 CONTINUED

(k) PROB. 4.144 (CONTINUED)



$+\circlearrowleft \Sigma M_B = 0$:

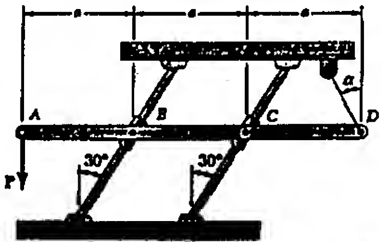
$$-Pa - \frac{7}{25}T(12) - \frac{24}{25}T(a+5) + T(12) = 0$$

$$T = \frac{Pa}{12 - \frac{84}{25} - \frac{24}{25}a - \frac{120}{25}}$$

$$T = \frac{Pa}{3.84 - \frac{24}{25}a}$$

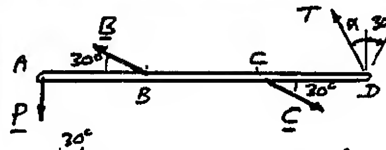
T BECOMES INFINITE WHEN $3.84 - \frac{24}{25}a = 0$

IMPROPER CONSTRAINT: $a = 4$ in.



(d) PROB. 4.151

FREE-BODY DIAGRAM

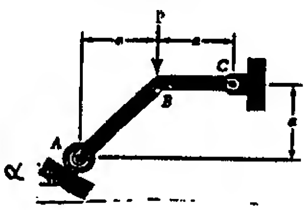


$30°$ $\nearrow \Sigma F = 0$: $\quad -P\cos 30° + T\cos(\alpha + 30°) = 0$

$$T = \frac{P\cos 30°}{\cos(\alpha+30°)}$$

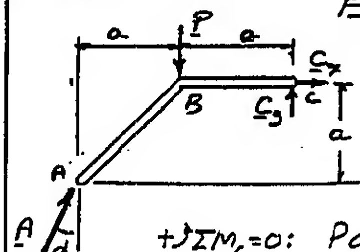
T BECOMES INFINITE WHEN $\cos(\alpha + 30°) = 0$

IMPROPER CONSTRAINT: $\alpha + 30° = 90°$, $\quad \alpha = 60°$



(e) PROB. 4.152 b

FREE-BODY DIAGRAM



$+\circlearrowleft \Sigma M_C = 0$: $\quad Pa + (A\sin\alpha)a - (A\cos\alpha)2a = 0$

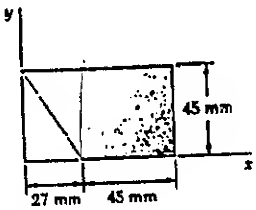
$$A = \frac{Pa}{a(2\cos\alpha - \sin\alpha)}$$

A BECOMES INFINITE WHEN $2\cos\alpha - \sin\alpha = 0$

$\tan\alpha = 2 \qquad \alpha = 63.43°$

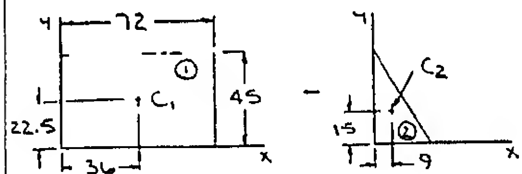
IMPROPER CONSTRAINT: $\alpha = 63.4°$

5.1



GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$



DIMENSIONS IN mm

	A, mm^2	$\bar{x}, mm$	$\bar{y}, mm$	$\bar{x}A, mm^3$	$\bar{y}A, mm^3$
1	$72 \times 45 = 3240$	36	22.5	116 640	72 900
2	$-\frac{1}{2} \times 27 \times 45 = -607.5$	9	15	-5467.5	-9112.5
Σ	2632.5			111 172.5	63 787.5

THEN $\bar{X}\Sigma A = \Sigma \bar{x}A$

$\bar{X}(2632.5) = 111\,172.5$

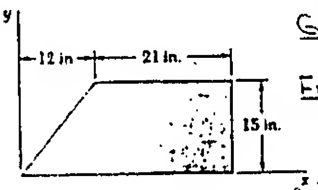
OR $\bar{X} = 42.2$ mm ◀

AND $\bar{Y}\Sigma A = \Sigma \bar{y}A$

$\bar{Y}(2632.5) = 63\,787.5$

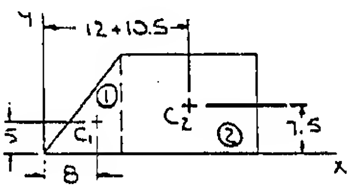
OR $\bar{Y} = 24.2$ mm ◀

5.2



GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$



DIMENSIONS IN IN.

	A, IN^2	$\bar{x}, IN.$	$\bar{y}, IN.$	$\bar{x}A, IN^3$	$\bar{y}A, IN^3$
1	$\frac{1}{2} \times 12 \times 15 = 90$	8	5	720	450
2	$21 \times 15 = 315$	22.5	7.5	7087.5	2362.5
Σ	405			7807.5	2812.5

THEN $\bar{X}\Sigma A = \Sigma \bar{x}A$

$\bar{X}(405) = 7807.5$

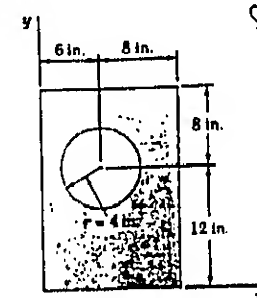
OR $\bar{X} = 19.28$ IN. ◀

AND $\bar{Y}\Sigma A = \Sigma \bar{y}A$

$\bar{Y}(405) = 2812.5$

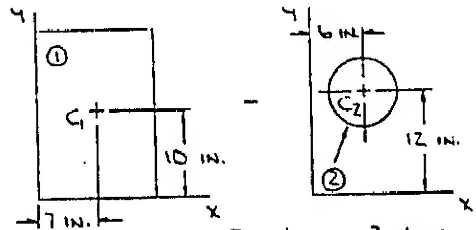
OR $\bar{Y} = 6.94$ IN. ◀

5.3



GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$



	A, IN^2	$\bar{x}, IN.$	$\bar{y}, IN.$	$\bar{x}A, IN^3$	$\bar{y}A, IN^3$
1	$14 \times 20 = 280$	7	10	1960	2800
2	$-\pi(4)^2 = -16\pi$	6	12	-301.59	-603.19
Σ	229.73			1658.41	2196.8

THEN $\bar{X}\Sigma A = \Sigma \bar{x}A$

$\bar{X}(229.73) = 1658.41$

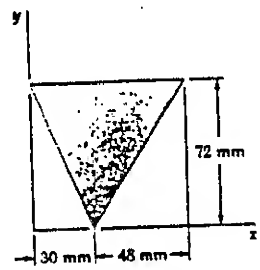
OR $\bar{X} = 7.22$ IN. ◀

AND $\bar{Y}\Sigma A = \Sigma \bar{y}A$

$\bar{Y}(229.73) = 2196.8$

OR $\bar{Y} = 9.56$ IN. ◀

5.4



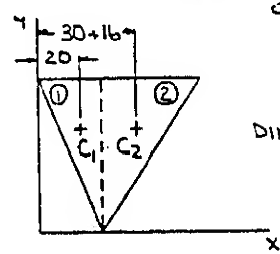
GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$

FOR THE AREA AS A WHOLE, IT CAN BE CONCLUDED BY OBSERVATION THAT

$\bar{Y} = \frac{2}{3}(72 \text{ mm})$

OR $\bar{Y} = 48.0$ mm ◀



DIMENSIONS IN mm

	A, mm^2	$\bar{x}, mm$	$\bar{x}A, mm^3$
1	$\frac{1}{2} \times 30 \times 72 = 1080$	20	21 600
2	$\frac{1}{2} \times 48 \times 72 = 1728$	46	79 488
Σ	2808		101 088

THEN $\bar{X}\Sigma A = \Sigma \bar{x}A$

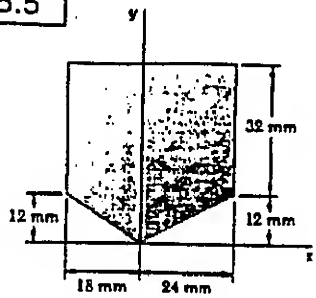
$\bar{X}(2808) = 101\,088$

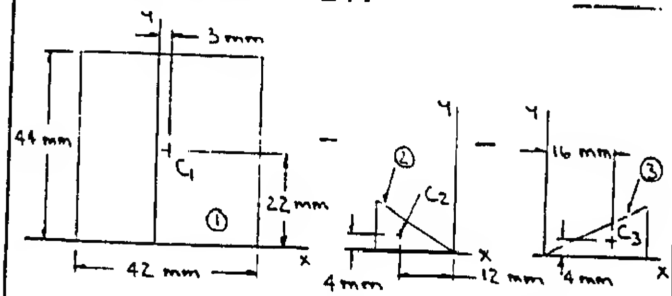
OR $\bar{X} = 36.0$ mm ◀

5.5

GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$





	A, mm^2	$\bar{x}, mm$	$\bar{y}, mm$	$\bar{x}A, mm^3$	$\bar{y}A, mm^3$
1	$42 \times 44 = 1848$	3	22	5544	40 656
2	$-\frac{1}{2} \times 18 \times 12 = -108$	−12	4	1296	−432
3	$-\frac{1}{2} \times 24 \times 12 = -144$	16	4	−2304	−576
Σ	1596			4536	39 648

THEN $\bar{X}\Sigma A = \Sigma \bar{x}A$

$\bar{X}(1596) = 4536$

OR $\bar{X} = 2.84$ mm ◀

AND $\bar{Y}\Sigma A = \Sigma \bar{y}A$

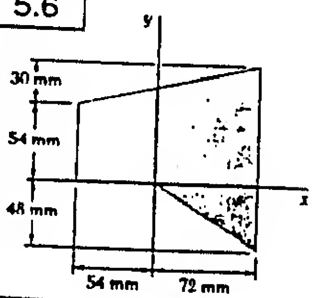
$\bar{Y}(1596) = 39\,648$

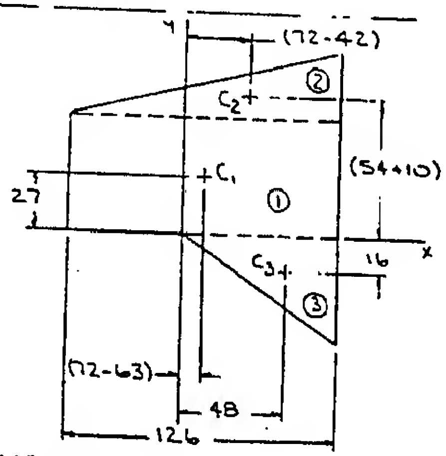
OR $\bar{Y} = 24.8$ mm ◀

5.6

GIVEN PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$





DIMENSIONS IN mm

(CONTINUED)

5.6 CONTINUED

	A, mm^2	$\bar{x}, mm$	$\bar{y}, mm$	$\bar{x}A, mm^3$	$\bar{y}A, mm^3$
1	$126 \times 54 = 6804$	9	27	61 236	183 708
2	$\frac{1}{2} \times 126 \times 30 = 1890$	30	64	56 700	120 960
3	$\frac{1}{2} \times 72 \times 48 = 1728$	48	−16	82 944	−27 648
Σ	10 422			200 880	277 020

THEN $\bar{X}\Sigma A = \Sigma \bar{x}A$

$\bar{X}(10\,422) = 200\,880$

OR $\bar{X} = 19.27$ mm ◀

AND $\bar{Y}\Sigma A = \Sigma \bar{y}A$

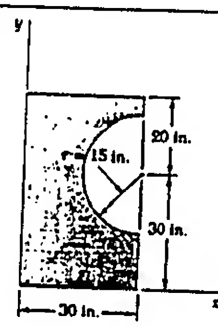
$\bar{Y}(10\,422) = 277\,020$

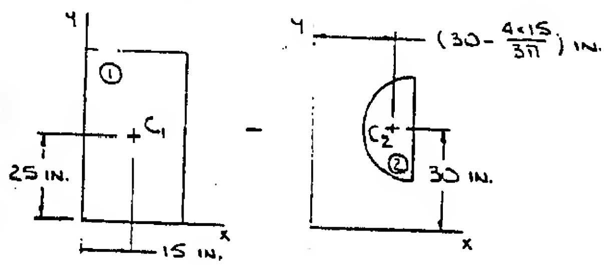
OR $\bar{Y} = 26.6$ mm ◀

5.7

GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$





	A, IN^2	$\bar{x}, IN.$	$\bar{y}, IN.$	$\bar{x}A, IN^3$	$\bar{y}A, IN^3$
1	$30 \times 50 = 1500$	15	25	22 500	37 500
2	$-\frac{\pi}{2}(15)^2 = -353.43$	23.634	30	−8353.0	−10 602.9
Σ	1146.57			14 147.0	26 897

THEN $\bar{X}\Sigma A = \Sigma \bar{x}A$

$\bar{X}(1146.57) = 14\,147.0$

OR $\bar{X} = 12.34$ IN. ◀

AND $\bar{Y}\Sigma A = \Sigma \bar{y}A$

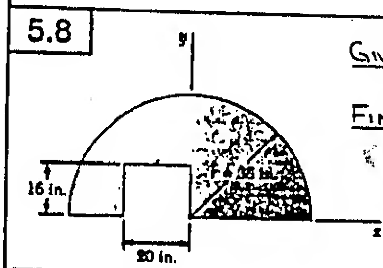
$\bar{Y}(1146.57) = 26\,897$

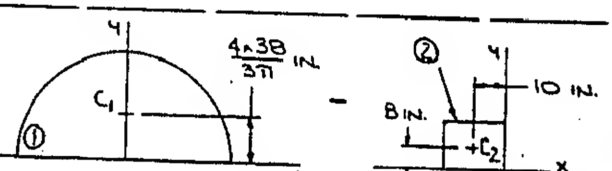
OR $\bar{Y} = 23.5$ IN. ◀

5.8

GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$

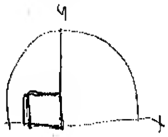




(CONTINUED)

5.8 CONTINUED

	A, in^2	$\bar{x}, in.$	$\bar{y}, in.$	$\bar{x}A, in^3$	$\bar{y}A, in^3$
1	$\frac{\pi}{2}(38)^2 = 2268.2$	0	16.1277	0	36 581
2	$-20\times16 = -320$	-10	8	3200	$-2\ 560$
Σ	1948.23			3200	34 021



THEN $\bar{X}\Sigma A = \Sigma\bar{x}A$

$\bar{X}(1948.23) = 3200$

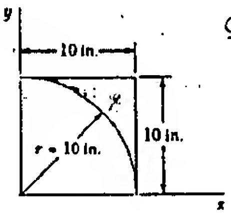
OR $\bar{X} = 1.643$ IN. ◀

AND $\bar{Y}\Sigma A = \Sigma\bar{y}A$

$\bar{Y}(1948.23) = 34\ 021$

OR $\bar{Y} = 17.46$ IN. ◀

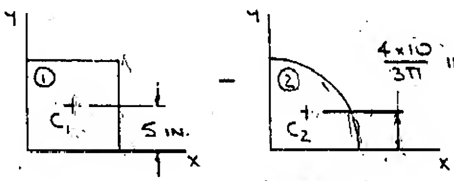
5.9



GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$

FIRST NOTE THAT SYMMETRY IMPLIES $\bar{X} = \bar{Y}$



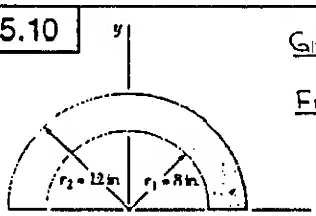
	A, in^2	$\bar{y}, in.$	$\bar{y}A, in^3$
1	$10\times10 = 100$	5	500
2	$-\frac{\pi}{4}(10)^2 = -78.540$	4.2441	-333.33
Σ	21.460		166.67

THEN $\bar{Y}\Sigma A = \Sigma\bar{y}A$

$\bar{Y}(21.460) = 166.67$

OR $\bar{X} = \bar{Y} = 7.77$ IN. ◀

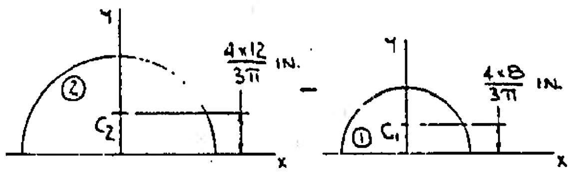
5.10



GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$

FIRST NOTE THAT SYMMETRY IMPLIES $\bar{X} = 0$ ◀



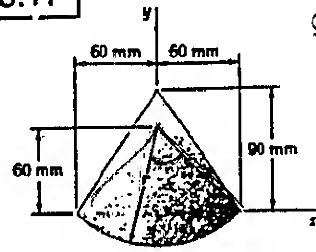
	A, in^2	$\bar{y}, in.$	$\bar{y}A, in^3$
1	$-\frac{\pi}{2}(8)^2 = -100.531$	3.3953	-341.33
2	$\frac{\pi}{2}(12)^2 = 226.19$	5.0930	1151.99
Σ	125.659		810.66

THEN $\bar{Y}\Sigma A = \Sigma\bar{y}A$

$\bar{Y}(125.659) = 810.66$

OR $\bar{Y} = 6.45$ IN. ◀

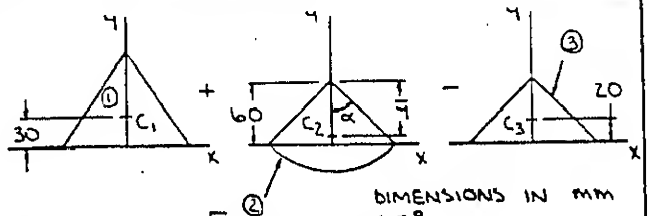
5.11



GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$

FIRST NOTE THAT SYMMETRY IMPLIES $\bar{X} = 0$ ◀



DIMENSIONS IN mm

NOTE: $r = 60\sqrt{2}$ mm $\quad \alpha = 45°$

$$\bar{y} = \frac{2r\sin\alpha}{3\alpha} = \frac{2(60\sqrt{2}\text{ mm})\sin 45°}{3\times\frac{\pi}{4}} \quad \text{(FIG. 5.8A)}$$

$= 50.930$ mm

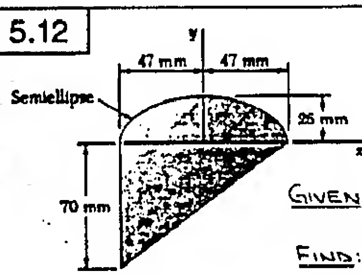
	A, mm^2	$\bar{y}, mm$	$\bar{y}A, mm^3$
1	$\frac{1}{2}\times120\times90 = 5400$	30	162 000
2	$\frac{\pi}{4}(60\sqrt{2})^2 = 5654.9$	$60-50.930 = 9.07$	51 290
3	$-\frac{1}{2}\times120\times60 = -3600$	20	$-72\ 000$
Σ	7454.9		141 290

THEN $\bar{Y}\Sigma A = \Sigma\bar{y}A$

$\bar{Y}(7454.9) = 141\ 290$

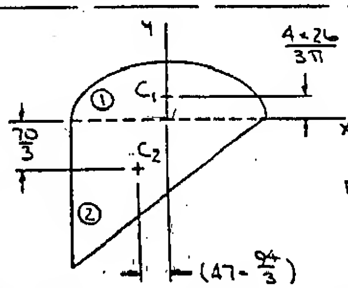
OR $\bar{Y} = 18.95$ mm ◀

5.12



GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$



(FIG. 5.8A)

DIMENSIONS IN mm

	A, mm^2	$\bar{x}, mm$	$\bar{y}, mm$	$\bar{x}A, mm^3$	$\bar{y}A, mm^3$
1	$\frac{\pi}{2}\times47\times26 = 1919.51$	0	11.0347	0	21 181
2	$\frac{1}{2}\times94\times70 = 3290$	-15.6667	-23.333	$-51\ 543$	$-76\ 766$
Σ	5209.5			$-51\ 543$	$-55\ 584$

THEN $\bar{X}\Sigma A = \Sigma\bar{x}A$

$\bar{X}(5209.5) = -51\ 543$

OR $\bar{X} = -9.89$ mm ◀

AND $\bar{Y}\Sigma A = \Sigma\bar{y}A$

$\bar{Y}(5209.5) = -55\ 584$

OR $\bar{Y} = -10.67$ mm ◀

5.16 CONTINUED

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{1}{4}\cdot 60 \cdot 150 = 2250$	48	42.857	108 000	96 429
2	$-\frac{1}{4}\cdot 30 \cdot 18.75 = -140.625$	24	5.3571	-3375	-753.35
Σ	2109.4			104 625	95 675

THEN $\bar{X}\Sigma A = \Sigma \bar{x}A$

$\bar{X}(2109.4) = 104\,625$

OR $\bar{X} = 49.6$ mm ◀

AND $\bar{Y}\Sigma A = \Sigma \bar{y}A$

$\bar{Y}(2109.4) = 95\,675$

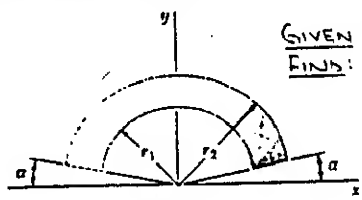
OR $\bar{Y} = 45.4$ mm ◀

5.17 and 5.18

5.17

GIVEN: PLANE AREA SHOWN

FIND: $\bar{Y}$



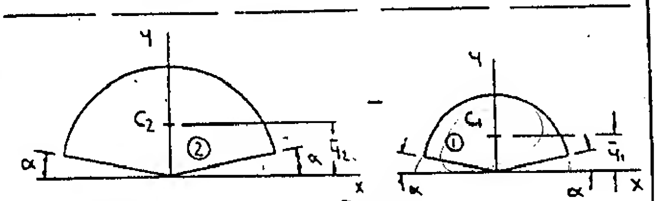


FIG. 5.8A: $\bar{y}_2 = \frac{2}{3}r_2 \dfrac{\sin(\frac{\pi}{2}-\alpha)}{(\frac{\pi}{2}-\alpha)}$ $\quad A_2 = (\frac{\pi}{2}-\alpha)r_2^2$

$= \frac{2}{3}r_2 \dfrac{\cos\alpha}{(\frac{\pi}{2}-\alpha)}$

SIMILARLY... $\bar{y}_1 = \frac{2}{3}r_1 \dfrac{\cos\alpha}{(\frac{\pi}{2}-\alpha)}$ $\quad A_1 = (\frac{\pi}{2}-\alpha)r_1^2$

THEN.. $\Sigma \bar{y}A = \frac{2}{3}r_2 \dfrac{\cos\alpha}{(\frac{\pi}{2}-\alpha)}\left[(\frac{\pi}{2}-\alpha)r_2^2\right] - \frac{2}{3}r_1\dfrac{\cos\alpha}{(\frac{\pi}{2}-\alpha)}\left[(\frac{\pi}{2}-\alpha)r_1^2\right]$

$= \frac{2}{3}(r_2^3 - r_1^3)\cos\alpha$

AND $\Sigma A = (\frac{\pi}{2}-\alpha)r_2^2 - (\frac{\pi}{2}-\alpha)r_1^2$

$= (\frac{\pi}{2}-\alpha)(r_2^2 - r_1^2)$

NOW $\bar{Y}\Sigma A = \Sigma \bar{y}A$

$\bar{Y}\left[(\frac{\pi}{2}-\alpha)(r_2^2-r_1^2)\right] = \frac{2}{3}(r_2^3 - r_1^3)\cos\alpha$

$$\bar{Y} = \frac{2}{3}\,\frac{r_2^3-r_1^3}{r_2^2-r_1^2}\,\frac{\cos\alpha}{\frac{\pi}{2}-\alpha}$$ ◀

5.18 GIVEN: PLANE AREA SHOWN

SHOW: $\bar{Y}$ APPROACHES $\bar{Y}$ OF AN ARC OF RADIUS $\frac{1}{2}(r_1+r_2)$ AS $r_1 \to r_2$

USING FIG. 5.8B, $\bar{Y}$ OF AN ARC OF RADIUS $\frac{1}{2}(r_1+r_2)$ IS..

$$\bar{Y} = \frac{1}{2}(r_1+r_2)\frac{\sin(\frac{\pi}{2}-\alpha)}{(\frac{\pi}{2}-\alpha)}$$

$$= \frac{1}{2}(r_1+r_2)\frac{\cos\alpha}{(\frac{\pi}{2}-\alpha)} \qquad (1)$$

(CONTINUED)

5.17 and 5.18 CONTINUED

FROM THE SOLUTION TO PROBLEM 5.17 HAVE

$$\bar{Y} = \frac{2}{3}\,\frac{r_2^3-r_1^3}{r_2^2-r_1^2}\,\frac{\cos\alpha}{\frac{\pi}{2}-\alpha}$$

NOW.. $\dfrac{r_2^3-r_1^3}{r_2^2-r_1^2} = \dfrac{(r_2-r_1)(r_2^2+r_1r_2+r_1^2)}{(r_2-r_1)(r_2+r_1)}$

$= \dfrac{r_2^2+r_1r_2+r_1^2}{r_2+r_1}$

LET $r_2 = r+\Delta$

$r_1 = r-\Delta$

THEN $r = \frac{1}{2}(r_1+r_2)$

AND $\dfrac{r_2^3-r_1^3}{r_2^2-r_1^2} = \dfrac{(r+\Delta)^2+(r+\Delta)(r-\Delta)+(r-\Delta)^2}{(r+\Delta)+(r-\Delta)}$

$= \dfrac{3r^2+\Delta^2}{2r}$

IN THE LIMIT AS $r_1 \to r_2$, $\Delta \to 0$. THEN

$\dfrac{r_2^3-r_1^3}{r_2^2-r_1^2} = \dfrac{3}{2}r$

$= \dfrac{3}{2}\times\dfrac{1}{2}(r_1+r_2)$

SO THAT $\bar{Y} = \dfrac{2}{3}\times\dfrac{3}{4}(r_1+r_2)\dfrac{\cos\alpha}{\frac{\pi}{2}-\alpha}$

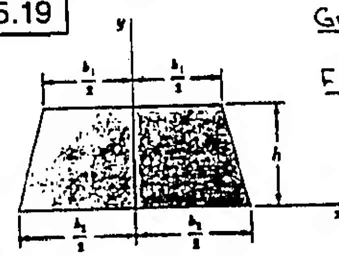
OR $\bar{Y} = \dfrac{1}{2}(r_1+r_2)\dfrac{\cos\alpha}{\frac{\pi}{2}-\alpha}$ ◀

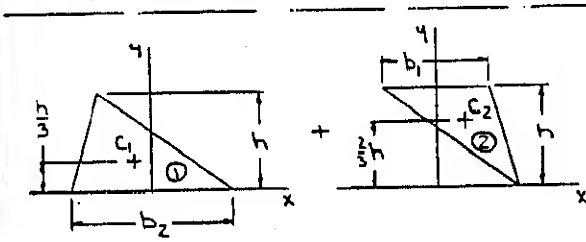
WHICH AGREES WITH EQ. (1).

5.19

GIVEN: PLANE AREA SHOWN

FIND: $\bar{Y}$





	A	$\bar{y}$	$\bar{y}A$
1	$\frac{1}{2}b_2h$	$\frac{1}{3}h$	$\frac{1}{6}b_2h^2$
2	$\frac{1}{2}b_1h$	$\frac{2}{3}h$	$\frac{1}{3}b_1h^2$
Σ	$\frac{1}{2}(b_1+b_2)h$		$\frac{1}{6}(2b_1+b_2)h$

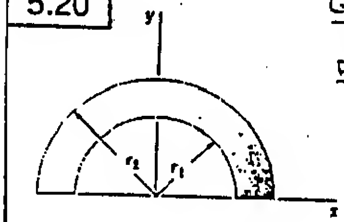
THEN $\bar{Y}\Sigma A = \Sigma\bar{y}A$

$\bar{Y}\left[\frac{1}{2}(b_1+b_2)h\right] = \frac{1}{6}(2b_1+b_2)h^2$

OR $\bar{Y} = \dfrac{2b_1+b_2}{b_1+b_2}\,\dfrac{h}{3}$ ◀

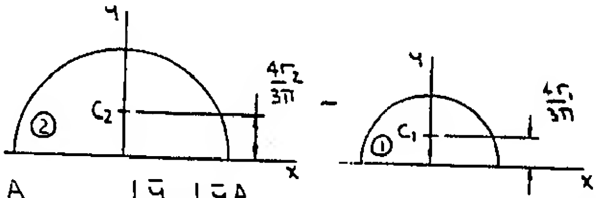
GIVEN: PLANE AREA

5.20



GIVEN: PLANE AREA SHOWN, $\bar{y} = \frac{3}{4} r_1$

FIND: r_2/r_1



	A	$\bar{y}$	$\bar{y}A$
1	$-\frac{\pi}{2} r_1^2$	$\frac{4r_1}{3\pi}$	$-\frac{2}{3} r_1^3$
2	$\frac{\pi}{2} r_2^2$	$\frac{4r_2}{3\pi}$	$\frac{2}{3} r_2^3$
Σ	$\frac{\pi}{2}(r_2^2 - r_1^2)$		$\frac{2}{3}(r_2^3 - r_1^3)$

THEN $\bar{Y}\Sigma A = \Sigma \bar{y}A$

OR $\frac{3}{4} r_1 \times \frac{\pi}{2}(r_2^2 - r_1^2) = \frac{2}{3}(r_2^3 - r_1^3)$

$$\frac{9\pi}{16}\left[\left(\frac{r_2}{r_1}\right)^2 - 1\right] = \left(\frac{r_2}{r_1}\right)^3 - 1$$

LET $p = \frac{r_2}{r_1}$

$$\frac{9\pi}{16}[(p+1)(p-1)] = (p-1)(p^2+p+1)$$

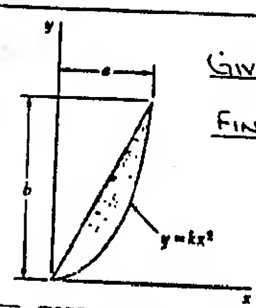
OR $16p^2 + (16-9\pi)p + (16-9\pi) = 0$

THEN $p = \frac{-(16-9\pi) \pm \sqrt{(16-9\pi)^2 - 4(16)(16-9\pi)}}{2(16)}$

OR $p = -0.5726$ $p = 1.3397$

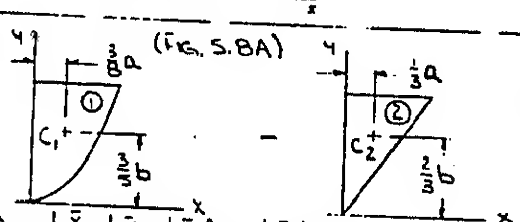
TAKING THE POSITIVE ROOT.. $\frac{r_2}{r_1} = 1.340$ ◀

5.21



GIVEN: PLANE AREA SHOWN, $\bar{x} = \bar{y}$

FIND: a/b



	A	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
1	$\frac{2}{3}ab$	$\frac{3}{8}a$	$\frac{3}{5}b$	$a^2b/4$	$2ab^2/5$
2	$-\frac{1}{2}ab$	$\frac{1}{3}a$	$\frac{2}{3}b$	$-a^2b/6$	$-ab^2/3$
Σ	$\frac{1}{6}ab$			$a^2b/12$	$ab^2/15$

(CONTINUED)

5.21 CONTINUED

THEN $\bar{X}\Sigma A = \Sigma \bar{x}A$ $\qquad$ $\bar{Y}\Sigma A = \Sigma \bar{y}A$

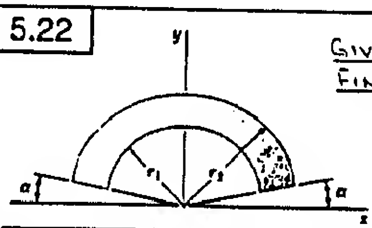
$\bar{X}(\frac{1}{6}ab) = a^2b/12$ $\qquad$ $\bar{Y}(\frac{1}{6}ab) = ab^2/15$

OR $\bar{X} = \frac{1}{2}a$ $\qquad$ OR $\bar{Y} = \frac{2}{5}b$

NOW $\bar{X} = \bar{Y} \Rightarrow$ $\frac{1}{2}a = \frac{2}{5}b$

OR $\frac{a}{b} = \frac{4}{5}$ ◀

5.22



GIVEN: $\bar{y}$, $\alpha = 60°$

FIND: r_2/r_1 IF $\bar{y} = r_1$

FROM THE SOLUTION TO PROBLEM 5.17 HAVE

$$\bar{y} = \frac{2}{3}\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\frac{\cos\alpha}{\frac{\pi}{2} - \alpha}$$

WHEN $\bar{y} = r_1$ AND $\alpha = 60°$ $(\frac{\pi}{3})$

$$r_1 = \frac{2}{3}\frac{r_1^3[(r_2/r_1)^3 - 1]}{r_1^2[(r_2/r_1)^2 - 1]}\frac{\cos\frac{\pi}{3}}{\frac{\pi}{2} - \frac{\pi}{3}}$$

$1 = \frac{2}{\pi}\frac{p^3 - 1}{p^2 - 1}$ WHERE $p = \frac{r_2}{r_1}$

THEN .. $\frac{\pi}{2} = \frac{(p-1)(p^2+p+1)}{(p+1)(p-1)}$

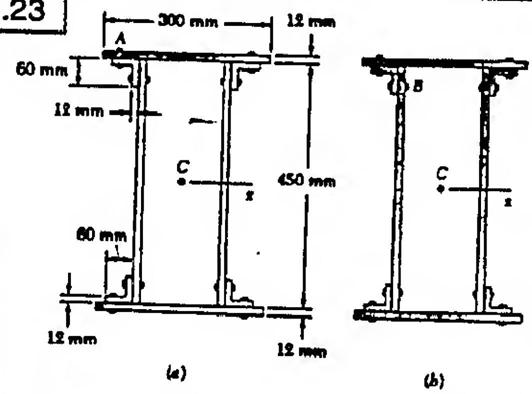
OR $2p^2 + (2-\pi)p + (2-\pi) = 0$

THEN.. $p = \frac{-(2-\pi) \pm \sqrt{(2-\pi)^2 - 4(2)(2-\pi)}}{2(2)}$

OR $p = -0.522$ $p = 1.093$

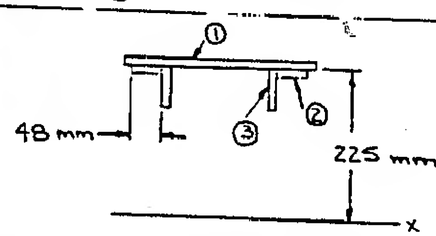
TAKING THE POSITIVE ROOT .. $\frac{r_2}{r_1} = 1.093$ ◀

5.23



GIVEN: $F_A \propto (Q_x)_A$, $F_B \propto (Q_x)_B$, $F_A = 280$ N

FIND: F_B



FROM THE PROBLEM STATEMENT, $F \propto Q_x$ SO THAT $\frac{F_A}{(Q_x)_A} = \frac{F_B}{(Q_x)_B}$

(CONTINUED)

5.23 CONTINUED

OR $F_B = \dfrac{(Q_x)_B}{(Q_x)_A} F_A$

NOW.. $Q_x = \Sigma \bar{y} A$

THEN $(Q_x)_A = [(225+6)\,mm](300\times 12)\,mm^2$
$= 831.6 \times 10^3\ mm^3$

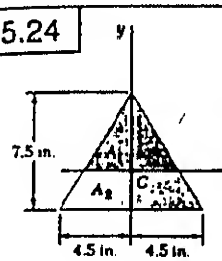
AND $(Q_x)_B = (Q_x)_A + 2[(225-6)\,mm](48\times 12)\,mm^2$
$+ 2\{(225-30)\,mm\}(60\times 12)\,mm^2$
$= 1364.688 \times 10^3\ mm^3$

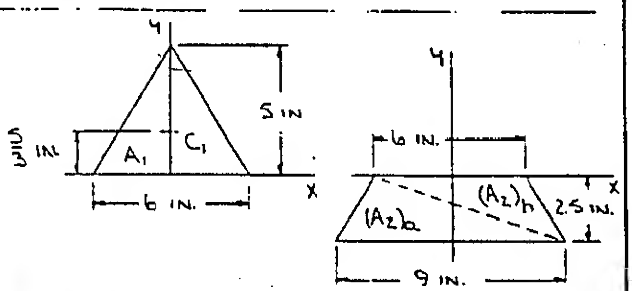
FINALLY.. $F_B = \dfrac{1364.688\times 10^3\ mm^3}{831.6\times 10^3\ mm^3} \times 280\ N$

OR $F_B = 459\ N$ ◀

5.24

GIVEN: PLANE AREA SHOWN
FIND: $(Q_x)_1$, $(Q_x)_2$
EXPLAIN RESULTS





HAVE. $Q_x = \Sigma \bar{y} A$

THEN $(Q_x)_1 = (\frac{5}{3}\ IN.)(\frac{1}{2}\times 6\times 5)\,IN^2$

$(Q_x)_1 = 25\ IN^3$ ◀

AND $(Q_x)_2 = (-\frac{2}{3}\times 2.5\ IN.)(\frac{1}{2}\times 9\times 2.5)\,IN^2$
$+ (-\frac{1}{3}\times 2.5\ IN.)(\frac{1}{2}\times 6\times 2.5)\,IN^2$

$(Q_x)_2 = -25\ IN^3$ ◀

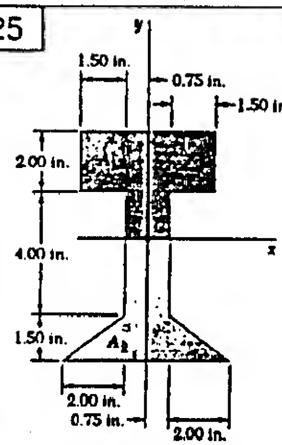
NOW.. $Q_x = (Q_x)_1 + (Q_x)_2 = 0$

THIS RESULT IS EXPECTED SINCE x IS A CENTROIDAL AXIS (THUS $\bar{Y} = 0$)

AND $Q_x = \Sigma \bar{y} A = \bar{Y} \Sigma A$ $(\bar{Y} = 0 \Rightarrow Q_x = 0)$

5.25

GIVEN: PLANE AREA SHOWN
FIND: $(Q_x)_1$, $(Q_x)_2$
EXPLAIN RESULTS



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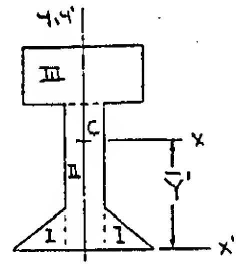
5.25 CONTINUED



FIRST DETERMINE THE LOCATION OF THE CENTROID C. HAVE..

	$A,\ IN^2$	$\bar{y}',\ IN.$	$\bar{y}'A,\ IN^3$
I	$2(\frac{1}{2}\times 2\times 1.5) = 3$	0.5	1.5
II	$1.5\times 5.5 = 8.25$	2.75	22.6875
III	$4.5\times 2 = 9$	6.5	58.5
Σ	20.25		82.6875

THEN $\bar{Y}'\Sigma A = \Sigma \bar{y}' A$
$\bar{Y}'(20.25) = 82.6875$
OR $\bar{Y}' = 4.0833\ IN.$

NOW $Q_x = \Sigma \bar{y} A$

THEN $(Q_x)_1 = [\frac{1}{2}(5.5-4.0833)\,IN.][(1.5)(5.5-4.0833)]\,IN^2$
$+ \{(6.5-4.0833)\,IN.\}[(4.5)(2)]\,IN^2$

OR $(Q_x)_1 = 23.3\ IN^3$ ◀

AND $(Q_x)_2 = -[\frac{1}{2}(4.0833\ IN.)][(1.5)(4.0833)]\,IN^2$
$-[(4.0833-0.5)\,IN.]\times 2[(\frac{1}{2}\times 2\times 1.5)\,IN^2]$

OR $(Q_x)_2 = -23.3\ IN^3$ ◀

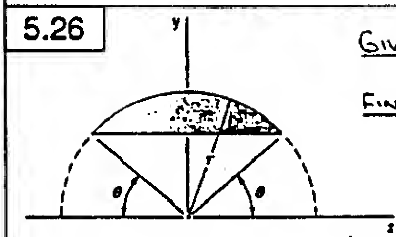
NOW.. $Q_x = (Q_x)_1 + (Q_x)_2 = 0$

THIS RESULT IS EXPECTED SINCE x IS A CENTROIDAL AXIS (THUS $\bar{Y} = 0$)

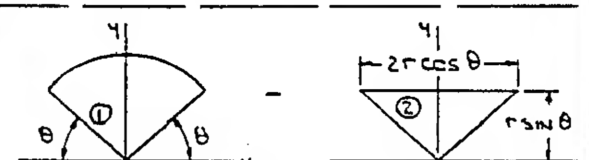
AND $Q_x = \Sigma \bar{y} A = \bar{Y}\Sigma A$ $(\bar{Y} = 0 \Rightarrow Q_x = 0)$

5.26

GIVEN: PLANE AREA SHOWN
FIND: (a) Q_x
(b) θ AND Q_x FOR THE MAXIMUM VALUE OF Q_x



(a)



HAVE $Q_x = \Sigma \bar{y} A$ AND USING FIG. 5.8A ...

$Q_x = \left(\frac{2}{3} r \dfrac{\sin(\pi/2-\theta)}{\frac{\pi}{2}-\theta}\right)\left[\left(\frac{\pi}{2}-\theta\right) r^2\right]$
$- \left(\frac{2}{3} r\sin\theta\right)\left(\frac{1}{2}\times 2r\cos\theta \times r\sin\theta\right)$
$= \frac{2}{3} r^3 (\cos\theta - \cos\theta\sin^2\theta)$

OR $Q_x = \frac{2}{3} r^3 \cos^3\theta$ ◀

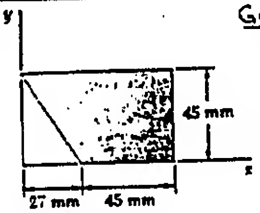
(b) BY OBSERVATION, Q_x IS MAXIMUM FOR

$\theta = 0$ ◀

AND THEN $(Q_x)_{MAX} = \frac{2}{3} r^3$ ◀

OR X = 18.45 IN. ◀
AND $\bar{Y}\Sigma L = \Sigma \bar{y} L$

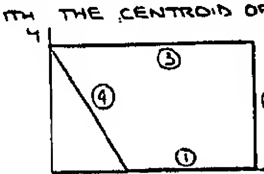
5.27



GIVEN: WIRE HAVING THE SHAPE OF THE PERIMETER OF THE PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$

FIRST NOTE THAT BECAUSE THE WIRE IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.



	L, mm	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}L$, mm^3	$\bar{y}L$, mm^3
1	45	49.5	0	2227.5	0
2	45	72	22.5	3240	1012.5
3	72	36	45	2592	3240
4	$\sqrt{27^2+45^2}=52.479$	13.5	22.5	708.47	1180.78
Σ	214.479			8768.0	5433.3

THEN $\bar{X}\Sigma L = \Sigma \bar{x}L$

$\bar{X}(214.479) = 8768.0$

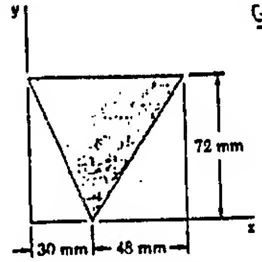
OR $\bar{X} = 40.9$ mm ◀

AND $\bar{Y}\Sigma L = \Sigma \bar{y}L$

$\bar{Y}(214.479) = 5433.3$

OR $\bar{Y} = 25.3$ mm ◀

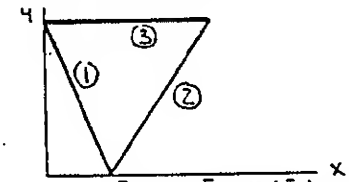
5.29



GIVEN: WIRE HAVING THE SHAPE OF THE PERIMETER OF THE PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$

FIRST NOTE THAT BECAUSE THE WIRE IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.



	L, mm	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}L$, mm^3	$\bar{y}L$, mm^3
1	$\sqrt{30^2+72^2}=78$	15	36	1170	2808
2	$\sqrt{48^2+72^2}=86.533$	54	36	4672.8	3115.2
3	78	39	72	3042	5616
Σ	242.53			8884.8	11 539.2

THEN $\bar{X}\Sigma L = \Sigma \bar{x}L$

$\bar{X}(242.53) = 8884.3$

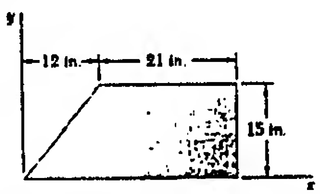
OR $\bar{X} = 36.6$ mm ◀

AND $\bar{Y}\Sigma L = \Sigma \bar{y}L$

$\bar{Y}(242.53) = 11\,539.2$

OR $\bar{Y} = 47.6$ mm ◀

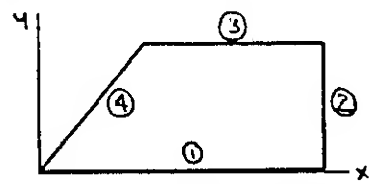
5.28



GIVEN: WIRE HAVING THE SHAPE OF THE PERIMETER OF THE PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$

FIRST NOTE THAT BECAUSE THE WIRE IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.



	L, IN.	$\bar{x}$, IN.	$\bar{y}$, IN.	$\bar{x}L$, IN^3	$\bar{y}L$, IN^3
1	33	16.5	0	544.5	0
2	15	33	7.5	495	112.5
3	21	22.5	15	472.5	315
4	$\sqrt{12^2+15^2}=19.2093$	6	7.5	115.256	144.070
Σ	88.209			1627.26	571.57

THEN $\bar{X}\Sigma L = \Sigma \bar{x}L$

$\bar{X}(88.209) = 1627.26$

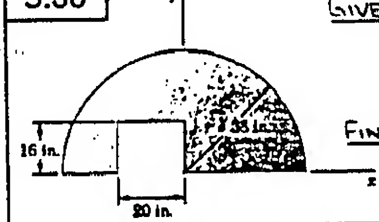
OR $\bar{X} = 18.45$ IN. ◀

AND $\bar{Y}\Sigma L = \Sigma \bar{y}L$

$\bar{Y}(88.209) = 571.57$

OR $\bar{Y} = 6.48$ IN. ◀

5.30



GIVEN: WIRE HAVING THE SHAPE OF THE PERIMETER OF THE PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$

FIRST NOTE THAT BECAUSE THE WIRE IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.

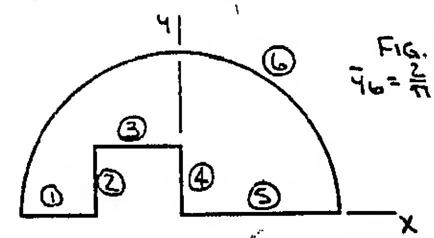


FIG. 5.8A

$\bar{y}_6 = \frac{2}{\pi}(38\text{ IN.})$

	L, IN.	$\bar{x}$, IN.	$\bar{y}$, IN	$\bar{x}L$, IN^3	$\bar{y}L$, IN^3
1	18	−29	0	−522	0
2	16	−20	8	−320	128
3	20	−10	16	−200	320
4	16	0	8	0	128
5	38	19	0	722	0
6	$\pi(38)=119.381$	0	24.192	0	2888.1
Σ	227.38			−320	3464.1

5.30 CONTINUED

THEN $\bar{X}\Sigma L = \Sigma \bar{x} L$

$\bar{X}(227.38) = -320$

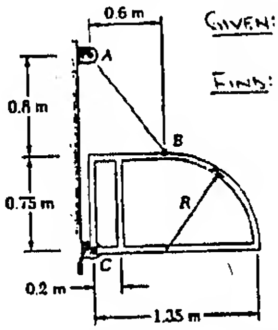
OR $\bar{X} = -1.407$ IN. ◀

AND $\bar{Y}\Sigma L = \Sigma \bar{y} L$

$\bar{Y}(227.38) = 3464.1$

OR $\bar{Y} = 15.23$ IN. ◀

5.31



GIVEN: MASS/LENGTH $m' = 4.73$ kg/m

FIND: (a) T_{BA}

(b) REACTION $\underline{C}$ AT PIN C

FIRST NOTE THAT BECAUSE THE FRAME IS FABRICATED FROM UNIFORM BAR STOCK, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.

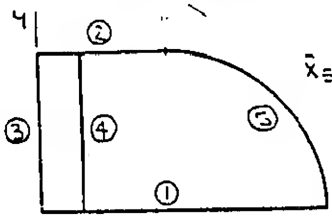


FIG. 5.8B

$\bar{x}_5 = 0.6\,m + \frac{2}{\pi}(0.75\,m)$

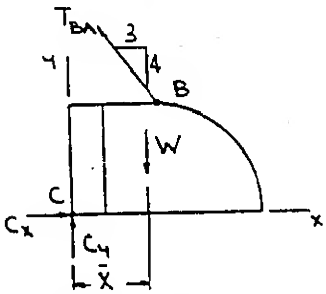
	L, m	$\bar{x}$, m	$\bar{x}L$, m^3
1	1.35	0.675	0.911 25
2	0.6	0.3	0.18
3	0.75	0	0
4	0.75	0.2	0.15
5	$\frac{\pi}{2}(0.75) = 1.17810$	1.07746	1.26936
Σ	4.628 10		2.5106

THEN $\bar{X}\Sigma L = \Sigma \bar{x} L$

$\bar{X}(4.628\,10) = 2.5106$

OR $\bar{X} = 0.542\,47$ m

THE FREE-BODY DIAGRAM OF THE FRAME IS THEN...



WHERE $W = (m'\Sigma L)g$

$= 4.73\ \text{kg/m} \times 4.628\,10\ \text{m} \times 9.81\ \frac{\text{m}}{\text{s}^2}$

$= 214.75$ N

EQUILIBRIUM THEN REQUIRES..

(CONTINUED)

5.31 CONTINUED

(a) $\Sigma M_C = 0$: $(1.55\,m)(\frac{3}{5}T_{BA}) - (0.542\,47\,m)(214.75\,N) = 0$

OR $T_{BA} = 125.264$ N

OR $T_{BA} = 125.3$ N ◀

(b) $\Sigma F_x = 0$: $C_x - \frac{3}{5}(125.264\,N) = 0$

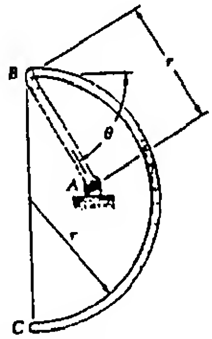
OR $\underline{C}_x = 75.158$ N ⟶

$\Sigma F_y = 0$: $C_y + \frac{4}{5}(125.264\,N) - (214.75\,N) = 0$

OR $\underline{C}_y = 114.539$ N ↑

THEN.. $\underline{C} = 137.0$ N ∠56.7° ◀

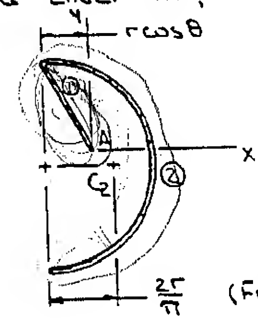
5.32



GIVEN: HOMOGENEOUS WIRE

FIND: θ FOR EQUILIBRIUM

FIRST NOTE THAT FOR EQUILIBRIUM, THE CENTER OF GRAVITY OF THE WIRE MUST LIE ON A VERTICAL LINE THROUGH A. FURTHER, BECAUSE THE WIRE IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE. THUS,



(FIG. 5.8B)

$\bar{X} = 0$

SO THAT $\Sigma \bar{x} L = 0$

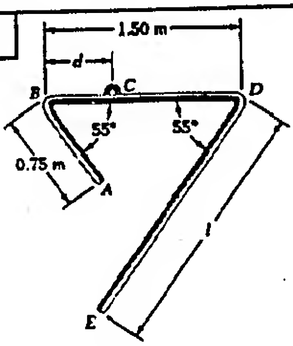
THEN.. $(-\frac{1}{2}r\cos\theta)(r) + (\frac{2r}{\pi} - r\cos\theta)(\pi r) = 0$

OR $\cos\theta = \frac{4}{1+2\pi}$

$= 0.549\,21$

OR $\theta = 56.7°$ ◀

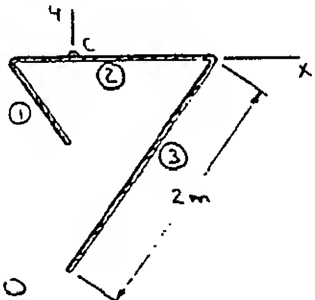
5.33



GIVEN: UNIFORM TUBING, $l = 2$ m, BCD IS HORIZONTAL

FIND: d

FIRST NOTE THAT FOR EQUILIBRIUM, THE CENTER OF GRAVITY OF THE COMPONENT MUST LIE ON A VERTICAL LINE THROUGH C. FURTHER, BECAUSE THE TUBING IS UNIFORM, THE CENTER OF GRAVITY OF THE COMPONENT WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE. THUS,



$\bar X = 0$

SO THAT.. $\Sigma \bar x L = 0$

THEN.. $-(d - \frac{0.75}{2}\cos 55°)\text{m} \times (0.75\text{ m})$

$+ (0.75 - d)\text{m} \times (1.5\text{ m})$

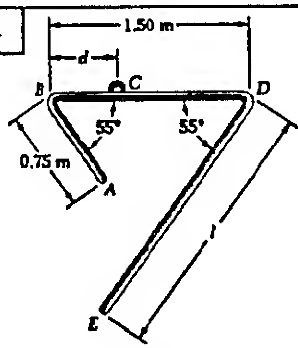
$+ [(1.5 - d)\text{m} - (\frac{1}{2} \times 2\text{m} \times \cos 55°)] \times (2\text{m})$

$= 0$

OR $(0.75 + 1.5 + 2)d = [\frac{1}{2}(0.75)^2 - 2]\cos 55° + (0.75)(1.5) + 3$

OR $d = 0.739$ m

5.34



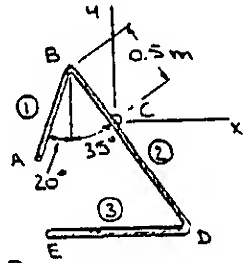
GIVEN: UNIFORM TUBING, $d = 0.5$ m, DE IS HORIZONTAL

FIND: l

FIRST NOTE THAT FOR EQUILIBRIUM, THE CENTER OF GRAVITY OF THE COMPONENT MUST LIE ON A VERTICAL LINE THROUGH C. FURTHER, BECAUSE THE TUBING IS UNIFORM, THE CENTER OF GRAVITY OF THE COMPONENT WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE. THUS,

(CONTINUED)

5.34 CONTINUED



$\bar X = 0$

SO THAT $\Sigma \bar x L = 0$

OR $-(\frac{0.75}{2}\sin 20° + 0.5\sin 35°)\text{m} \times (0.75\text{ m})$

$+ (0.25\text{ m} \times \sin 35°) \times (1.5\text{ m})$

$+ (1.0 \times \sin 35° - \frac{l}{2})\text{m} \times (l\text{ m}) = 0$

OR $\underbrace{-0.096193}_{(\bar x L)_{AB} + (\bar x L)_{BD}} + \underbrace{(\sin 35° - \frac{1}{2} l) l}_{(\bar x L)_{DE}} = 0$

THIS EQUATION IMPLIES THAT THE CENTER OF GRAVITY OF DE MUST BE TO THE RIGHT OF C.

THEN.. $l^2 - 1.14715\,l + 0.192386 = 0$

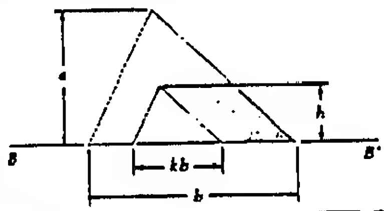
OR $l = \frac{1.14715 \pm \sqrt{(-1.14715)^2 - 4(0.192386)}}{2}$

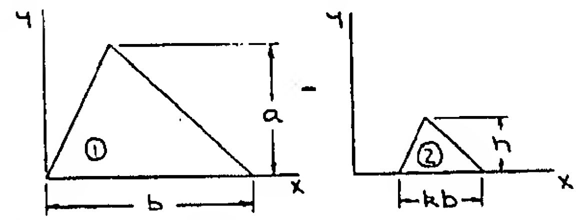
OR $l = 0.204$ m AND $l = 0.943$ m

NOTE THAT $\sin 35° - \frac{1}{2} l > 0$ FOR BOTH VALUES OF l SO BOTH VALUES ARE ACCEPTABLE.

5.35 and 5.36

GIVEN: PLANE AREA SHOWN





	A	$\bar y$	$\bar y A$
1	$\frac{1}{2}ba$	$\frac{1}{3}a$	$\frac{1}{6}a^2 b$
2	$-\frac{1}{2}(kb)h$	$\frac{1}{3}h$	$-\frac{1}{6}kbh^2$
Σ	$\frac{b}{2}(a - kh)$		$\frac{b}{6}(a^2 - kh^2)$

THEN $\bar Y \Sigma A = \Sigma \bar y A$

$\bar Y[\frac{b}{2}(a - kh)] = \frac{b}{6}(a^2 - kh^2)$

OR $\bar Y = \frac{a^2 - kh^2}{3(a - kh)}$ (1)

AND $\frac{d\bar Y}{dh} = \frac{1}{3}\,\frac{-2kh(a - kh) - (a^2 - kh^2)(-k)}{(a - kh)^2} = 0$

OR $2h(a - kh) - a^2 + kh^2 = 0$ (2)

5.35 FIND: h SO THAT $\bar Y$ IS MAXIMUM

(a) $k = 0.10$

(b) $k = 0.80$

(CONTINUED)

5.35 and 5.36 CONTINUED

SIMPLIFYING EQ. (2) YIELDS...

$$kh^2 - 2ah + a^2 = 0$$

THEN $h = \dfrac{2a \pm \sqrt{(-2a)^2 - 4(k)(a^2)}}{2k}$

$$= \frac{a}{k}\left[1 \pm \sqrt{1-k}\,\right]$$

NOTE THAT ONLY THE NEGATIVE ROOT IS ACCEPTABLE SINCE $h < a$. THEN...

(a) $k = 0.10$

$$h = \frac{a}{0.10}\left[1 - \sqrt{1 - 0.10}\right]$$

OR $h = 0.513a$ ◀

(b) $k = 0.80$

$$h = \frac{a}{0.80}\left[1 - \sqrt{1-0.80}\right]$$

OR $h = 0.691a$ ◀

5.36 SHOW: $\bar{Y} = \frac{2}{3}h$ FOR THE VALUE OF h WHICH MAXIMIZES $\bar{Y}$

REARRANGING EQ. (2) (WHICH DEFINES THE VALUE OF h WHICH MAXIMIZES $\bar{Y}$) YIELDS

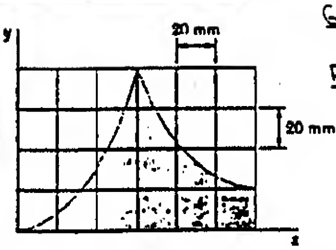
$$a^2 - kh^2 = 2h(a - kh)$$

THEN SUBSTITUTING INTO EQ. (1) (WHICH DEFINES $\bar{Y}$)...

$$\bar{Y} = \frac{1}{3(a-kh)} \cdot 2h(a-kh)$$

OR $\bar{Y} = \frac{2}{3}h$ ◀

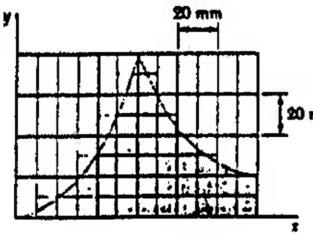
5.37 and 5.38



GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ (5.37) AND $\bar{Y}$ (5.38) USING APPROXIMATE MEANS

THE AREA IS FIRST DIVIDED INTO TWELVE VERTICAL STRIPS, EACH 10 mm WIDE, AND THEN THE AREA AND THE LOCATION OF THE CENTROID ARE APPROXIMATED FOR EACH STRIP. A 10×10-mm GRID IS USED TO FACILITATE APPROXIMATING THE VALUES.



(CONTINUED)

5.37 and 5.38 CONTINUED

STRIP	A, mm^2	$\bar{x}, mm$	$\bar{y}, mm$	$\bar{x}A, mm^3$	$\bar{y}A, mm^3$
1	15	7	1	105	15
2	65	16	3	1040	195
3	150	26	7	3900	1050
4	250	36	14	9000	3500
5	400	47	21	18 800	8400
6	650	57	33	37 050	21 450
7	700	63	36	44 100	25 200
8	520	74	27	38 480	14 040
9	390	83	18	32 370	7020
10	295	94	15	27 730	4425
11	240	104	12	24 960	2880
12	210	113	11	23 730	2310
Σ	3885			261 265	90 485

5.37 HAVE.. $\bar{X}\Sigma A = \Sigma \bar{x}A$

$$\bar{X}(3885) = 261\,265$$

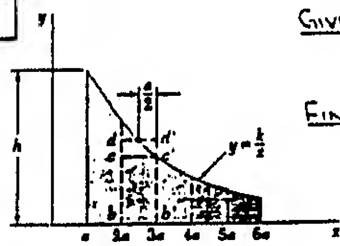
OR $\bar{X} = 67.2$ mm ◀

5.38 HAVE.. $\bar{Y}\Sigma A = \Sigma \bar{y}A$

$$\bar{Y}(3885) = 90\,485$$

OR $\bar{Y} = 23.3$ mm ◀

5.39



GIVEN: PLANE AREA SHOWN, $\bar{x} = 5a/\ln 6$

FIND: $\bar{X}$ USING APPROXIMATE MEANS BASED ON RECTANGLES $bcc'b'$

HAVE $y = \frac{k}{x}$

THEN AT $x = a$, $y = h$: $h = \frac{k}{a}$ OR $k = ah$

SO THAT $y = \frac{ah}{x}$

RECTANGLE	x_b'	y_c'	A	$\bar{x}$	$\bar{x}A$
1	2a	h/2	ah/2	1.5a	$0.75a^2h$
2	3a	h/3	ah/3	2.5a	$0.833a^2h$
3	4a	h/4	ah/4	3.5a	$0.875a^2h$
4	5a	h/5	ah/5	4.5a	$0.9a^2h$
5	6a	h/6	ah/6	5.5a	$0.917a^2h$
Σ			1.45ah		$4.275a^2h$

THEN $\bar{X}\Sigma A = \Sigma \bar{x}A$

$$\bar{X}(1.45ah) = 4.275a^2h$$

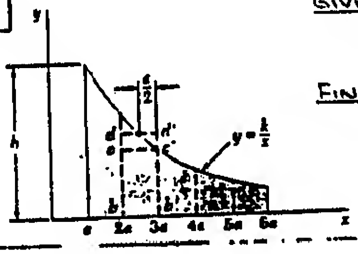
OR $\bar{X} = 2.9483a$

OR $\bar{X} = 2.95a$ ◀

$$\%\ \text{ERROR} = \frac{\left|\frac{5a}{\ln 6} - 2.9483a\right|}{\frac{5a}{\ln 6}} \times 100\%$$

OR % ERROR = 5.65% ◀

5.40



GIVEN: PLANE AREA SHOWN, $\bar{x} = 5a/\ln 6$

FIND: $\bar{X}$ USING APPROXIMATE MEANS BASED ON RECTANGLES bdd'b'

HAVE $y = \frac{k}{x}$

THEN AT $x = a,\ y = h$: $h = \frac{k}{a}$ OR $k = ah$

SO THAT $y = \frac{ah}{x}$

RECTANGLE	x_{AV}	$y_{x_{AV}}$	A	$\bar{x}$	$\bar{x}A$
1	1.5a	h/1.5	ah/1.5	1.5a	a^2h
2	2.5a	h/2.5	ah/2.5	2.5a	a^2h
3	3.5a	h/3.5	ah/3.5	3.5a	a^2h
4	4.5a	h/4.5	ah/4.5	4.5a	a^2h
5	5.5a	h/5.5	ah/5.5	5.5a	a^2h
Σ			1.75642ah		$5a^2h$

THEN: $\bar{X}\Sigma A = \Sigma \bar{x} A$

$\bar{X}(1.75642\,ah) = 5a^2h$

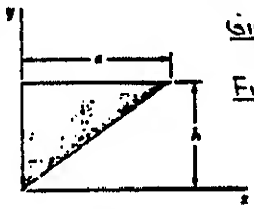
OR $\bar{X} = 2.8467a$

OR $\bar{X} = 2.85a$ ◀

$$\%\ \text{ERROR} = \frac{\left|\frac{5a}{\ln 6} - 2.8467a\right|}{\frac{5a}{\ln 6}} \times 100\%$$

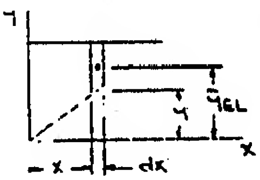
OR % ERROR = 2.01% ◀

5.41



GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$ USING DIRECT INTEGRATION



HAVE.. $y = \frac{h}{a}x$

AND $dA = (h - y)dx$
$= h(1 - \frac{x}{a})dx$

$\bar{x}_{EL} = x$

$\bar{y}_{EL} = \frac{1}{2}(h + y)$
$= \frac{h}{2}(1 + \frac{x}{a})$

THEN.. $A = \int dA = \int_0^a h(1 - \frac{x}{a})dx = h\left[x - \frac{x^2}{2a}\right]_0^a$

$= \frac{1}{2}ah$

AND.. $\int \bar{x}_{EL}\,dA = \int_0^a x\left[h(1 - \frac{x}{a})dx\right] = h\left[\frac{x^2}{2} - \frac{x^3}{3a}\right]_0^a$

$= \frac{1}{6}a^2h$

$\int \bar{y}_{EL}\,dA = \int_0^a \frac{h}{2}(1 + \frac{x}{a})\left[h(1 - \frac{x}{a})dx\right]$

$= \frac{h^2}{2}\int_0^a (1 - \frac{x^2}{a^2})dx = \frac{h^2}{2}\left[x - \frac{x^3}{3a^2}\right]_0^a$

$= \frac{1}{3}ah^2$

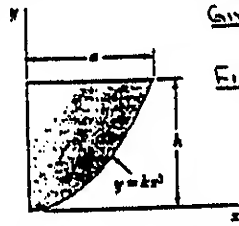
$\bar{x}A = \int \bar{x}_{EL}\,dA$: $\bar{x}(\frac{1}{2}ah) = \frac{1}{6}a^2h$ $\qquad \bar{x} = \frac{1}{3}a$ ◀

(CONTINUED)

5.41 CONTINUED

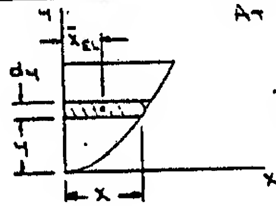
$\bar{y}A = \int \bar{y}_{EL}\,dA$: $\bar{y}(\frac{1}{2}ah) = \frac{1}{3}ah^2$ $\qquad \bar{y} = \frac{2}{3}h$ ◀

5.42



GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$ USING DIRECT INTEGRATION



AT $x = a,\ y = h$: $h = ka^3$

OR $k = \frac{h}{a^3}$

THEN $x = \frac{a}{h^{1/3}}y^{1/3}$

NOW.. $dA = x\,dy$
$= \frac{a}{h^{1/3}}y^{1/3}dy$

$\bar{x}_{EL} = \frac{1}{2}x = \frac{1}{2}\frac{a}{h^{1/3}}y^{1/3}$

$\bar{y}_{EL} = y$

THEN.. $A = \int dA = \int_0^h \frac{a}{h^{1/3}}y^{1/3}dy = \frac{3}{4}\frac{a}{h^{1/3}}\left[y^{4/3}\right]_0^h = \frac{3}{4}ah$

AND.. $\int \bar{x}_{EL}\,dA = \int_0^h \frac{1}{2}\frac{a}{h^{1/3}}y^{1/3}\left(\frac{a}{h^{1/3}}y^{1/3}dy\right) = \frac{1}{2}\frac{a^2}{h^{2/3}}\left[\frac{3}{5}y^{5/3}\right]_0^h$

$= \frac{3}{10}a^2h$

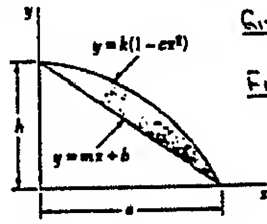
$\int \bar{y}_{EL}\,dA = \int_0^h y\left[\frac{a}{h^{1/3}}y^{1/3}dy\right] = \frac{a}{h^{1/3}}\left[\frac{3}{7}y^{7/3}\right]_0^h$

$= \frac{3}{7}ah^2$

$\bar{x}A = \int \bar{x}_{EL}\,dA$: $\bar{x}(\frac{3}{4}ah) = \frac{3}{10}a^2h$ $\qquad \bar{x} = \frac{2}{5}a$ ◀

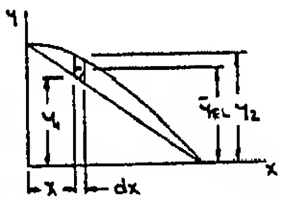
$\bar{y}A = \int \bar{y}_{EL}\,dA$: $\bar{y}(\frac{3}{4}ah) = \frac{3}{7}ah^2$ $\qquad \bar{y} = \frac{4}{7}h$ ◀

5.43



GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$ USING DIRECT INTEGRATION



BY OBSERVATION..

$y_1 = -\frac{h}{a}x + h$
$= h(1 - \frac{x}{a})$

FOR y_2..

AT $x = 0,\ y = h$: $h = k(1 - 0)$

OR $k = h$

AT $x = a,\ y = 0$: $0 = h(1 - ca^2)$

OR $c = \frac{1}{a^2}$

THEN.. $y_2 = h(1 - \frac{x^2}{a^2})$

NOW.. $dA = (y_2 - y_1)dx = h\left[(1 - \frac{x^2}{a^2}) - (1 - \frac{x}{a})\right]dx$
$= h(\frac{x}{a} - \frac{x^2}{a^2})dx$

$\bar{x}_{EL} = x$ $\qquad \bar{y}_{EL} = \frac{1}{2}(y_1 + y_2) = \frac{h}{2}\left[(1 - \frac{x}{a}) + (1 - \frac{x^2}{a^2})\right]$

$= \frac{h}{2}(2 - \frac{x}{a} - \frac{x^2}{a^2})$

THEN.. $A = \int dA = \int_0^a h(\frac{x}{a} - \frac{x^2}{a^2})dx = h\left[\frac{x^2}{2a} - \frac{x^3}{3a^2}\right]_0^a$

$= \frac{1}{6}ah$

(CONTINUED)

5.43 CONTINUED

AND.. $\int \bar{x}_{EL}\,dA = \int_0^a x\left[h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)dx\right] = h\left[\frac{x^3}{3a} - \frac{x^4}{4a^2}\right]_0^a$

$= \frac{1}{12}a^2h$

$\int \bar{y}_{EL}\,dA = \int_0^a \frac{h}{2}\left(2 - \frac{x}{a} - \frac{x^2}{a^2}\right)\left[h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)dx\right]$

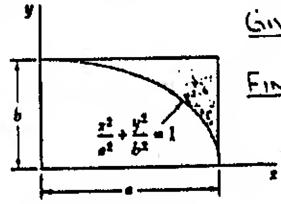
$= \frac{h^2}{2}\int_0^a \left(2\frac{x}{a} - 3\frac{x^2}{a^2} + \frac{x^4}{a^4}\right)dx$

$= \frac{h^2}{2}\left[\frac{x^2}{a} - \frac{x^3}{a^2} + \frac{x^5}{5a^4}\right]_0^a = \frac{1}{10}ah^2$

$\bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x}\left(\frac{1}{6}ah\right) = \frac{1}{12}a^2h \qquad \bar{x} = \frac{1}{2}a$

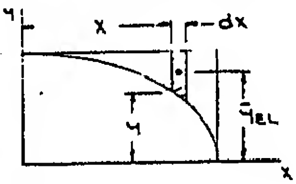
$\bar{y}A = \int \bar{y}_{EL}\,dA: \quad \bar{y}\left(\frac{1}{6}ah\right) = \frac{1}{10}a^2h \qquad \bar{y} = \frac{3}{5}h$

5.44



GIVEN: PLANE AREA SHOWN

FIND: $\bar{x}$ AND $\bar{y}$ USING DIRECT INTEGRATION



HAVE.. $y = \frac{b}{a}\sqrt{a^2 - x^2}$

AND

$dA = (b - y)dx$

$= \frac{b}{a}\left(a - \sqrt{a^2 - x^2}\right)dx$

$\bar{x}_{EL} = x$

$\bar{y}_{EL} = \frac{1}{2}(y + b)$

$= \frac{b}{2a}\left(a + \sqrt{a^2 - x^2}\right)$

THEN.. $A = \int dA = \int_0^a \frac{b}{a}\left(a - \sqrt{a^2 - x^2}\right)dx$

LET $x = a\sin\theta$: $\sqrt{a^2 - x^2} = a\cos\theta$

$dx = a\cos\theta\,d\theta$

THEN. $A = \int_0^{\frac{\pi}{2}} \frac{b}{a}(a - a\cos\theta)(a\cos\theta\,d\theta)$

$= \frac{b}{a}\left[a^2\sin\theta - a^2\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\right]_0^{\frac{\pi}{2}}$

$= ab\left(1 - \frac{\pi}{4}\right)$

AND.. $\int \bar{x}_{EL}\,dA = \int_0^a x\left[\frac{b}{a}\left(a - \sqrt{a^2 - x^2}\right)dx\right]$

$= \frac{b}{a}\left[\frac{a}{2}x^2 + \frac{1}{3}(a^2 - x^2)^{3/2}\right]_0^a = \frac{1}{6}a^2b$

$\int \bar{y}_{EL}\,dA = \int_0^a \frac{b}{2a}\left(a + \sqrt{a^2 - x^2}\right)\left[\frac{b}{a}\left(a - \sqrt{a^2 - x^2}\right)dx\right]$

$= \frac{b^2}{2a^2}\int_0^a (x^2)dx = \frac{b^2}{2a^2}\left[\frac{1}{3}x^3\right]_0^a$

$= \frac{1}{6}ab^2$

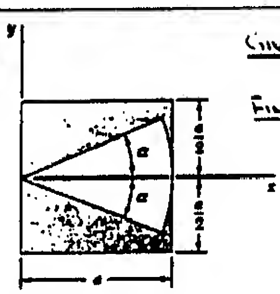
$\bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x}\left[ab\left(1 - \frac{\pi}{4}\right)\right] = \frac{1}{6}a^2b$

OR $\bar{x} = \frac{2a}{3(4 - \pi)}$

$\bar{y}A = \int \bar{y}_{EL}\,dA: \quad \bar{y}\left[ab\left(1 - \frac{\pi}{4}\right)\right] = \frac{1}{6}ab^2$

OR $\bar{y} = \frac{2b}{3(4 - \pi)}$

5.45



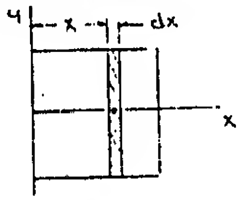
GIVEN: PLANE AREA SHOWN

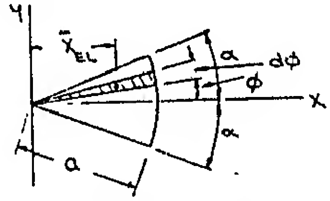
FIND: $\bar{x}$ AND $\bar{y}$ USING DIRECT INTEGRATION

(CONTINUED)

5.45 CONTINUED

FIRST NOTE THAT SYMMETRY IMPLIES $\bar{y} = 0$





$dA = a\,dx$

$\bar{x}_{EL} = x$

$dA = \frac{1}{2}a(a\,d\phi)$

$\bar{x}_{EL} = \frac{2}{3}a\cos\phi$

THEN.. $A = \int dA = \int_0^a a\,dx - \int_{-\alpha}^{\alpha} \frac{1}{2}a^2\,d\phi$

$= a[x]_0^a - \frac{a^2}{2}[\phi]_{-\alpha}^{\alpha} = a^2(1 - \alpha)$

AND.. $\int \bar{x}_{EL}\,dA = \int_0^a x(a\,dx) - \int_{-\alpha}^{\alpha} \frac{2}{3}a\cos\phi\left(\frac{1}{2}a^2\,d\phi\right)$

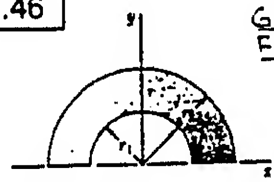
$= a\left[\frac{x^2}{2}\right]_0^a - \frac{1}{3}a^3[\sin\phi]_{-\alpha}^{\alpha}$

$= a^3\left(\frac{1}{2} - \frac{2}{3}\sin\alpha\right)$

$\bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x}[a^2(1 - \alpha)] = a^3\left(\frac{1}{2} - \frac{2}{3}\sin\alpha\right)$

OR $\bar{x} = \frac{3 - 4\sin\alpha}{6(1 - \alpha)}a$

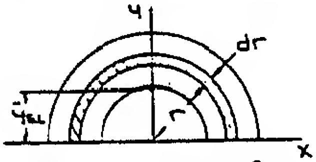
5.46



GIVEN: PLANE AREA SHOWN

FIND: $\bar{x}$ AND $\bar{y}$ USING DIRECT INTEGRATION

FIRST NOTE THAT SYMMETRY IMPLIES $\bar{x} = 0$



HAVE.. $dA = \pi r\,dr$

AND $\bar{y}_{EL} = \frac{2r}{\pi}$

(FIG. 5.8B)

THEN.. $A = \int dA = \int_{r_1}^{r_2} \pi r\,dr = \frac{\pi}{2}[r^2]_{r_1}^{r_2} = \frac{\pi}{2}(r_2^2 - r_1^2)$

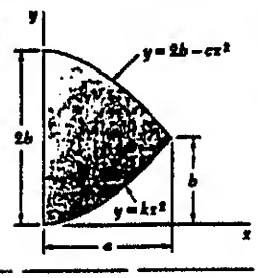
AND.. $\int \bar{y}_{EL}\,dA = \int_{r_1}^{r_2} \frac{2r}{\pi}(\pi r\,dr) = 2\left[\frac{1}{3}r^3\right]_{r_1}^{r_2}$

$= \frac{2}{3}(r_2^3 - r_1^3)$

$\bar{y}A = \int \bar{y}_{EL}\,dA: \quad \bar{y}\left[\frac{\pi}{2}(r_2^2 - r_1^2)\right] = \frac{2}{3}(r_2^3 - r_1^3)$

OR $\bar{y} = \frac{4}{3\pi}\,\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$

5.47



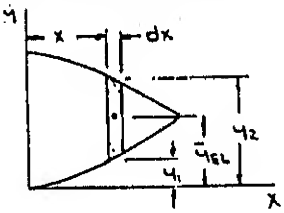
GIVEN: PLANE AREA SHOWN

FIND: $\bar{x}$ AND $\bar{y}$ USING DIRECT INTEGRATION

FIRST NOTE THAT SYMMETRY IMPLIES $\bar{y} = b$

(CONTINUED)

5.47 CONTINUED



At $x = a,\ y = b$

y_1: $b = ka^2$ or $k = \frac{b}{a^2}$

Then $y_1 = \frac{b}{a^2}x^2$

y_2: $b = 2b - ca^2$ or $c = \frac{b}{a^2}$

Then $y_2 = b(2 - \frac{x^2}{a^2})$

Now.. $dA = (y_2 - y_1)dx = [b(2 - \frac{x^2}{a^2}) - \frac{b}{a^2}x^2]dx = 2b(1 - \frac{x^2}{a^2})dx$

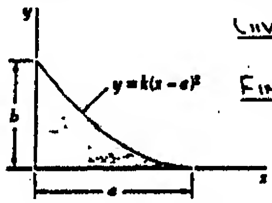
And $\bar{x}_{EL} = x$

Then.. $A = \int dA = \int_0^a 2b(1 - \frac{x^2}{a^2})dx = 2b[x - \frac{x^3}{3a^2}]_0^a = \frac{4}{3}ab$

And $\int \bar{x}_{EL}dA = \int_0^a x[2b(1 - \frac{x^2}{a^2})dx] = 2b[\frac{x^2}{2} - \frac{x^4}{4a^2}]_0^a = \frac{1}{2}a^2b$

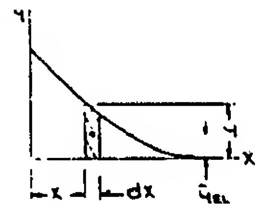
$\bar{x}A = \int \bar{x}_{EL}dA$: $\bar{x}(\frac{4}{3}ab) = \frac{1}{2}a^2b$ $\quad \bar{x} = \frac{3}{8}a$ ◀

5.48



Given: Plane area shown

Find: $\bar{x}$ and $\bar{y}$ using direct integration



At $x = 0,\ y = b$

$b = k(0 - a)^2$ or $k = \frac{b}{a^2}$

Then $y = \frac{b}{a^2}(x - a)^2$

Now.. $\bar{x}_{EL} = x$

$\bar{y}_{EL} = \frac{1}{2}y = \frac{b}{2a^2}(x - a)^2$

And $dA = y\,dx = \frac{b}{a^2}(x - a)^2dx$

Then.. $A = \int dA = \int_0^a \frac{b}{a^2}(x-a)^2dx = \frac{b}{3a^2}[(x-a)^3]_0^a$

$= \frac{1}{3}ab$

And.. $\int \bar{x}_{EL}dA = \int_0^a x[\frac{b}{a^2}(x-a)^2dx] = \frac{b}{a^2}\int_0^a(x^3 - 2ax^2 + a^2x)dx$

$= \frac{b}{a^2}[\frac{x^4}{4} - \frac{2}{3}ax^3 + \frac{1}{2}a^2x^2]_0^a = \frac{1}{12}a^2b$

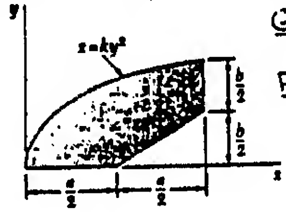
$\int \bar{y}_{EL}dA = \int_0^a \frac{b}{2a^2}(x-a)^2[\frac{b}{a^2}(x-a)^2dx]$

$= \frac{b^2}{2a^4}[\frac{1}{5}(x-a)^5]_0^a = \frac{1}{10}ab^2$

$\bar{x}A = \int \bar{x}_{EL}dA$: $\bar{x}(\frac{1}{3}ab) = \frac{1}{12}a^2b$ $\quad \bar{x} = \frac{1}{4}a$ ◀

$\bar{y}A = \int \bar{y}_{EL}dA$: $\bar{y}(\frac{1}{3}ab) = \frac{1}{10}ab^2$ $\quad \bar{y} = \frac{3}{10}b$ ◀

5.49



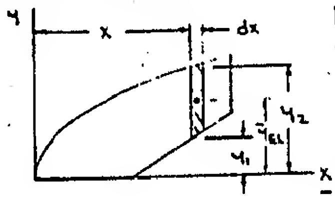
Given: Plane area shown

Find: $\bar{x}$ and $\bar{y}$ using direct integration

By observation.. $y_1 = \frac{b}{a}x - \frac{b}{2} = b(\frac{x}{a} - \frac{1}{2})$

(continued)

5.49 CONTINUED



For y_2 at $x = a,\ y = b$

$a = kb^2$ or $k = \frac{a}{b^2}$

Then $y_2 = b\frac{x^{1/2}}{\sqrt{a}}$

Now.. $\bar{x}_{EL} = x$

and for $0 \le x \le \frac{a}{2}$:

$\bar{y}_{EL} = \frac{1}{2}y_2 = \frac{b}{2}\frac{x^{1/2}}{\sqrt{a}}$ $\quad dA = y_2dx = b\frac{x^{1/2}}{\sqrt{a}}dx$

For $\frac{a}{2} \le x \le a$: $\bar{y}_{EL} = \frac{1}{2}(y_1 + y_2) = \frac{b}{2}(\frac{x}{a} - \frac{1}{2} + \frac{x^{1/2}}{\sqrt{a}})$

$dA = (y_2 - y_1)dx = b(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2})dx$

Then.. $A = \int dA = \int_0^{a/2} b\frac{x^{1/2}}{\sqrt{a}}dx + \int_{a/2}^a b(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2})dx$

$= \frac{b}{\sqrt{a}}[\frac{2}{3}x^{3/2}]_0^{a/2} + b[\frac{2}{3}\frac{x^{3/2}}{\sqrt{a}} - \frac{x^2}{2a} + \frac{1}{2}x]_{a/2}^a$

$= \frac{2}{3}\frac{b}{\sqrt{a}}[(\frac{a}{2})^{3/2} + (a)^{3/2} - (\frac{a}{2})^{3/2}]$

$+ b\{-\frac{1}{2a}[(a)^2 - (\frac{a}{2})^2] + \frac{1}{2}[(a) - (\frac{a}{2})]\}$

$= \frac{13}{24}ab$

And.. $\int \bar{x}_{EL}dA = \int_0^{a/2} x[b\frac{x^{1/2}}{\sqrt{a}}dx] + \int_{a/2}^a x[b(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2})dx]$

$= \frac{b}{\sqrt{a}}[\frac{2}{5}x^{5/2}]_0^{a/2} + b[\frac{2}{5}\frac{x^{5/2}}{\sqrt{a}} - \frac{x^3}{3a} + \frac{x^2}{4}]_{a/2}^a$

$= \frac{2}{5}\frac{b}{\sqrt{a}}[(\frac{a}{2})^{5/2} + (a)^{5/2} - (\frac{a}{2})^{5/2}]$

$+ b\{-\frac{1}{3a}[(a)^3 - (\frac{a}{2})^3] + \frac{1}{4}[(a)^2 - (\frac{a}{2})^2]\}$

$= \frac{71}{240}a^2b$

$\int \bar{y}_{EL}dA = \int_0^{a/2}\frac{b}{2}\frac{x^{1/2}}{\sqrt{a}}[b\frac{x^{1/2}}{\sqrt{a}}dx]$

$+ \int_{a/2}^a \frac{b}{2}(\frac{x}{a} - \frac{1}{2} + \frac{x^{1/2}}{\sqrt{a}})[b(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2})dx]$

$= \frac{b^2}{2a}[\frac{1}{2}x^2]_0^{a/2} + \frac{b^2}{2}[(\frac{x^2}{2a}) - \frac{1}{3a}(\frac{x}{a} - \frac{1}{2})^3]_{a/2}^a$

$= \frac{b^2}{4a}[(\frac{a}{2})^2 + (a)^2 - (\frac{a}{2})^2] - \frac{b^2}{6a}(\frac{a}{a} - \frac{1}{2})^3$

$= \frac{11}{48}ab^2$

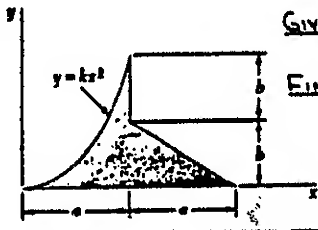
$\bar{x}A = \int \bar{x}_{EL}dA$: $\bar{x}(\frac{13}{24}ab) = \frac{71}{240}a^2b$ $\quad \bar{x} = \frac{71}{130}a$

or $\bar{x} = 0.546a$ ◀

$\bar{y}A = \int \bar{y}_{EL}dA$: $\bar{y}(\frac{13}{24}ab) = \frac{11}{48}ab^2$ $\quad \bar{y} = \frac{11}{26}b$

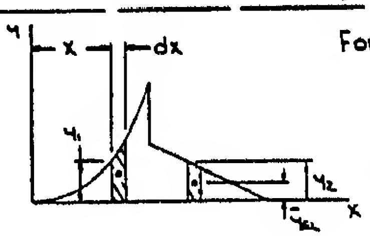
or $\bar{y} = 0.423b$ ◀

5.50



Given: Plane area shown

Find: $\bar{x}$ and $\bar{y}$ using direct integration



For y_1 at $x = a,\ y = 2b$

$2b = ka^2$ or $k = \frac{2b}{a^2}$

Then.. $y_1 = \frac{2b}{a^2}x^2$

By observation

$y_2 = -\frac{b}{a}x + 2b$

$= b(2 - \frac{x}{a})$

(continued)

5.50 CONTINUED

Now $\bar{x}_{EL} = x$

And for $0 \le x \le a$: $\bar{y}_{EL} = \frac{1}{2}y_1 = \frac{b}{a^2}x^2$

$dA = y_1\,dx = \frac{2b}{a^2}x^2\,dx$

For $a \le x \le 2a$: $\bar{y}_{EL} = \frac{1}{2}y_2 = \frac{b}{2}(2-\frac{x}{a})$

$dA = y_2\,dx = b(2-\frac{x}{a})dx$

Then.. $A = \int dA = \int_0^a \frac{2b}{a^2}x^2dx + \int_a^{2a} b(2-\frac{x}{a})dx$

$= \frac{2b}{a^2}[\frac{1}{3}x^3]_0^a + b[-\frac{a}{2}(2-\frac{x}{a})^2]_a^{2a}$

$= \frac{7}{6}ab$

And.. $\int \bar{x}_{EL}\,dA = \int_0^a x(\frac{2b}{a^2}x^2dx) + \int_a^{2a} x[b(2-\frac{x}{a})dx]$

$= \frac{2b}{a^2}[\frac{1}{4}x^4]_0^a + b[x^2 - \frac{x^3}{3a}]_a^{2a}$

$= \frac{1}{2}a^2b + b\{[(2a)^2-(a)^2] - \frac{1}{3a}[(2a)^3-(a)^3]\}$

$= \frac{7}{6}a^2b$

$\int \bar{y}_{EL}\,dA = \int_0^a \frac{b}{a^2}x^2[\frac{2b}{a^2}x^2dx] + \int_a^{2a}\frac{b}{2}(2-\frac{x}{a})[b(2-\frac{x}{a})dx]$

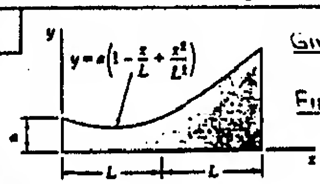
$= \frac{2b^2}{a^4}[\frac{1}{5}x^5]_0^a + \frac{b^2}{2}[-\frac{a}{3}(2-\frac{x}{a})^3]_a^{2a}$

$= \frac{17}{30}ab^2$

$\bar{x}A = \int \bar{x}_{EL}\,dA$: $\bar{x}(\frac{7}{6}ab) = \frac{7}{6}a^2b$ $\qquad \bar{x} = a$ ◀

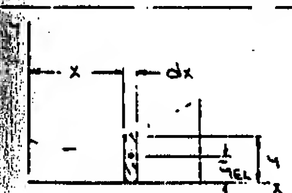
$\bar{y}A = \int \bar{y}_{EL}\,dA$: $\bar{y}(\frac{7}{6}ab) = \frac{17}{30}ab^2$ $\qquad \bar{y} = \frac{17}{35}b$ ◀

5.51



GIVEN: PLANE AREA SHOWN

FIND: $\bar{x}$ AND $\bar{y}$ USING DIRECT INTEGRATION



Have.. $\bar{x}_{EL} = x$

$\bar{y}_{EL} = \frac{1}{2}y = \frac{a}{2}(1-\frac{x}{L}+\frac{x^2}{L^2})$

$dA = y\,dx$

$= a(1-\frac{x}{L}+\frac{x^2}{L^2})dx$

Then. $A = \int dA = \int_0^{2L} a(1-\frac{x}{L}+\frac{x^2}{L^2})dx$

$= a[x - \frac{x^2}{2L} + \frac{x^3}{3L^2}]_0^{2L} = \frac{8}{3}aL$

And. $\int \bar{x}_{EL}\,dA = \int_0^{2L} x[a(1-\frac{x}{L}+\frac{x^2}{L^2})dx]$

$= a[\frac{x^2}{2} - \frac{x^3}{3L} + \frac{x^4}{4L^2}]_0^{2L} = \frac{10}{3}aL^2$

$\int \bar{y}_{EL}\,dA = \int_0^{2L} \frac{a}{2}(1-\frac{x}{L}+\frac{x^2}{L^2})[a(1-\frac{x}{L}+\frac{x^2}{L^2})dx]$

$= \frac{a^2}{2}\int_0^{2L}(1 - 2\frac{x}{L} + 3\frac{x^2}{L^2} - 2\frac{x^3}{L^3} + \frac{x^4}{L^4})dx$

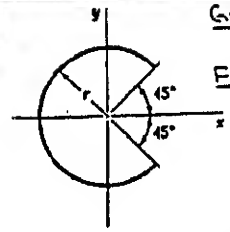
$= \frac{a^2}{2}[x - \frac{x^2}{L} + \frac{x^3}{L^2} - \frac{x^4}{2L^3} + \frac{x^5}{5L^4}]_0^{2L}$

$= \frac{11}{5}a^2L$

$\bar{x}A = \int \bar{x}_{EL}\,dA$: $\bar{x}(\frac{8}{3}aL) = \frac{10}{3}aL^2$ $\qquad \bar{x} = \frac{5}{4}L$ ◀

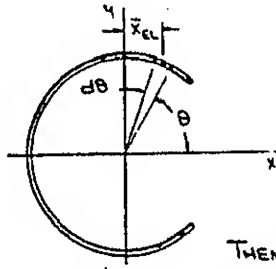
$\bar{y}A = \int \bar{y}_{EL}\,dA$: $\bar{y}(\frac{8}{3}aL) = \frac{11}{5}a^2L$ $\qquad \bar{y} = \frac{33}{40}a$ ◀

5.52



GIVEN: HOMOGENEOUS WIRE SHOWN

FIND: $\bar{x}$ USING DIRECT INTEGRATION



FIRST NOTE THAT BECAUSE THE WIRE IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.

Now.. $\bar{x}_{EL} = r\cos\theta$

And $dL = r\,d\theta$

Then.. $L = \int dL = \int_{\pi/4}^{7\pi/4} r\,d\theta = r[\theta]_{\pi/4}^{7\pi/4}$

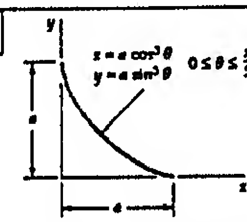
$= \frac{3}{2}\pi r$

And.. $\int \bar{x}\,dL = \int_{\pi/4}^{7\pi/4} r\cos\theta(r\,d\theta) = r^2[\sin\theta]_{\pi/4}^{7\pi/4}$

$= r^2(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}) = -r^2\sqrt{2}$

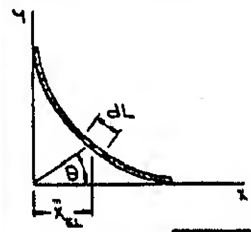
$\bar{x}L = \int \bar{x}\,dL$: $\bar{x}(\frac{3}{2}\pi r) = -r^2\sqrt{2}$ $\qquad \bar{x} = -\frac{2\sqrt{2}}{3\pi}r$ ◀

5.53



GIVEN: HOMOGENEOUS WIRE SHOWN

FIND: $\bar{x}$ USING DIRECT INTEGRATION



FIRST NOTE THAT BECAUSE THE WIRE IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.

Now.. $\bar{x}_{EL} = a\cos^3\theta$

And $dL = \sqrt{dx^2+dy^2}$

Where.. $x = a\cos^3\theta$: $dx = -3a\cos^2\theta\sin\theta\,d\theta$

$y = a\sin^3\theta$: $dy = 3a\sin^2\theta\cos\theta\,d\theta$

Then.. $dL = [(-3a\cos^2\theta\sin\theta\,d\theta)^2 + (3a\sin^2\theta\cos\theta\,d\theta)^2]^{\frac{1}{2}}$

$= 3a\cos\theta\sin\theta[\cos^2\theta + \sin^2\theta]^{\frac{1}{2}}d\theta$

$= 3a\cos\theta\sin\theta\,d\theta$

$\therefore$ $L = \int dL = \int_0^{\pi/2} 3a\cos\theta\sin\theta\,d\theta = 3a[\frac{1}{2}\sin^2\theta]_0^{\pi/2}$

$= \frac{3}{2}a$

And.. $\int \bar{x}_{EL}\,dL = \int_0^{\pi/2} a\cos^3\theta(3a\cos\theta\sin\theta\,d\theta)$

$= 3a^2[-\frac{1}{5}\cos^5\theta]_0^{\pi/2} = \frac{3}{5}a^2$

$\bar{x}L = \int \bar{x}_{EL}\,dL$: $\bar{x}(\frac{3}{2}a) = \frac{3}{5}a^2$ $\qquad \bar{x} = \frac{2}{5}a$ ◀

ALTERNATIVE SOLUTION

$x = a\cos^3\theta \Rightarrow \cos^2\theta = (\frac{x}{a})^{2/3}$

$y = a\sin^3\theta \Rightarrow \sin^2\theta = (\frac{y}{a})^{2/3}$

$\therefore$ $(\frac{x}{a})^{2/3} + (\frac{y}{a})^{2/3} = 1$ OR $y = (a^{2/3} - x^{2/3})^{3/2}$

Then $\frac{dy}{dx} = (a^{2/3} - x^{2/3})^{\frac{1}{2}}(-x^{-1/3})$

(CONTINUED)

5.53 CONTINUED

Now.. $\bar{x}_{EL} = x$

AND $dL = \sqrt{1+(\frac{dy}{dx})^2}\,dx = \{1+[(a^{2/3}-x^{2/3})^{1/2}(-x^{-1/3})]^2\}^{1/2}dx$

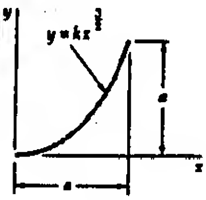
$= \frac{a^{1/3}}{x^{1/3}}dx$

THEN.. $L = \int dL = \int_0^a \frac{a^{1/3}}{x^{1/3}}dx = a^{1/3}[\frac{3}{2}x^{2/3}]_0^a = \frac{3}{2}a$

AND.. $\int \bar{x}_{EL}dL = \int_0^a x(\frac{a^{1/3}}{x^{1/3}}dx) = a^{1/3}[\frac{3}{5}x^{5/3}]_0^a = \frac{3}{5}a^2$

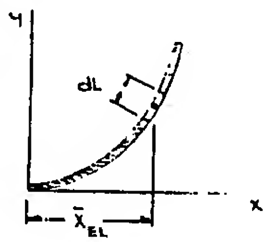
$\bar{x}L = \int \bar{x}_{EL}dL \cdot \quad \bar{x}(\frac{3}{2}a) = \frac{3}{5}a^2 \qquad \bar{x} = \frac{2}{5}a$ ◀

* 5.54



GIVEN: HOMOGENEOUS WIRE SHOWN

FIND: $\bar{x}$ USING DIRECT INTEGRATION



FIRST NOTE THAT BECAUSE THE WIRE IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.

HAVE AT $x = a$, $y = a$
$a = ka^{3/2}$ OR $k = \frac{1}{\sqrt{a}}$

THEN $y = \frac{1}{\sqrt{a}}x^{3/2}$

AND $\frac{dy}{dx} = \frac{3}{2\sqrt{a}}x^{1/2}$

Now.. $\bar{x}_{EL} = x$

AND. $dL = \sqrt{1+(\frac{dy}{dx})^2}\,dx = [1+(\frac{3}{2\sqrt{a}}x^{1/2})^2]^{1/2}dx$

$= \frac{1}{2\sqrt{a}}\sqrt{4a+9x}\,dx$

THEN.. $L = \int dL = \int_0^a \frac{1}{2\sqrt{a}}\sqrt{4a+9x}\,dx$

$= \frac{1}{2\sqrt{a}}[\frac{2}{3}\cdot\frac{1}{9}(4a+9x)^{3/2}]_0^a = \frac{a}{27}[(13)^{3/2}-8]$

$= 1.43971a$

AND. $\int \bar{x}_{EL}dL = \int_0^a x[\frac{1}{2\sqrt{a}}\sqrt{4a+9x}\,dx]$

USE INTEGRATION BY PARTS WITH

$u = x \qquad dv = \sqrt{4a+9x}\,dx$

$du = dx \qquad v = \frac{2}{27}(4a+9x)^{3/2}$

THEN.. $\int \bar{x}_{EL}dL = \frac{1}{2\sqrt{a}}\{[x\cdot\frac{2}{27}(4a+9x)^{3/2}]_0^a - \int_0^a \frac{2}{27}(4a+9x)^{3/2}dx\}$

$= \frac{(13)^{3/2}}{27}a^2 - \frac{1}{27\sqrt{a}}[\frac{2}{45}(4a+9x)^{5/2}]_0^a$

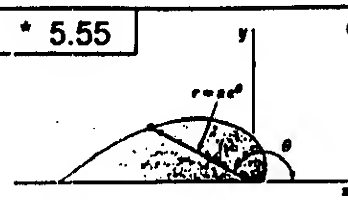
$= \frac{a^2}{27}\{(13)^{3/2} - \frac{2}{45}[(13)^{5/2}-32]\}$

$= 0.78566a^2$

$\bar{x}L = \int \bar{x}_{EL}dL$: $\bar{x}(1.43971a) = 0.78566a^2$

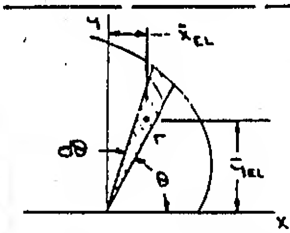
OR $\bar{x} = 0.546a$ ◀

* 5.55



GIVEN: PLANE AREA SHOWN

FIND: $\bar{x}$ AND $\bar{y}$ USING DIRECT INTEGRATION



HAVE.. $\bar{x}_{EL} = \frac{2}{3}r\cos\theta$
$= \frac{2}{3}ae^{\theta}\cos\theta$

$\bar{y}_{EL} = \frac{2}{3}r\sin\theta$
$= \frac{2}{3}ae^{\theta}\sin\theta$

AND $dA = \frac{1}{2}\cdot r\cdot r\,d\theta$
$= \frac{1}{2}a^2e^{2\theta}d\theta$

THEN.. $A = \int dA = \int_0^{\pi}\frac{1}{2}a^2e^{2\theta}d\theta = \frac{1}{2}a^2[\frac{1}{2}e^{2\theta}]_0^{\pi}$

$= \frac{1}{4}a^2(e^{2\pi}-1) = 133.623a^2$

AND $\int \bar{x}_{EL}dA = \int_0^{\pi}\frac{2}{3}ae^{\theta}\cos\theta(\frac{1}{2}a^2e^{2\theta}d\theta)$

$= \frac{1}{3}a^3\int_0^{\pi}e^{3\theta}\cos\theta\,d\theta$

USE INTEGRATION BY PARTS WITH

$u = e^{3\theta} \qquad dv = \cos\theta\,d\theta$

$du = 3e^{3\theta}d\theta \qquad v = \sin\theta$

THEN.. $\int e^{3\theta}\cos\theta\,d\theta = e^{3\theta}\sin\theta - \int\sin\theta(3e^{3\theta}d\theta)$

Now LET $u = e^{3\theta} \qquad dv = \sin\theta\,d\theta$

$du = 3e^{3\theta}d\theta \qquad v = -\cos\theta$

THEN.. $\int e^{3\theta}\cos\theta\,d\theta = e^{3\theta}\sin\theta - 3[-e^{3\theta}\cos\theta - \int(-\cos\theta)(3e^{3\theta}d\theta)]$

SO THAT $\int e^{3\theta}\cos\theta\,d\theta = \frac{e^{3\theta}}{10}(\sin\theta + 3\cos\theta)$

$\therefore \int \bar{x}_{EL}dA = \frac{1}{3}a^3[\frac{e^{3\theta}}{10}(\sin\theta+3\cos\theta)]_0^{\pi}$

$= \frac{a^3}{30}(-3e^{3\pi}-3) = -1239.26a^3$

ALSO.. $\int \bar{y}_{EL}dA = \int_0^{\pi}\frac{2}{3}ae^{\theta}\sin\theta(\frac{1}{2}a^2e^{2\theta}d\theta)$

$= \frac{1}{3}a^3\int_0^{\pi}e^{3\theta}\sin\theta\,d\theta$

USE INTEGRATION BY PARTS WITH

$u = e^{3\theta} \qquad dv = \int\sin\theta\,d\theta$

$du = 3e^{3\theta}d\theta \qquad v = -\cos\theta$

THEN.. $\int e^{3\theta}\sin\theta\,d\theta = -e^{3\theta}\cos\theta - \int(-\cos\theta)(3e^{3\theta}d\theta)$

Now LET $u = e^{3\theta} \qquad dv = \cos\theta\,d\theta$

$du = 3e^{3\theta}d\theta \qquad v = \sin\theta$

THEN.. $\int e^{3\theta}\sin\theta\,d\theta = -e^{3\theta}\cos\theta + 3[e^{3\theta}\sin\theta - \int(\sin\theta)(3e^{3\theta}d\theta)]$

SO THAT $\int e^{3\theta}\sin\theta\,d\theta = \frac{e^{3\theta}}{10}(-\cos\theta + 3\sin\theta)$

$\therefore \int \bar{y}_{EL}dA = \frac{1}{3}a^3[\frac{e^{3\theta}}{10}(-\cos\theta+3\sin\theta)]_0^{\pi}$

$= \frac{a^3}{30}(e^{3\pi}+1) = 413.09a^3$

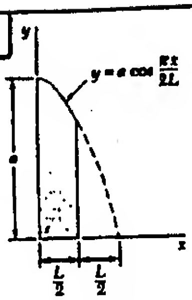
$\bar{x}A = \int \bar{x}_{EL}dA$: $\bar{x}(133.623a^2) = -1239.26a^3$

OR $\bar{x} = -9.27a$ ◀

$\bar{y}A = \int \bar{y}_{EL}dA$: $\bar{y}(133.623a^2) = 413.09a^3$

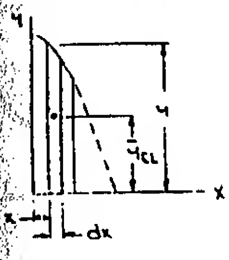
OR $\bar{y} = 3.09a$ ◀

5.56



GIVEN: PLANE AREA SHOWN

FIND: $\bar{x}$ AND $\bar{y}$ USING DIRECT INTEGRATION



HAVE.. $\bar{x}_{EL} = x$

$\bar{y}_{EL} = \frac{1}{2}y = \frac{a}{2}\cos\frac{\pi x}{2L}$

AND.. $dA = y\,dx = a\cos\frac{\pi x}{2L}dx$

THEN.. $A = \int dA = \int_0^{L/2} a\cos\frac{\pi x}{2L}dx$

$= a\left[\frac{2L}{\pi}\sin\frac{\pi x}{2L}\right]_0^{L/2}$

$= \frac{\sqrt{2}}{\pi}aL$

AND $\int \bar{x}_{EL}\,dA = \int x\left(a\cos\frac{\pi x}{2L}dx\right)$

USE INTEGRATION BY PARTS WITH

$u = x$ $\quad dv = \cos\frac{\pi x}{2L}dx$

$du = dx$ $\quad v = \frac{2L}{\pi}\sin\frac{\pi x}{2L}$

THEN.. $\int x\cos\frac{\pi x}{2L}dx = \frac{2L}{\pi}x\sin\frac{\pi x}{2L} - \int\frac{2L}{\pi}\sin\frac{\pi x}{2L}dx$

$= \frac{2L}{\pi}\left(x\sin\frac{\pi x}{2L} + \frac{2L}{\pi}\cos\frac{\pi x}{2L}\right)$

$\therefore \int \bar{x}_{EL}\,dA = a\frac{2L}{\pi}\left[x\sin\frac{\pi x}{2L} + \frac{2L}{\pi}\cos\frac{\pi x}{2L}\right]_0^{L/2}$

$= a\frac{2L}{\pi}\left[\left(\frac{L}{2\sqrt{2}} + \frac{\sqrt{2}}{\pi}L\right) - \frac{2L}{\pi}\right]$

$= 0.106374\,aL^2$

ALSO.. $\int \bar{y}_{EL}\,dA = \int_0^{L/2}\frac{a}{2}\cos\frac{\pi x}{2L}\left(a\cos\frac{\pi x}{2L}dx\right)$

$= \frac{a^2}{2}\left[\frac{x}{2} + \frac{\sin\frac{\pi x}{L}}{\frac{2\pi}{L}}\right]_0^{L/2} = \frac{a^2}{2}\left(\frac{L}{4} + \frac{L}{2\pi}\right)$

$= 0.20458\,a^2L$

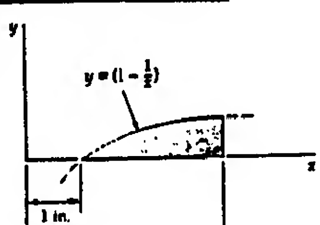
$\bar{x}A = \int \bar{x}_{EL}\,dA$: $\quad \bar{x}\left(\frac{\sqrt{2}}{\pi}aL\right) = 0.106374\,aL^2$

OR $\bar{x} = 0.236\,L$ ◄

$\bar{y}A = \int \bar{y}_{EL}\,dA$: $\quad \bar{y}\left(\frac{\sqrt{2}}{\pi}aL\right) = 0.20458\,a^2L$

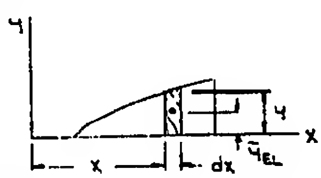
OR $\bar{y} = 0.454\,a$ ◄

5.57 and 5.58



GIVEN: PLANE AREA SHOWN

FIND: $\bar{x}$ AND $\bar{y}$



HAVE.. $\bar{x}_{EL} = x$

$\bar{y}_{EL} = \frac{1}{2}y = \frac{1}{2}\left(1 - \frac{1}{x}\right)$

AND $dA = y\,dx = \left(1 - \frac{1}{x}\right)dx$

(CONTINUED)

5.57 and 5.58 CONTINUED

THEN.. $A = \int dA = \int_1^a\left(1 - \frac{1}{x}\right)dx = \left[x - \ln x\right]_1^a$

$= (a - \ln a - 1)\ \text{in}^2$

AND $\int \bar{x}_{EL}\,dA = \int_1^a x\left[\left(1 - \frac{1}{x}\right)dx\right] = \left[\frac{x^2}{2} - x\right]_1^a$

$= \left(\frac{a^2}{2} - a + \frac{1}{2}\right)\ \text{in}^3$

$\int \bar{y}_{EL}\,dA = \int_1^a \frac{1}{2}\left(1 - \frac{1}{x}\right)\left[\left(1 - \frac{1}{x}\right)dx\right] = \frac{1}{2}\int_1^a\left(1 - \frac{2}{x} + \frac{1}{x^2}\right)dx$

$= \frac{1}{2}\left[x - 2\ln x - \frac{1}{x}\right]_1^a = \frac{1}{2}\left(a - 2\ln a - \frac{1}{a}\right)\ \text{in}^3$

$\bar{x}A = \int \bar{x}_{EL}\,dA$: $\quad \bar{x} = \dfrac{\frac{a^2}{2} - a + \frac{1}{2}}{a - \ln a - 1}$ in.

$\bar{y}A = \int \bar{y}_{EL}\,dA$: $\quad \bar{y} = \dfrac{a - 2\ln a - \frac{1}{a}}{2(a - \ln a - 1)}$ in.

5.57 FIND: $\bar{x}$ AND $\bar{y}$ WHEN $a = 2$ in.

HAVE.. $\bar{x} = \dfrac{\frac{1}{2}(2)^2 - 2 + \frac{1}{2}}{2 - \ln 2 - 1}$ OR $\bar{x} = 1.629$ in ◄

AND $\bar{y} = \dfrac{2 - 2\ln 2 - \frac{1}{2}}{2(2 - \ln 2 - 1)}$ OR $\bar{y} = 0.1853$ in ◄

5.58 FIND: a SO THAT $\frac{\bar{x}}{\bar{y}} = 9$

HAVE.. $\dfrac{\bar{x}}{\bar{y}} = \dfrac{\bar{x}A}{\bar{y}A} = \dfrac{\int \bar{x}_{EL}\,dA}{\int \bar{y}_{EL}\,dA}$

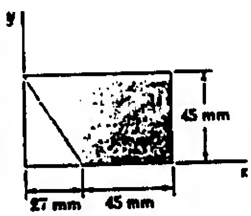
THEN.. $\dfrac{\frac{1}{2}a^2 - a + \frac{1}{2}}{\frac{1}{2}\left(a - 2\ln a - \frac{1}{a}\right)} = 9$

OR $a^3 - 11a^2 + a + 18a\ln a + 9 = 0$

USING TRIAL AND ERROR OR NUMERICAL METHODS AND IGNORING THE TRIVIAL SOLUTION $a = 1$ in., FIND..

$a = 1.901$ in. AND $a = 3.74$ in. ◄

5.59



GIVEN: PLANE AREA SHOWN

FIND: VOLUME AND SURFACE AREA OF SOLID OBTAINED BY ROTATING THE AREA ABOUT

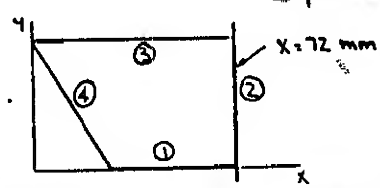
(a) THE X AXIS

(b) THE LINE $x = 72$ mm

FROM THE SOLUTION TO PROBLEM 5.1 HAVE

$A = 2632.5\ \text{mm}^2$ $\quad \Sigma\bar{x}A = 111\,172.5\ \text{mm}^3$

$\Sigma\bar{y}A = 63\,787.5\ \text{mm}^3$



APPLYING THE THEOREMS OF PAPPUS-GULDINUS HAVE..

(a) ROTATION ABOUT THE X AXIS:

VOLUME $= 2\pi\bar{Y}_{AREA}A = 2\pi(\Sigma\bar{y}A) = 2\pi(63\,787.5\ \text{mm}^3)$

OR VOLUME $= 401 \times 10^3\ \text{mm}^3$ ◄

AREA $= 2\pi\bar{Y}_{LINE}L = 2\pi\Sigma(\bar{y}_{LINE})L$

(CONTINUED)

5.59 CONTINUED

$\text{AREA} = 2\pi(\bar{y}_2L_2 + \bar{y}_3L_3 + \bar{y}_4L_4)$

$= 2\pi[(22.5)(45) + (45)(72) + (22.5)(\sqrt{27^2+45^2})]$

OR $\text{AREA} = 34.1\times10^3\ \text{mm}^2$ ◀

(b) ROTATION ABOUT THE LINE $x = 72$ mm:

$\text{VOLUME} = 2\pi(72 - \bar{X}_{AREA})A = 2\pi(72A - \Sigma\bar{x}A)$

$= 2\pi[(72\ \text{mm})(2632.5\ \text{mm}^2) - (111\ 172.5\ \text{mm}^3)]$

OR $\text{VOLUME} = 492\times10^3\ \text{mm}^3$ ◀

$\text{AREA} = 2\pi\bar{X}_{LINE}L = 2\pi\Sigma(\bar{x}_{LINE})L$

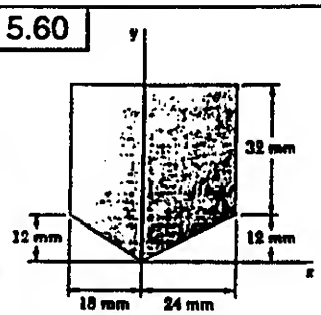
$= 2\pi(\bar{x}_1L_1 + \bar{x}_3L_3 + \bar{x}_4L_4)$

WHERE $\bar{x}_1$, $\bar{x}_3$, AND $\bar{x}_4$ ARE MEASURED WITH RESPECT TO THE LINE $x = 72$ mm. THEN

$\text{AREA} = 2\pi[(22.5)(45) + (36)(72) + (\frac{45+72}{2})(\sqrt{27^2+45^2})]$

OR $\text{AREA} = 41.9\times10^3\ \text{mm}^2$ ◀

5.60



GIVEN: PLANE AREA SHOWN

FIND: VOLUME AND SURFACE AREA OF SOLID OBTAINED BY ROTATING THE AREA ABOUT

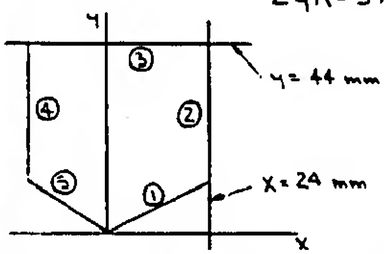
(a) THE LINE $y = 44$ mm

(b) THE LINE $x = 24$ mm

FROM THE SOLUTION TO PROBLEM 5.5 HAVE

$A = 1596\ \text{mm}^2$ $\quad \Sigma\bar{x}A = 4536\ \text{mm}^3$

$\Sigma\bar{y}A = 39\ 648\ \text{mm}^3$



APPLYING THE THEOREMS OF PAPPUS-GULDINUS HAVE

(a) ROTATION ABOUT THE LINE $y = 44$ mm:

$\text{VOLUME} = 2\pi(44 - \bar{Y}_{AREA})A = 2\pi(44A - \Sigma\bar{y}A)$

$= 2\pi[(44\ \text{mm})(1596\ \text{mm}^2) - (39\ 648\ \text{mm}^3)]$

OR $\text{VOLUME} = 192.1\times10^3\ \text{mm}^3$ ◀

$\text{AREA} = 2\pi\bar{Y}_{LINE}L = 2\pi\Sigma(\bar{y}_{LINE})L$

$= 2\pi(\bar{y}_1L_1 + \bar{y}_2L_2 + \bar{y}_4L_4 + \bar{y}_5L_5)$

WHERE $\bar{y}_1, \ldots, \bar{y}_5$ ARE MEASURED WITH RESPECT TO THE LINE $y = 44$ mm. THEN..

$\text{AREA} = 2\pi[(38)(\sqrt{24^2+12^2}) + (16)(32) + (16)(32) + (38)(\sqrt{18^2+12^2})]$

OR $\text{AREA} = 18.01\times10^3\ \text{mm}^2$ ◀

(b) ROTATION ABOUT THE LINE $x = 24$ mm:

$\text{VOLUME} = 2\pi(24 - \bar{X})A = 2\pi(24A - \Sigma\bar{x}A)$

$= 2\pi[(24\ \text{mm})(1596\ \text{mm}^2) - (4536\ \text{mm}^3)]$

OR $\text{VOLUME} = 212\times10^3\ \text{mm}^3$ ◀

$\text{AREA} = 2\pi\bar{X}_{LINE}L = 2\pi\Sigma(\bar{x}_{LINE})L$

$= 2\pi(\bar{x}_1L_1 + \bar{x}_3L_3 + \bar{x}_4L_4 + \bar{x}_5L_5)$

WHERE $\bar{x}_1, \ldots, \bar{x}_5$ ARE MEASURED WITH RESPECT

(CONTINUED)

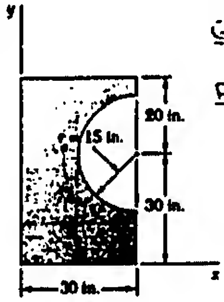
5.60 CONTINUED

TO THE LINE $x = 24$ mm. THEN..

$\text{AREA} = 2\pi[(12)\sqrt{24^2+12^2} + (21)(42) + (42)(32) + (33)(\sqrt{18^2+12^2})]$

OR $\text{AREA} = 20.5\times10^3\ \text{mm}^2$ ◀

5.61



GIVEN: PLANE AREA SHOWN

FIND: VOLUME AND SURFACE AREA OF SOLID OBTAINED BY ROTATING THE AREA ABOUT

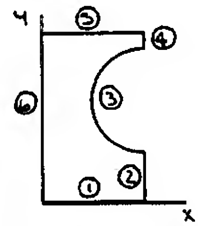
(a) THE x AXIS

(b) THE y AXIS

FROM THE SOLUTION TO PROBLEM 5.7 HAVE

$A = 1146.57\ \text{in}^2$ $\quad \Sigma\bar{x}A = 14\ 147.0\ \text{in}^3$

$\Sigma\bar{y}A = 26\ 897\ \text{in}^3$



APPLYING THE THEOREMS OF PAPPUS-GULDINUS HAVE--

(a) ROTATION ABOUT THE x AXIS:

$\text{VOLUME} = 2\pi\bar{Y}_{AREA}A = 2\pi\Sigma\bar{y}A = 2\pi(26\ 897\ \text{in}^3)$

OR $\text{VOLUME} = 169.0\times10^3\ \text{in}^3$ ◀

$\text{AREA} = 2\pi\bar{Y}_{LINE}A = 2\pi\Sigma(\bar{y}_{LINE})A$

$= 2\pi(\bar{y}_2L_2 + \bar{y}_3L_3 + \bar{y}_4L_4 + \bar{y}_5L_5 + \bar{y}_6L_6)$

$= 2\pi[(7.5)(15) + (30)(\pi\times15) + (47.5)(5) + (50)(30) + (25)(50)]$

OR $\text{AREA} = 28.4\times10^3\ \text{in}^2$ ◀

(b) ROTATION ABOUT THE y AXIS:

$\text{VOLUME} = 2\pi\bar{X}_{AREA}A = 2\pi\Sigma\bar{x}A = 2\pi(14\ 147.0\ \text{in}^3)$

OR $\text{VOLUME} = 88.9\times10^3\ \text{in}^3$ ◀

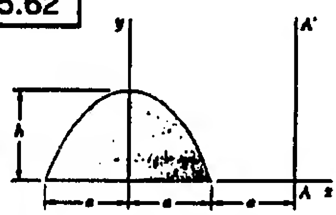
$\text{AREA} = 2\pi\bar{X}_{LINE}L = 2\pi\Sigma(\bar{x}_{LINE})L$

$= 2\pi(\bar{x}_1L_1 + \bar{x}_2L_2 + \bar{x}_3L_3 + \bar{x}_4L_4 + \bar{x}_5L_5)$

$= 2\pi[(15)(30) + (30)(15) + (30 - \frac{2\times15}{\pi})(\pi\times15) + (30)(5) + (15)(30)]$

OR $\text{AREA} = 15.48\times10^3\ \text{in}^2$ ◀

5.62



GIVEN: PLANE PARABOLIC AREA SHOWN

FIND: VOLUME OF SOLID OBTAINED BY ROTATING THE AREA ABOUT

(a) THE x AXIS

(b) THE LINE AA'

FIRST, FROM FIG. 5.8A HAVE.. $A = \frac{4}{3}ah$

$\bar{y} = \frac{2}{5}h$

APPLYING THE SECOND THEOREM OF PAPPUS-GULDINUS HAVE..

(a) ROTATION ABOUT THE x AXIS:

(CONTINUED)

5.62 CONTINUED

$\text{VOLUME} = 2\pi\bar{y}A = 2\pi\left(\frac{2}{5}h\right)\left(\frac{4}{3}ah\right)$

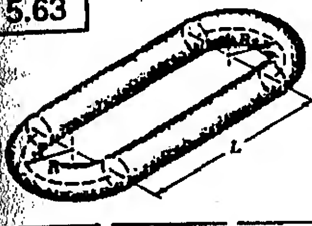
OR $\text{VOLUME} = \frac{16}{15}\pi ah^2$ ◄

(b) ROTATION ABOUT THE LINE AA':

$\text{VOLUME} = 2\pi(2a)A = 2\pi(2a)\left(\frac{4}{3}ah\right)$

OR $\text{VOLUME} = \frac{16}{3}\pi a^2 h$ ◄

5.63



GIVEN: $d = 6$ mm, $R = 10$ mm, $L = 30$ mm

FIND: VOLUME V AND SURFACE AREA A_s OF THE LINK

FIRST NOTE THAT THE AREA A AND THE CIRCUMFERENCE C OF THE CROSS SECTION OF THE BAR ARE

$A = \frac{\pi}{4}d^2 \qquad C = \pi d$

OBSERVING THAT THE SEMICIRCULAR ENDS OF THE LINK CAN BE OBTAINED BY ROTATING THE CROSS SECTION THROUGH A HORIZONTAL SEMICIRCULAR ARC OF RADIUS R. THEN, APPLYING THE THEOREMS OF PAPPUS-GULDINUS HAVE..

VOLUME: $V = 2(V_{SIDE}) + 2(V_{END})$

$= 2(AL) + 2(\pi RA) = 2(L + \pi R)A$

$= 2[(30\text{ mm}) + \pi(10\text{ mm})][\frac{\pi}{4}(6\text{ mm})^2]$

OR $V = 3470\text{ mm}^3$ ◄

AREA: $A_s = 2(A_{SIDE}) + 2(A_{END})$

$= 2(CL) + 2(\pi RC) = 2(L + \pi R)C$

$= 2[(30\text{ mm}) + \pi(10\text{ mm})][\pi(6\text{ mm})]$

OR $A_s = 2320\text{ mm}^2$ ◄

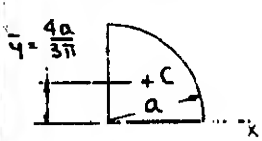
5.64

GIVEN: FIRST FOUR SHAPES OF FIG. 5.21

FIND: VOLUME OF EACH SHAPE

FOLLOWING THE SECOND THEOREM OF PAPPUS-GULDINUS, IN EACH CASE A SPECIFIC GENERATING AREA A WILL BE ROTATED ABOUT THE x AXIS TO PRODUCE THE GIVEN SHAPE. VALUES OF $\bar{y}$ ARE FROM FIG. 5.8 A.

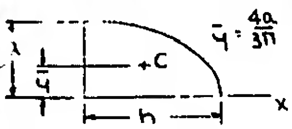
(1) HEMISPHERE: THE GENERATING AREA IS A QUARTER CIRCLE



HAVE. $V = 2\pi\bar{y}A = 2\pi\left(\frac{4a}{3\pi}\right)\left(\frac{\pi}{4}a^2\right)$

OR $V = \frac{2}{3}\pi a^3$ ◄

(2) SEMIELLIPSOID OF REVOLUTION: THE GENERATING AREA IS A QUARTER ELLIPSE



HAVE.. $V = 2\pi\bar{y}A$

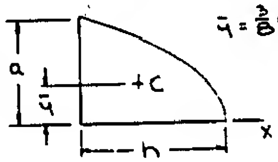
$= 2\pi\left(\frac{4a}{3\pi}\right)\left(\frac{\pi}{4}ha\right)$

OR $V = \frac{2}{3}\pi a^2 h$ ◄

(CONTINUED)

5.64 CONTINUED

(3) PARABOLOID OF REVOLUTION: THE GENERATING AREA IS A QUARTER PARABOLA



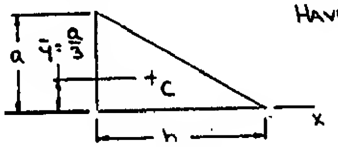
$\bar{y} = \frac{3}{8}a$

HAVE.. $V = 2\pi\bar{y}A$

$= 2\pi\left(\frac{3}{8}a\right)\left(\frac{2}{3}ah\right)$

OR $V = \frac{1}{2}\pi a^2 h$ ◄

(4) CONE: THE GENERATING AREA IS A TRIANGLE

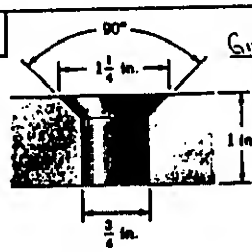


HAVE.. $V = 2\pi\bar{y}A$

$= 2\pi\left(\frac{a}{3}\right)\left(\frac{1}{2}ha\right)$

OR $V = \frac{1}{3}\pi a^2 h$ ◄

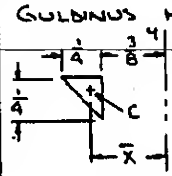
5.65



GIVEN: COUNTERSUNK HOLE SHOWN

FIND: VOLUME OF STEEL REMOVED DURING COUNTERSINKING PROCESS

THE REQUIRED VOLUME CAN BE GENERATED BY ROTATING THE AREA SHOWN ABOUT THE y AXIS. APPLYING THE SECOND THEOREM OF PAPPUS-GULDINUS HAVE..



$V = 2\pi\bar{x}A$

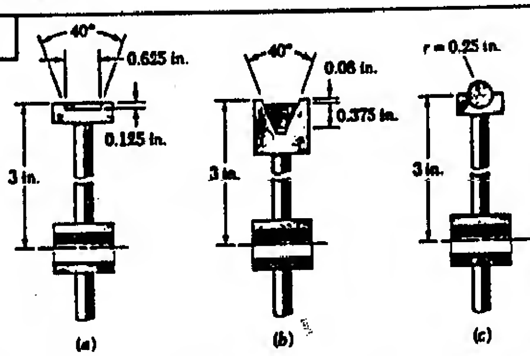
$= 2\pi\left[\frac{3}{8} + \frac{1}{3}\left(\frac{1}{4}\right)\right]\text{in.}$

$\times\left[\frac{1}{2}\times\frac{1}{4}\text{ in.}\times\frac{1}{4}\text{ in.}\right]$

OR $V = 0.0900\text{ in}^3$ ◄

ALL DIMENSIONS ARE IN INCHES

5.66



GIVEN: THREE DRIVE BELT PROFILES, EACH BELT CONTACTS ONE-HALF OF THE CIRCUMFERANCE OF ITS PULLEY

FIND: CONTACT AREA BETWEEN EACH BELT AND ITS PULLEY

APPLYING THE FIRST THEOREM OF PAPPUS-GULDINUS, THE CONTACT AREA A_c OF A BELT

(CONTINUED)

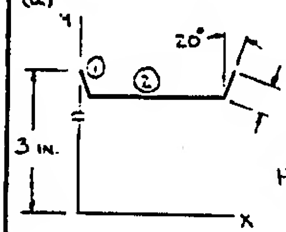
5.66 CONTINUED

IS GIVEN BY

$$A_c = \pi \bar{Y} L = \pi \Sigma \bar{y} L$$

WHERE THE INDIVIDUAL LENGTHS ARE THE LENGTHS OF THE BELT CROSS SECTION WHICH ARE IN CONTACT WITH THE PULLEY.

(a)

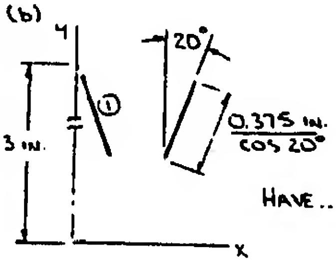


HAVE.. $A_c = \pi[2(\bar{y}_1 L_1) + \bar{y}_2 L_2]$

$$= \pi\left[2\left(3 - \frac{0.125}{2}\right)\left(\frac{0.125}{\cos 20°}\right) + (3 - 0.125)(0.625)\right]$$

OR $A_c = 8.10 \text{ in}^2$ ◀

(b)

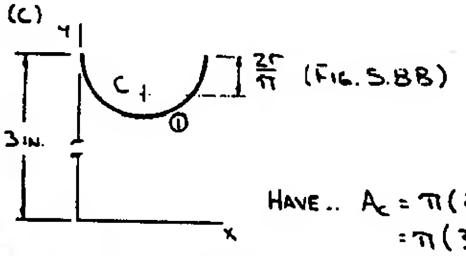


HAVE.. $A_c = \pi[2(\bar{y}_1 L_1)]$

$$= 2\pi\left(3 - 0.08 - \frac{0.375}{2}\right) \times \left(\frac{0.375}{\cos 20°}\right)$$

OR $A_c = 6.85 \text{ in}^2$ ◀

(c)

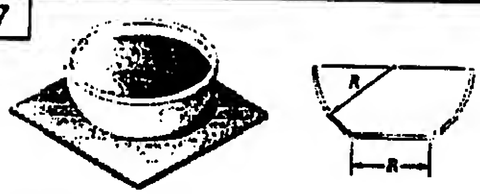


HAVE.. $A_c = \pi(\bar{y}_1 L_1)$

$$= \pi\left(3 - \frac{2 \times 0.25}{\pi}\right) \times (\pi \times 0.25)$$

OR $A_c = 7.01 \text{ in}^2$ ◀

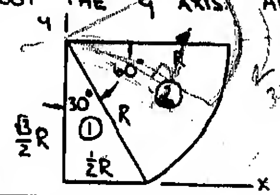
5.67



GIVEN: BOWL SHOWN, $R = 250$ mm

FIND: VOLUME V IN LITERS

THE VOLUME CAN BE GENERATED BY ROTATING THE TRIANGLE AND CIRCULAR SECTOR SHOWN ABOUT THE y AXIS. APPLYING THE SECOND THEOREM OF PAPPUS-GULDINUS AND USING FIG. 5.8A, HAVE..



(CONTINUED)

5.67 CONTINUED

$$V = 2\pi \bar{X} A = 2\pi \Sigma \bar{x} A = 2\pi(\bar{x}_1 A_1 + \bar{x}_2 A_2)$$

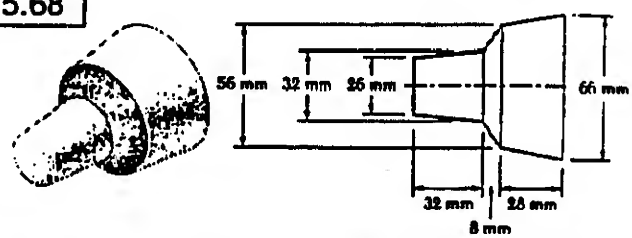
$$= 2\pi\left[\left(\tfrac{1}{3} \cdot \tfrac{1}{2}R\right)\left(\tfrac{1}{2} \cdot \tfrac{1}{2}R \cdot \tfrac{\sqrt{3}}{2}R\right) + \left(\frac{2R \sin 30°}{3 \cdot \frac{\pi}{6}} \cos 30°\right)\left(\frac{\pi}{6}R^2\right)\right]$$

$$= 2\pi\left(\frac{R^3}{16\sqrt{3}} + \frac{R^3}{2\sqrt{3}}\right) = \frac{3\sqrt{3}}{8}\pi R^3$$

$$= \frac{3\sqrt{3}}{8}\pi(0.25 \text{ m})^3 = 0.031\,883 \text{ m}^3 \times \frac{10^3 \, \ell}{1 \text{ m}^3}$$

OR $V = 31.9 \, \ell$ ◀

5.68



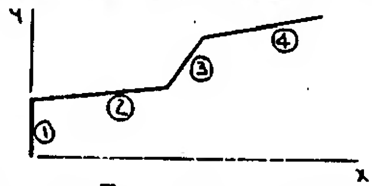
GIVEN: LAMP SHADE SHOWN, DENSITY $\rho = 2800 \frac{\text{kg}}{\text{m}^3}$, THICKNESS $t = 1$ mm

FIND: MASS m

THE MASS OF THE SHADE IS GIVEN BY

$$m = \rho V = \rho A t$$

WHERE A IS THE SURFACE AREA OF THE SHADE. THIS AREA CAN BE GENERATED BY ROTATING THE LINE SHOWN ABOUT THE x AXIS. APPLYING THE FIRST THEOREM OF PAPPUS-GULDINUS HAVE..



$$A = 2\pi \bar{Y} L = 2\pi \Sigma \bar{y} L = 2\pi(\bar{y}_1 L_1 + \bar{y}_2 L_2 + \bar{y}_3 L_3 + \bar{y}_4 L_4)$$

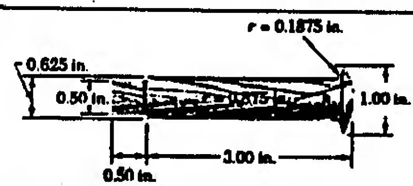
$$= 2\pi\left[\left(\tfrac{13}{2}\right)(13) + \left(\tfrac{13+16}{2}\right)\left(\sqrt{32^2 + 3^2}\right) + \left(\tfrac{16+28}{2}\right)\left(\sqrt{8^2 + 12^2}\right) + \left(\tfrac{28+33}{2}\right)\left(\sqrt{28^2 + 5^2}\right)\right]$$

$$= 2\pi(1735.33 \text{ mm}^2)$$

THEN.. $m = 2800 \frac{\text{kg}}{\text{m}^3} \times [2\pi(1735.33 \text{ mm}^2)] \times 1 \text{ mm} \times \frac{1 \text{ m}^3}{10^9 \text{ mm}^3}$

OR $m = 0.0305$ kg ◀

5.69



GIVEN: 20,000 PEGS HAVING SHAPE SHOWN, 2 COATS OF PAINT, 1 GALLON PAINT / 100 ft²

FIND: NUMBER OF GALLONS NEEDED

THE NUMBER OF GALLONS OF PAINT NEEDED IS GIVEN BY

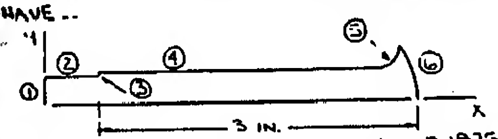
NUMBER OF GALLONS = (NUMBER OF PEGS)(SURFACE AREA OF 1 PEG)$\left(\frac{1 \text{ GALLON}}{100 \text{ ft}^2}\right)$ (2 COATS)

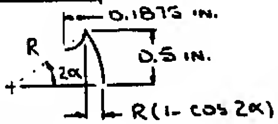
(CONTINUED)

5.69 CONTINUED

OR NUMBER OF GALLONS = 400 A_s $(A_s \sim ft^2)$

WHERE A_s IS THE SURFACE AREA OF ONE PEG. A_s CAN BE GENERATED BY ROTATING THE LINE SHOWN ABOUT THE X AXIS. USING THE FIRST THEOREM OF PAPPUS-GULDINUS AND FIG. 5.8B, HAVE..





R = 0.875 IN.

$\sin 2\alpha = \frac{0.5}{0.875}$

OR $2\alpha = 34.850°$ $\alpha = 17.425°$

$A_s = 2\pi \bar{Y} L = 2\pi \Sigma \bar{y} L$

	L, IN.	$\bar{y}$, IN.	$\bar{y}L$, IN^2
1	0.25	0.125	0.03125
2	0.5	0.25	0.125
3	0.0625	$\frac{0.25+0.3125}{2} = 0.28125$	0.0175781
4	$3 - 0.875(1-\cos 34.850°) - 0.1875 = 2.6556$	0.3125	0.82988
5	$\frac{\pi}{2} \cdot 0.1875 = 0.29452$	$0.5 - \frac{2 \times 0.1875}{\pi} = 0.38063$	0.112103
6	$2\alpha(0.875)$	$\frac{0.875 \sin 17.425°}{\alpha} = \sin 17.425°$	0.137314

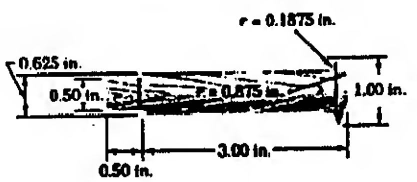
$\Sigma \bar{y} L = 1.25312\ IN^2$

THEN.. $A_s = 2\pi(1.25312\ IN^2) \times \frac{1\ ft^2}{144\ IN^2} = 0.054678\ ft^2$

FINALLY.. NUMBER OF GALLONS = 400 × 0.054678
= 21.87 GALLONS

∴ ORDER 22 GALLONS ◀

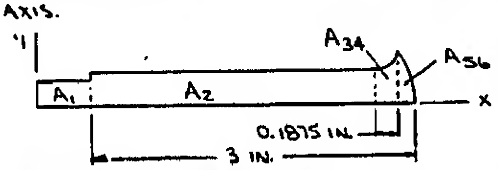
5.70



<u>GIVEN</u>: PEG HAVING THE SHAPE SHOWN, INITIAL SIZE OF DOWEL.. 1 IN. DIA. × 4 IN. LONG

<u>FIND</u>: PERCENT (VOLUME) OF DOWEL THAT BECOMES WASTE

TO OBTAIN THE SOLUTION IT IS FIRST NECESSARY TO DETERMINE THE VOLUME OF THE PEG. THAT VOLUME CAN BE GENERATED BY ROTATING THE AREA SHOWN ABOUT THE X AXIS.

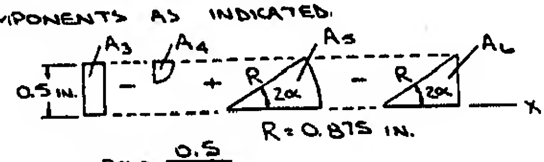


THE GENERATING AREA IS NEXT DIVIDED INTO SIX

(CONTINUED)

5.70 CONTINUED

COMPONENTS AS INDICATED.



$\sin 2\alpha = \frac{0.5}{0.875}$

OR $2\alpha = 34.850°$ $\alpha = 17.425°$

APPLYING THE SECOND THEOREM OF PAPPUS-GULDINUS AND THEN USING FIG. 5.8A, HAVE..

$V_{PEG} = 2\pi \bar{Y} A = 2\pi \Sigma \bar{y} A$

	A, IN^2	$\bar{y}$, IN.	$\bar{y}A$, IN^3
1	$0.5 \times 0.25 = 0.125$	0.125	0.015625
2	$[3 - 0.875(1-\cos 34.850°) - 0.1875] \times (0.3125) = 0.82987$	0.15625	0.129667
3	$0.1875 \times 0.5 = 0.09375$	0.25	0.023438
4	$-\frac{\pi}{4}(0.1875)^2 = -0.027612$	$0.5 - \frac{4 \times 0.1875}{3\pi} = 0.42042$	-0.011609
5	$\alpha(0.875)^2$	$\frac{2 \times 0.875 \sin 17.425°}{3\alpha} = \sin 17.425°$	0.04005
6	$-\frac{1}{2}(0.875 \cos 34.850°)(0.5) = -0.179517$	$\frac{1}{3}(0.5) = 0.166667$	-0.029920

$\Sigma \bar{y} L = 0.167252\ IN^3$

THEN.. $V_{PEG} = 2\pi(0.167252\ IN^3) = 1.05088\ IN^3$

NOW.. $V_{DOWEL} = \frac{\pi}{4}(\text{DIAMETER})^2(\text{LENGTH}) = \frac{\pi}{4}(1\ IN.)^2(4\ IN.)$
$= 3.14159\ IN^3$

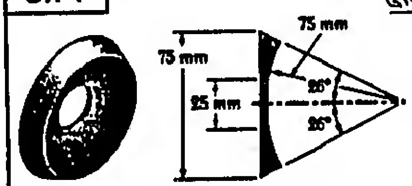
THEN.. % WASTE $= \frac{V_{WASTE}}{V_{DOWEL}} \times 100\%$

$= \frac{V_{DOWEL} - V_{PEG}}{V_{DOWEL}} \times 100\%$

$= \left(1 - \frac{1.05088}{3.14159}\right) \times 100\%$

OR % WASTE = 66.5% ◀

5.71



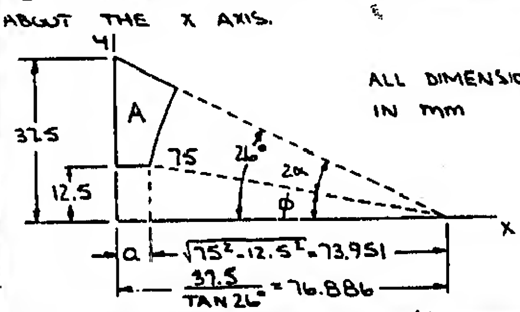
<u>GIVEN</u>: BRASS PLATE, DENSITY $\rho = 8470\ kg/m^3$

<u>FIND</u>: MASS m

THE MASS OF THE ESCUTCHEON IS GIVEN BY

$m = \rho V$

WHERE V IS THE VOLUME OF THE PLATE. V CAN BE GENERATED BY ROTATING THE AREA A ABOUT THE X AXIS.



ALL DIMENSIONS IN mm

(CONTINUED)

5.71 CONTINUED

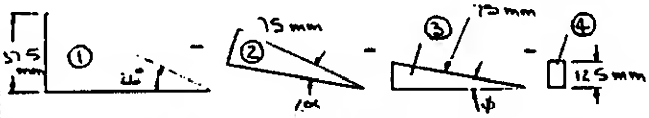
HAVE.. $a = 76.886 - 73.951 = 2.935$ mm

AND.. $\sin\phi = \frac{12.5}{75} \Rightarrow \phi = 9.5941^\circ$

THEN $2\alpha = 26^\circ - 9.5941^\circ = 16.4059^\circ$

AND $\alpha = 8.2030^\circ$

THE AREA A CAN BE OBTAINED BY COMBINED THE FOLLOWING FOUR AREAS AS INDICATED.



APPLYING THE SECOND THEOREM OF PAPPUS-GULDINUS AND THEN USING FIG. 5.8A, HAVE..

$$V = 2\pi \bar{Y}A = 2\pi \Sigma \bar{y}A$$

	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$\frac{1}{2} \times 76.886 \times 37.5 = 1441.61$	$\frac{1}{3}(37.5) = 12.5$	18 020.13
2	$-\alpha(75)^2$	$\frac{2(75)\sin 8.203^\circ}{3\alpha} \times \sin(8.203^\circ + 9.5941^\circ)$	$-12\ 265.30$
3	$-\frac{1}{2} \times 73.951 \times 12.5 = -462.19$	$\frac{1}{3}(12.5) = 4.1667$	-1925.81
4	$-2.935 \times 12.5 = -36.688$	$\frac{1}{2}(12.5) = 6.25$	-229.30

$\Sigma \bar{y}A = 3599.72$ mm^3

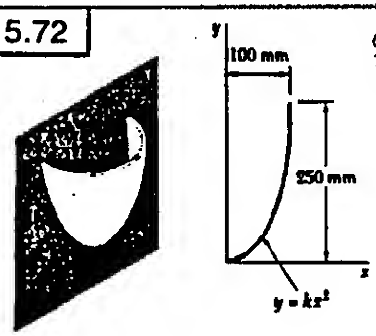
THEN.. $V = 2\pi(3599.72 \text{ mm}^3) = 22\ 617.7 \text{ mm}^3$

SO THAT $m = 8470 \frac{kg}{m^3} \times 22\ 617.7 \times 10^{-9} \text{ m}^3$

$= 0.1916$ kg

OR $m = 191.6$ g ◀

* 5.72

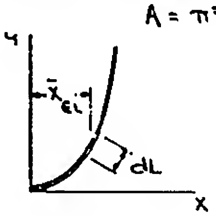


GIVEN: SHADE SHOWN

FIND: OUTER SURFACE AREA

FIRST NOTE THAT THE REQUIRED SURFACE AREA A CAN BE GENERATED BY ROTATING THE PARABOLIC CROSS SECTION THROUGH π RADIANS ABOUT THE y AXIS. APPLYING THE FIRST THEOREM OF PAPPUS-GULDINUS HAVE

$$A = \pi \bar{x} L$$



NOW.. AT $x = 100$ mm, $y = 250$ mm

$250 = k(100)^2$

OR $k = 0.025$ mm^{-1}

AND $\bar{x}_{EL} = x$

$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$

WHERE $\frac{dy}{dx} = 2kx$

THEN.. $dL = \sqrt{1 + 4k^2x^2}\,dx$

HAVE.. $\bar{x}L = \int \bar{x}_{EL}\,dL = \int_0^{100} x\left(\sqrt{1 + 4k^2x^2}\,dx\right)$

(CONTINUED)

5.72 CONTINUED

$\bar{x}L = \left[\frac{1}{3}\frac{1}{4k^2}(1 + 4k^2x^2)^{3/2}\right]_0^{100}$

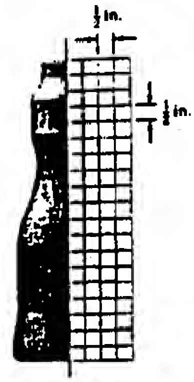
$= \frac{1}{12}\frac{1}{(0.025)^2}\left\{\left[1 + 4(0.025)^2(100)^2\right]^{3/2} - (1)^{3/2}\right\}$

$= 17\ 543.3$ mm^2

FINALLY... $A = \pi(17\ 543.3 \text{ mm}^2)$

OR $A = 55.1 \times 10^3$ mm^2 ◀

5.73



GIVEN: BOTTLE OF CROSS SECTION SHOWN, $W = 0.131$ lb, SPECIFIC WEIGHT $\gamma = 59.0$ lb/ft^3

FIND: AVERAGE WALL THICKNESS t

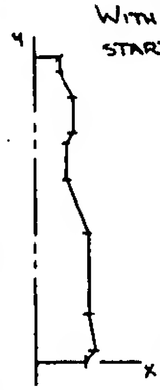
THE WEIGHT OF THE BOTTLE IS GIVEN BY

$$W = \gamma V = \gamma A_s t$$

WHERE A_s IS THE SURFACE AREA OF THE BOTTLE. A_s CAN BE GENERATED BY ROTATING THE CURVE BOUNDING THE CROSS SECTION ABOUT THE VERTICAL AXIS OF SYMMETRY. APPROXIMATING THE PORTION OF THIS CURVE TO THE RIGHT OF THE VERTICAL AXIS WITH A SERIES OF SHORT, STRAIGHT LINE SEGMENTS AND THEN APPROXIMATING THE LENGTH AND THE VALUE OF $\bar{x}$ FOR EACH SEGMENT USING THE GIVEN GRID, A_s IS THEN DETERMINED USING THE FIRST THEOREM OF PAPPUS-GULDINUS.

$$A_s = 2\pi \bar{X} L = 2\pi \Sigma \bar{x} L$$

WITH THE ELEVEN SEGMENTS NUMBERED STARTING AT THE TOP, HAVE..



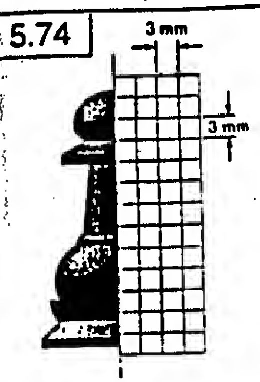
	L, in.	$\bar{x}$, in.	$\bar{x}L$, in^2
1	0.76	0.38	0.2888
2	0.48	0.76	0.3648
3	0.88	0.98	0.8624
4	1.06	1.20	1.272
5	0.36	1.08	0.3888
6	1.12	0.98	1.0976
7	1.78	1.32	2.3496
8	2.50	1.66	4.15
9	1.12	1.74	1.9488
10	0.48	1.68	0.8064
11	1.56	0.78	1.2168
Σ			14.7460

THEN.. $A_s = 2\pi(14.7460 \text{ in}^2) = 92.652 \text{ in}^2$

FINALLY.. $0.131 \text{ lb} = 59.0 \frac{lb}{ft^3} \times 92.652 \text{ in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3 t$

OR $t = 0.0414$ in. ◀

5.74



GIVEN: PAWN OF CROSS SECTION SHOWN, DENSITY $\rho = 7310\ \mathrm{kg/m^3}$

FIND: MASS m

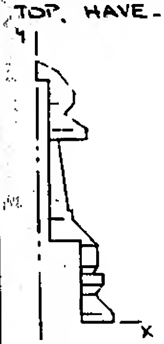
THE MASS OF THE PAWN IS GIVEN BY

$$m = \rho V$$

WHERE V IS THE VOLUME OF THE PEWTER. V CAN BE GENERATED BY ROTATING THE CROSS-SECTIONAL AREA OF THE PEWTER ABOUT THE VERTICAL AXIS OF SYMMETRY. APPROXIMATING THIS AREA WITH A TRIANGLE AND A SERIES OF RECTANGLES AND TRAPEZOIDS AND APPROXIMATING THE DIMENSIONS OF THESE ELEMENTS USING THE GIVEN GRID, V IS THEN DETERMINED USING THE SECOND THEOREM OF PAPPUS-GULDINUS.

$$V = 2\pi \bar{X} A = 2\pi \Sigma \bar{x} A$$

WITH THE AREAS TAKEN STARTING AT THE TOP, HAVE..



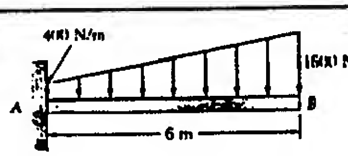
	A, mm²	$\bar{x}$, mm	$\bar{x}A$, mm³
1	$\frac{1}{2} \times 3 \times 1.5 = 2.25$	1.0	2.25
2	$\frac{1.05}{2}(3+3.9) = 3.62$	1.75	6.34
3	$\frac{1.8}{2}(3.9+3.6) = 6.75$	3.4	22.95
4	$3.6 \times 1.2 = 4.32$	3.3	14.26
5	$\frac{1.65}{2}(3.6+2.25) = 4.83$	3.0	14.49
6	$\frac{2.25}{2}(2.25+5.15) = 8.44$	3.5	29.54
7	$5.25 \times 1.2 = 6.3$	4.1	25.83
8	$\frac{9.90}{2}(1.35+2.55) = 19.31$	2.5	48.28
9	$3.15 \times 0.9 = 2.84$	3.1	6.80
10	$\frac{3.15}{2}(3.15+5.85) = 14.18$	3.8	53.88
11	$\frac{2.75}{2}(1.65+2.25) = 1.46$	6.7	9.78
12	$2.25 \times 2.85 = 6.41$	6.8	45.59
13	$\frac{1.05}{2}(2.25+1.65) = 2.05$	6.7	13.74
14	$3 \times 1.5 = 4.5$	7.2	32.40
15	$\frac{1.35}{2}(3+1.95) = 3.34$	6.95	23.21
16	$\frac{2.70}{2}(1.95+4.35) = 8.51$	7.35	62.55
17	$4.35 \times 1.20 = 5.22$	7.9	41.24
Σ	104.33		453.12

THEN.. $V = 2\pi(453.12\ \mathrm{mm^3}) = 2847.0\ \mathrm{mm^3}$

FINALLY.. $m = 7310\ \frac{\mathrm{kg}}{\mathrm{m^3}} \times 2847.0 \times 10^{-9}\ \mathrm{m^3}$

OR $m = 0.0208\ \mathrm{kg}$

5.75



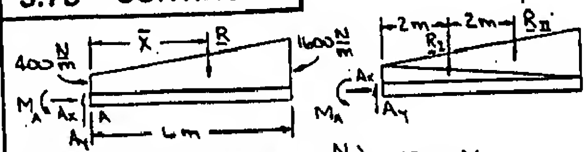
GIVEN: BEAM AND LOADING SHOWN

FIND: (a) RESULTANT R

(b) REACTIONS AT A

(CONTINUED)

5.75 CONTINUED



(a) HAVE.. $R_I = \frac{1}{2}(6\,\mathrm{m})(400\,\frac{\mathrm{N}}{\mathrm{m}}) = 1200\,\mathrm{N}$

$R_{II} = \frac{1}{2}(6\,\mathrm{m})(1600\,\frac{\mathrm{N}}{\mathrm{m}}) = 4800\,\mathrm{N}$

THEN.. ΣF_y: $-R = -R_I - R_{II}$

OR $R = 1200 + 4800 = 6000\,\mathrm{N}$

AND ΣM_A: $-\bar{X}(6000) = -2(1200) - 4(4800)$

OR $\bar{X} = 3.6\,\mathrm{m}$

$\therefore$ $\mathbf{R} = 6000\,\mathrm{N}\downarrow$, $\bar{X} = 3.6\,\mathrm{m}$

(b) REACTIONS

$\xrightarrow{+} \Sigma F_x = 0$: $A_x = 0$

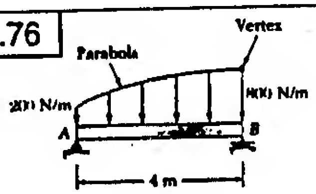
$+\uparrow \Sigma F_y = 0$: $A_y - 6000\,\mathrm{N} = 0$ $\quad A_y = 6000\,\mathrm{N}$

$\therefore$ $\mathbf{A} = 6000\,\mathrm{N}\uparrow$

$+\circlearrowleft \Sigma M_A = 0$: $M_A - (3.6\,\mathrm{m})(6000\,\mathrm{N}) = 0$

OR $\mathbf{M}_A = 21.6\,\mathrm{kN \cdot m}\circlearrowleft$

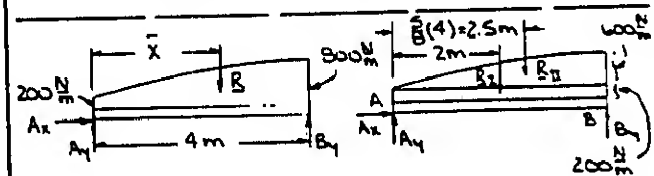
5.76



GIVEN: BEAM AND LOADING SHOWN

FIND: (a) RESULTANT R

(b) REACTIONS AT SUPPORTS



(a) HAVE.. $R_I = (4\,\mathrm{m})(200\,\frac{\mathrm{N}}{\mathrm{m}}) = 800\,\mathrm{N}$

$R_{II} = \frac{2}{3}(4\,\mathrm{m})(600\,\frac{\mathrm{N}}{\mathrm{m}}) = 1600\,\mathrm{N}$

THEN.. ΣF_y: $-R = -R_I - R_{II}$

OR $R = 800 + 1600 = 2400\,\mathrm{N}$

AND ΣM_A: $-\bar{X}(2400) = -2(800) - 2.5(1600)$

OR $\bar{X} = \frac{7}{3}\,\mathrm{m}$

$\therefore$ $\mathbf{R} = 2400\,\mathrm{N}\downarrow$, $\bar{X} = 2.33\,\mathrm{m}$

(b) REACTIONS

$\xrightarrow{+} \Sigma F_x = 0$: $A_x = 0$

$+\circlearrowleft \Sigma M_A = 0$: $(4\,\mathrm{m})B_y - (\frac{7}{3}\,\mathrm{m})(2400\,\mathrm{N}) = 0$

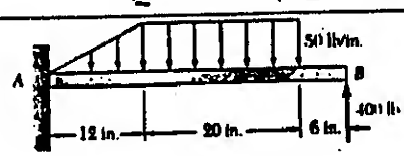
OR $B_y = 1400\,\mathrm{N}$

$+\uparrow \Sigma F_y = 0$: $A_y + 1400\,\mathrm{N} - 2400\,\mathrm{N} = 0$

OR $A_y = 1000\,\mathrm{N}$

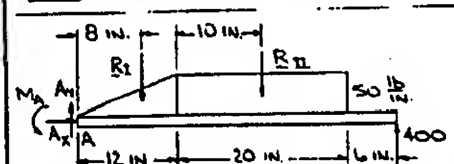
$\therefore$ $\mathbf{A} = 1000\,\mathrm{N}\uparrow$, $\mathbf{B} = 1400\,\mathrm{N}\uparrow$

5.77



GIVEN: BEAM AND LOADING SHOWN

FIND: REACTIONS AT A



(CONTINUED)

5.77 CONTINUED

HAVE.. $R_I = \frac{1}{2}(12\text{ in.})(50\ \frac{lb}{in.}) = 300\text{ lb}$

$R_{II} = (20\text{ in.})(50\ \frac{lb}{in.}) = 1000\text{ lb}$

THEN. $\xrightarrow{+}\Sigma F_x = 0$: $A_x = 0$

$+\uparrow\Sigma F_y = 0$: $A_y - 300\text{ lb} - 1000\text{ lb} + 400\text{ lb} = 0$

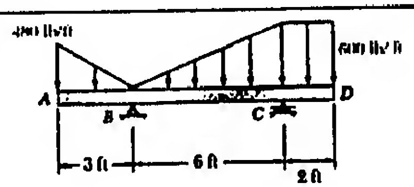
OR $A_y = 900\text{ lb}$ $\quad \underline{A} = 900\text{ lb}\uparrow$ ◀

$+\circlearrowleft\Sigma M_A = 0$: $M_A - (8\text{ in.})(300\text{ lb}) - (22\text{ in.})(1000\text{ lb})$
$+ (38\text{ in.})(400\text{ lb}) = 0$

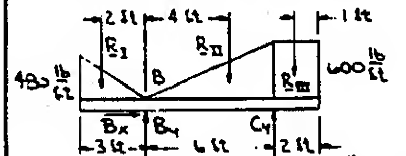
OR $M_A = 9200\text{ lb}\cdot\text{in}$

$\underline{M_A} = 9200\text{ lb}\cdot\text{in.}\circlearrowleft$ ◀

5.78



GIVEN: BEAM AND LOADING SHOWN
FIND: REACTIONS AT SUPPORTS



HAVE.. $R_I = \frac{1}{2}(3\text{ ft})(480\ \frac{lb}{ft}) = 720\text{ lb}$

$R_{II} = \frac{1}{2}(6\text{ ft})(600\ \frac{lb}{ft}) = 1800\text{ lb}$

$R_{III} = (2\text{ ft})(600\ \frac{lb}{ft}) = 1200\text{ lb}$

THEN.. $\xrightarrow{+}\Sigma F_x = 0$: $B_x = 0$

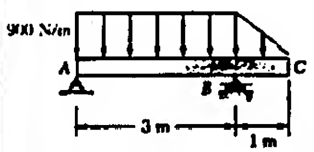
$+\circlearrowleft\Sigma M_B = 0$: $(2\text{ ft})(720\text{ lb}) - (4\text{ ft})(1800\text{ lb})$
$+ (6\text{ ft})C_y - (7\text{ ft})(1200\text{ lb}) = 0$

OR $C_y = 2360\text{ lb}$ $\quad \underline{C} = 2360\text{ lb}\uparrow$ ◀

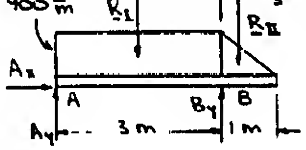
$+\uparrow\Sigma F_y = 0$: $-720\text{ lb} + B_y - 1800\text{ lb} + 2360\text{ lb}$
$- 1200\text{ lb} = 0$

OR $B_y = 1360\text{ lb}$ $\quad \underline{B} = 1360\text{ lb}\uparrow$ ◀

5.79



GIVEN: BEAM AND LOADING SHOWN
FIND: REACTIONS AT SUPPORTS



$R_I = (3\text{ m})(900\ \frac{N}{m})$
$= 2700\text{ N}$

$R_{II} = \frac{1}{2}(1\text{ m})(900\ \frac{N}{m})$
$= 450\text{ N}$

NOW.. $\xrightarrow{+}\Sigma F_x = 0$: $A_x = 0$

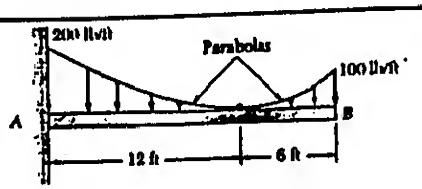
$+\circlearrowleft\Sigma M_B = 0$: $-(3\text{ m})A_y + (1.5\text{ m})(2700\text{ N})$
$- (\frac{1}{3}\text{ m})(450\text{ N}) = 0$

OR $A_y = 1300\text{ N}$ $\quad \underline{A} = 1300\text{ N}\uparrow$ ◀

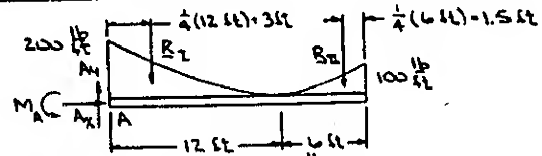
$+\uparrow\Sigma F_y = 0$: $1300\text{ N} - 2700\text{ N} + B_y - 450\text{ N} = 0$

OR $B_y = 1850\text{ N}$ $\quad \underline{B} = 1850\text{ N}\uparrow$ ◀

5.80



GIVEN: BEAM AND LOADING SHOWN
FIND: REACTIONS AT A



HAVE.. $R_I = \frac{1}{3}(12\text{ ft})(200\ \frac{lb}{ft}) = 800\text{ lb}$

$R_{II} = \frac{1}{3}(6\text{ ft})(100\ \frac{lb}{ft}) = 200\text{ lb}$

THEN.. $\xrightarrow{+}\Sigma F_x = 0$: $A_x = 0$

$+\uparrow\Sigma F_y = 0$: $A_y - 800\text{ lb} - 200\text{ lb} = 0$

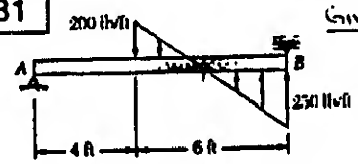
OR $A_y = 1000\text{ lb}$ $\quad \underline{A} = 1000\text{ lb}\uparrow$ ◀

$+\circlearrowleft\Sigma M_A = 0$: $M_A - (3\text{ ft})(800\text{ lb}) - (16.5\text{ ft})(200\text{ lb}) = 0$

OR $M_A = 5700\text{ lb}\cdot\text{ft}$

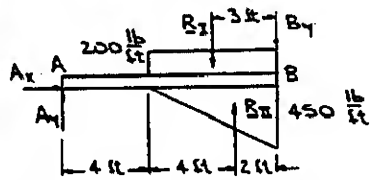
$\underline{M_A} = 5700\text{ lb}\cdot\text{ft}\circlearrowleft$ ◀

5.81



GIVEN: BEAM AND LOADING SHOWN
FIND: REACTIONS AT SUPPORTS

FIRST REPLACE THE GIVEN LOADING WITH THE LOADING SHOWN BELOW. THE TWO LOADINGS ARE EQUIVALENT BECAUSE BOTH ARE DEFINED BY A LINEAR RELATION BETWEEN LOAD AND DISTANCE AND THE VALUES AT THE END POINTS ARE THE SAME.



HAVE -- $R_I = (6\text{ ft})(200\ \frac{lb}{ft}) = 1200\text{ lb}$

$R_{II} = \frac{1}{2}(6\text{ ft})(450\ \frac{lb}{ft}) = 1350\text{ lb}$

THEN.. $\xrightarrow{+}\Sigma F_x = 0$: $A_x = 0$

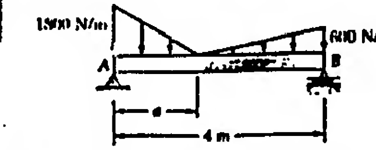
$+\circlearrowleft\Sigma M_B = 0$: $-(10\text{ ft})A_y + (3\text{ ft})(1200\text{ lb})$
$- (2\text{ ft})(1350\text{ lb}) = 0$

OR $A_y = 90\text{ lb}$ $\quad \underline{A} = 90\text{ lb}\uparrow$ ◀

$+\uparrow\Sigma F_y = 0$: $90\text{ lb} - 1200\text{ lb} + 1350\text{ lb} - B_y = 0$

OR $B_y = 240\text{ lb}$ $\quad \underline{B} = 240\text{ lb}\downarrow$ ◀

5.86

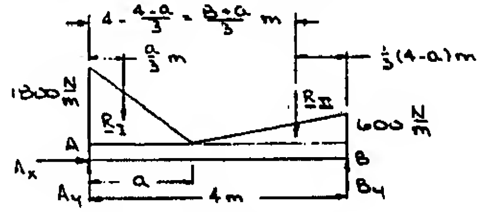


GIVEN: BEAM AND LOADING SHOWN

FIND: (a) a SO THAT B_y IS MINIMUM

(b) REACTIONS AT SUPPORTS

(a)



HAVE.. $R_I = \frac{1}{2}(a\ \text{m})(1800\ \frac{N}{m}) = 900a\ \text{N}$

$R_{II} = \frac{1}{2}[(4-a)\text{m}](600\ \frac{N}{m}) = 300(4-a)\ \text{N}$

THEN. $\circlearrowleft \Sigma M_A = 0: -(\frac{a}{3}\text{m})(900a\ \text{N})$
$- (\frac{8+a}{3}\text{m})[300(4-a)\text{N}] + (4\text{m})B_y = 0$

OR $B_y = 50a^2 - 100a + 800$ (1)

THEN.. $\frac{dB_y}{da} = 100a - 100 = 0$

OR $a = 1.00\text{m}$ ◀

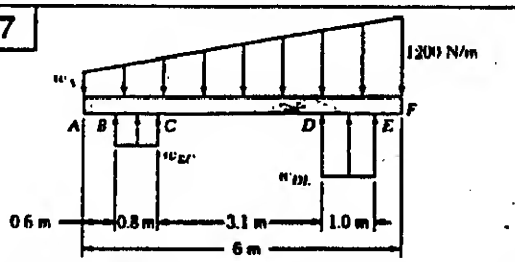
(b) EQ. (1).. $B_y = 50(1)^2 - 100(1) + 800$
$= 750\ \text{N}$ $\quad \mathbf{B} = 750\ \text{N}\uparrow$ ◀

AND.. $\xrightarrow{+} \Sigma F_x = 0$ $\quad A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y - 900(1)\text{N} - 300(4-1)\text{N}$
$+ 750\ \text{N} = 0$

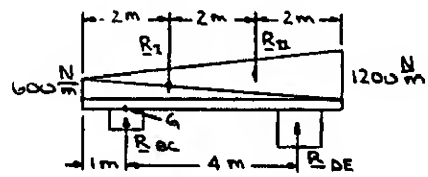
OR $A_y = 1050\ \text{N}$ $\quad \mathbf{A} = 1050\ \text{N}\uparrow$ ◀

5.87



GIVEN: BEAM AND LOADING SHOWN, $w_A = 600\ \frac{N}{m}$

FIND: w_{BC} AND w_{DE}



HAVE. $R_I = \frac{1}{2}(6\text{m})(600\ \frac{N}{m}) = 1800\ \text{N}$

$R_{II} = \frac{1}{2}(6\text{m})(1200\ \frac{N}{m}) = 3600\ \text{N}$

$R_{BC} = (0.8\text{m})(w_{BC}\ \frac{N}{m}) = (0.8 w_{BC})\ \text{N}$

$R_{DE} = (1.0\text{m})(w_{DE}\ \frac{N}{m}) = (w_{DE})\ \text{N}$

THEN. $\circlearrowleft \Sigma M_G = 0: -(1\text{m})(1800\text{N}) - (3\text{m})(3600\text{N})$
$+ (4\text{m})(w_{DE}\ \text{N}) = 0$

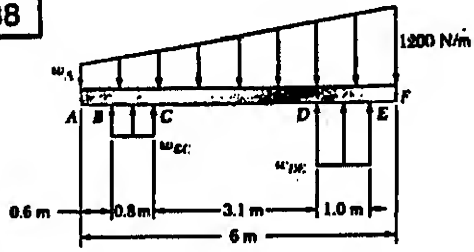
OR $w_{DE} = 3150\ \frac{N}{m}$ ◀

AND.. $+\uparrow \Sigma F_y = 0$: $(0.8 w_{BC})\text{N} - 1800\ \text{N} - 3600\ \text{N}$
$+ 3150\ \text{N} = 0$

OR $w_{BC} = 2812.5\ \frac{N}{m}$

$w_{BC} = 2810\ \frac{N}{m}$ ◀

5.88

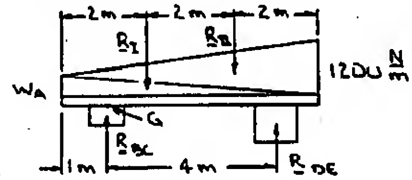


GIVEN: BEAM AND LOADING SHOWN

FIND: (a) w_A SO THAT $w_{BC} = w_{DE}$

(b) w_{BC} AND w_{DE}

(a)



HAVE.. $R_I = \frac{1}{2}(6\text{m})(w_A\ \frac{N}{m}) = (3w_A)\ \text{N}$

$R_{II} = \frac{1}{2}(6\text{m})(1200\ \frac{N}{m}) = 3600\ \text{N}$

$R_{BC} = (0.8\text{m})(w_{BC}\ \frac{N}{m}) = (0.8 w_{BC})\ \text{N}$

$R_{DE} = (1\text{m})(w_{DE}\ \frac{N}{m}) = (w_{DE})\ \text{N}$

THEN.. $+\uparrow \Sigma F_y = 0$: $(0.8 w_{BC})\text{N} - (3w_A)\text{N} - 3600\ \text{N}$
$+ (w_{DE})\text{N} = 0$

OR $0.8 w_{BC} + w_{DE} = 3600 + 3w_A$

NOW.. $w_{BC} = w_{DE} \Rightarrow w_{BC} = w_{DE} = 2000 + \frac{5}{3}w_A$ (1)

ALSO.. $\circlearrowleft \Sigma M_G = 0: -(1\text{m})(3w_A\ \text{N}) - (3\text{m})(3600\ \text{N})$
$+ (4\text{m})(w_{DE}\ \text{N}) = 0$

OR $w_{DE} = 2700 + \frac{3}{4}w_A$ (2)

EQUATING EQS. (1) AND (2)..

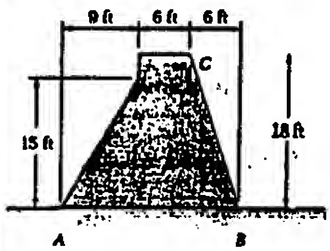
$2000 + \frac{5}{3}w_A = 2700 + \frac{3}{4}w_A$

OR $w_A = \frac{8400}{11}\ \frac{N}{m}$ $\quad w_A = 764\ \frac{N}{m}$ ◀

(b) EQ. (1) ⇒ $w_{BC} = w_{DE} = 2000 + \frac{5}{3}(\frac{8400}{11})$

OR $w_{BC} = w_{DE} = 3270\ \frac{N}{m}$ ◀

5.89

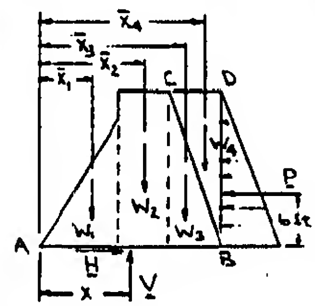


GIVEN: DAM CROSS SECTION SHOWN, WIDTH = 1 ft

FIND: (a) REACTION FORCES EXERTED ON BASE OF DAM

(b) POINT OF APPLICATION OF REACTION FORCES

(c) RESULTANT FORCE ON FACE OF DAM



THE FREE BODY SHOWN CONSISTS OF A 1-ft THICK SECTION OF THE DAM AND THE TRIANGULAR SECTION BCD OF WATER ABOVE THE DAM.

NOTE: $\bar{x}_1 = 6\ \text{ft}$

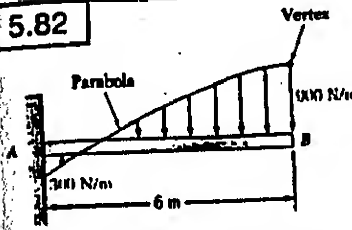
$\bar{x}_2 = (9+3)\text{ft} = 12\ \text{ft}$

$\bar{x}_3 = (15+2)\text{ft} = 17\ \text{ft}$

$\bar{x}_4 = (15+4)\text{ft} = 19\ \text{ft}$

(CONTINUED)

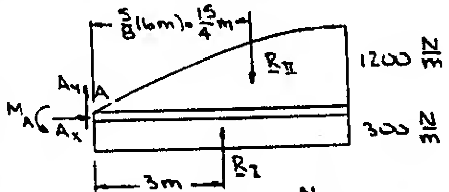
5.82



GIVEN: BEAM AND LOADING SHOWN

FIND: REACTIONS AT A

FIRST REPLACE THE GIVEN LOADING WITH THE LOADING SHOWN BELOW. THE TWO LOADINGS ARE EQUIVALENT BECAUSE BOTH ARE DEFINED BY A PARABOLIC RELATION BETWEEN LOAD AND DISTANCE AND THE VALUES AT THE END POINTS ARE THE SAME.



HAVE.. $R_I = (6\,m)(300\,\frac{N}{m}) = 1800\ N$

$R_{II} = \frac{2}{3}(6\,m)(1200\,\frac{N}{m}) = 4800\ N$

THEN.. $\xrightarrow{+}\Sigma F_x = 0$: $A_x = 0$

$+\uparrow\Sigma F_y = 0$: $A_y + 1800N - 4800N = 0$

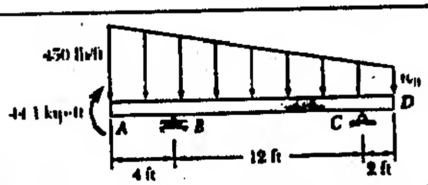
OR $A_y = 3000\ N$ $\quad \underline{A} = 3000 N \uparrow$

$+\circlearrowleft\Sigma M_A = 0$: $M_A + (3m)(1800N) - (\frac{15}{4}m)(4800N) = 0$

OR $M_A = 12.6\ kN\cdot m$

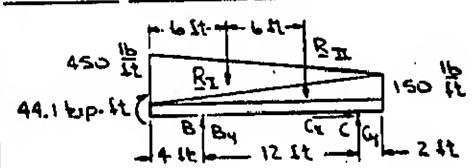
$\underline{M}_A = 12.6\ kN\cdot m \circlearrowleft$

5.83



GIVEN: BEAM AND LOADING SHOWN, $w_0 = 150\ \frac{lb}{ft}$

FIND: REACTIONS AT SUPPORTS



HAVE.. $R_I = \frac{1}{2}(18\,ft)(450\,\frac{lb}{ft}) = 4050\ lb$

$R_{II} = \frac{1}{2}(18\,ft)(150\,\frac{lb}{ft}) = 1350\ lb$

THEN.. $\xrightarrow{+}\Sigma F_x = 0$: $C_x = 0$

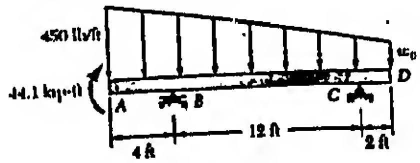
$+\circlearrowleft\Sigma M_B = 0$: $-(44{,}100\ kip\cdot ft) - (2ft)(4050 lb) - (8ft)(1350 lb) + (12ft)C_y = 0$

OR $C_y = 5250\ lb$ $\quad \underline{C} = 5250 lb \uparrow$

$+\uparrow\Sigma F_y = 0$: $B_y - 4050\ lb - 1350\ lb + 5250\ lb = 0$

OR $B_y = 150\ lb$ $\quad \underline{B} = 150 lb \uparrow$

5.84

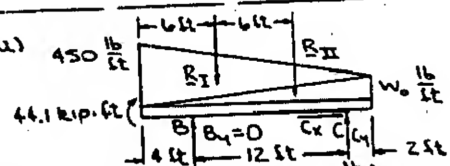


GIVEN: BEAM AND LOADING SHOWN

FIND: (a) w_0 SO THAT $\underline{B} = 0$

(b) REACTION AT C

(a)



HAVE.. $R_I = \frac{1}{2}(18ft)(450\,\frac{lb}{ft}) = 4050\ lb$

$R_{II} = \frac{1}{2}(18ft)(w_0\,\frac{lb}{ft}) = 9w_0\ lb$

THEN.. $+\circlearrowleft\Sigma M_C = 0$: $-(44{,}100\ lb\cdot ft) + (10\,ft)(4050\,lb) + (4\,ft)(9w_0\,lb) = 0$

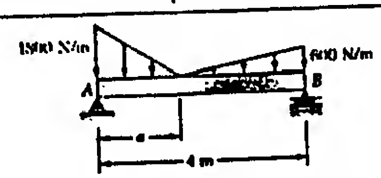
OR $w_0 = 100\ \frac{lb}{ft}$

(b) $\xrightarrow{+}\Sigma F_x = 0$: $C_x = 0$

$+\uparrow\Sigma F_y = 0$: $-4050\,lb - (9\times100)lb + C_y = 0$

OR $C_y = 4950\,lb$ $\quad \underline{C} = 4950 lb \uparrow$

5.85

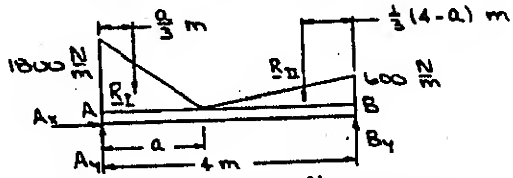


GIVEN: BEAM AND LOADING SHOWN

FIND: (a) a SO THAT $A_y = B_y$

(b) REACTIONS AT SUPPORTS

(a)



HAVE.. $R_I = \frac{1}{2}(a\,m)(1800\,\frac{N}{m}) = 900a\ N$

$R_{II} = \frac{1}{2}[(4-a)m](600\,\frac{N}{m}) = 300(4-a)\ N$

THEN.. $+\uparrow\Sigma F_y = 0$: $A_y - 900a - 300(4-a) + B_y = 0$

OR $A_y + B_y = 1200 + 600a$

NOW $A_y = B_y \Rightarrow A_y = B_y = 600 + 300a\ (N)$ (1)

ALSO.. $+\circlearrowleft\Sigma M_B = 0$: $-(4m)A_y + [(4 - \frac{a}{3})m][(900a)N] + [\frac{1}{3}(4-a)m][300(4-a)N] = 0$

OR $A_y = 400 + 700a - 50a^2$ (2)

EQUATING EQS. (1) AND (2)

$600 + 300a = 400 + 700a - 50a^2$

OR $a^2 - 8a + 4 = 0$

THEN.. $a = \dfrac{8 \pm \sqrt{(-8)^2 - 4(1)(4)}}{2}$

OR $a = 0.53590\,m$ $\quad a = 7.4641\,m$

NOW $a \le 4\,m \Rightarrow$ $\quad a = 0.536\,m$

(b) HAVE.. $\xrightarrow{+}\Sigma F_x = 0$: $A_x = 0$

EQ. (1).. $A_y = B_y = 600 + 300(0.53590)$

$= 761\ N$

$\therefore$ $\quad \underline{A} = \underline{B} = 761\ N\uparrow$

5.89 CONTINUED

(a) Now.. $W = \gamma V$ so that

$W_1 = (150\ \frac{lb}{ft^3})[\frac{1}{2}(9\ ft)(15\ ft)(1\ ft)] = 10,125\ lb$

$W_2 = (150\ \frac{lb}{ft^3})[(6\ ft)(18\ ft)(1\ ft)] = 16,200\ lb$

$W_3 = (150\ \frac{lb}{ft^3})[\frac{1}{2}(6\ ft)(18\ ft)(1\ ft)] = 8100\ lb$

$W_4 = (62.4\ \frac{lb}{ft^3})[\frac{1}{2}(6\ ft)(18\ ft)(1\ ft)] = 3369.6\ lb$

Also.. $P = \frac{1}{2}Ap = \frac{1}{2}[(18\ ft)(1\ ft)][(62.4\ \frac{lb}{ft^3})(18\ ft)]$
$= 10,108.8\ lb$

Then.. $\xrightarrow{+} \Sigma F_x = 0$: $H - 10,108.8\ lb = 0$

or $\underline{H} = 10.11\ kips \longrightarrow$ ◀

$+\!\uparrow \Sigma F_y = 0$: $V - 10,125\ lb - 16,200\ lb - 8100\ lb - 3369.6\ lb = 0$

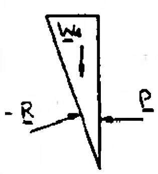
or $V = 37,794.6\ lb$ $\quad \underline{V} = 37.8\ kips \uparrow$ ◀

(b) Have.. $+\circlearrowleft \Sigma M_A = 0$: $x(37,794.6\ lb) - (6\ ft)(10,125\ lb) - (12\ ft)(16,200\ lb) - (17\ ft)(8100\ lb) - (19\ ft)(3369.6\ lb) + (6\ ft)(10,108.8\ lb) = 0$

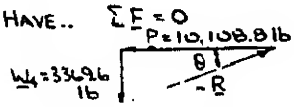
or.. $37,794.6x - 60,750 - 194,400 - 137,700 - 64,022.4 + 60,652.8 = 0$

or $x = 10.48\ ft$ ◀

(c) Consider water section BCD as the free body



Have.. $\Sigma F = 0$

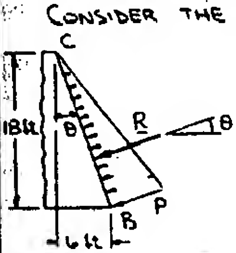


Then.. $-\underline{R} = 10.66\ kips \measuredangle 18.43°$

or $\underline{R} = 10.66\ kips \measuredangle 18.43°$ ◀

ALTERNATIVE SOLUTION

Consider the face BC of the dam.



Have.. $BC = \sqrt{6^2 + 18^2} = 18.9737\ ft$

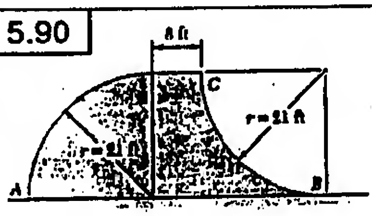
$\tan\theta = \frac{6}{18}$ $\quad \theta = 18.43°$

and.. $p = \gamma h = 62.4\ \frac{lb}{ft^3} \times 18\ ft$
$= 1123.2\ \frac{lb}{ft^2}$

Then.. $R = \frac{1}{2}Ap$
$= \frac{1}{2}[(18.9737\ ft)(1\ ft)] \times (1123.2\ \frac{lb}{ft^2})$
$= 10,655.6\ lb$

$\therefore \underline{R} = 10.66\ kips \measuredangle 18.43°$ ◀

5.90



Given: Dam cross section shown, width = 1 ft

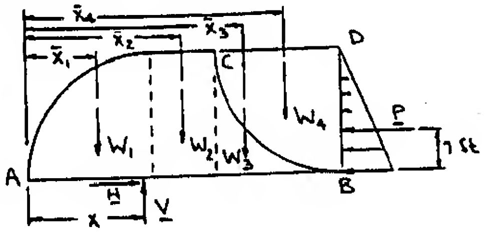
Find: (a) Reaction forces exerted on base of dam

(b) Point of application of reaction forces

(c) Resultant force on face of dam

The free body shown (top of next column) consists of a 1-ft thick section of the dam and the quarter circular section of water above the dam. (continued)

5.90 CONTINUED

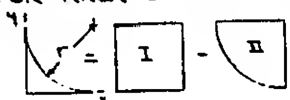


Note: $\bar{x}_1 = (21 - \frac{4 \times 21}{3\pi})\ ft = 12.0873\ ft$

$\bar{x}_2 = (21 + 4)\ ft = 25\ ft$

$\bar{x}_4 = (50 - \frac{4 \times 21}{3\pi})\ ft = 41.087\ ft$

For area 3 first note..



	A	$\bar{x}$
I	r^2	$\frac{1}{2}r$
II	$-\frac{\pi}{4}r^2$	$r - \frac{4r}{3\pi}$

Then.. $\bar{x}_3 = 29\ ft + [\frac{\frac{1}{2}(21)(21)^2 + (21 - \frac{4 \times 21}{3\pi})(-\frac{\pi}{4} \times 21^2)}{(21)^2 - \frac{\pi}{4}(21)^2}]\ ft$

$= (29 + 4.6907)\ ft = 33.691\ ft$

(a) Now.. $W = \gamma V$ so that

$W_1 = (150\ \frac{lb}{ft^3})[\frac{\pi}{4}(21\ ft)^2(1\ ft)] = 51,954\ lb$

$W_2 = (150\ \frac{lb}{ft^3})[(8\ ft)(21\ ft)(1\ ft)] = 25,200\ lb$

$W_3 = (150\ \frac{lb}{ft^3})[(21^2 - \frac{\pi}{4} \cdot 21^2)\ ft^2 \times (1\ ft)] = 14,196\ lb$

$W_4 = (62.4\ \frac{lb}{ft^3})[\frac{\pi}{4}(21\ ft)^2(1\ ft)] = 21,613\ lb$

Also $P = \frac{1}{2}Ap = \frac{1}{2}[(21\ ft)(1\ ft)][(62.4\ \frac{lb}{ft^3})(21\ ft)]$
$= 13,759\ lb$

Then.. $\xrightarrow{+} \Sigma F_x = 0$: $H - 13,759\ lb = 0$

or $\underline{H} = 13.76\ kips \longrightarrow$ ◀

$+\!\uparrow \Sigma F_y = 0$: $V - 51,954\ lb - 25,200\ lb - 14,196\ lb - 21,613\ lb = 0$

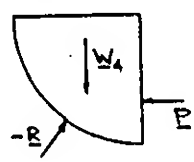
or $V = 112,963\ lb$ $\quad \underline{V} = 113.0\ kips \uparrow$ ◀

(b) Have.. $+\circlearrowleft \Sigma M_A = 0$: $x(112,963\ lb) - (12.0873\ ft)(51,954\ lb) - (25\ ft)(25,200\ lb) - (33.691\ ft)(14,196\ lb) - (41.087\ ft)(21,613\ lb) + (7\ ft)(13,759\ lb) = 0$

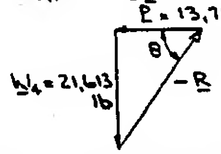
or $112,963x - 627,980 - 630,000 - 478,280 - 888,010 + 96,313 = 0$

or $x = 22.4\ ft$ ◀

(c) Consider water section BCD as the free body



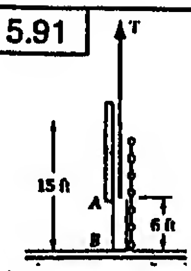
Have.. $\Sigma F = 0$



Then.. $-\underline{R} = 25.6\ kips \measuredangle 57.5°$

or $\underline{R} = 25.6\ kips \measuredangle 57.5°$ ◀

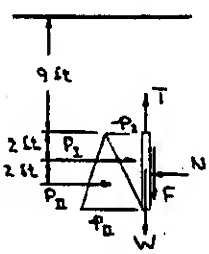
5.91



GIVEN: 6×6-ft GATE, W = 1000 lb, FRICTION FORCE F = 0.1 × RESULTANT PRESSURE FORCE P, $\gamma = 62.4\ \frac{lb}{ft^3}$

FIND: T

CONSIDER THE FREE-BODY DIAGRAM OF THE GATE. NOW..



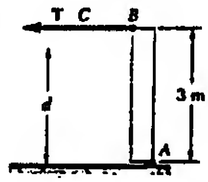
$P_I = \frac{1}{2}A p_I = \frac{1}{2}[(6\times6)ft^2][(62.4\ \frac{lb}{ft^3})(9\ ft)]$
$= 10,108.8$ lb

$P_{II} = \frac{1}{2}A p_{II} = \frac{1}{2}[(6\times6)ft^2][(62.4\ \frac{lb}{ft^3})(15\ ft)]$
$= 16,848$ lb

THEN.. $F = 0.1P = 0.1(P_I + P_{II})$
$= 0.1(10,108.8 + 16,848)$ lb
$= 2695.7$ lb

FINALLY..
$+\uparrow\Sigma F_y = 0$: $T - 2695.7\ lb - 1000\ lb = 0$
OR $\underline{T = 3.70\ kips}$

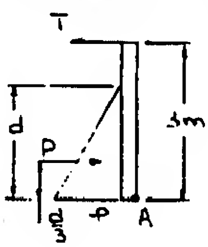
5.92



GIVEN: 3×4-m SIDE, $T_{MAX} = 0.2(200\ kN)$, $\rho = 10^3\ \frac{kg}{m^3}$

FIND: d_{MAX}

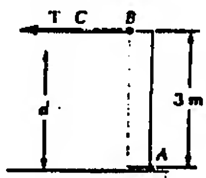
CONSIDER THE FREE-BODY DIAGRAM OF THE SIDE.



HAVE.. $P = \frac{1}{2}Ap = \frac{1}{2}A(\rho g d)$
NOW.. $\circlearrowright\Sigma M_A = 0$: $hT - \frac{d}{3}P = 0$
WHERE $h = 3$ m
THEN FOR d_{MAX}:
$(3\,m)(0.2\times200\times10^3\,N)$
$-\frac{d_{max}}{3}[\frac{1}{2}(4\,m\times d_{max})\times(10^3\,\frac{kg}{m^3}\times9.81\,\frac{m}{s^2}\times d_{max})] = 0$

OR $120\ N\cdot m - 6.54\ d_{max}^3\ \frac{N}{m^2} = 0$
OR $\underline{d_{MAX} = 2.64\,m}$

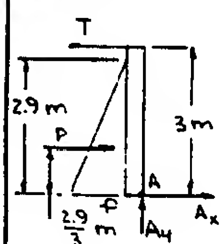
5.93



GIVEN 3×4-m SIDE, $\rho = 1263\ \frac{kg}{m^3}$, $d = 2.9$ m

FIND: T, REACTION AT A

CONSIDER THE FREE-BODY DIAGRAM OF THE SIDE.



HAVE.. $P = \frac{1}{2}Ap = \frac{1}{2}A(\rho g d)$
$= \frac{1}{2}[(2.9\,m)(4\,m)]$
$\cdot[(1263\,\frac{kg}{m^3})(9.81\,\frac{m}{s^2})(2.9\,m)]$
$= 208.40$ kN

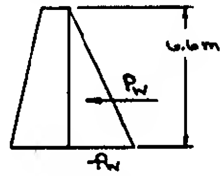
THEN.. $+\uparrow\Sigma F_y = 0$: $A_y = 0$
$\circlearrowright\Sigma M_A = 0$: $(3\,m)T - (\frac{2.9}{3}\,m)(208.4\,kN) = 0$
OR $T = 67.151$ kN $\underline{T = 67.2\ kN \leftarrow}$
$\xrightarrow{+}\Sigma F_x = 0$: $A_x + 208.40\,kN - 67.151\,kN = 0$
OR $A_x = -141.249$ kN $\underline{A = 141.2\ kN \leftarrow}$

5.94

GIVEN: $\rho_s = 1.76\times10^3\ \frac{kg}{m^3}$, WIDTH = 1 m, $d_s = 2$ m

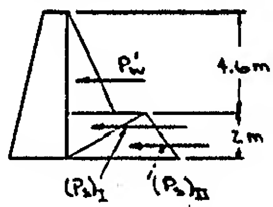
FIND: PERCENTAGE INCREASE OF FORCE ON DAM FACE BECAUSE OF SILT

FIRST DETERMINE THE FORCE ON THE DAM FACE WITHOUT THE SILT. HAVE..



$P_w = \frac{1}{2}A p_w = \frac{1}{2}A(\rho g h)$
$= \frac{1}{2}[(6.6\,m)(1\,m)]$
$\cdot[(10^3\,\frac{kg}{m^3})(9.81\,\frac{m}{s^2})(6.6\,m)]$
$= 213.66$ kN

NEXT DETERMINE THE FORCE ON THE DAM FACE WITH THE SILT. HAVE..



$P'_w = \frac{1}{2}[(4.6\,m)(1\,m)]$
$\cdot[(10^3\,\frac{kg}{m^3})(9.81\,\frac{m}{s^2})(4.6\,m)]$
$= 103.79$ kN

$(P_s)_I = \frac{1}{2}[(2\,m)(1\,m)][(1.76\times10^3\,\frac{kg}{m^3})$
$\cdot(9.81\,\frac{m}{s^2})(4.6\,m)]$
$= 79.42$ kN

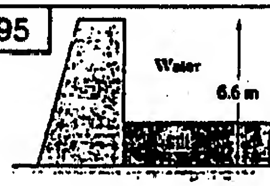
$(P_s)_{II} = \frac{1}{2}[(2\,m)(1\,m)][(1.76\times10^3\,\frac{kg}{m^3})$
$\times(9.81\,\frac{m}{s^2})(6.6\,m)]$
$= 113.95$ kN

THEN.. $P' = P'_w + (P_s)_I + (P_s)_{II} = 297.16$ kN

THE PERCENTAGE INCREASE % INC. IS THEN GIVEN BY..
$\%\,inc. = \frac{P' - P_w}{P_w}\times100\% = \frac{(297.16 - 213.66)}{213.66}\times100\%$

OR $\%\,inc. = 39.1\%$

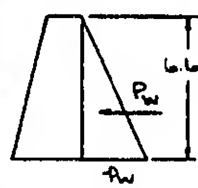
5.95



GIVEN: $(F_{BASE})_{MAX} = 1.2\times$ FORCE OF WATER, $\rho_s = 1.76\times10^3\ \frac{kg}{m^3}$, WIDTH = 1 m, RATE r_s AT WHICH SILT IS DEPOSITED = 12 mm/YEAR

FIND: NUMBER OF YEARS N UNTIL DAM BECOMES UNSAFE

FIRST DETERMINE THE FORCE ON THE DAM FACE BEFORE ANY SILT IS DEPOSITED. HAVE..



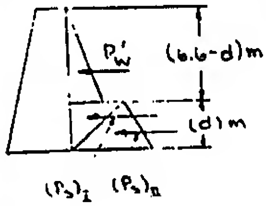
$P_w = \frac{1}{2}A p_w = \frac{1}{2}A(\rho g h)$
$= \frac{1}{2}[(6.6\,m)(1\,m)]$
$\cdot[(10^3\,\frac{kg}{m^3})(9.81\,\frac{m}{s^2})(6.6\,m)]$
$= 213.66$ kN

THE MAXIMUM ALLOWED FORCE P_{ALLOW} ON THE DAM IS THEN..
$P_{ALLOW} = 1.2P_w = 1.2(213.66\,kN) = 256.39$ kN

NEXT DETERMINE THE FORCE P' ON THE DAM FACE AFTER A DEPTH d OF SILT HAS SETTLED.

(CONTINUED)

5.95 CONTINUED



HAVE..

$P_{W'} = \frac{1}{2}[(6.6-d)m \cdot (1\,m)] \cdot [(10^3 \frac{kg}{m^3})(9.81 \frac{m}{s^2})(6.6-d)m]$
$= 4.905(6.6-d)^2$ kN

$(P_s)_I = \frac{1}{2}[(d)m \cdot (1\,m)] \cdot [(1.76 \times 10^3 \frac{kg}{m^3})(9.81 \frac{m}{s^2}) \cdot (6.6-d)m]$
$= 8.6328(6.6d-d^2)$ kN

$(P_s)_{II} = \frac{1}{2}[(d)m \cdot (1\,m)] \cdot [(1.76 \times 10^3 \frac{kg}{m^3})(9.81 \frac{m}{s^2}) \cdot (6.6\,m)]$
$= 56.976\,d$ kN

THEN $P' = P_{W'} + (P_s)_I + (P_s)_{II}$
$= [4.905(6.6-d)^2 + 8.6328(6.6d-d^2) + 56.976d]$ kN
$= (213.66 + 49.206d - 3.7278d^2)$ kN

NOW REQUIRE THAT $P' = P_{ALLOW}$ TO DETERMINE THE MAXIMUM VALUE OF d.

$\therefore (213.66 + 49.206d - 3.7278d^2)$ kN $= 256.39$ kN

OR $3.7278d^2 - 49.206d + 42.73 = 0$

THEN.. $d = \dfrac{49.206 \pm \sqrt{(-49.206)^2 - 4(3.7278)(42.73)}}{2(3.7278)}$

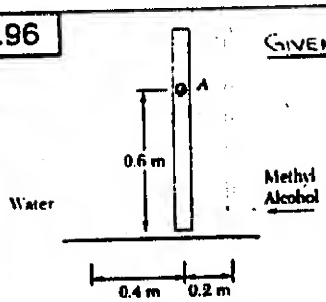
OR $d = 0.934\,56$ m AND $d = 12.2652$ m

NOW, $d \le 6.6$ m AND $d = r_s N$

THEN $0.934\,56\,m = 12 \times 10^{-3} \frac{m}{YEAR} \cdot N$

OR $N = 77.9$ YEARS ◀

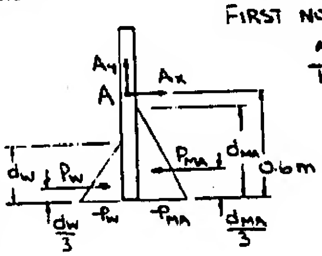
5.96



GIVEN: 1×1-m GATE, $M_R = 490$ N·m, $r_W = 0.1 \frac{m^3}{min}$, $r_{MA} = 0.2 \frac{m^3}{min}$, $\rho_{MA} = 789$ kg/m³

FIND: TIME t_R WHEN GATE ROTATES, DIRECTION OF ROTATION

CONSIDER THE FREE-BODY DIAGRAM OF THE GATE.



FIRST NOTE.. $V = A_{BASE}\,d$ AND $V = rt$

THEN.. $d_W = \dfrac{0.1 \frac{m^3}{min} \times t(min)}{(0.4\,m)(1\,m)}$
$= 0.25t$, (m)

$d_{MA} = \dfrac{0.2 \frac{m^3}{min} \times t(min)}{(0.2\,m)(1\,m)}$
$= t$ (m)

NOW.. $P = \frac{1}{2}A\,p = \frac{1}{2}A(\rho g h)$ SO THAT

$P_W = \frac{1}{2}[(0.25t)m \times (1m)][(10^3 \frac{kg}{m^3})(9.81 \frac{m}{s^2})(0.25t)m]$
$= 306.56\,t^2$ N

$P_{MA} = \frac{1}{2}[(t)m \times (1\,m)][(789 \frac{kg}{m^3})(9.81 \frac{m}{s^2})(t)\,m]$
$= 3870\,t^2$ N

NOW ASSUME THAT THE GATE WILL ROTATE CLOCKWISE AND WHEN $d_{MA} \le 0.6$ m. WHEN

(CONTINUED)

5.96 CONTINUED

ROTATION OF THE GATE IS IMPENDING, REQUIRE

ΣM_A: $M_R = (0.6m - \frac{1}{3}d_{MA})P_{MA} - (0.6m - \frac{1}{3}d_W)P_W$

SUBSTITUTING..

490 N·m $= (0.6 - \frac{1}{3}t)m \times (3870t^2)$ N
$- (0.6 - \frac{1}{3} \times 0.25t)\,m \times (306.56t^2)$ N

SIMPLIFYING.. $1264.45t^3 - 2138.1t^2 + 490 = 0$

SOLVING (POSITIVE ROOTS ONLY)..

$t = 0.594\,51$ min AND $t = 1.524\,11$ min

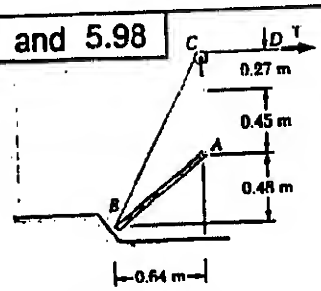
NOW CHECK ASSUMPTION USING THE SMALLER ROOT. HAVE..

$d_{MA} = (t)m = 0.594\,51$ m < 0.6 m

$\therefore t = 0.594\,51$ min $= 35.7$ s ◀

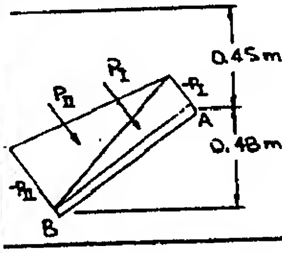
AND THE GATE ROTATES CLOCKWISE ◀

5.97 and 5.98



GIVEN: 0.5×0.8-m GATE, WATER, FRICTIONLESS STOP AT B

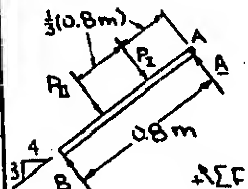
FIRST CONSIDER THE FORCE OF THE WATER ON THE GATE. HAVE $P = \frac{1}{2}A\,p = \frac{1}{2}A(\rho g h)$ SO THAT..



$P_I = \frac{1}{2}[(0.5m)(0.8m)] \cdot [(10^3 \frac{kg}{m^3})(9.81 \frac{m}{s^2})(0.45m)]$
$= 882.9$ N

$P_{II} = \frac{1}{2}[(0.5m)(0.8m)] \times [(10^3 \frac{kg}{m^3})(9.81 \frac{m}{s^2})(0.93m)]$
$= 1824.66$ N

5.97 FIND: REACTIONS AT A AND B WHEN T=0



HAVE..

$+\circlearrowleft \Sigma M_A = 0$: $\frac{1}{3}(0.8m)(882.9\,N) + \frac{2}{3}(0.8m)(1824.66\,N) - (0.8\,m)B = 0$

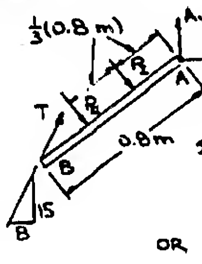
OR $B = 1510.74$ N

OR $\underline{B} = 1511$ N $\measuredangle 53.1°$ ◀

$+\nwarrow\Sigma F = 0$: $A + 1510.74N - 882.9N - 1824.66\,N = 0$

OR $A = 1197$ N $\measuredangle 53.1°$ ◀

5.98 FIND: T TO OPEN GATE



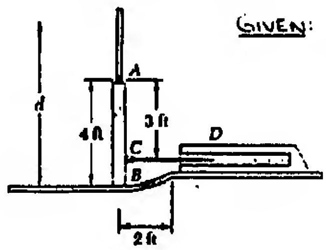
FIRST NOTE THAT WHEN THE GATE BEGINS TO OPEN, THE REACTION AT B → 0. THEN..

$+\circlearrowleft \Sigma M_A = 0$: $\frac{1}{3}(0.8m)(882.9\,N) + \frac{2}{3}(0.8\,m)(1824.66\,N) - (0.45 + 0.27)m \times (\frac{8}{17}T) = 0$

OR $235.44 + 973.152 - 0.33882\,T = 0$

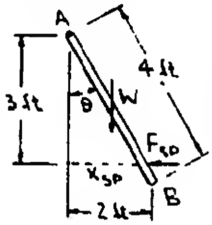
OR $T = 3570$ N ◀

5.99 and 5.100



GIVEN: 4×2-ft GATE, $k = 828\ \frac{lb}{ft}$, SPRING IS UNDEFORMED WHEN GATE IS VERTICAL, WATER

FIRST DETERMINE THE FORCES EXERTED ON THE GATE BY THE SPRING AND THE WATER WHEN B IS AT THE END OF THE CYLINDRICAL PORTION OF THE FLOOR.

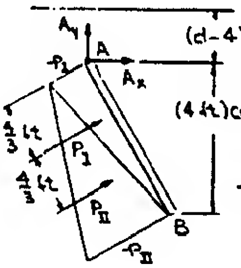


HAVE.. $\sin\theta = \frac{2}{4}$ $\quad \theta = 30°$

THEN $x_{SP} = (3\ ft)\tan 30°$

AND $F_{SP} = k x_{SP}$
$= 828\ \frac{lb}{ft} \times 3\ ft \times \tan 30°$
$= 1434.14\ lb$

ASSUME $d \geq 4\ ft$



HAVE.. $P = \frac{1}{2}Ap = \frac{1}{2}A(\gamma h)$

THEN.. $P_I = \frac{1}{2}[(4\ ft)(2\ ft)] \cdot [(62.4\ \frac{lb}{ft^3})(d-4)\ ft]$
$= 249.6(d-4)\ lb$

$P_{II} = \frac{1}{2}[(4\ ft)(2\ ft)] \cdot [(62.4\ \frac{lb}{ft^3})(d-4+4\cos 30°)]$
$= 249.6(d-0.53590)\ lb$

5.99 FIND: d, W = 0

USING THE ABOVE FREE-BODY DIAGRAMS OF THE GATE, HAVE..

$$\circlearrowleft \Sigma M_A = 0:\ (\tfrac{4}{3}\ ft)[249.6(d-4)\ lb] + (\tfrac{8}{3}\ ft)[249.6(d-0.53590)\ lb] - (3\ ft)(1434.14\ lb) = 0$$

OR $(332.8d - 1331.2) + (665.6d - 356.70) - 4302.4 = 0$

OR $d = 6.00\ ft$

$d \geq 4\ ft \Rightarrow$ ASSUMPTION CORRECT $\quad \therefore d = 6.00\ ft$

5.100 FIND: d, W = 1000 lb

USING THE ABOVE FREE-BODY DIAGRAMS OF THE GATE, HAVE..

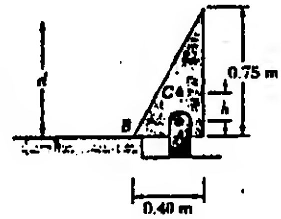
$$\circlearrowleft \Sigma M_A = 0:\ (\tfrac{4}{3}\ ft)[249.6(d-4)\ lb] + (\tfrac{8}{3}\ ft)[249.6(d-0.53590)\ lb] - (3\ ft)(1434.14\ lb) - (1\ ft)(1000\ lb) = 0$$

OR $(332.8d - 1331.2) + (665.6d - 356.70) - 4302.4 - 1000 = 0$

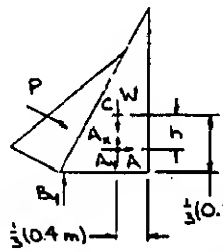
OR $d = 7.00\ ft$

$d \geq 4\ ft \Rightarrow$ ASSUMPTION CORRECT $\quad \therefore d = 7.00\ ft$

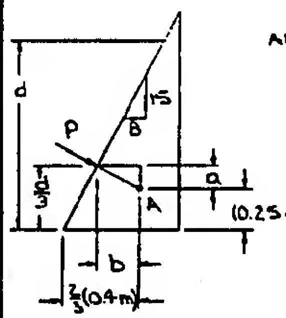
5.101 and 5.102



GIVEN: PRISMATICALLY SHAPED GATE, WATER



FIRST NOTE THAT WHEN THE GATE IS ABOUT TO OPEN (CLOCKWISE ROTATION IS IMPENDING), $B_y \to 0$ AND THE LINE OF ACTION OF THE RESULTANT $\underline{P}$ OF THE PRESSURE FORCES PASSES THROUGH THE PIN AT A. IN ADDITION, IF IT IS ASSUMED THAT THE GATE IS HOMOGENEOUS, THEN ITS CENTER OF GRAVITY C COINCIDES WITH THE CENTROID OF THE TRIANGULAR AREA. THEN...



$a = \frac{d}{3} - (0.25-h)$

AND $b = \frac{2}{3}(0.4) - \frac{8}{15}\left(\frac{d}{3}\right)$

NOW $\frac{a}{b} = \frac{8}{15}$

SO THAT

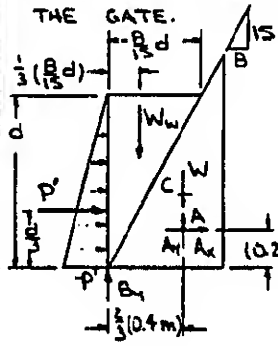
$$\frac{\frac{d}{3} - (0.25-h)}{\frac{2}{3}(0.4) - \frac{8}{15}\left(\frac{d}{3}\right)} = \frac{8}{15}$$

SIMPLIFYING YIELDS..

$$\frac{289}{45}d + 15h = \frac{70.6}{12} \qquad (1)$$

ALTERNATIVE SOLUTION

CONSIDER A FREE BODY CONSISTING OF A 1-m THICK SECTION OF THE GATE AND THE TRIANGULAR SECTION BDE OF WATER ABOVE THE GATE.



NOW...

$P' = \frac{1}{2}Ap' = \frac{1}{2}(d \times 1\ m)(\rho g d)$
$= \frac{1}{2}\rho g d^2 \quad (N)$

$W' = \rho g V = \rho g(\frac{1}{2} \times \frac{8}{15}d \times d \times 1\ m)$
$= \frac{4}{15}\rho g d^2 \quad (N)$

THEN WITH $B_y = 0$ (AS EXPLAINED ABOVE), HAVE..

$$\circlearrowleft \Sigma M_A = 0:\ [\tfrac{2}{3}(0.4) - \tfrac{1}{3}(\tfrac{8}{15}d)](\tfrac{4}{15}\rho g d^2) - [\tfrac{d}{3} - (0.25-h)](\tfrac{1}{2}\rho g d^2) = 0$$

SIMPLIFYING YIELDS.. $\frac{289}{45}d + 15h = \frac{70.6}{12}$ AS ABOVE.

(CONTINUED)

5.101 and 5.102 CONTINUED

5.101 FIND: d, $h = 0.10$ m

SUBSTITUTING INTO EQ. (1)..

$$\frac{289}{45}d + 15(0.10) = \frac{70.6}{12}$$

OR $d = 0.683$ m ◀

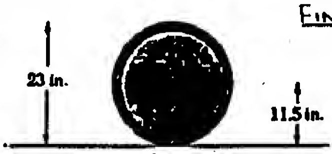
5.102 FIND: h, $d = 0.75$ m

SUBSTITUTING INTO EQ. (1)..

$$\frac{289}{45}(0.75) + 15h = \frac{70.6}{12}$$

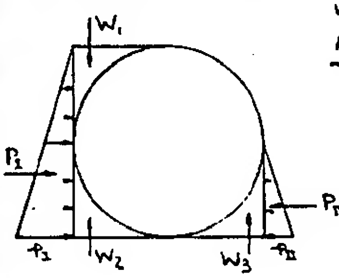
OR $h = 0.0711$ m ◀

5.103



GIVEN: WIDTH = 30 IN., WATER
FIND: RESULTANT $\underline{R}$ OF PRESSURE FORCES ACTING ON DRUM

CONSIDER THE ELEMENTS OF WATER SHOWN. THE RESULTANT OF THE WEIGHTS OF WATER ABOVE EACH SECTION OF THE DRUM AND THE RESULTANTS OF THE PRESSURE FORCES ACTING ON THE VERTICAL SURFACES OF THE ELEMENTS IS EQUAL TO THE RESULTANT HYDROSTATIC FORCE ACTING ON THE DRUM.



THEN..

$$P_I = \tfrac{1}{2}A p_I = \tfrac{1}{2}A(\gamma h)$$
$$= \tfrac{1}{2}\left[\left(\tfrac{30}{12}\right)\text{ft} \times \left(\tfrac{23}{12}\right)\text{ft}\right] \times \left[\left(62.4\tfrac{\text{lb}}{\text{ft}^3}\right)\left(\tfrac{23}{12}\text{ ft}\right)\right]$$
$$= 286.542 \text{ lb}$$

$$P_{II} = \tfrac{1}{2}A p_{II} = \tfrac{1}{2}A(\gamma h)$$
$$= \tfrac{1}{2}\left[\left(\tfrac{30}{12}\right)\text{ft} \times \left(\tfrac{11.5}{12}\right)\text{ft}\right] \times \left[\left(62.4\tfrac{\text{lb}}{\text{ft}^3}\right)\left(\tfrac{11.5}{12}\text{ ft}\right)\right]$$
$$= 71.635 \text{ lb}$$

$$W_1 = \gamma V_1 = \left(62.4\tfrac{\text{lb}}{\text{ft}^3}\right)\left[\left(\tfrac{11.5}{12}\right)^2\text{ft}^2 - \tfrac{\pi}{4}\left(\tfrac{11.5}{12}\right)^2\text{ft}^2\right]\left(\tfrac{30}{12}\text{ ft}\right)$$
$$= 30.746 \text{ lb}$$

$$W_2 = \gamma V_2 = \left(62.4\tfrac{\text{lb}}{\text{ft}^3}\right)\left[\left(\tfrac{11.5}{12}\right)^2\text{ft}^2 + \tfrac{\pi}{4}\left(\tfrac{11.5}{12}\right)^2\text{ft}^2\right]\left(\tfrac{30}{12}\text{ ft}\right)$$
$$= 255.80 \text{ lb}$$

$$W_3 = \gamma V_3 = \left(62.4\tfrac{\text{lb}}{\text{ft}^3}\right)\left[\tfrac{\pi}{4}\left(\tfrac{11.5}{12}\right)^2\text{ft}^2\right]\left(\tfrac{30}{12}\text{ ft}\right)$$
$$= 112.525 \text{ lb}$$

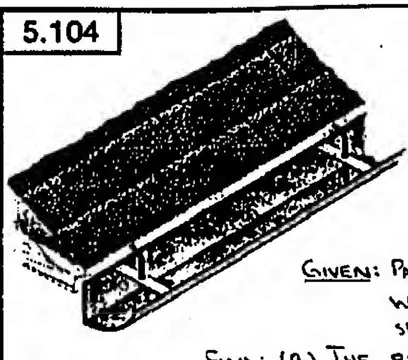
THEN.. $\xrightarrow{+}\Sigma F_x$: $R_x = (286.542 - 71.635)\text{ lb} = 214.91$ lb

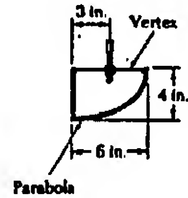
$+\!\uparrow\Sigma F_y$: $R_y = (-30.746 + 255.80 + 112.525)$ lb $= 337.58$ lb

FINALLY.. $R = \sqrt{R_x^2 + R_y^2} = 400.18$ lb $\tan\theta = \dfrac{R_y}{R_x}$ $\theta = 57.5°$

$\therefore\ \underline{R} = 400$ lb $\measuredangle\,57.5°$ ◀

5.104



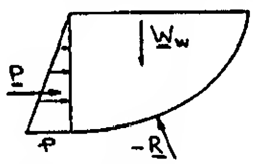


GIVEN: PARABOLIC GUTTER, WATER, HANGERS SPACED 2 ft APART

FIND: (a) THE RESULTANT $\underline{R}$ OF THE PRESSURE FORCES EXERTED ON A 2-ft SECTION OF GUTTER

(b) THE FORCE-COUPLE SYSTEM EXERTED ON A HANGER AT THE GUTTER

(a) CONSIDER A 2-ft-LONG PARABOLIC SECTION OF WATER. THEN..



$$P = \tfrac{1}{2}Ap = \tfrac{1}{2}A(\gamma h)$$
$$= \tfrac{1}{2}\left[\left(\tfrac{4}{12}\text{ft}\right)(2\text{ ft})\right]\left[\left(62.4\tfrac{\text{lb}}{\text{ft}^3}\right)\left(\tfrac{4}{12}\text{ ft}\right)\right]$$
$$= 6.9333 \text{ lb}$$

$$W_w = \gamma V$$
$$= \left(62.4\tfrac{\text{lb}}{\text{ft}^3}\right)\left[\tfrac{2}{3}\left(\tfrac{6}{12}\text{ft}\right)\left(\tfrac{4}{12}\text{ft}\right)(2\text{ ft})\right]$$
$$= 13.8667 \text{ lb}$$

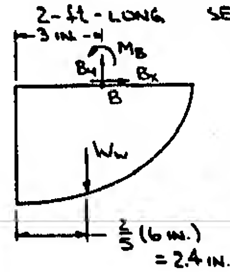
NOW.. $\Sigma\underline{F} = 0$: $(-\underline{R}) + \underline{P} + \underline{W} = 0$

SO THAT $R = \sqrt{P^2 + W^2}$ $\tan\theta = \dfrac{W}{P}$

$= 15.5034$ lb $\theta = 63.4°$

$\therefore\ \underline{R} = 15.50$ lb $\measuredangle\,63.4°$ ◀

(b) CONSIDER THE FREE-BODY DIAGRAM OF A 2-ft-LONG SECTION OF WATER AND GUTTER.



THEN..

$\xrightarrow{+}\Sigma F_x = 0$: $B_x = 0$

$+\!\uparrow\Sigma F_y = 0$: $B_y - 13.8667 \text{ lb} = 0$

OR $B_y = 13.8667$ lb

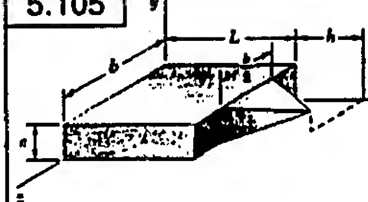
$+\circlearrowleft\Sigma M_B = 0$: $M_B + (3 - 2.4)\text{ in.} \times (13.8667 \text{ lb}) = 0$

OR $M_B = -8.320$ lb·in.

THE FORCE-COUPLE SYSTEM EXERTED ON THE HANGER IS THEN..

13.87 lb $\downarrow$, 8.32 lb·in. $\circlearrowright$ ◀

5.105



GIVEN: COMPOSITE BODY SHOWN

FIND: (a) $\bar{x}$, $h = \frac{1}{2}L$

(b) $\frac{h}{L}$, $\bar{x} = L$

	V	$\bar{x}$	$\bar{x}V$
RECTANGULAR PRISM	Lab	$\frac{1}{2}L$	$\frac{1}{2}L^2ab$
PYRAMID	$\frac{1}{3}a\left(\frac{b}{2}\right)h$	$L + \frac{1}{4}h$	$\frac{1}{6}abh\left(L + \frac{1}{4}h\right)$

THEN.. $\Sigma V = ab\left(L + \frac{1}{6}h\right)$ $\Sigma\bar{x}V = \frac{1}{6}ab\left[3L^2 + h\left(L + \frac{1}{4}h\right)\right]$

(CONTINUED)

5.105 CONTINUED

Now.. $\bar{X}\Sigma V = \Sigma \bar{x} V$ so that

$\bar{X}[ab(L+\frac{1}{6}h)] = \frac{1}{6}ab(3L^2+hL+\frac{1}{4}h^2)$

or $\bar{X}(1+\frac{1}{6}\frac{h}{L}) = \frac{1}{6}L(3+\frac{h}{L}+\frac{1}{4}\frac{h^2}{L^2})$ (1)

(a) $\bar{X} = ?$ when $h = \frac{1}{2}L$

Substituting $\frac{h}{L} = \frac{1}{2}$ into Eq. (1)..

$\bar{X}[1+\frac{1}{6}(\frac{1}{2})] = \frac{1}{6}L[3+(\frac{1}{2})+\frac{1}{4}(\frac{1}{2})^2]$

or $\bar{X} = \frac{57}{104}L$ $\bar{X} = 0.548L$ ◄

(b) $\frac{h}{L} = ?$ when $\bar{X} = L$

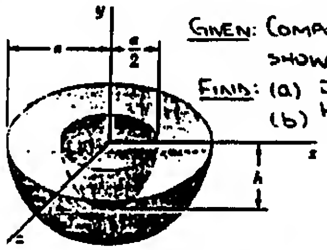
Substituting into Eq. (1)..

$L(1+\frac{1}{6}\frac{h}{L}) = \frac{1}{6}L(3+\frac{h}{L}+\frac{1}{4}\frac{h^2}{L^2})$

or.. $1+\frac{1}{6}\frac{h}{L} = \frac{1}{2}+\frac{1}{6}\frac{h}{L}+\frac{1}{24}\frac{h^2}{L^2}$

or $\frac{h^2}{L^2} = 12$ $\therefore \frac{h}{L} = 2\sqrt{3}$ ◄

5.106



Given: Composite body shown

Find: (a) $\bar{y}$, $h = \frac{a}{2}$

(b) h/a, $\bar{y} = -0.4a$

	V	$\bar{y}$	$\bar{y}V$
Hemisphere	$\frac{2}{3}\pi a^3$	$-\frac{3}{8}a$	$-\frac{1}{4}\pi a^4$
Semiellipsoid	$-\frac{2}{3}\pi(\frac{a}{2})^2 h = -\frac{1}{6}\pi a^2 h$	$-\frac{3}{8}h$	$+\frac{1}{16}\pi a^2 h^2$

Then.. $\Sigma V = \frac{\pi}{6}a^2(4a-h)$ $\Sigma \bar{y}V = -\frac{\pi}{16}a^2(4a^2-h^2)$

Now.. $\bar{Y}\Sigma V = \Sigma \bar{y} V$ so that

$\bar{Y}[\frac{\pi}{6}a^2(4a-h)] = -\frac{\pi}{16}a^2(4a^2-h^2)$

or $\bar{Y}(4-\frac{h}{a}) = -\frac{3}{8}a[4-(\frac{h}{a})^2]$ (1)

(a) $\bar{Y} = ?$ when $h = \frac{a}{2}$

Substituting $\frac{h}{a} = \frac{1}{2}$ into Eq. (1)..

$\bar{Y}(4-\frac{1}{2}) = -\frac{3}{8}a[4-(\frac{1}{2})^2]$

or $\bar{Y} = -\frac{45}{112}a$ $\bar{Y} = -0.402a$ ◄

(b) $\frac{h}{a} = ?$ when $\bar{Y} = -0.4a$

Substituting into Eq. (1)..

$(-0.4a)(4-\frac{h}{a}) = -\frac{3}{8}a[4-(\frac{h}{a})^2]$

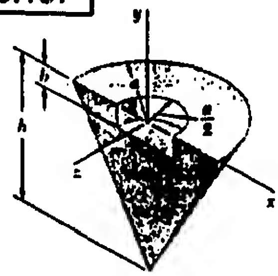
or $3(\frac{h}{a})^2 - 3.2(\frac{h}{a}) + 0.8 = 0$

Then.. $\frac{h}{a} = \frac{3.2 \pm \sqrt{(-3.2)^2 - 4(3)(0.8)}}{2(3)}$

$= \frac{3.2 \pm 0.8}{6}$

or $\frac{h}{a} = \frac{2}{5}$ and $\frac{h}{a} = \frac{2}{3}$ ◄

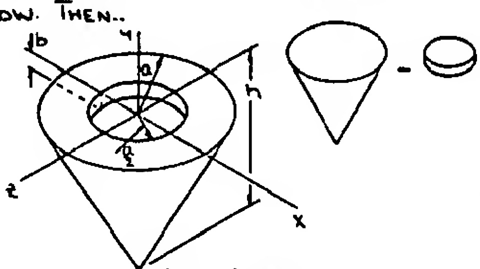
5.107



Given: Composite body shown

Find: $\bar{Y}$

First note that the values of $\bar{Y}$ will be the same for the given body and the body shown below. Then..



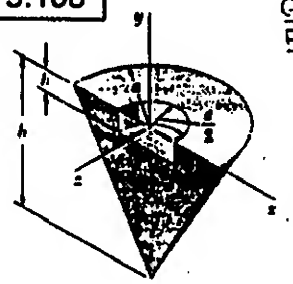
	V	$\bar{y}$	$\bar{y}V$
Cone	$\frac{1}{3}\pi a^2 h$	$-\frac{1}{4}h$	$-\frac{1}{12}\pi a^2 h^2$
Cylinder	$-\pi(\frac{a}{2})^2 b = -\frac{1}{4}\pi a^2 b$	$-\frac{1}{2}b$	$\frac{1}{8}\pi a^2 b^2$
Σ	$\frac{\pi}{12}a^2(4h-3b)$		$-\frac{\pi}{24}a^2(2h^2-3b^2)$

Have.. $\bar{Y}\Sigma V = \Sigma \bar{y} V$

Then.. $\bar{Y}[\frac{\pi}{12}a^2(4h-3b)] = -\frac{\pi}{24}a^2(2h^2-3b^2)$

or $\bar{Y} = -\frac{2h^2-3b^2}{2(4h-3b)}$ ◄

5.108



Given: Composite body shown

Find: $\bar{Z}$

First note that the body can be formed by removing a 'half-cylinder' from a 'half-cone.'

	V	$\bar{z}$	$\bar{z}V$
Half-cone	$\frac{1}{6}\pi a^2 h$	$-\frac{a}{\pi}$*	$-\frac{1}{6}a^3 h$
Half-cylinder	$-\frac{\pi}{2}(\frac{a}{2})^2 b = -\frac{\pi}{8}a^2 b$	$-\frac{4}{3\pi}(\frac{a}{2}) = -\frac{2a}{3\pi}$	$\frac{1}{12}a^3 b$
Σ	$\frac{\pi}{24}a^2(4h-3b)$		$-\frac{1}{12}a^3(2h-b)$

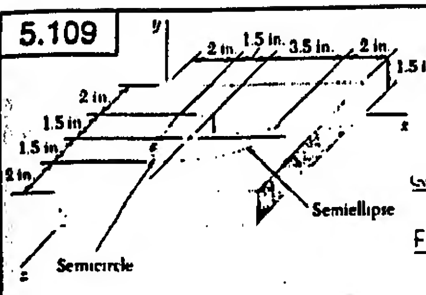
*From Sample Problem 5.13

Have.. $\bar{Z}\Sigma V = \Sigma \bar{z} V$

Then.. $\bar{Z}[\frac{\pi}{24}a^2(4h-3b)] = -\frac{1}{12}a^3(2h-b)$

or $\bar{Z} = -\frac{2a}{\pi}\frac{2h-b}{4h-3b}$ ◄

5.109



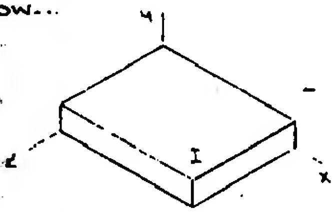
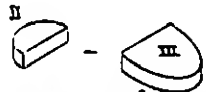
GIVEN: SAND MOLD SHOWN

FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE MOLD IS HOMOGENEOUS SO THAT ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING VOLUME. SYMMETRY THEN IMPLIES

$\bar{Z} = 3.5$ IN. ◀

NOW...

$\bar{x}_{II} = 3.5 - \frac{4 \times 1.5}{3\pi} = 2.8634$ IN.

$\bar{x}_{III} = 3.5 + \frac{4 \times 3.5}{3\pi} = 4.9854$ IN.

	V, IN³	$\bar{x}$, IN	$\bar{y}$, IN	$\bar{x}V$, IN⁴	$\bar{y}V$, IN⁴
I	$(9)(1.5)(7) = 94.5$	4.5	0.75	425.25	70.875
II	$-\frac{\pi}{2}(1.5)^2(0.75) = -2.6507$	2.8634	1.125	−7.5900	−2.9820
III	$-\frac{\pi}{2}(3.5)(1.5)(0.75) = -6.1850$	4.9854	1.125	−30.835	−6.9581
Σ	85.664			386.83	60.935

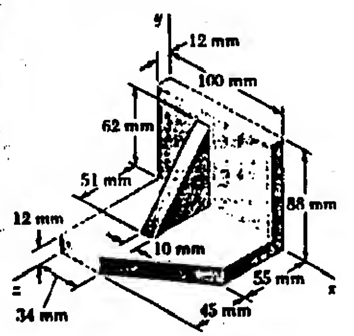
HAVE... $\bar{X}\Sigma V = \Sigma \bar{x} V$: $\bar{X}(85.664 \text{ in}^3) = 386.83 \text{ in}^4$

OR $\bar{X} = 4.52$ in. ◀

AND $\bar{Y}\Sigma V = \Sigma \bar{y} V$: $\bar{Y}(85.664 \text{ in}^3) = 60.935 \text{ in}^4$

OR $\bar{Y} = 0.711$ in. ◀

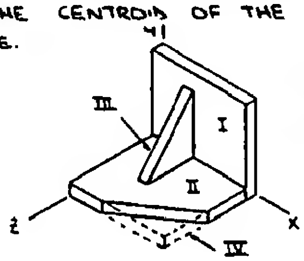
5.110 and 5.111



GIVEN: STOP BRACKET SHOWN

FIND: $\bar{X}$ (5.110)
$\bar{Z}$ (5.111)

FIRST ASSUME THAT THE BRACKET IS HOMOGENEOUS SO THAT ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING VOLUME.



$\bar{x}_{III} = 34 + \frac{1}{2}(10) = 39$ mm

$\bar{z}_{III} = 12 + \frac{1}{3}(88) = 56$ mm

$\bar{x}_{IV} = 34 + \frac{2}{3}(66) = 78$ mm

$\bar{z}_{IV} = 55 + \frac{2}{3}(45) = 85$ mm

(CONTINUED)

5.110 and 5.111 CONTINUED

	V, mm³	$\bar{x}$, mm	$\bar{z}$, mm	$\bar{x}V$, mm⁴	$\bar{z}V$, mm⁴
I	$(100)(88)(12) = 105\,600$	50	6	5 280 000	633 600
II	$(100)(12)(88) = 105\,600$	50	56	5 280 000	5 913 600
III	$\frac{1}{2}(10)(62)(51) = 15\,810$	39	29	616 590	458 490
IV	$-\frac{1}{2}(66)(12)(45) = -17\,820$	78	85	−1 389 960	−1 514 700
Σ	209 190			9 786 630	5 490 990

5.110 HAVE... $\bar{X}\Sigma V = \Sigma \bar{x} V$

$\bar{X}(209\,190 \text{ mm}^3) = 9\,786\,630 \text{ mm}^4$

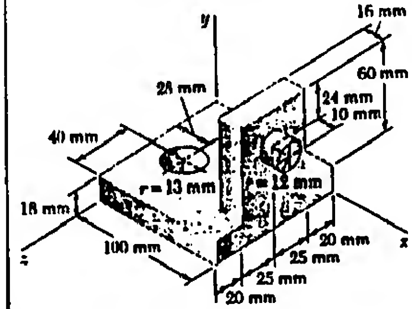
OR $\bar{X} = 46.8$ mm ◀

5.111 HAVE... $\bar{Z}\Sigma V = \Sigma \bar{z} V$

$\bar{Z}(209\,190 \text{ mm}^3) = 5\,490\,990 \text{ mm}^4$

OR $\bar{Z} = 26.2$ mm ◀

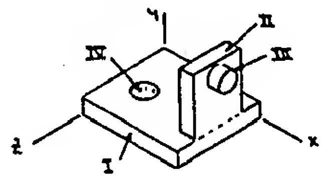
5.112 and 5.115



GIVEN: MACHINE ELEMENT SHOWN

FIND: $\bar{X}$ (5.112)
$\bar{Y}$ (5.115)

FIRST ASSUME THAT THE MACHINE ELEMENT IS HOMOGENEOUS SO THAT ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING VOLUME.



	V, mm³	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}V$, mm⁴	$\bar{y}V$, mm⁴
I	$(100)(18)(90) = 162\,000$	50	9	8 100 000	1 458 000
II	$(16)(60)(50) = 48\,000$	92	48	4 416 000	2 304 000
III	$\pi(12)^2(10) = 4523.9$	105	54	475 010	244 290
IV	$-\pi(13)^2(18) = -9556.7$	28	9	−267 590	−86 010
Σ	204 967.2			12 723 420	3 920 280

5.112 HAVE... $\bar{X}\Sigma V = \Sigma \bar{x} V$

$\bar{X}(204\,967.2 \text{ mm}^3) = 12\,723\,420 \text{ mm}^4$

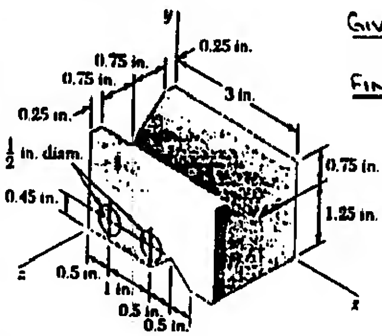
OR $\bar{X} = 62.1$ mm ◀

5.115 HAVE... $\bar{Y}\Sigma V = \Sigma \bar{y} V$

$\bar{Y}(204\,967.2 \text{ mm}^3) = 3\,920\,280 \text{ mm}^4$

OR $\bar{Y} = 19.13$ mm ◀

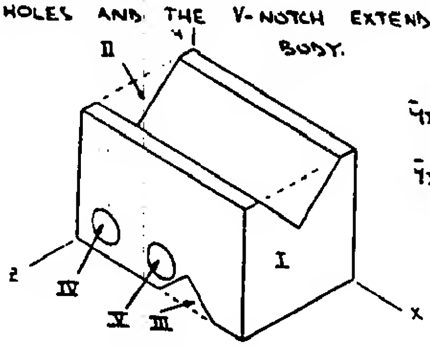
5.113 and 5.114



GIVEN: MACHINE ELEMENT SHOWN

FIND: $\bar{X}$ (5.113)
$\bar{Y}$ (5.114)

FIRST ASSUME THAT THE MACHINE ELEMENT IS HOMOGENEOUS SO THAT ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING VOLUME. ALSO NOTE THAT THE TWO HOLES AND THE V-NOTCH EXTEND THROUGH THE BODY.



$\bar{y}_{II} = 1.25 + \frac{2}{3}(0.75)$
$= 1.75$ IN.
$\bar{y}_{III} = \frac{1}{3}(0.45)$
$= 0.15$ IN.

	V, IN^3	$\bar{x}$, IN.	$\bar{y}$, IN.	$\bar{x}V$, IN^4	$\bar{y}V$, IN^4
I	$(3)(2)(2) = 12$	1.5	1	18	12
II	$-\frac{1}{2}(1.5)(0.75)(3) = -1.6875$	1.5	1.75	-2.53125	-2.9531
III	$-\frac{1}{2}(1)(0.45)(2) = -0.45$	2	0.15	-0.90	-0.0675
IV	$-\pi(\frac{1}{4})^2(2) = -0.39270$	0.5	0.45	-0.19635	-0.17672
V	$-\pi(\frac{1}{4})^2(2) = -0.39270$	1.5	0.45	-0.58905	-0.17672
Σ	9.0771			13.7834	8.6260

5.113 HAVE.. $\bar{X}\Sigma V = \Sigma \bar{x} V$

$\bar{X}(9.0771 \text{ IN}^3) = 13.7834 \text{ IN}^4$

OR $\bar{X} = 1.518$ IN. ◄

5.114 HAVE.. $\bar{Y}\Sigma V = \Sigma \bar{y} V$

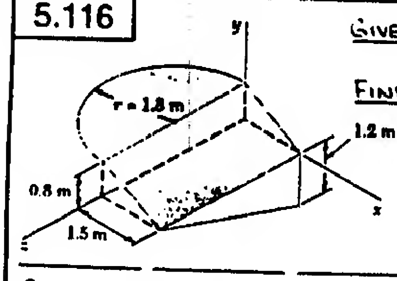
$\bar{Y}(9.0771 \text{ IN}^3) = 8.6260 \text{ IN}^4$

OR $\bar{Y} = 0.950$ IN. ◄

5.115

SEE SOLUTION TO PROBLEM 5.112

5.116



GIVEN: SHEET-METAL FORM SHOWN

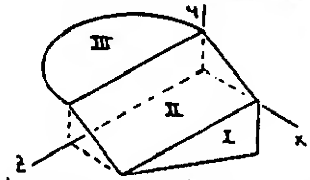
FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE SHEET METAL IS

(CONTINUED)

5.116 CONTINUED

HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE FORM WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING AREA.



$\bar{y}_I = -\frac{1}{3}(1.2) = -0.4$ m
$\bar{z}_I = \frac{1}{3}(3.6) = 1.2$ m
$\bar{x}_{III} = -\frac{4(1.8)}{3\pi} = -\frac{2.4}{\pi}$ m

	A, m^2	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}A$, m^3	$\bar{y}A$, m^3	$\bar{z}A$, m^3
I	$\frac{1}{2}(3.6)(1.2) = 2.16$	1.5	-0.4	1.2	3.24	-0.864	2.592
II	$(3.6)(1.7) = 6.12$	0.75	0.4	1.8	4.59	2.448	11.016
III	$\frac{\pi}{2}(1.8)^2 = 5.0894$	$-\frac{2.4}{\pi}$	0.8	1.8	-3.888	4.0715	9.1609
Σ	13.3694				3.942	5.6555	22.769

HAVE.. $\bar{X}\Sigma V = \Sigma \bar{x} V$: $\bar{X}(13.3694 \text{ m}^2) = 3.942 \text{ m}^3$

OR $\bar{X} = 0.295$ m ◄

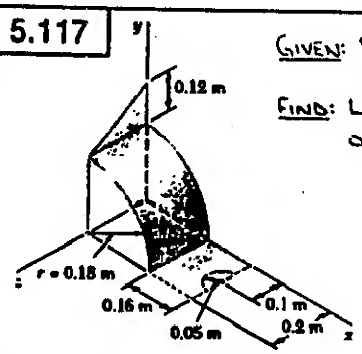
$\bar{Y}\Sigma V = \Sigma \bar{y} V$: $\bar{Y}(13.3694 \text{ m}^2) = 5.6555 \text{ m}^3$

OR $\bar{Y} = 0.423$ m ◄

$\bar{Z}\Sigma V = \Sigma \bar{z} V$: $\bar{Z}(13.3694 \text{ m}^2) = 22.769 \text{ m}^3$

OR $\bar{Z} = 1.703$ m ◄

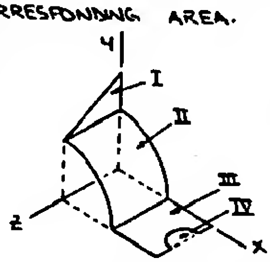
5.117



GIVEN: SHEET-METAL FORM SHOWN

FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE SHEET METAL IS HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE FORM WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING AREA.



$\bar{y}_I = 0.18 + \frac{1}{3}(0.12) = 0.22$ m
$\bar{z}_I = \frac{1}{3}(0.2 \text{ m})$
$\bar{x}_{II} = \bar{y}_{II} = \frac{2 \times 0.18}{\pi} = \frac{0.36}{\pi}$ m
$\bar{x}_{IV} = 0.34 - \frac{4 \times 0.05}{3\pi}$
$= 0.31878$ m

	A, m^2	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}A$, m^3	$\bar{y}A$, m^3	$\bar{z}A$, m^3
I	$\frac{1}{2}(0.2)(0.12) = 0.012$	0	0.22	$\frac{0.2}{3}$	0	0.00264	0.0008
II	$\frac{\pi}{2}(0.18)(0.2) = 0.018\pi$	$\frac{0.36}{\pi}$	$\frac{0.36}{\pi}$	0.1	0.00648	0.00648	0.005655
III	$(0.16)(0.2) = 0.032$	0.26	0	0.1	0.00832	0	0.0032
IV	$-\frac{\pi}{2}(0.05)^2 = -0.00125\pi$	0.31878	0	0.1	-0.001258	0	-0.000393
Σ	0.096622				0.013548	0.00912	0.009262

HAVE.. $\bar{X}\Sigma V = \Sigma \bar{x} V$: $\bar{X}(0.096\,622 \text{ m}^2) = 0.013\,548 \text{ m}^3$

OR $\bar{X} = 0.1402$ m ◄

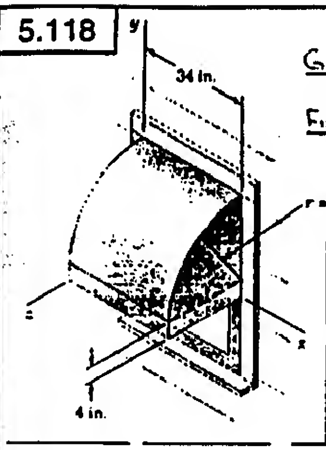
$\bar{Y}\Sigma V = \Sigma \bar{y} V$: $\bar{Y}(0.096\,622 \text{ m}^2) = 0.00912 \text{ m}^3$

OR $\bar{Y} = 0.0944$ m ◄

$\bar{Z}\Sigma V = \Sigma \bar{z} V$: $\bar{Z}(0.096\,622 \text{ m}^2) = 0.009\,262 \text{ m}^3$

OR $\bar{Z} = 0.0959$ m ◄

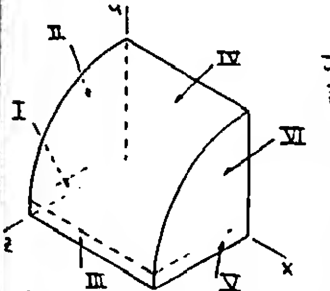
5.118



GIVEN: SHEET-METAL AWNING SHOWN

FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE SHEET METAL IS HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE AWNING WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING AREA.



$\bar{y}_{II} = \bar{y}_{VI} = 4 + \frac{4 \times 25}{3\pi} = 14.6103$ IN.

$\bar{z}_{II} = \bar{z}_{VI} = \frac{4 \times 25}{3\pi} = \frac{100}{3\pi}$ IN.

$\bar{y}_{IV} = 4 + \frac{2 \times 25}{\pi} = 19.9155$ IN.

$\bar{z}_{IV} = \frac{2 \times 25}{\pi} = \frac{50}{\pi}$ IN.

$A_{II} = A_{VI} = \frac{\pi}{4}(25)^2 = 156.25\pi$ IN^2

$A_{IV} = \frac{\pi}{2}(25)(34) = 425\pi$ IN^2

	A, IN^2	$\bar{y}$, IN.	$\bar{z}$, IN.	$\bar{y}A$, IN^3	$\bar{z}A$, IN^3
I	$(4)(25) = 100$	2	12.5	200	1250
II	$156.25\pi = 490.87$	14.6103	$\frac{100}{3\pi}$	7171.8	5208.3
III	$(4)(34) = 136$	2	25	272	3400
IV	$425\pi = 1335.18$	19.9155	$\frac{50}{\pi}$	26,591	21,250
V	$(4)(25) = 100$	2	12.5	200	1250
VI	$156.25\pi = 490.87$	14.6103	$\frac{100}{3\pi}$	7171.8	5208.3
Σ	2652.9			41,606.6	37,566.6

NOW.. SYMMETRY IMPLIES $\bar{X} = 17.00$ IN. ◀

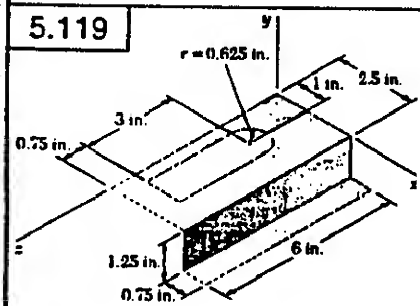
AND $\bar{Y}\Sigma A = \Sigma \bar{y}A$: $\bar{Y}(2652.9 \text{ IN}^2) = 41,606.6 \text{ IN}^3$

OR $\bar{Y} = 15.68$ IN. ◀

$\bar{Z}\Sigma A = \Sigma \bar{z}A$: $\bar{Z}(2652.9 \text{ IN}^2) = 37,566.6 \text{ IN}^3$

OR $\bar{Z} = 14.16$ IN. ◀

5.119



GIVEN: SHEET-METAL BRACKET SHOWN

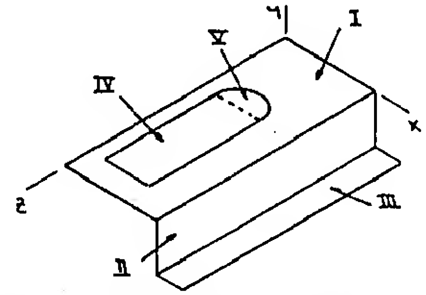
FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE SHEET METAL IS HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE BRACKET WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING AREA. THEN (SEE DIAGRAM AT THE TOP OF NEXT COLUMN)

$\bar{z}_V = 2.25 - \frac{4 \times 0.625}{3\pi}$
$= 1.98474$ IN.

$A_V = -\frac{\pi}{2}(0.625)^2$
$= -0.61359$ IN^2

5.119 CONTINUED



	A, IN^2	$\bar{x}$, IN.	$\bar{y}$, IN.	$\bar{z}$, IN.	$\bar{x}A$, IN^3	$\bar{y}A$, IN^3	$\bar{z}A$, IN^3
I	$(2.5)(6) = 15$	1.25	0	3	18.75	0	45
II	$(1.25)(6) = 7.5$	2.5	-0.625	3	18.75	-4.6875	22.5
III	$(0.75)(6) = 4.5$	2.875	-1.25	3	12.9375	-5.625	13.5
IV	$-(\frac{5}{4})(3) = -3.75$	1	0	3.75	-3.75	0	-14.0625
V	-0.61359	1	0	1.98474	-0.61359	0	-1.21782
Σ	22.6364				46.0739	-10.3125	65.7197

HAVE.. $\bar{X}\Sigma A = \Sigma \bar{x}A$: $\bar{X}(22.6364 \text{ IN}^2) = 46.0739 \text{ IN}^3$

OR $\bar{X} = 2.04$ IN. ◀

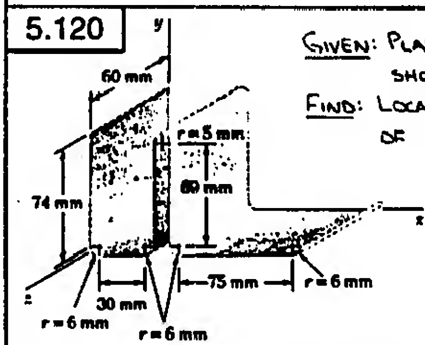
$\bar{Y}\Sigma A = \Sigma \bar{y}A$: $\bar{Y}(22.6364 \text{ IN}^2) = -10.3125 \text{ IN}^3$

OR $\bar{Y} = -0.456$ IN. ◀

$\bar{Z}\Sigma A = \Sigma \bar{z}A$: $\bar{Z}(22.6364 \text{ IN}^2) = 65.7197 \text{ IN}^3$

OR $\bar{Z} = 2.90$ IN. ◀

5.120

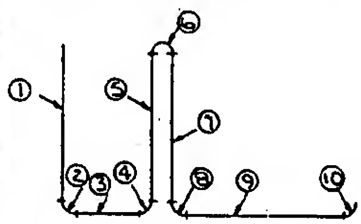


GIVEN: PLASTIC ORGANIZER SHOWN

FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE PLASTIC IS HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE ORGANIZER WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING AREA. NOW NOTE THAT SYMMETRY IMPLIES

$\bar{Z} = 30$ mm ◀



$\bar{x}_2 = 6 - \frac{2 \times 6}{\pi} = 2.1803$ mm

$\bar{x}_4 = 36 + \frac{2 \times 6}{\pi} = 39.820$ mm

$\bar{x}_8 = 58 - \frac{2 \times 6}{\pi} = 54.180$ mm

$\bar{x}_{10} = 133 + \frac{2 \times 6}{\pi} = 136.820$ mm

$\bar{y}_2 = \bar{y}_4 = \bar{y}_8 = \bar{y}_{10} = 6 - \frac{2 \times 6}{\pi} = 2.1803$ mm

$\bar{y}_6 = 75 + \frac{2 \times 5}{\pi} = 78.183$ mm

(CONTINUED)

5.120 CONTINUED

$A_2 = A_4 = A_8 = A_{10} = \frac{\pi}{2}\cdot 6 \cdot 60 = 565.49\ \text{mm}^2$

$A_6 = \pi \cdot 5 \cdot 60 = 942.48\ \text{mm}^2$

	A, mm^2	$\bar{x}, \text{mm}$	$\bar{y}, \text{mm}$	$\bar{x}A, \text{mm}^3$	$\bar{y}A, \text{mm}^3$
1	$(74)(60) = 4440$	0	43	0	190 920
2	565.49	2.1803	2.1803	1233	1 233
3	$(30)(60) = 1800$	21	0	37 800	0
4	565.49	39.820	2.1803	22 518	1233
5	$(69)(60) = 4140$	42	40.5	173 880	167 670
6	942.48	47	78.183	44 297	73 686
7	$(69)(60) = 4140$	52	40.5	215 280	167 670
8	565.49	54.180	2.1803	30 638	1233
9	$(75)(60) = 4500$	95.5	0	429 750	0
10	565.49	136.820	2.1803	77 370	1233
Σ	22 224.44			1 032 766	604 818

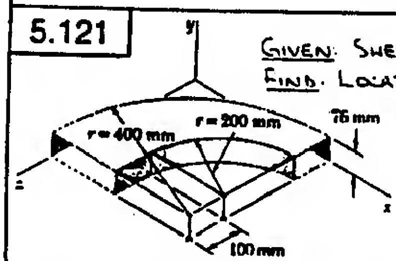
HAVE.. $\bar{X}\Sigma A = \Sigma \bar{x}A$: $\bar{X}(22\,224.44\ \text{mm}^2) = 1\,032\,766\ \text{mm}^3$

OR $\bar{X} = 46.5\ \text{mm}$ ◀

$\bar{Y}\Sigma A = \Sigma \bar{y}A$: $\bar{Y}(22\,224.44\ \text{mm}^2) = 604\,818\ \text{mm}^3$

OR $\bar{Y} = 27.2\ \text{mm}$ ◀

5.121

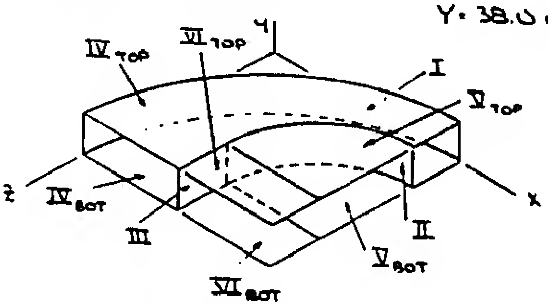


GIVEN: SHEET-METAL DUCT SHOWN

FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE SHEET METAL IS HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE DUCT WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING AREA. NOW NOTE THAT SYMMETRY IMPLIES

$\bar{Y} = 38.0\ \text{mm}$ ◀



$\bar{x}_I = \bar{z}_I = 400 - \frac{2\times400}{\pi} = 145.352\ \text{mm}$

$\bar{x}_{II} = 400 - \frac{2\times200}{\pi} = 272.68\ \text{mm}$ $\quad \bar{z}_{II} = 300 - \frac{2\times200}{\pi} = 172.676\ \text{mm}$

$\bar{x}_{IV} = \bar{z}_{IV} = 400 - \frac{4\times400}{3\pi} = 230.23\ \text{mm}$

$\bar{x}_{V} = 400 - \frac{4\times200}{3\pi} = 315.12\ \text{mm}$ $\quad \bar{z}_{V} = 300 - \frac{4\times200}{3\pi} = 215.12\ \text{mm}$

ALSO NOTE THAT THE CORRESPONDING TOP AND BOTTOM AREAS WILL CONTRIBUTE EQUALLY WHEN DETERMINING $\bar{X}$ AND $\bar{Z}$. THUS..

(CONTINUED)

5.121 CONTINUED

	A, mm^2	$\bar{x}, \text{mm}$	$\bar{z}, \text{mm}$	$\bar{x}A, \text{mm}^3$	$\bar{z}A, \text{mm}^3$
I	$\frac{\pi}{2}(400)(76) = 47\,752$	145.352	145.352	6 940 850	6 940 850
II	$\frac{\pi}{2}(200)(76) = 23\,876$	272.68	172.676	6 510 510	4 122 810
III	$(100)(76) = 7600$	200	350	1 520 000	2 660 000
IV	$2\cdot\frac{\pi}{4}(400)^2 = 251\,327$	230.23	230.23	57 863 020	57 863 020
V	$-2\cdot\frac{\pi}{4}(200)^2 = -62\,832$	315.12	215.12	−19 799 620	−13 516 420
VI	$-2(100\times200) = -40\,000$	300	350	−12 000 000	−14 000 000
Σ	227 723			41 034 760	44 070 260

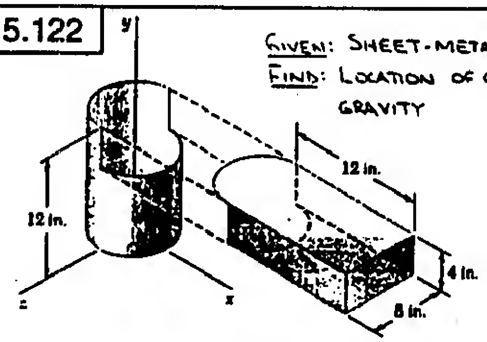
HAVE.. $\bar{X}\Sigma A = \Sigma \bar{x}A$: $\bar{X}(227\,723\ \text{mm}^2) = 41\,034\,760\ \text{mm}^3$

OR $\bar{X} = 180.2\ \text{mm}$ ◀

$\bar{Z}\Sigma A = \Sigma \bar{z}A$: $\bar{Z}(227\,723\ \text{mm}^2) = 44\,070\,260\ \text{mm}^3$

OR $\bar{Z} = 193.5\ \text{mm}$ ◀

5.122

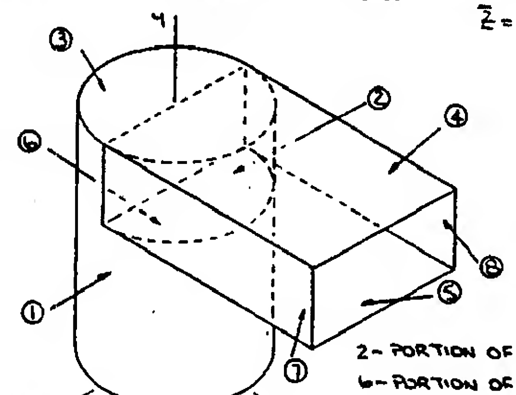


GIVEN: SHEET-METAL DUCT SHOWN

FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE SHEET METAL IS HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE DUCT ASSEMBLY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING AREA. NOW NOTE THAT SYMMETRY IMPLIES

$\bar{Z} = 0$ ◀



2 – PORTION OF 1 REMOVED

6 – PORTION OF 5 REMOVED

	A, in^2	$\bar{x}, \text{in.}$	$\bar{y}, \text{in.}$	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	$\pi(8)(12) = 301.59$	0	6	0	1809.54
2	$-\frac{\pi}{2}(8)(4) = -50.27$	$\frac{2\times4}{\pi} = 2.5465$	10	−128	−502.7
3	$\frac{\pi}{2}(4)^2 = 25.13$	$-\frac{4\times4}{3\pi} = -1.69765$	12	−42.667	301.56
4	$(12)(8) = 96$	6	12	576	1152
5	$(12)(8) = 96$	6	8	576	768
6	$-\frac{\pi}{2}(4)^2 = -25.13$	$\frac{4\times4}{3\pi} = 1.69765$	8	−42.667	−201.04
7	$(12)(4) = 48$	6	10	288	480
8	$(12)(4) = 48$	6	10	288	480
Σ	539.32			1514.666	4287.36

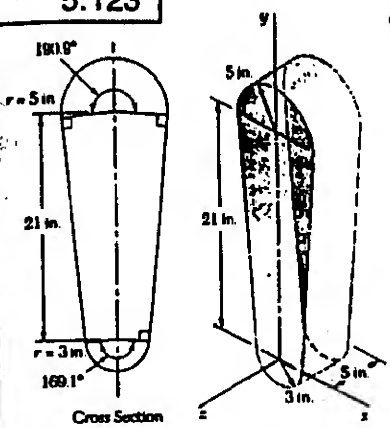
HAVE.. $\bar{X}\Sigma A = \Sigma \bar{x}A$: $\bar{X}(539.32\ \text{in}^2) = 1514.666\ \text{in}^2$

OR $\bar{X} = 2.81\ \text{in.}$ ◀

$\bar{Y}\Sigma A = \Sigma \bar{y}A$: $\bar{Y}(539.32\ \text{in}^2) = 4287.36\ \text{in}^2$

OR $\bar{Y} = 7.95\ \text{in.}$ ◀

* 5.123



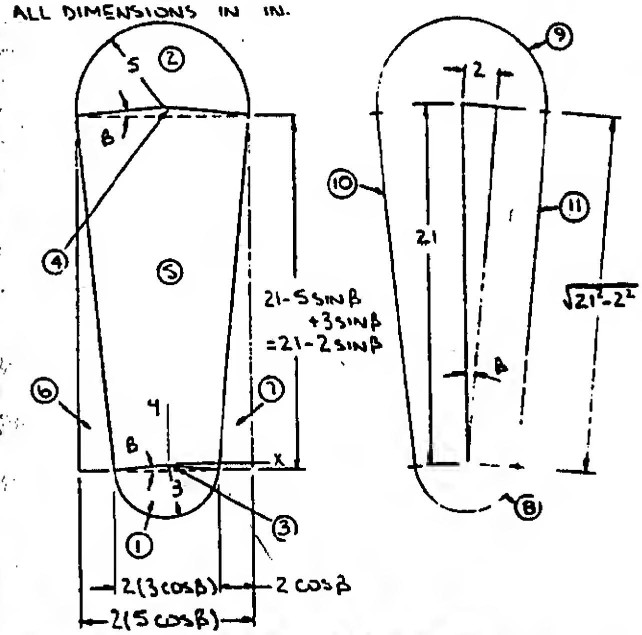
GIVEN: SHEET-METAL COVER SHOWN

FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE SHEET METAL IS HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE COVER WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING AREA. NOW NOTE THAT SYMMETRY IMPLIES

$$\bar{X} = 0$$

ALL DIMENSIONS IN IN.



FIRST NOTE.. $\beta = 90° - \frac{169.1°}{2} = 5.45°$

$\bar{y}_1 = -\dfrac{2(3)\sin\left(\frac{169.1°}{2}\right)}{3\left(\frac{169.1°}{2}\cdot\frac{\pi}{180°}\right)} = -1.3492$ IN. $\qquad A_1 = \left(\frac{169.1°}{2}\cdot\frac{\pi}{180°}\right)(3)^2 = 13.281$ IN^2

$\bar{y}_2 = 21 + \dfrac{2(5)\sin\left(\frac{190.9°}{2}\right)}{3\left(\frac{190.9°}{2}\cdot\frac{\pi}{180°}\right)} = 22.99$ IN. $\qquad A_2 = \left(\frac{190.9°}{2}\cdot\frac{\pi}{180°}\right)(5)^2 = 41.65$ IN^2

$\bar{y}_3 = -\frac{2}{3}(3\sin 5.45°) = -0.18995$ IN. $\qquad A_3 = -\frac{1}{2}[2(3\cos 5.45°)] \cdot (3\sin 5.45°) = -0.8509$ IN^2

$\bar{y}_4 = 21 - \frac{2}{3}(5\sin 5.45°) = 20.68$ IN. $\qquad A_4 = \frac{1}{2}[2(5\cos 5.45°)] \cdot (5\sin 5.45°)$

$A_4 = 2.364$ IN^2 (CONTINUED)

5.123 CONTINUED

$\bar{y}_5 = \frac{1}{2}(21 - 2\sin 5.45°) - 3\sin 5.45° = 10.120$ IN $\qquad A_5 = (21 - 2\sin 5.45°) \times 2(5\cos 5.45°) = 207.2$ IN^2

$\bar{y}_6 = \bar{y}_7 = \frac{1}{3}(21 - 2\sin 5.45°) - 3\sin 5.45° = 6.652$ IN. $\qquad A_6 = A_7 = -\frac{1}{2}(2\cos 5.45°) \times (21 - 2\sin 5.45°) = -20.72$ IN^2

$\bar{y}_8 = -\dfrac{3\sin\left(\frac{169.1°}{2}\right)}{\left(\frac{169.1°}{2}\cdot\frac{\pi}{180°}\right)} = -2.024$ IN. $\qquad A_8 = \left[\left(169.1°\cdot\frac{\pi}{180°}\right)(3)\right](5) = 44.27$ IN^2

$\bar{y}_9 = 21 + \dfrac{5\sin\left(\frac{190.9°}{2}\right)}{\left(\frac{190.9°}{2}\cdot\frac{\pi}{180°}\right)} = 23.99$ IN. $\qquad A_9 = \left[\left(190.9°\cdot\frac{\pi}{180°}\right)(5)\right](5) = 83.30$ IN^2

$\bar{y}_{10} = \bar{y}_{11} = \bar{y}_5 = 10.120$ IN. $\qquad A_{10} = A_{11} = \left(\sqrt{21^2 - 2^2}\right)(5) = 104.52$ IN^2

	A, IN^2	$\bar{y}$, IN.	$\bar{z}$, IN.	$\bar{y}A$, IN^3	$\bar{z}A$, IN^3
1	13.281	-1.3492	-5	-17.919	-66.41
2	41.65	22.99	-5	957.5	-208.3
3	-0.8509	-0.18995	-5	0.16162	4.255
4	2.364	20.68	-5	48.89	-11.820
5	207.2	10.120	-5	2097	-1036.0
6	-20.72	6.652	-5	-137.83	103.60
7	-20.72	6.652	-5	-137.83	103.60
8	44.27	-2.024	-2.5	-89.60	-110.68
9	83.30	23.99	-2.5	1998.4	-208.3
10	104.52	10.120	-2.5	1057.7	-261.3
11	104.52	10.120	-2.5	1057.7	-261.3
Σ	558.8			6834	-1952.7

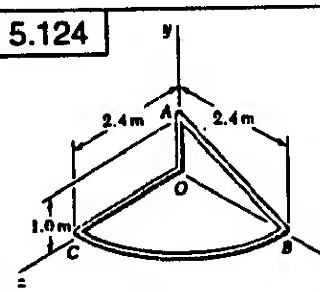
HAVE.. $\bar{Y}\Sigma A = \Sigma\bar{y}A$: $\bar{Y}(558.8\text{ IN}^2) = 6834\text{ IN}^3$

OR $\bar{Y} = 12.23$ IN.

$\bar{Z}\Sigma A = \Sigma\bar{z}A$: $\bar{Z}(558.8\text{ IN}^2) = -1952.7\text{ IN}^3$

OR $\bar{Z} = -3.49$ IN.

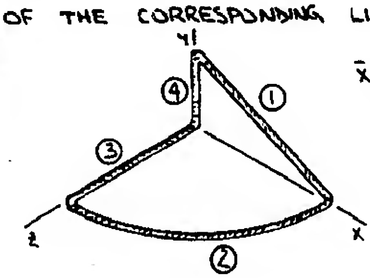
5.124



GIVEN: UNIFORM WIRE BENT INTO THE SHAPE SHOWN

FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE WIRE IS HOMOGENEOUS SO THAT ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.



$\bar{x}_2 = \bar{z}_2 = \frac{2 \times 2.4}{\pi} = \frac{4.8}{\pi}$ m

(CONTINUED)

5.124 CONTINUED

	L, m	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}L$, m^2	$\bar{y}L$, m^2	$\bar{z}L$, m^2
1	2.6	1.2	0.5	0	3.12	1.3	0
2	$\frac{\pi}{2}\times 2.4 = 1.2\pi$	$\frac{4.8}{\pi}$	0	$\frac{4.8}{\pi}$	5.76	0	5.76
3	2.4	0	0	1.2	0	0	2.88
4	1.0	0	0.5	0	0	0.5	0
Σ	9.7699				8.88	1.8	8.64

HAVE.. $\bar{X}\Sigma L = \Sigma \bar{x}L$: $\bar{X}(9.7699\text{ m}) = 8.88\text{ m}^2$

OR $\bar{X} = 0.909\text{ m}$ ◀

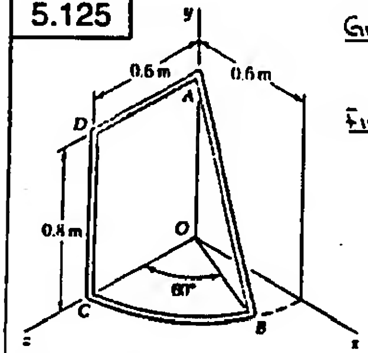
$\bar{Y}\Sigma L = \Sigma \bar{y}L$: $\bar{Y}(9.7699\text{ m}) = 1.8\text{ m}^2$

OR $\bar{Y} = 0.1842\text{ m}$ ◀

$\bar{Z}\Sigma L = \Sigma \bar{z}L$: $\bar{Z}(9.7699\text{ m}) = 8.64\text{ m}^2$

OR $\bar{Z} = 0.884\text{ m}$ ◀

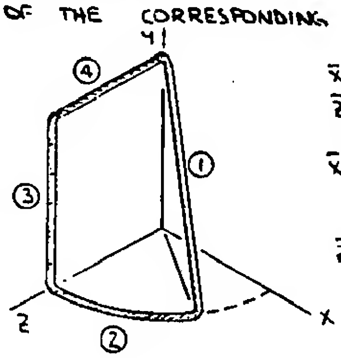
5.125



GIVEN: UNIFORM WIRE BENT INTO THE SHAPE SHOWN

FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE WIRE IS HOMOGENEOUS SO THAT ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.



$\bar{x}_1 = 0.3\sin 60° = 0.15\sqrt{3}\text{ m}$

$\bar{z}_1 = 0.3\cos 60° = 0.15\text{ m}$

$\bar{x}_2 = \left(\frac{0.6\sin 30°}{\pi/6}\right)\sin 30° = \frac{0.9}{\pi}\text{ m}$

$\bar{z}_2 = \left(\frac{0.6\sin 30°}{\pi/6}\right)\cos 30° = \frac{0.9}{\pi}\sqrt{3}\text{ m}$

$L_2 = \left(\frac{\pi}{3}\right)(0.6) = 0.2\pi\text{ m}$

	L, m	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}L$, m^2	$\bar{y}L$, m^2	$\bar{z}L$, m^2
1	1.0	$0.15\sqrt{3}$	0.4	0.15	0.25981	0.4	0.15
2	0.2π	$\frac{0.9}{\pi}$	0	$\frac{0.9\sqrt{3}}{\pi}$	0.18	0	0.31177
3	0.8	0	0.4	0.6	0	0.32	0.48
4	0.6	0	0.8	0.3	0	0.48	0.18
Σ	3.0283				0.43981	1.20	1.12177

HAVE.. $\bar{X}\Sigma L = \Sigma \bar{x}L$: $\bar{X}(3.0283\text{ m}) = 0.43981\text{ m}^2$

OR $\bar{X} = 0.1452\text{ m}$ ◀

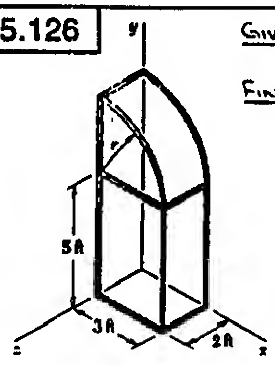
$\bar{Y}\Sigma L = \Sigma \bar{y}L$: $\bar{Y}(3.0283\text{ m}) = 1.20\text{ m}^2$

OR $\bar{Y} = 0.396\text{ m}$ ◀

$\bar{Z}\Sigma L = \Sigma \bar{z}L$: $\bar{Z}(3.0283\text{ m}) = 1.12177\text{ m}^2$

OR $\bar{Z} = 0.370\text{ m}$ ◀

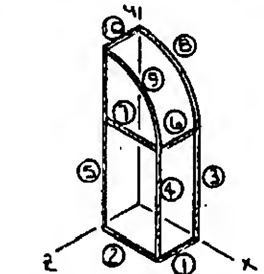
5.126



GIVEN: PORTION OF GREENHOUSE FRAME SHOWN

FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE CHANNELS ARE HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE FRAME WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE.



$\bar{x}_8 = \bar{x}_9 = \frac{2\times 3}{\pi} = \frac{6}{\pi}\text{ ft}$

$\bar{y}_8 = \bar{y}_9 = 5 + \frac{2\times 3}{\pi} = 6.9099\text{ ft}$

	L, ft	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{z}$, ft	$\bar{x}L$, ft^2	$\bar{y}L$, ft^2	$\bar{z}L$, ft^2
1	2	3	0	1	6	0	2
2	3	1.5	0	2	4.5	0	6
3	5	3	2.5	0	15	12.5	0
4	5	3	2.5	2	15	12.5	10
5	8	0	4	2	0	32	16
6	2	3	5	1	6	10	2
7	3	1.5	5	2	4.5	15	6
8	$\frac{\pi}{2}\cdot 3 = 4.7124$	$\frac{6}{\pi}$	6.9099	0	9	32.562	0
9	$\frac{\pi}{2}\cdot 3 = 4.7124$	$\frac{6}{\pi}$	6.9099	2	9	32.562	9.4248
10	2	0	8	1	0	16	2
Σ	39.4248				69	163.124	53.4248

HAVE.. $\bar{X}\Sigma L = \Sigma \bar{x}L$: $\bar{X}(39.4248\text{ ft}) = 69\text{ ft}^2$

OR $\bar{X} = 1.750\text{ ft}$ ◀

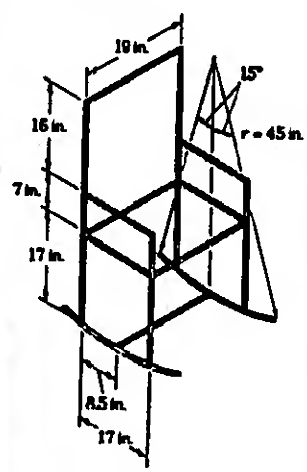
$\bar{Y}\Sigma L = \Sigma \bar{y}L$: $\bar{Y}(39.4248\text{ ft}) = 163.124\text{ ft}^2$

OR $\bar{Y} = 4.14\text{ ft}$ ◀

$\bar{Z}\Sigma L = \Sigma \bar{z}L$: $\bar{Z}(39.4248\text{ ft}) = 53.4248\text{ ft}^2$

OR $\bar{Z} = 1.355\text{ ft}$ ◀

* 5.127



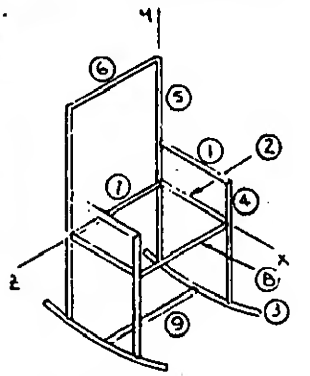
GIVEN: ROCKING CHAIR FRAME SHOWN

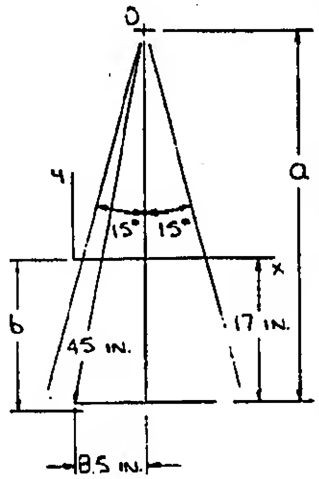
FIND: ANGLE BETWEEN CHAIR BACK AND VERTICAL

FIRST ASSUME THAT THE TUBING IS HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE FRAME WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE. ALSO, NOTE THAT THE CENTER OF GRAVITY MUST LIE ON A VERTICAL LINE THAT PASSES THROUGH THE POINT OF CONTACT OF A ROCKER AND THE GROUND.

(CONTINUED)

5.127 CONTINUED





$a = \sqrt{45^2 - 8.5^2}$ IN.

$b = 45 - (a - 17)$

$= 17.8101$ IN.

$\bar{y}_3 = -\left[17 + \left(\frac{45 \sin 15^\circ}{\pi/12} - a\right)\right]$

$= -17.2978$ IN.

$L_3 = \frac{\pi}{6}(45)$

$= 23.562$ IN.

NOTE. TO ACCOUNT FOR THE TWO SIDES OF THE CHAIR, THE LENGTHS OF MEMBERS 1-5 WILL BE COUNTED TWICE

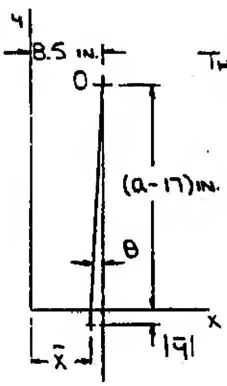
	L, IN.	$\bar{x}$, IN.	$\bar{y}$, IN.	$\bar{x}L$, IN^2	$\bar{y}L$, IN^2
1	2(17)	8.5	7	289	238
2	2(17)	8.5	0	289	0
3	2(23.562)	8.5	-17.2978	400.55	-815.14
4	2(24)	17	-5	816	-240
5	2(40)	0	3	0	240
6	19	0	23	0	437
7	19	0	0	0	0
8	19	17	0	323	0
9	19	8.5	-17.8101	161.5	-338.39
Σ	319.124			2279.1	-478.53

HAVE.. $\bar{X}\Sigma L = \Sigma \bar{x} L$: $\bar{X}(319.124 \text{ in.}) = 2279.1 \text{ in}^2$

OR $\bar{X} = 7.1417$ IN.

$\bar{Y}\Sigma L = \Sigma \bar{y} L$: $\bar{Y}(319.124 \text{ in.}) = -478.53 \text{ in}^2$

OR $\bar{Y} = -1.49951$ IN.



THEN.. $\tan\theta = \frac{8.5 - 7.1417}{(\sqrt{45^2 - 8.5^2} - 17) + 1.49951}$

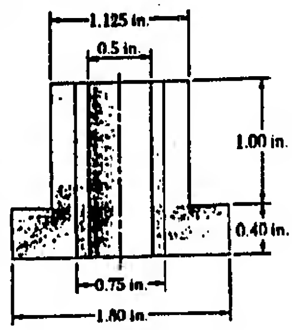
$= 0.047345$

OR $\theta = 2.7106°$

∴ THE ANGLE FORMED BY THE BACK OF THE CHAIR AND THE VERTICAL IS

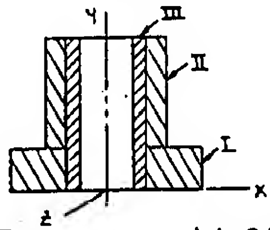
2.71°

5.128



GIVEN: BRONZE BUSHING AND STEEL SLEEVE SHOWN, $\gamma_{BR} = 0.318 \text{ lb/in}^3$, $\gamma_{ST} = 0.284 \text{ lb/in}^3$

FIND: LOCATION OF CENTER OF GRAVITY



FIRST NOTE THAT SYMMETRY IMPLIES $\bar{X} = \bar{Z} = 0$

NOW.. $W = \gamma V$

$\bar{y}_I = 0.20$ IN. $\quad W_I = 0.284 \frac{\text{lb}}{\text{in}^3} \times \frac{\pi}{4}(1.8^2 - 0.75^2)\text{in}^2 \times 0.4 \text{ in.}$

$= 0.238\ 89$ lb

$\bar{y}_{II} = 0.90$ IN. $\quad W_{II} = 0.284 \frac{\text{lb}}{\text{in}^3} \times \frac{\pi}{4}(1.125^2 - 0.75^2)\text{in}^2 \times 1$ in.

$= 0.1568\ 34$ lb

$\bar{y}_{III} = 0.70$ IN. $\quad W_{III} = 0.318 \frac{\text{lb}}{\text{in}^3} \times \frac{\pi}{4}(0.75^2 - 0.5^2)\text{in}^2 \times 1.4$ in.

$= 0.1092\ 69$ lb

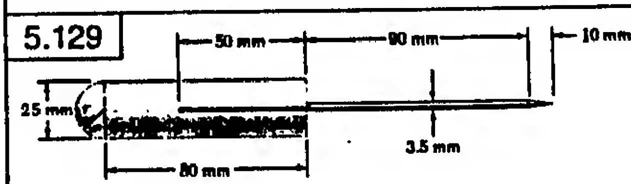
	W, lb	$\bar{y}$, in.	$\bar{y}W$, in·lb
I	0.238 89	0.20	0.047 778
II	0.1568 34	0.90	0.141 151
III	0.1092 69	0.70	0.076 488
Σ	0.50499		0.265 42

HAVE.. $\bar{Y}\Sigma W = \Sigma \bar{y} W$: $\bar{Y}(0.50499 \text{ lb})$

$= 0.265\ 42$ in·lb

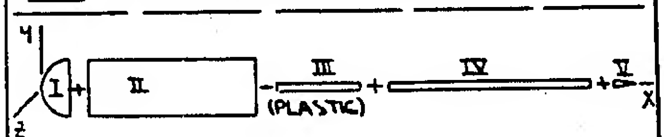
OR $\bar{Y} = 0.526$ IN. (ABOVE BASE)

5.129



GIVEN: AWL HAVING PLASTIC HANDLE AND STEEL BLADE, $\rho_{PL} = 1030 \text{ kg/m}^3$, $\rho_{ST} = 7860 \text{ kg/m}^3$

FIND: LOCATION OF CENTER OF GRAVITY



FIRST NOTE THAT SYMMETRY IMPLIES $\bar{Y} = \bar{Z} = 0$

$\bar{x}_I = \frac{5}{8}(12.5)$

$= 7.8125$ mm

$m_I = \rho_{PL} V_I = 1030 \frac{\text{kg}}{\text{m}^3} \times \frac{2}{3}\pi(0.0125 \text{ m})^3$

$= 4.2133 \times 10^{-3}$ kg

$\bar{x}_{II} = 52.5$ mm

$m_{II} = \rho_{PL} V_{II} = 1030 \frac{\text{kg}}{\text{m}^3} \times \frac{\pi}{4}(0.025 \text{ m})^2(0.08 \text{ m})$

$= 40.448 \times 10^{-3}$ kg

$\bar{x}_{III} = 92.5 - 25$

$= 67.5$ mm

$m_{III} = -\rho_{PL} V_{III} = -1030 \frac{\text{kg}}{\text{m}^3} \times \frac{\pi}{4}(0.0035 \text{ m})^2(0.05 \text{ m})$

$= -0.495\ 49 \times 10^{-3}$ kg

(CONTINUED)

5.129 CONTINUED

$\bar{x}_{IV} = 182.5 - 70 = 112.5$ mm

$m_{IV} = \rho_{ST} V_{IV} = 7860 \frac{kg}{m^3} \cdot \frac{\pi}{4}(0.0035\ m)^2(0.14\ m) = 10.5871\times10^{-3}$ kg

$\bar{x}_V = 182.5 + \frac{1}{4}(10) = 185$ mm

$m_V = \rho_{ST} V_V = 7860 \frac{kg}{m^3} \cdot \frac{\pi}{3}(0.00175\ m)^2(0.01\ m) = 0.25207\times10^{-3}$ kg

	m, kg	$\bar{x}$, mm	$\bar{x}m$, kg·mm
I	4.2133×10^{-3}	7.8125	32.916×10^{-3}
II	40.448×10^{-3}	52.5	2123.5×10^{-3}
III	-0.49549×10^{-3}	67.5	-33.447×10^{-3}
IV	10.5871×10^{-3}	112.5	1191.05×10^{-3}
V	0.25207×10^{-3}	185	46.633×10^{-3}
Σ	55.005×10^{-3}		3360.7×10^{-3}

HAVE.. $\bar{X}\Sigma m = \Sigma \bar{x} m$: $\bar{X}(55.005\times10^{-3}\ kg) = 3360.7\times10^{-3}$ kg·mm

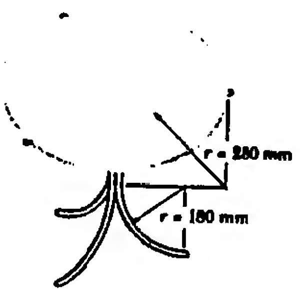
OR $\bar{X} = 61.098$ mm

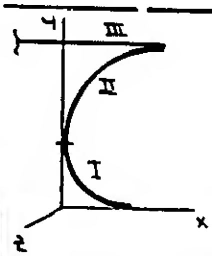
∴ THE CENTER OF GRAVITY IS 61.1 mm FROM THE END OF THE HANDLE.

5.130

GIVEN: TABLE WITH GLASS TOP ($\rho_{GL} = 2190\ kg/m^3$) AND STEEL TUBING LEGS ($\rho_{ST} = 7860\ kg/m^3$), $d_{TOP} = 600$ mm, $t_{TOP} = 10$ mm, $A_{TUBING} = 150\ mm^2$, $(d_o)_{TUBING} = 24$ mm

FIND: LOCATION OF CENTER OF GRAVITY





FIRST NOTE THAT SYMMETRY IMPLIES $\bar{X} = \bar{Z} = 0$

ALSO, TO ACCOUNT FOR THE THREE LEGS, THE MASSES OF COMPONENTS I AND II WILL EACH BE MULTIPLIED BY THREE.

$\bar{y}_I = 12 + 180 - \frac{2\times180}{\pi} = 77.408$ mm

$m_I = \rho_{ST} V_I = 7860\ kg/m^3 \times (150\times10^{-6}\ m^2) \times \frac{\pi}{2}(0.180\ m) = 0.33335$ kg

$\bar{y}_{II} = 12 + 180 + \frac{2\times280}{\pi} = 370.25$ mm

$m_{II} = \rho_{ST} V_{II} = 7860\ kg/m^3 \times (150\times10^{-6}\ m^2) \times \frac{\pi}{2}(0.280\ m) = 0.51855$ kg

$\bar{y}_{III} = 24 + 180 + 280 + 5 = 489$ mm

$m_{III} = \rho_{GL} V_{III} = 2190\ kg/m^3 \times \frac{\pi}{4}(0.6\ m)^2 \times (0.010\ m) = 6.1921$ kg

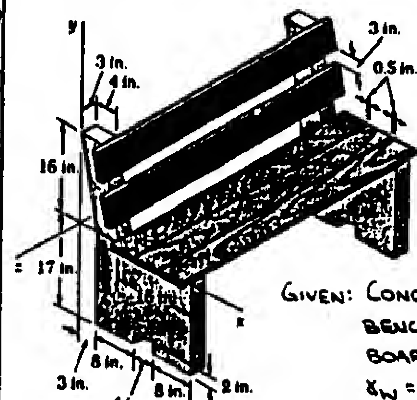
	m, kg	$\bar{y}$, mm	$\bar{y}m$, kg·mm
I	3(0.33335)	77.408	77.412
II	3(0.51855)	370.25	575.98
III	6.1921	489	3027.9
Σ	8.7478		3681.3

HAVE.. $\bar{Y}\Sigma m = \Sigma \bar{y} m$: $\bar{Y}(8.7478\ kg) = 3681.3$ kg·mm

OR $\bar{Y} = 420.8$ mm

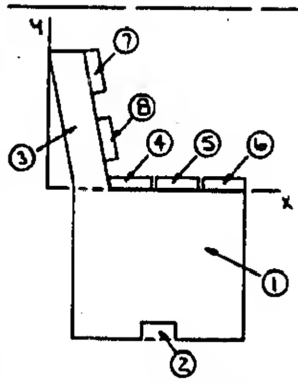
∴ THE CENTER OF GRAVITY IS 421 mm ABOVE THE FLOOR.

* 5.131



GIVEN: CONCRETE AND WOOD BENCH, $1\frac{1}{2}\times5\times48$-IN. BOARDS, $\gamma_C = 0.084\ lb/in^3$, $\gamma_W = 0.017\ lb/in^3$

FIND: X AND Y COORDINATES OF CENTER OF GRAVITY



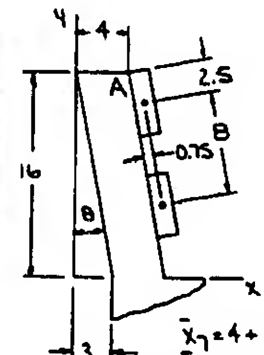
FIRST NOTE TO ACCOUNT FOR THE TWO CONCRETE ENDS, THE WEIGHTS OF COMPONENTS 1-3 WILL BE COUNTED TWICE.

$W_1 = \gamma_C V_1 = 0.084 \frac{lb}{in^3} \times (20\times17\times3)\ in^3 = 85.68$ lb

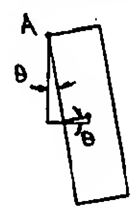
$W_2 = -\gamma_C V_2 = -0.084 \frac{lb}{in^3} \times (4\times2\times3)\ in^3 = -2.016$ lb

$W_3 = \gamma_C V_3 = 0.084 \frac{lb}{in^3} \times (4\times16\times3)\ in^3 = 16.128$ lb

$W_4 = W_5 = W_6 = W_7 = W_8 = \gamma_W V_{BOARD} = 0.017\ lb/in^3 \times (5\times1\frac{1}{2}\times48)\ in^3 = 6.12$ lb



ALL DIMENSIONS IN IN.



$\tan\theta = \frac{3}{16}$

$\theta = 10.61966°$

$\bar{x}_7 = 4 + 2.5\sin\theta + 0.75\cos\theta = 5.1979$ in.

$\bar{y}_7 = 16 - 2.5\cos\theta + 0.75\sin\theta = 13.6810$ in.

$\bar{x}_8 = \bar{x}_7 + 8\sin\theta = 6.6722$ in. $\quad \bar{y}_8 = \bar{y}_7 - 8\cos\theta = 5.8180$ in.

	W, lb	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}W$, in·lb	$\bar{y}W$, in·lb
1	2(85.68)	13	-8.5	2227.7	-1456.56
2	2(-2.016)	13	-16	-52.416	64.512
3	2(16.128)	3.5	8	112.896	258.05
4	6.12	9.5	0.75	58.14	4.59
5	6.12	15	0.75	91.8	4.59
6	6.12	20.5	0.75	125.46	4.59
7	6.12	5.1979	13.6810	31.811	83.728
8	6.12	6.6722	5.8180	40.834	35.606

(CONTINUED)

5.131 CONTINUED

THEN.. $\Sigma W = 230.18$ lb

$\Sigma \bar{x} W = 2636.2$ in·lb $\quad \Sigma \bar{y} W = -1000.89$ in·lb

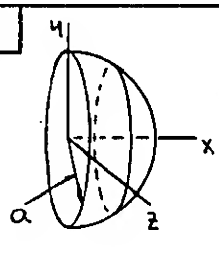
NOW.. $\bar{X}\Sigma W = \Sigma \bar{x} W$: $\bar{X}(230.18 \text{ lb}) = 2636.2$ in·lb

OR $\bar{X} = 11.45$ in. ◀

$\bar{Y}\Sigma W = \Sigma \bar{y} W$: $\bar{Y}(230.18 \text{ lb}) = -1000.89$ in·lb

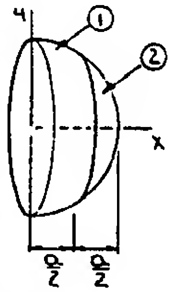
OR $\bar{Y} = -4.35$ in. ◀

5.132



GIVEN: A HEMISHERE WHICH IS CUT INTO TWO COMPONENTS OF EQUAL WIDTH AS SHOWN

FIND: $\bar{x}$ OF EACH COMPONENT



CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS r AND THICKNESS dx. THEN

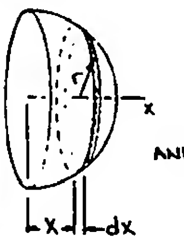
$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$

THE EQUATION OF THE GENERATING CURVE IS

$x^2 + y^2 = a^2$ SO THAT

$r^2 = a^2 - x^2$ AND THEN

$dV = \pi(a^2 - x^2)dx$



COMPONENT 1

$V_1 = \int_0^{a/2} \pi(a^2 - x^2)dx = \pi\left[a^2x - \frac{x^3}{3}\right]_0^{a/2}$

$= \frac{11}{24}\pi a^3$

AND.. $\int_1 \bar{x}_{EL} dV = \int_0^{a/2} x[\pi(a^2 - x^2)dx]$

$= \pi\left[a^2\frac{x^2}{2} - \frac{x^4}{4}\right]_0^{a/2}$

$= \frac{7}{64}\pi a^4$

NOW.. $\bar{x}_1 V_1 = \int \bar{x}_{EL} dV$: $\bar{x}_1\left(\frac{11}{24}\pi a^3\right) = \frac{7}{64}\pi a^4$

OR $\bar{x}_1 = \frac{21}{88}a$ ◀

COMPONENT 2

$V_2 = \int_{a/2}^{a} \pi(a^2 - x^2)dx = \pi\left[a^2x - \frac{x^3}{3}\right]_{a/2}^{a}$

$= \pi\left\{\left[a^2(a) - \frac{a^3}{3}\right] - \left[a^2\left(\frac{a}{2}\right) - \frac{\left(\frac{a}{2}\right)^3}{3}\right]\right\}$

$= \frac{5}{24}\pi a^3$

AND $\int_2 \bar{x}_{EL} dV = \int_{a/2}^{a} x\{\pi(a^2 - x^2)dx\} = \pi\left[a^2\frac{x^2}{2} - \frac{x^4}{4}\right]_{a/2}^{a}$

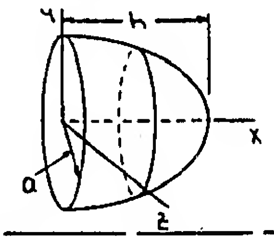
$= \pi\left\{\left[a^2\frac{(a)^2}{2} - \frac{(a)^4}{4}\right] - \left[a^2\frac{\left(\frac{a}{2}\right)^2}{2} - \frac{\left(\frac{a}{2}\right)^4}{4}\right]\right\}$

$= \frac{9}{64}\pi a^4$

NOW.. $\bar{x}_2 V_2 = \int_2 \bar{x}_{EL} dV$: $\bar{x}_2\left(\frac{5}{24}\pi a^3\right) = \frac{9}{64}\pi a^4$

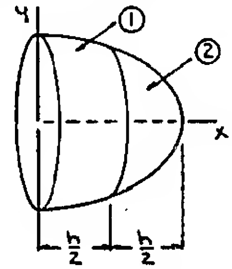
OR $\bar{x}_2 = \frac{27}{40}a$ ◀

5.133



GIVEN: A SEMIELLIPSOID OF REVOLUTION WHICH IS CUT INTO TWO COMPONENTS OF EQUAL WIDTH AS SHOWN

FIND: $\bar{x}$ OF EACH COMPONENT



CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS r AND THICKNESS dx. THEN

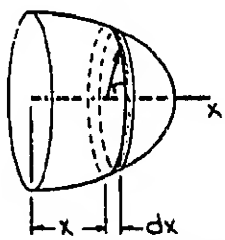
$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$

THE EQUATION OF THE GENERATING CURVE IS

$\frac{x^2}{h^2} + \frac{y^2}{a^2} = 1$ SO THAT

$r^2 = \frac{a^2}{h^2}(h^2 - x^2)$ AND THEN

$dV = \pi\frac{a^2}{h^2}(h^2 - x^2)dx$



COMPONENT 1

$V_1 = \int_0^{h/2} \pi\frac{a^2}{h^2}(h^2 - x^2)dx$

$= \pi\frac{a^2}{h^2}\left[h^2x - \frac{x^3}{3}\right]_0^{h/2}$

$= \frac{11}{24}\pi a^2 h$

AND $\int_1 \bar{x}_{EL} dV = \int_0^{h/2} x\left[\pi\frac{a^2}{h^2}(h^2 - x^2)dx\right]$

$= \pi\frac{a^2}{h^2}\left[h^2\frac{x^2}{2} - \frac{x^4}{4}\right]_0^{h/2}$

$= \frac{7}{64}\pi a^2 h^2$

NOW.. $\bar{x}_1 V_1 = \int_1 \bar{x}_{EL} dV$: $\bar{x}_1\left(\frac{11}{24}\pi a^2 h\right) = \frac{7}{64}\pi a^2 h^2$

OR $\bar{x}_1 = \frac{21}{88}h$ ◀

COMPONENT 2

$V_2 = \int_{h/2}^{h} \pi\frac{a^2}{h^2}(h^2 - x^2)dx = \pi\frac{a^2}{h^2}\left[h^2x - \frac{x^3}{3}\right]_{h/2}^{h}$

$= \pi\frac{a^2}{h^2}\left\{\left[h^2(h) - \frac{(h)^3}{3}\right] - \left[h^2\left(\frac{h}{2}\right) - \frac{(h/2)^3}{3}\right]\right\}$

$= \frac{5}{24}\pi a^2 h$

AND $\int_2 \bar{x}_{EL} dV = \int_{h/2}^{h} x\left[\pi\frac{a^2}{h^2}(h^2 - x^2)dx\right]$

$= \pi\frac{a^2}{h^2}\left[h^2\frac{x^2}{2} - \frac{x^4}{4}\right]_{h/2}^{h}$

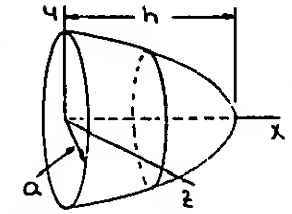
$= \pi\frac{a^2}{h^2}\left\{\left[h^2\frac{(h)^2}{2} - \frac{(h)^4}{4}\right] - \left[h^2\frac{(h/2)^2}{2} - \frac{(h/2)^4}{4}\right]\right\}$

$= \frac{9}{64}\pi a^2 h^2$

NOW.. $\bar{x}_2 V_2 = \int_2 \bar{x}_{EL} dV$: $\bar{x}_2\left(\frac{5}{24}\pi a^2 h\right) = \frac{9}{64}\pi a^2 h^2$

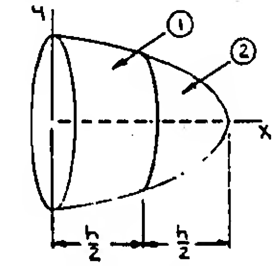
OR $\bar{x}_2 = \frac{27}{40}h$ ◀

5.134



GIVEN: A PARABOLOID OF REVOLUTION WHICH IS CUT INTO TWO COMPONENTS OF EQUAL WIDTH AS SHOWN

FIND: $\bar{x}$ OF EACH COMPONENT



CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS r AND THICKNESS dx. THEN

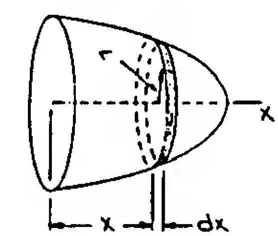
$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$

THE EQUATION OF THE GENERATING CURVE IS

$x = h - \frac{h}{a^2}y^2$ SO THAT

$r^2 = \frac{a^2}{h}(h-x)$ AND THEN

$dV = \pi\frac{a^2}{h}(h-x)dx$



COMPONENT 1

$$V_1 = \int_0^{h/2}\pi\frac{a^2}{h}(h-x)dx = \pi\frac{a^2}{h}\left[hx - \frac{x^2}{2}\right]_0^{h/2} = \frac{3}{8}\pi a^2 h$$

AND.. $\int_1 \bar{x}_{EL}dV = \int_0^{h/2} x\left[\pi\frac{a^2}{h}(h-x)dx\right] = \pi\frac{a^2}{h}\left[h\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{h/2} = \frac{1}{12}\pi a^2h^2$

NOW.. $\bar{x}_1V_1 = \int_1\bar{x}_{EL}dV$: $\quad \bar{x}_1\left(\frac{3}{8}\pi a^2h\right) = \frac{1}{12}\pi a^2h^2$

OR $\bar{x}_1 = \frac{2}{9}h$

COMPONENT 2

$$V_2 = \int_{h/2}^h \pi\frac{a^2}{h}(h-x)dx = \pi\frac{a^2}{h}\left[hx-\frac{x^2}{2}\right]_{h/2}^h$$

$$= \pi\frac{a^2}{h}\left\{\left[h(h)-\frac{(h)^2}{2}\right] - \left[h\left(\frac{h}{2}\right) - \frac{(h/2)^2}{2}\right]\right\}$$

$$= \frac{1}{8}\pi a^2 h$$

AND.. $\int_2 \bar{x}_{EL}dV = \int_{h/2}^h x\left[\pi\frac{a^2}{h}(h-x)dx\right] = \pi\frac{a^2}{h}\left[h\frac{x^2}{2}-\frac{x^3}{3}\right]_{h/2}^h$

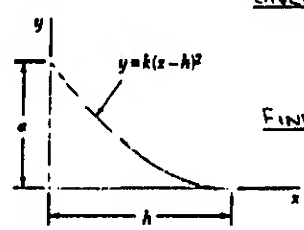
$$= \pi\frac{a^2}{h}\left\{\left[h\frac{(h)^2}{2} - \frac{(h)^3}{3}\right] - \left[h\frac{(h/2)^2}{2} - \frac{(h/2)^3}{3}\right]\right\}$$

$$= \frac{1}{12}\pi a^2h^2$$

NOW.. $\bar{x}_2V_2 = \int_2\bar{x}_{EL}dV$: $\quad \bar{x}_2\left(\frac{1}{8}\pi a^2h\right) = \frac{1}{12}\pi a^2h^2$

OR $\bar{x}_2 = \frac{2}{3}h$

5.135



GIVEN: VOLUME GENERATED BY ROTATING THE AREA SHOWN ABOUT THE X AXIS

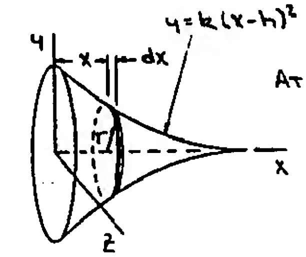
FIND: LOCATION OF THE CENTROID OF THE VOLUME

FIRST NOTE THAT SYMMETRY IMPLIES

(CONTINUED)

5.135 CONTINUED

$\bar{y} = 0$ AND $\bar{z} = 0$



AT $x=0, y=a$: $a = k(-h)^2$

OR $k = a/h^2$

CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS r AND THICKNESS dx. THEN

$dV = \pi r^2dx, \quad \bar{x}_{EL}=x$

NOW $r = \frac{a}{h^2}(x-h)^2$ SO THAT

$dV = \pi\frac{a^2}{h^4}(x-h)^4dx$

THEN.. $V = \int_0^h \pi\frac{a^2}{h^4}(x-h)^4dx = \frac{\pi}{5}\frac{a^2}{h^4}\left[(x-h)^5\right]_0^h$

$$= \frac{1}{5}\pi a^2 h$$

AND $\int\bar{x}_{EL}dV = \int_0^h x\left[\pi\frac{a^2}{h^4}(x-h)^4dx\right]$

$$= \pi\frac{a^2}{h^4}\int_0^h(x^5 - 4hx^4 + 6h^2x^3 - 4h^3x^2 + h^4x)dx$$

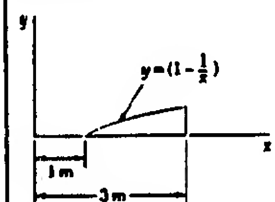
$$= \pi\frac{a^2}{h^4}\left[\frac{1}{6}x^6 - \frac{4}{5}hx^5 + \frac{3}{2}h^2x^4 - \frac{4}{3}h^3x^3 + \frac{1}{2}h^4x^2\right]_0^h$$

$$= \frac{1}{30}\pi a^2h^2$$

NOW.. $\bar{x}V = \int\bar{x}_{EL}dV$: $\quad \bar{x}\left(\frac{\pi}{5}a^2h\right) = \frac{\pi}{30}a^2h^2$

OR $\bar{x} = \frac{1}{6}h$

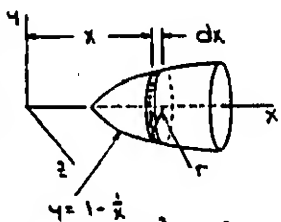
5.136



GIVEN: VOLUME GENERATED BY ROTATING THE AREA SHOWN ABOUT THE X AXIS

FIND: LOCATION OF THE CENTROID OF THE VOLUME

FIRST NOTE THAT SYMMETRY IMPLIES $\bar{y}=0$, $\bar{z}=0$



CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS r AND THICKNESS dx. THEN

$dV = \pi r^2dx, \quad \bar{x}_{EL}=x$

NOW $r = 1 - \frac{1}{x}$ SO THAT

$dV = \pi\left(1-\frac{1}{x}\right)^2dx = \pi\left(1 - \frac{2}{x} + \frac{1}{x^2}\right)dx$

THEN.. $V = \int_1^3\pi\left(1-\frac{2}{x}+\frac{1}{x^2}\right)dx = \pi\left[x - 2\ln x - \frac{1}{x}\right]_1^3$

$$= \pi\left[\left(3 - 2\ln 3 - \frac{1}{3}\right) - \left(1 - 2\ln 1 - \frac{1}{1}\right)\right]$$

$$= 0.46944\pi \text{ m}^3$$

AND $\int\bar{x}_{EL}dV = \int_1^3 x\left[\pi\left(1-\frac{2}{x}+\frac{1}{x^2}\right)dx\right] = \pi\left[\frac{x^2}{2} - 2x + \ln x\right]_1^3$

$$= \pi\left\{\left[\frac{3^2}{2} - 2(3) + \ln 3\right] - \left[\frac{1^2}{2} - 2(1) + \ln 1\right]\right\}$$

$$= 1.09861\pi \text{ m}$$

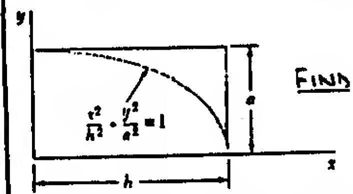
NOW.. $\bar{x}V = \int\bar{x}_{EL}dV$: $\quad \bar{x}(0.46944\pi\text{ m}^3) = 1.09861\pi\text{ m}^4$

OR $\bar{x} = 2.34$ m

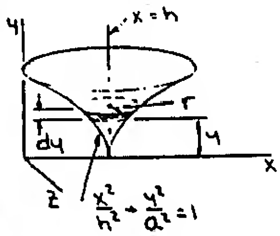
5.137

GIVEN: Volume generated by rotating the area shown about the line $x=h$

FIND: Location of the centroid of the volume



First note that symmetry implies $\bar{x}=h$, $\bar{z}=0$



Choose as the element of volume a disk of radius r and thickness dy. Then

$dV=\pi r^2 dy$, $\bar{y}_{EL}=y$

Now $x^2=\frac{h^2}{a^2}(a^2-y^2)$

so that $r=h-\frac{h}{a}\sqrt{a^2-y^2}$

Then $dV=\pi\frac{h^2}{a^2}(a-\sqrt{a^2-y^2})^2dy$

and $V=\int_0^a \pi\frac{h^2}{a^2}(a-\sqrt{a^2-y^2})^2 dy$

Let $y=a\sin\theta \Rightarrow dy=a\cos\theta\, d\theta$

Then $V=\pi\frac{h^2}{a^2}\int_0^{\pi/2}(a-\sqrt{a^2-a^2\sin^2\theta})^2 a\cos\theta\, d\theta$

$=\pi\frac{h^2}{a^2}\int_0^{\pi/2}[a^2-2a(a\cos\theta)+(a^2-a^2\sin^2\theta)]a\cos\theta\, d\theta$

$=\pi a h^2\int_0^{\pi/2}(2\cos\theta-2\cos^2\theta-\sin^2\theta\cos\theta)d\theta$

$=\pi a h^2[2\sin\theta-2(\frac{\theta}{2}+\frac{\sin 2\theta}{4})-\frac{1}{3}\sin^3\theta]_0^{\pi/2}$

$=\pi a h^2[2-2(\frac{\pi/2}{2})-\frac{1}{3}]$

$=0.095\,870\pi a h^2$

and $\int \bar{y}_{EL}dV=\int_0^a y[\pi\frac{h^2}{a^2}(a-\sqrt{a^2-y^2})^2dy]$

$=\pi\frac{h^2}{a^2}\int_0^a(2a^2y-2ay\sqrt{a^2-y^2}-y^3)dy$

$=\pi\frac{h^2}{a^2}[a^2y^2+\frac{2}{3}a(a^2-y^2)^{3/2}-\frac{1}{4}y^4]_0^a$

$=\pi\frac{h^2}{a^2}\{[a^2(a)^2-\frac{1}{4}a^4]-[\frac{2}{3}a(a^2)^{3/2}]\}$

$=\frac{1}{12}\pi a^2h^2$

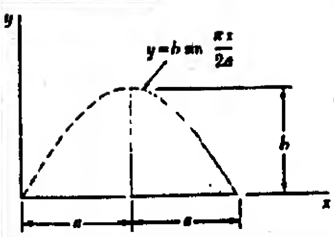
Now... $\bar{y}V=\int\bar{y}_{EL}dV$: $\bar{y}(0.095\,870\pi a h^2)=\frac{1}{12}\pi a^2h^2$

or $\bar{y}=0.869a$

* 5.138

GIVEN: Volume generated by rotating the area shown about the x axis

FIND: Location of the centroid of the volume

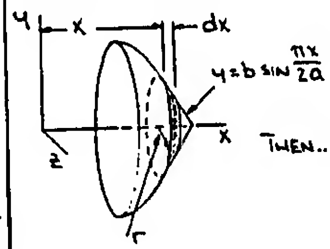


First note that symmetry implies $\bar{y}=0$, $\bar{z}=0$

Choose as the element of volume a disk of radius r and thickness dx. Then

(CONTINUED)

5.138 CONTINUED



$dV=\pi r^2dx$, $\bar{x}_{EL}=x$

Now $r=b\sin\frac{\pi x}{2a}$

so that $dV=\pi b^2\sin^2\frac{\pi x}{2a}dx$

Then... $V=\int_a^{2a}\pi b^2\sin^2\frac{\pi x}{2a}dx$

$=\pi b^2[\frac{x}{2}-\frac{\sin\frac{\pi x}{a}}{2\frac{\pi}{a}}]_a^{2a}$

$=\pi b^2[(\frac{2a}{2})-(\frac{a}{2})]$

$=\frac{1}{2}\pi a b^2$

And $\int\bar{x}dV=\int_a^{2a}x[\pi b^2\sin^2\frac{\pi x}{2a}dx]$

Use integration by parts with

$u=x$ $\quad dv=\sin^2\frac{\pi x}{2a}$

$du=dx$ $\quad v=\frac{x}{2}-\frac{\sin\frac{\pi x}{a}}{\frac{2\pi}{a}}$

Then $\int\bar{x}dV=\pi b^2\{[x(\frac{x}{2}-\frac{\sin\pi x/a}{2\pi/a})]_a^{2a}-\int_a^{2a}(\frac{x}{2}-\frac{\sin\pi x/a}{2\pi/a})dx\}$

$=\pi b^2\{[2a(\frac{2a}{2})-a(\frac{a}{2})]-[\frac{1}{4}x^2+\frac{a^2}{2\pi^2}\cos\frac{\pi x}{a}]_a^{2a}\}$

$=\pi b^2\{(\frac{3}{2}a^2)-[\frac{1}{4}(2a)^2+\frac{a^2}{2\pi^2}-\frac{1}{4}(a)^2+\frac{a^2}{2\pi^2}]\}$

$=\pi a^2b^2(\frac{3}{4}-\frac{1}{\pi^2})$

$=0.648\,68\pi a^2b^2$

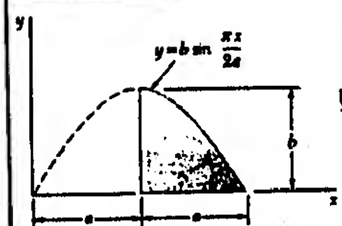
Now... $\bar{x}V=\int\bar{x}_{EL}dV$: $\bar{x}(\frac{1}{2}\pi ab^2)=0.648\,68\pi a^2b^2$

or $\bar{x}=1.297a$

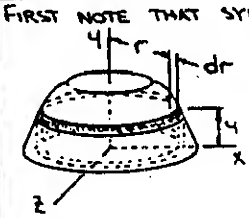
* 5.139

GIVEN: Volume generated by rotating the area shown about the y axis

FIND: Location of the centroid of the volume



First note that symmetry implies $\bar{x}=0$, $\bar{z}=0$



Choose as the element of volume a cylindrical shell of radius r, thickness dr, and height y. Then

$dV=(2\pi r)(y)(dr)$, $\bar{y}_{EL}=\frac{1}{2}y$

Now $y=b\sin\frac{\pi r}{2a}$ so that

$dV=2\pi b r\sin\frac{\pi r}{2a}dr$

Then $V=\int_a^{2a}2\pi br\sin\frac{\pi r}{2a}dr$

Use integration by parts with

$u=r$ $\quad dv=\sin\frac{\pi r}{2a}dr$

$du=dr$ $\quad v=-\frac{2a}{\pi}\cos\frac{\pi r}{2a}$

Then $V=2\pi b\{[(r)(-\frac{2a}{\pi}\cos\frac{\pi r}{2a})]_a^{2a}-\int_a^{2a}(-\frac{2a}{\pi}\cos\frac{\pi r}{2a})dr\}$

$=2\pi b\{-\frac{2a}{\pi}[(2a)(-1)]+[\frac{4a^2}{\pi^2}\sin\frac{\pi r}{2a}]_a^{2a}\}$

(CONTINUED)

5.139 CONTINUED

$$V = 2\pi b\left(\frac{4a^2}{\pi} - \frac{4a^2}{\pi^2}\right)$$
$$= 8a^2b\left(1 - \frac{1}{\pi}\right)$$
$$= 5.4535\, a^2 b$$

ALSO $\int \bar{y}_{EL}\, dV = \int_a^{2a}\left(\frac{1}{2} b \sin\frac{\pi r}{2a}\right)\left(2\pi b r \sin\frac{\pi r}{2a}\, dr\right)$

$$= \pi b^2 \int_a^{2a} r \sin^2\frac{\pi r}{2a}\, dr$$

USE INTEGRATION BY PARTS WITH

$u = r$ $\quad dv = \sin^2\frac{\pi r}{2a}\, dr$

$du = dr$ $\quad v = \frac{r}{2} - \frac{\sin \pi r/a}{2\pi/a}$

THEN.. $\int \bar{y}_{EL}\, dV = \pi b^2 \left\{\left[(r)\left(\frac{r}{2} - \frac{\sin \pi r/a}{2\pi/a}\right)\right]_a^{2a} - \int_a^{2a}\left(\frac{r}{2} - \frac{\sin\frac{\pi r}{a}}{2\pi/a}\right)dr\right\}$

$$= \pi b^2 \left\{\left[(2a)\left(\frac{2a}{2}\right) - (a)\left(\frac{a}{2}\right)\right] - \left[\frac{r^2}{4} + \frac{a^2}{2\pi^2}\cos\frac{\pi r}{a}\right]_a^{2a}\right\}$$
$$= \pi b^2 \left\{\frac{3}{2}a^2 - \left[\frac{(2a)^2}{4} + \frac{a^2}{2\pi^2} - \frac{(a)^2}{4} + \frac{a^2}{2\pi^2}\right]\right\}$$
$$= \pi a^2 b^2\left(\frac{3}{4} - \frac{1}{\pi^2}\right)$$
$$= 2.0379\, a^2 b^2$$

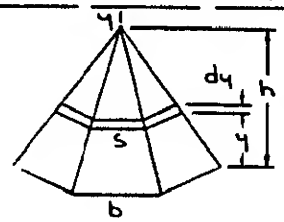
NOW.. $\bar{y}V = \int \bar{y}_{EL}\, dV$: $\bar{y}(5.4535\, a^2 b) = 2.0379\, a^2 b^2$

OR $\bar{y} = 0.374 b$ ◀

*** 5.140**

GIVEN: A REGULAR PYRAMID OF HEIGHT h AND N SIDES

SHOW: $\bar{y} = \frac{h}{4}$ ABOVE THE BASE



CHOOSE AS THE ELEMENT OF VOLUME A HORIZONTAL SLICE OF THICKNESS dy. FOR ANY NUMBER N OF SIDES, THE AREA OF THE BASE OF THE PYRAMID IS GIVEN BY

$$A_{BASE} = k b^2$$

WHERE $k = k(N)$; SEE NOTE BELOW. USING SIMILAR TRIANGLES HAVE

$$\frac{s}{b} = \frac{h-y}{h}$$

OR $s = \frac{b}{h}(h-y)$

THEN.. $dV = A_{SLICE}\, dy = ks^2\, dy = k\frac{b^2}{h^2}(h-y)^2 dy$

AND $V = \int_0^h k\frac{b^2}{h^2}(h-y)^2 dy = k\frac{b^2}{h^2}\left[-\frac{1}{3}(h-y)^3\right]_0^h$

$$= \frac{1}{3}k b^2 h$$

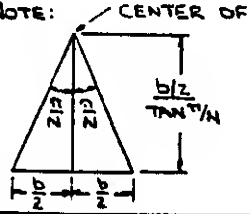
ALSO.. $\bar{y}_{EL} = y$ SO THEN

$$\int \bar{y}_{EL}\, dV = \int_0^h y\left(k\frac{b^2}{h^2}(h-y)^2 dy\right) = k\frac{b^2}{h^2}\int_0^h (h^2y - 2hy^2 + y^3)dy$$
$$= k\frac{b^2}{h^2}\left[\frac{1}{2}h^2y^2 - \frac{2}{3}hy^3 + \frac{1}{4}y^4\right]_0^h = \frac{1}{12}kb^2h^2$$

NOW.. $\bar{y}V = \int \bar{y}_{EL}\, dV$: $\bar{y}\left(\frac{1}{3}kb^2h\right) = \frac{1}{12}kb^2h^2$

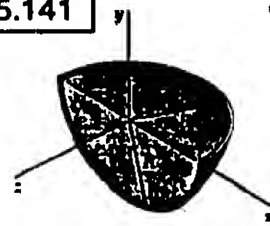
OR $y = \frac{1}{4}h$ Q.E.D. ◀

NOTE: CENTER OF BASE



$$A_{BASE} = N\left(\frac{1}{2} \cdot b \cdot \frac{b/2}{\tan \pi/N}\right)$$
$$= \frac{N}{4\tan \pi/N}\, b^2$$
$$= k(N)\, b^2$$

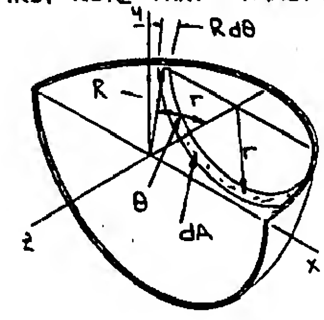
5.141



GIVEN: ONE-HALF OF A THIN, UNIFORM HEMISPHERICAL SHELL

FIND: LOCATION OF CENTROID USING DIRECT INTEGRATION

FIRST NOTE THAT SYMMETRY IMPLIES $\bar{x} = 0$ ◀



THE ELEMENT OF AREA dA OF THE SHELL SHOWN IS OBTAINED BY CUTTING THE SHELL WITH TWO PLANES PARALLEL TO THE xy PLANE. NOW

$$dA = (\pi r)(R\, d\theta)$$
$$\bar{y}_{EL} = -\frac{2r}{\pi}$$

WHERE $r = R\sin\theta$

SO THAT $dA = \pi R^2 \sin\theta\, d\theta$, $\quad \bar{y}_{EL} = -\frac{2R}{\pi}\sin\theta$

THEN $A = \int_0^{\pi/2} \pi R^2 \sin\theta\, d\theta = \pi R^2\left[-\cos\theta\right]_0^{\pi/2}$

$$= \pi R^2$$

AND $\int \bar{y}_{EL}\, dA = \int_0^{\pi/2}\left(-\frac{2R}{\pi}\sin\theta\right)(\pi R^2 \sin\theta\, d\theta)$

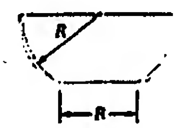
$$= -2R^3\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/2}$$
$$= -\frac{\pi}{2}R^3$$

NOW.. $\bar{y}A = \int \bar{y}_{EL}\, dA$: $\quad \bar{y}(\pi R^2) = -\frac{\pi}{2}R^3$

OR $\bar{y} = -\frac{1}{2}R$ ◀

SYMMETRY IMPLIES $\bar{z} = \bar{y}$ $\quad \therefore\ \bar{z} = -\frac{1}{2}R$ ◀

5.142



GIVEN: PUNCH BOWL OF UNIFORM WALL THICKNESS t, $R = 250$ mm, $t \ll R$

FIND: LOCATION OF THE CENTER OF GRAVITY OF

(a) THE BOWL

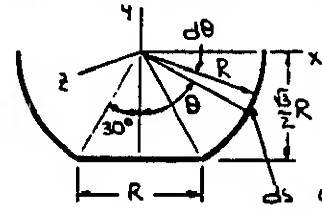
(b) THE PUNCH

(a) BOWL

FIRST NOTE THAT SYMMETRY IMPLIES $\bar{x} = 0$ ◀

$\bar{z} = 0$ ◀

FOR THE COORDINATE AXES SHOWN BELOW. NOW ASSUME THAT THE BOWL MAY BE TREATED AS A SHELL; THE CENTER OF GRAVITY OF THE BOWL WILL COINCIDE WITH THE CENTROID OF THE SHELL.



FOR THE WALLS OF THE BOWL, AN ELEMENT OF AREA IS OBTAINED BY ROTATING THE ARC ds ABOUT THE y AXIS. THEN

$$dA_{WALL} = (2\pi R\sin\theta)(R\, d\theta)$$

(CONTINUED)

5.142 CONTINUED

AND $(\bar{y}_{EL})_{WALL} = -R\cos\theta$

THEN $A_{WALL} = \int_{\pi/6}^{\pi/2} 2\pi R^2 \sin\theta\, d\theta = 2\pi R^2[-\cos\theta]_{\pi/6}^{\pi/2}$

$= \pi\sqrt{3}R^2$

AND $\bar{y}_{WALL}A_{WALL} = \int(\bar{y}_{EL})_{WALL}\, dA$

$= \int_{\pi/6}^{\pi/2}(-R\cos\theta)(2\pi R^2\sin\theta\, d\theta)$

$= \pi R^3[\cos^2\theta]_{\pi/6}^{\pi/2}$

$= -\frac{3}{4}\pi R^3$

BY OBSERVATION.. $A_{BASE} = \frac{\pi}{4}R^2$, $\bar{y}_{BASE} = -\frac{\sqrt{3}}{2}R$

NOW.. $\bar{y}\Sigma A = \Sigma \bar{y}A$

OR.. $\bar{y}(\pi\sqrt{3}R^2 + \frac{\pi}{4}R^2) = -\frac{3}{4}\pi R^3 + \frac{\pi}{4}R^2(-\frac{\sqrt{3}}{2}R)$

OR $\bar{y} = -0.48763R$ $R = 250$ mm

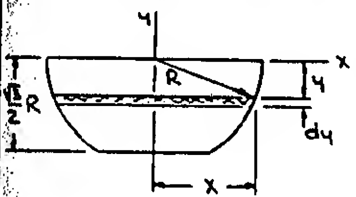
∴ $\bar{y} = -121.9$ mm

(b) PUNCH

FIRST NOTE THAT SYMMETRY IMPLIES $\bar{x} = 0$

$\bar{z} = 0$

AND THAT BECAUSE THE PUNCH IS HOMOGENEOUS, ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING VOLUME.



CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS x AND THICKNESS dy. THEN

$dV = \pi x^2 dy$, $\bar{y}_{EL} = y$

NOW.. $x^2 + y^2 = R^2$

SO THAT $dV = \pi(R^2 - y^2)dy$

THEN $V = \int_{-\frac{\sqrt{3}}{2}R}^{0} \pi(R^2-y^2)dy = \pi[R^2y - \frac{1}{3}y^3]_{-\frac{\sqrt{3}}{2}R}^{0}$

$= -\pi[R^2(-\frac{\sqrt{3}}{2}R) - \frac{1}{3}(-\frac{\sqrt{3}}{2}R)^3] = \frac{3}{8}\pi\sqrt{3}R^3$

AND $\int \bar{y}_{EL}dV = \int_{-\frac{\sqrt{3}}{2}R}^{0}(y)[\pi(R^2-y^2)dy] = \pi[\frac{1}{2}R^2y^2 - \frac{1}{4}y^4]_{-\frac{\sqrt{3}}{2}R}^{0}$

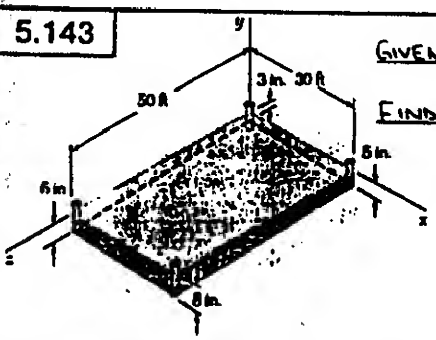
$= -\pi[\frac{1}{2}R^2(-\frac{\sqrt{3}}{2}R)^2 - \frac{1}{4}(-\frac{\sqrt{3}}{2}R)^4] = -\frac{15}{64}\pi R^4$

NOW. $\bar{y}V = \int \bar{y}_{EL}dV$: $\bar{y}(\frac{3}{8}\pi\sqrt{3}R^3) = -\frac{15}{64}\pi R^4$

OR $\bar{y} = -\frac{5}{8\sqrt{3}}R$ $R = 250$ mm

∴ $\bar{y} = -90.2$ mm

5.143



GIVEN: GRAVEL BASE, $y = a + bx + cz$

FIND: VOLUME OF GRAVEL, $\bar{x}$

FIRST DETERMINE THE CONSTANTS a, b, AND c

AT $x=0, z=0$: $y = -3$ in.: $-\frac{3}{12}$ ft $= a$ $a = -\frac{1}{4}$ ft

$x = 30$ ft, $z = 0$: $y = -5$ in.: $-\frac{5}{12}$ ft $= -\frac{1}{4}$ ft $+ b(30$ ft$)$

$b = -\frac{1}{180}$

$x = 0$, $z = 50$ ft: $y = -6$ in.: $-\frac{6}{12}$ ft $= -\frac{1}{4}$ ft $+ c(50$ ft$)$

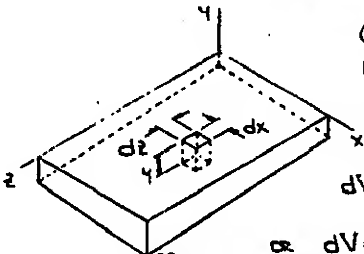
(CONTINUED)

5.143 CONTINUED

OR $c = -\frac{1}{200}$

∴ $y = -\frac{1}{4} - \frac{1}{180}x - \frac{1}{200}z$

$= -\frac{1}{4}(1 + \frac{1}{45}x + \frac{1}{50}z)$ WHERE ALL DIMENSIONS ARE IN FEET



CHOOSE AS THE ELEMENT OF VOLUME A FILAMENT OF BASE dx · dz AND HEIGHT |y|. THEN

$dV = |y|dxdz$, $\bar{x}_{EL} = x$

OR $dV = \frac{1}{4}(1 + \frac{1}{45}x + \frac{1}{50}z)dxdz$

THEN $V = \int_0^{50}\int_0^{30}\frac{1}{4}(1+\frac{1}{45}x+\frac{1}{50}z)dxdz$

$= \frac{1}{4}\int_0^{50}[x + \frac{1}{90}x^2 + \frac{z}{50}x]_0^{30}dz$

$= \frac{1}{4}\int_0^{50}[30 + \frac{(30)^2}{90} + \frac{z}{50}(30)]dz$

$= \frac{1}{4}[40z + \frac{3}{10}z^2]_0^{50} = \frac{1}{4}[40(50) + \frac{3}{10}(50)^2]$

$= 687.5$ ft^3 $V = 688$ ft^3

AND $\int \bar{x}_{EL}dV = \int_0^{50}\int_0^{30} x[\frac{1}{4}(1+\frac{1}{45}x+\frac{1}{50}z)dxdz]$

$= \frac{1}{4}\int_0^{50}[\frac{x^2}{2} + \frac{1}{135}x^3 + \frac{z}{100}x^2]_0^{30}dz$

$= \frac{1}{4}\int_0^{50}[\frac{(30)^2}{2} + \frac{(30)^3}{135} + \frac{z}{100}(30)^2]dz$

$= \frac{1}{4}[(450+200)z + \frac{9}{2}z^2]_0^{50}$

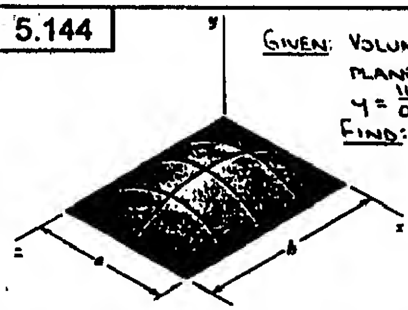
$= \frac{1}{4}[650(50) + \frac{9}{2}(50)^2]$

$= 10{,}937.5$ ft^4

NOW.. $\bar{x}V = \int \bar{x}_{EL}dV$: $\bar{x}(687.5$ ft$^3) = 10{,}937.5$ ft^4

OR $\bar{x} = 15.91$ ft

5.144

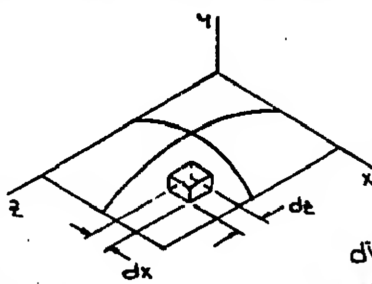


GIVEN: VOLUME BETWEEN THE xz PLANE AND THE SURFACE $y = \frac{16h}{a^2b^2}(ax - x^2)(bz - z^2)$

FIND: LOCATION OF THE CENTROID USING DIRECT INTEGRATION

FIRST NOTE THAT SYMMETRY IMPLIES $\bar{x} = \frac{a}{2}$

$\bar{z} = \frac{b}{2}$



CHOOSE AS THE ELEMENT OF VOLUME A FILAMENT OF BASE dx · dz AND HEIGHT y. THEN

$dV = y\,dxdz$, $\bar{y}_{EL} = \frac{1}{2}y$

OR $dV = \frac{16h}{a^2b^2}(ax - x^2)(bz - z^2)dxdz$

THEN $V = \int_0^b\int_0^a \frac{16h}{a^2b^2}(ax - x^2)(bz - z^2)dxdz$

(CONTINUED)

5.144 CONTINUED

$$V = \frac{16h}{a^2b^2}\int_0^b (bz - z^2)\left[\frac{a}{2}x^2 - \frac{1}{3}x^3\right]_0^a dz$$
$$= \frac{16h}{a^2b^2}\left[\frac{a}{2}(a)^2 - \frac{1}{3}(a)^3\right]\left[\frac{b}{2}z^2 - \frac{1}{3}z^3\right]_0^b$$
$$= \frac{8ah}{3b^2}\left[\frac{b}{2}(b)^2 - \frac{1}{3}(b)^3\right] = \frac{4}{9}abh$$

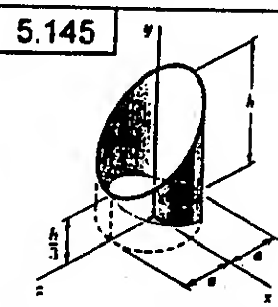
AND $\int \bar{y}_{EL}\,dV = \int_0^b\int_0^a \frac{1}{2}\times\frac{16h}{a^2b^2}(ax - x^2)(bz - z^2) \times \left[\frac{16h}{a^2b^2}(ax - x^2)(bz - z^2)\,dx\,dz\right]$

$$= \frac{128h^2}{a^4b^4}\int_0^b\int_0^a (a^2x^2 - 2ax^3 + x^4) \times (b^2z^2 - 2bz^3 + z^4)\,dx\,dz$$
$$= \frac{128h^2}{a^4b^4}\int_0^b (b^2z^2 - 2bz^3 + z^4) \cdot \left[\frac{a^2}{3}x^3 - \frac{a}{2}x^4 + \frac{1}{5}x^5\right]_0^a dz$$
$$= \frac{128h^2}{a^4b^4}\left[\frac{a^2}{3}(a)^3 - \frac{a}{2}(a)^4 + \frac{1}{5}(a)^5\right] \times \left[\frac{b^2}{3}z^3 - \frac{b}{2}z^4 + \frac{1}{5}z^5\right]_0^b$$
$$= \frac{64ah^2}{15b^4}\left[\frac{b^2}{3}(b)^3 - \frac{b}{2}(b)^4 + \frac{1}{5}(b)^5\right]$$
$$= \frac{32}{225}abh^2$$

Now.. $\bar{y}V = \int \bar{y}_{EL}\,dV$: $\quad \bar{y}\left(\frac{4}{9}abh\right) = \frac{32}{225}abh^2$

OR $\bar{y} = \frac{8}{25}h$ ◄

5.145

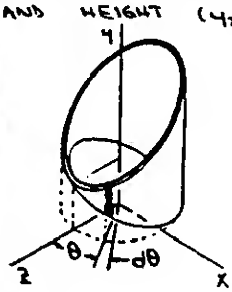


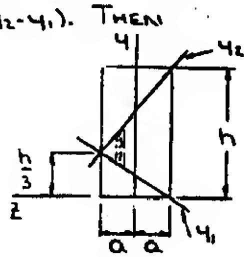
GIVEN: THE PORTION OF A CIRCULAR PIPE SHOWN

FIND: LOCATION OF THE CENTROID

FIRST NOTE THAT SYMMETRY IMPLIES $\bar{x} = 0$ ◄

ASSUME THAT THE PIPE HAS A UNIFORM WALL THICKNESS t AND CHOOSE AS THE ELEMENT OF VOLUME A VERTICAL STRIP OF WIDTH $a\,d\theta$ AND HEIGHT $(y_2 - y_1)$. THEN





$dV = (y_2 - y_1)t\,a\,d\theta, \quad \bar{y}_{EL} = \frac{1}{2}(y_1 + y_2) \quad \bar{z}_{EL} = z$

Now.. $y_1 = \frac{h/3}{2a}z + \frac{h}{6} = \frac{h}{6a}(z + a)$ $\qquad y_2 = -\frac{2h/3}{2a}z + \frac{2}{3}h = \frac{h}{3a}(-z + 2a)$

AND $z = a\cos\theta$

THEN $(y_2 - y_1) = \frac{h}{3a}(-a\cos\theta + 2a) - \frac{h}{6a}(a\cos\theta + a) = \frac{h}{2}(1 - \cos\theta)$

AND $(y_1 + y_2) = \frac{h}{6a}(a\cos\theta + a) + \frac{h}{3a}(-a\cos\theta + 2a) = \frac{h}{6}(5 - \cos\theta)$

(CONTINUED)

5.145 CONTINUED

$\therefore\ dV = \frac{aht}{2}(1 - \cos\theta)d\theta, \quad \bar{y}_{EL} = \frac{h}{12}(5 - \cos\theta), \quad \bar{z}_{EL} = a\cos\theta$

THEN $V = 2\int_0^\pi \frac{aht}{2}(1 - \cos\theta)d\theta = aht\left[\theta - \sin\theta\right]_0^\pi = \pi aht$

AND $\int \bar{y}_{EL}\,dV = 2\int_0^\pi \frac{h}{12}(5 - \cos\theta)\left[\frac{aht}{2}(1 - \cos\theta)d\theta\right]$

$$= \frac{ah^2t}{12}\int_0^\pi (5 - 6\cos\theta + \cos^2\theta)d\theta$$
$$= \frac{ah^2t}{12}\left[5\theta - 6\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^\pi$$
$$= \frac{11}{24}\pi ah^2t$$

$\int \bar{z}_{EL}\,dV = 2\int_0^\pi a\cos\theta\left[\frac{aht}{2}(1 - \cos\theta)d\theta\right]$

$$= a^2ht\left[\sin\theta - \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^\pi$$
$$= -\frac{1}{2}\pi a^2ht$$

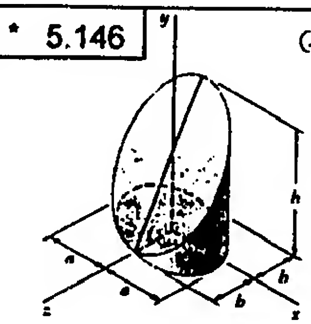
Now.. $\bar{y}V = \int \bar{y}_{EL}\,dV$: $\quad \bar{y}(\pi aht) = \frac{11}{24}\pi ah^2t$

OR $\bar{y} = \frac{11}{24}h$ ◄

AND $\bar{z}V = \int \bar{z}_{EL}\,dV$: $\quad \bar{z}(\pi aht) = -\frac{1}{2}\pi a^2ht$

OR $\bar{z} = -\frac{1}{2}a$ ◄

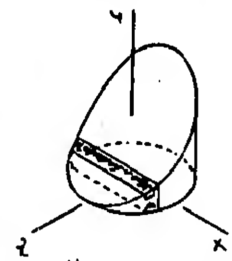
* 5.146

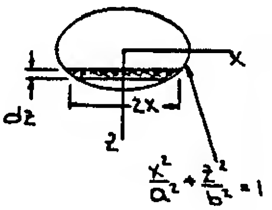


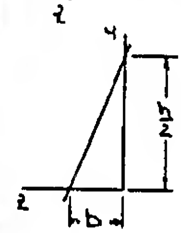
GIVEN: THE PORTION OF AN ELLIPTICAL CYLINDER SHOWN

FIND: LOCATION OF THE CENTROID

FIRST NOTE THAT SYMMETRY IMPLIES $\bar{x} = 0$ ◄







CHOOSE AS THE ELEMENT OF VOLUME A VERTICAL SLICE OF WIDTH $2x$, THICKNESS dz, AND HEIGHT y. THEN

$dV = 2xy\,dz, \quad \bar{y}_{EL} = \frac{1}{2}y, \quad \bar{z}_{EL} = z$

Now $x = \frac{a}{b}\sqrt{b^2 - z^2}$ AND $y = -\frac{h/2}{b}z + \frac{h}{2} = \frac{h}{2b}(b - z)$

THEN $V = \int_{-b}^{b}\left(2\frac{a}{b}\sqrt{b^2 - z^2}\right)\left[\frac{h}{2b}(b - z)\right]dz$

LET $z = b\sin\theta, \quad dz = b\cos\theta\,d\theta$

THEN $V = \frac{ah}{b^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(b\cos\theta)\left[b(1 - \sin\theta)\right]b\cos\theta\,d\theta$

$$= abh\int_{-\pi/2}^{\pi/2}(\cos^2\theta - \sin\theta\cos^2\theta)d\theta$$
$$= abh\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} + \frac{1}{3}\cos^3\theta\right]_{-\pi/2}^{\pi/2}$$

(CONTINUED)

5.146 CONTINUED

$V = \frac{1}{2}\pi abh$

AND $\int \bar{y}_{EL}\,dV = \int_{-b}^{b}\left[\frac{1}{2}\times\frac{h}{2b}(b-z)\right]\left\{\left(2\frac{a}{b}\sqrt{b^2-z^2}\right)\left[\frac{h}{2b}(b-z)\right]dz\right\}$

$= \frac{1}{4}\frac{ah^2}{b^3}\int_{-b}^{b}(b-z)^2\sqrt{b^2-z^2}\,dz$

LET $z = b\sin\theta$, $dz = b\cos\theta\,d\theta$

THEN $\int \bar{y}_{EL}\,dV = \frac{1}{4}\frac{ah^2}{b^3}\int_{-\pi/2}^{\pi/2}[b(1-\sin\theta)]^2(b\cos\theta)\times(b\cos\theta\,d\theta)$

$= \frac{1}{4}abh^2\int_{-\pi/2}^{\pi/2}(\cos^2\theta - 2\sin\theta\cos^2\theta + \sin^2\theta\cos^2\theta)\,d\theta$

NOW $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ $\quad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

SO THAT $\sin^2\theta\cos^2\theta = \frac{1}{4}(1-\cos^2 2\theta)$

THEN $\int \bar{y}_{EL}\,dV = \frac{1}{4}abh^2\int_{-\pi/2}^{\pi/2}\left[\cos^2\theta - 2\sin\theta\cos^2\theta + \frac{1}{4}(1-\cos^2 2\theta)\right]d\theta$

$= \frac{1}{4}abh^2\left[\left(\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right) + \frac{1}{3}\cos^3\theta + \frac{1}{4}\theta - \frac{1}{4}\left(\frac{\theta}{2}+\frac{\sin 4\theta}{8}\right)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

$= \frac{5}{32}\pi abh^2$

ALSO $\int \bar{z}_{EL}\,dV = \int_{-b}^{b} z\left\{2\frac{a}{b}\sqrt{b^2-z^2}\left[\frac{h}{2b}(b-z)\right]dz\right\}$

$= \frac{ah}{b^2}\int_{-b}^{b} z(b-z)\sqrt{b^2-z^2}\,dz$

LET $z = b\sin\theta$, $dz = b\cos\theta\,d\theta$

THEN $\int \bar{z}_{EL}\,dV = \frac{ah}{b^2}\int_{-\pi/2}^{\pi/2}(b\sin\theta)[b(1-\sin\theta)](b\cos\theta)\times(b\cos\theta\,d\theta)$

$= ab^2h\int_{-\pi/2}^{\pi/2}(\sin\theta\cos^2\theta - \sin^2\theta\cos^2\theta)\,d\theta$

USING $\sin^2\theta\cos^2\theta = \frac{1}{4}(1-\cos^2 2\theta)$ FROM ABOVE..

$\int \bar{z}_{EL}\,dV = ab^2h\int_{-\pi/2}^{\pi/2}\left[\sin\theta\cos^2\theta - \frac{1}{4}(1-\cos^2 2\theta)\right]d\theta$

$= ab^2h\left[-\frac{1}{3}\cos^3\theta - \frac{1}{4}\theta + \frac{1}{4}\left(\frac{\theta}{2}+\frac{\sin 4\theta}{8}\right)\right]_{-\pi/2}^{\pi/2}$

$= -\frac{1}{8}\pi ab^2h$

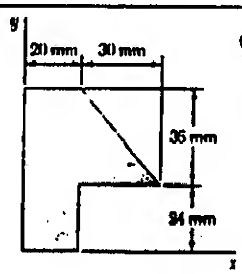
NOW.. $\bar{y}V = \int \bar{y}_{EL}\,dV$: $\quad \bar{y}\left(\frac{1}{2}\pi abh\right) = \frac{5}{32}\pi abh^2$

OR $\bar{y} = \frac{5}{16}h$ ◄

AND $\bar{z}V = \int \bar{z}_{EL}\,dV$: $\quad \bar{z}\left(\frac{1}{2}\pi abh\right) = -\frac{1}{8}\pi ab^2h$

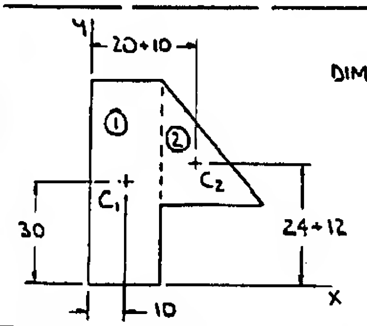
OR $\bar{z} = -\frac{1}{4}b$ ◄

5.147



GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$



DIMENSIONS IN MM

(CONTINUED)

5.147 CONTINUED

	A, mm²	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm³	$\bar{y}A$, mm³
1	$20\times60=1200$	10	30	12 000	36 000
2	$\frac{1}{2}\times30\times36=540$	30	36	16 200	19 440
Σ	1740			28 200	55440

THEN $\bar{X}\Sigma A = \Sigma\bar{x}A$

$\bar{X}(1740) = 28\,200$

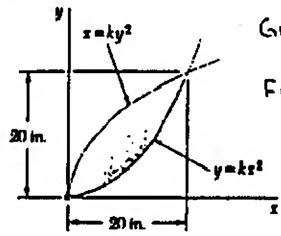
OR $\bar{X} = 16.21$ mm ◄

AND $\bar{Y}\Sigma A = \Sigma\bar{y}A$

$\bar{Y}(1740) = 55\,440$

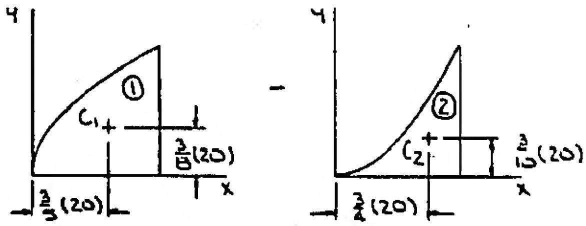
OR $\bar{Y} = 31.9$ mm ◄

5.148



GIVEN: PLANE AREA SHOWN

FIND: $\bar{X}$ AND $\bar{Y}$



DIMENSIONS IN IN.

	A, in²	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in³	$\bar{y}A$, in³
1	$\frac{2}{3}(20)(20) = 800/3$	12	7.5	3200	2000
2	$-\frac{1}{3}(20)(20) = -400/3$	15	6	−2000	−800
Σ	400/3			1200	1200

THEN $\bar{X}\Sigma A = \Sigma\bar{x}A$

$\bar{X}(400/3) = 1200$

OR $\bar{X} = 9.00$ in. ◄

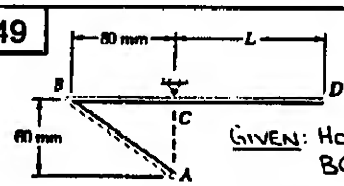
AND $\bar{Y}\Sigma A = \Sigma\bar{y}A$

$\bar{Y}(400/3) = 1200$

OR $\bar{Y} = 9.00$ in. ◄

NOTE: SYMMETRY IMPLIES $\bar{X} = \bar{Y}$, WHICH IS CONFIRMED BY THE ABOVE SOLUTION.

5.149



GIVEN: HOMOGENEOUS WIRE, BCD IS HORIZONTAL

FIND: L

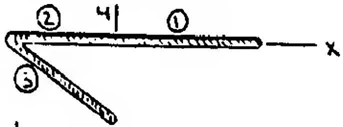
FIRST NOTE THAT FOR EQUILIBRIUM, THE CENTER OF GRAVITY OF THE WIRE MUST LIE ON A VERTICAL LINE THROUGH C. FURTHER, BECAUSE THE WIRE IS HOMOGENEOUS, THE CENTER OF GRAVITY OF THE WIRE WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING LINE. THUS,

$\bar{X} = 0$ (SEE SKETCH ON THE NEXT PAGE)

SO THAT $\Sigma\bar{x}L = 0$

(CONTINUED)

5.149 CONTINUED

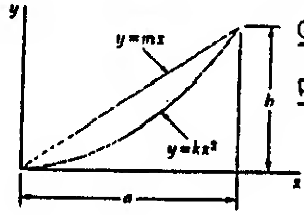


Then $\frac{L}{2}(L) + (-40\text{ mm})(80\text{ mm}) + (-40\text{ mm})(100\text{ mm}) = 0$

or $L^2 = 14\,400\text{ mm}^2$

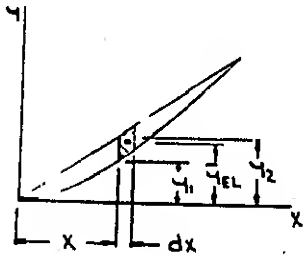
or $L = 120.0\text{ mm}$ ◄

5.150



GIVEN: PLANE AREA SHOWN

FIND: $\bar{x}$ AND $\bar{y}$ USING DIRECT INTEGRATION



AT (a, b)

y_1: $b = ka^2$ OR $k = \frac{b}{a^2}$

y_2: $b = ma$ OR $m = \frac{b}{a}$

NOW.. $\bar{x}_{EL} = x$

$\bar{y}_{EL} = \frac{1}{2}(y_1 + y_2)$

AND $dA = (y_2 - y_1)dx$

$= (\frac{b}{a}x - \frac{b}{a^2}x^2)dx$

$= \frac{b}{a^2}(ax - x^2)dx$

THEN $A = \int dA = \int_0^a \frac{b}{a^2}(ax - x^2)dx = \frac{b}{a^2}[\frac{a}{2}x^2 - \frac{1}{3}x^3]_0^a$

$= \frac{1}{6}ab$

AND $\int \bar{x}_{EL}\,dA = \int_0^a x[\frac{b}{a^2}(ax - x^2)dx] = \frac{b}{a^2}[\frac{a}{3}x^3 - \frac{1}{4}x^4]_0^a$

$= \frac{1}{12}a^2b$

$\int \bar{y}_{EL}\,dA = \int \frac{1}{2}(y_1 + y_2)[(y_2 - y_1)dx]$

$= \int \frac{1}{2}(y_2^2 - y_1^2)dx$

$= \frac{1}{2}\int_0^a (\frac{b^2}{a^2}x^2 - \frac{b^2}{a^4}x^4)dx$

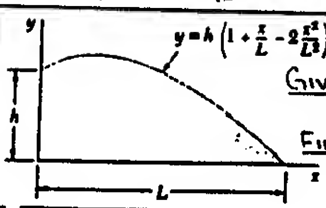
$= \frac{1}{2}\frac{b^2}{a^4}[\frac{a^2}{3}x^3 - \frac{1}{5}x^5]_0^a$

$= \frac{1}{15}ab^2$

$\bar{x}A = \int \bar{x}_{EL}\,dA$: $\bar{x}(\frac{1}{6}ab) = \frac{1}{12}a^2b$ $\qquad \bar{x} = \frac{1}{2}a$ ◄

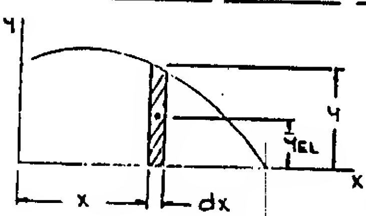
$\bar{y}A = \int \bar{y}_{EL}\,dA$: $\bar{y}(\frac{1}{6}ab) = \frac{1}{15}ab^2$ $\qquad \bar{y} = \frac{2}{5}b$ ◄

5.151



GIVEN: PLANE AREA SHOWN

FIND: $\bar{y}$ BY DIRECT INTEGRATION



(CONTINUED)

5.151 CONTINUED

HAVE $dA = y\,dx = h(1 + \frac{x}{L} - 2\frac{x^2}{L^2})dx$

AND $\bar{y}_{EL} = \frac{1}{2}y = \frac{1}{2}h(1 + \frac{x}{L} - 2\frac{x^2}{L^2})$

THEN $A = \int dA = \int_0^L h(1 + \frac{x}{L} - 2\frac{x^2}{L^2})dx$

$= h[x + \frac{1}{2L}x^2 - \frac{2}{3L^2}x^3]_0^L = \frac{5}{6}hL$

AND $\int \bar{y}_{EL}\,dA = \int_0^L \frac{1}{2}h(1 + \frac{x}{L} - 2\frac{x^2}{L^2})[h(1 + \frac{x}{L} - 2\frac{x^2}{L^2})dx]$

$= \frac{1}{2}h^2\int_0^L (1 + 2\frac{x}{L} - 3\frac{x^2}{L^2} - 4\frac{x^3}{L^3} + 4\frac{x^4}{L^4})dx$

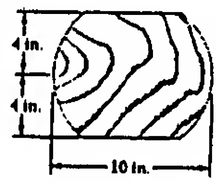
$= \frac{1}{2}h^2[x + \frac{1}{L}x^2 - \frac{1}{L^2}x^3 - \frac{1}{L^3}x^4 + \frac{4}{5L^4}x^5]_0^L$

$= \frac{2}{5}Lh^2$

$\bar{y}A = \int \bar{y}_{EL}\,dA$: $\bar{y}(\frac{5}{6}hL) = \frac{2}{5}Lh^2$ $\qquad \bar{y} = \frac{12}{25}h$

OR $\bar{y} = 0.48h$ ◄

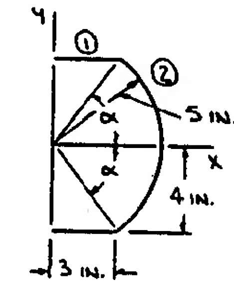
5.152



GIVEN: WOODEN SPHERE WITH TWO EQUAL CAPS REMOVED

FIND: SURFACE AREA OF BODY

THE SURFACE AREA CAN BE GENERATED BY ROTATING THE LINE SHOWN ABOUT THE y AXIS APPLYING THE FIRST THEOREM OF PAPPUS-GULDINUS HAVE



$A = 2\pi\bar{X}L = 2\pi\Sigma\bar{x}L$

$= 2\pi(2\bar{x}_1 L_1 + \bar{x}_2 L_2)$

NOW $\tan\alpha = \frac{4}{3}$

OR $\alpha = 53.130°$

THEN $\bar{x}_2 = \frac{5\text{ IN.}\cdot\sin 53.130°}{53.130°\cdot\frac{\pi}{180°}}$

$= 4.3136\text{ IN.}$

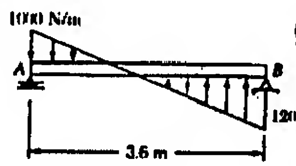
AND $L_2 = 2(53.130°\cdot\frac{\pi}{180°})(5\text{ IN.})$

$= 9.2729\text{ IN.}$

$\therefore A = 2\pi[2(\frac{3}{2}\text{ IN.})(3\text{ IN.}) + (4.3136\text{ IN.})(9.2729\text{ IN.})]$

OR $A = 308\text{ IN}^2$ ◄

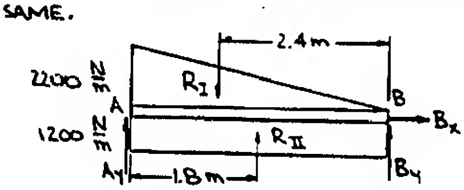
5.153



GIVEN: BEAM AND LOADING SHOWN

FIND: REACTIONS AT THE SUPPORTS

FIRST REPLACE THE GIVEN LOADING WITH THE LOADING SHOWN BELOW. THE TWO LOADINGS ARE EQUIVALENT BECAUSE BOTH ARE DEFINED BY A LINEAR RELATION BETWEEN LOAD AND DISTANCE AND THE VALUES AT THE END POINTS ARE THE SAME.



(CONTINUED)

5.153 CONTINUED

HAVE.. $R_I = \frac{1}{2}(3.6\,m)(2200\,\frac{N}{m}) = 3960\ N$

$R_{II} = (3.6\,m)(1200\,\frac{N}{m}) = 4320\ N$

THEN.. $\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$

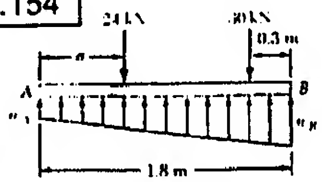
$\circlearrowleft \Sigma M_B = 0: \ -(3.6\,m)A_y + (2.4\,m)(3960\,N) - (1.8\,m)(4320\,N) = 0$

OR $A_y = 480\,N$ $\quad \underline{A} = 480\,N \uparrow$ ◀

$+\uparrow \Sigma F_y = 0: \ 480\,N - 3960\,N + 4320 + B_y = 0$

OR $B_y = -840\,N$ $\quad \underline{B} = 840\,N \downarrow$ ◀

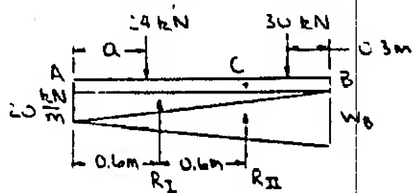
5.154



GIVEN: BEAM AND LOADING SHOWN, $w_A = 20\ kN/m$

FIND: (a) a

(b) w_B



HAVE.. $R_I = \frac{1}{2}(1.8\,m)(20\,\frac{kN}{m}) = 18\ kN$

$R_{II} = \frac{1}{2}(1.8\,m)(w_B\,\frac{kN}{m}) = 0.9\,w_B\ kN$

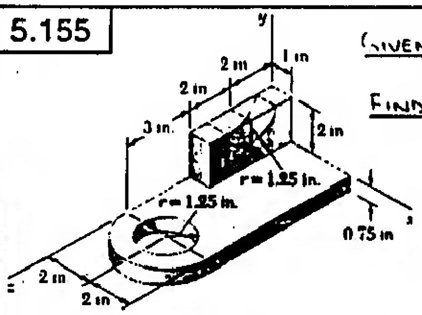
(a) $\circlearrowleft \Sigma M_C = 0: \ (1.2 - a)\,m \times 24\,kN - 0.6\,m \times 18\,kN - 0.3\,m \times 30\,kN = 0$

OR $a = 0.375\,m$ ◀

(b) $+\uparrow \Sigma F_y = 0: \ -24\,kN + 18\,kN + (0.9\,w_B)\,kN - 30\,kN = 0$

OR $w_B = 40\ \frac{kN}{m}$ ◀

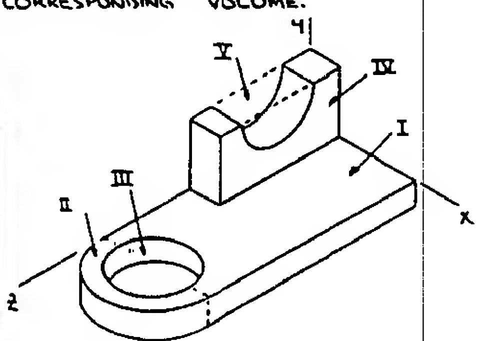
5.155



GIVEN: MACHINE ELEMENT SHOWN

FIND: $\bar{z}$

FIRST ASSUME THAT THE MACHINE ELEMENT IS HOMOGENEOUS SO THAT ITS CENTER OF GRAVITY WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING VOLUME.



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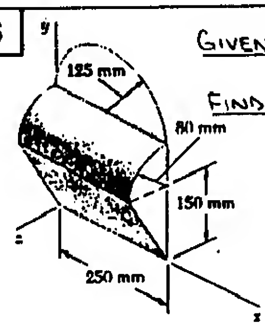
5.155 CONTINUED

	$V,\ in^3$	$\bar{z},\ in.$	$\bar{z}V,\ in^4$
I	$(4)(0.75)(7) = 21$	3.5	73.5
II	$\frac{\pi}{2}(2)^2(0.75) = 4.7124$	$7 + \frac{4 \times 2}{3\pi} = 7.8488$	36.987
III	$-\pi(1.25)^2(0.75) = -3.6816$	7	−25.771
IV	$(1)(2)(4) = 8$	2	16
V	$-\frac{\pi}{2}(1.25)^2(1) = -2.4544$	2	−4.9088
Σ	27.576		95.807

HAVE.. $\bar{Z}\Sigma V = \Sigma \bar{z} V: \quad \bar{Z}(27.576\ in^3) = 95.807\ in^4$

OR $\bar{Z} = 3.47\ in.$ ◀

5.156

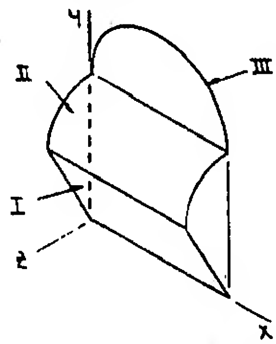


GIVEN: SHEET-METAL FORM SHOWN

FIND: LOCATION OF CENTER OF GRAVITY

FIRST ASSUME THAT THE SHEET METAL IS HOMOGENEOUS SO THAT THE CENTER OF GRAVITY OF THE FORM WILL COINCIDE WITH THE CENTROID OF THE CORRESPONDING AREA. NOW NOTE THAT SYMMETRY IMPLIES

$\bar{X} = 125\ mm$ ◀



$\bar{y}_{II} = 150 + \frac{2 \times 80}{\pi} = 200.93\ mm$

$\bar{z}_{II} = \frac{2 \times 80}{\pi} = 50.930\ mm$

$\bar{y}_{III} = 230 + \frac{4 \times 125}{3\pi} = 283.05\ mm$

	$A,\ mm^2$	$\bar{y},\ mm$	$\bar{z},\ mm$	$\bar{y}A,\ mm^3$	$\bar{z}A,\ mm^3$
I	$(250)(170) = 42\,500$	75	40	3 187 500	1 700 000
II	$\frac{\pi}{2}(80)(250) = 31\,416$	200.93	50.930	6 312 400	1 600 000
III	$\frac{\pi}{2}(125)^2 = 24\,544$	283.05	0	6 947 200	0
Σ	98 460			16 447 100	3 300 000

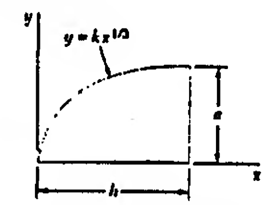
HAVE.. $\bar{Y}\Sigma A = \Sigma \bar{y} A: \quad \bar{Y}(98\,460\ mm^2) = 16\,447\,100\ mm^3$

OR $\bar{Y} = 167.0\ mm$ ◀

$\bar{Z}\Sigma A = \Sigma \bar{z} A: \quad \bar{Z}(98\,460\ mm^2) = 3.300 \times 10^6\ mm^3$

OR $\bar{Z} = 33.5\ mm$ ◀

5.157



GIVEN: VOLUME GENERATED BY ROTATING THE AREA SHOWN ABOUT THE X AXIS

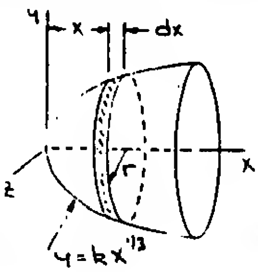
FIND: LOCATION OF THE CENTROID OF THE VOLUME

FIRST NOTE THAT SYMMETRY IMPLIES

$\bar{y} = 0$ ◀

$\bar{z} = 0$ ◀

CHOOSE AS THE ELEMENT OF VOLUME A DISK OF RADIUS r AND THICKNESS dx. THEN



$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$

NOW $r = kx^{1/3}$ SO THAT

$dV = \pi k^2 x^{2/3} dx$

AT $x = h, y = a$: $a = kh^{1/3}$

OR $k = a/h^{1/3}$

THEN $dV = \pi \frac{a^2}{h^{2/3}} x^{2/3} dx$

AND $V = \int_0^h \pi \frac{a^2}{h^{2/3}} x^{2/3} dx$

$= \pi \frac{a^2}{h^{2/3}} \left[\frac{3}{5} x^{5/3} \right]_0^h$

$= \frac{3}{5} \pi a^2 h$

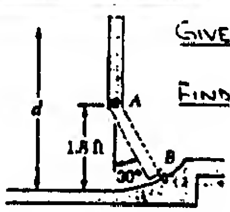
ALSO.. $\int \bar{x}_{EL} dV = \int_0^h x \left(\pi \frac{a^2}{h^{2/3}} x^{2/3} dx \right) = \pi \frac{a^2}{h^{2/3}} \left[\frac{3}{8} x^{8/3} \right]_0^h$

$= \frac{3}{8} \pi a^2 h^2$

NOW.. $\bar{x} V = \int \bar{x} dV$: $\bar{x} \left(\frac{3}{5} \pi a^2 h \right) = \frac{3}{8} \pi a^2 h^2$

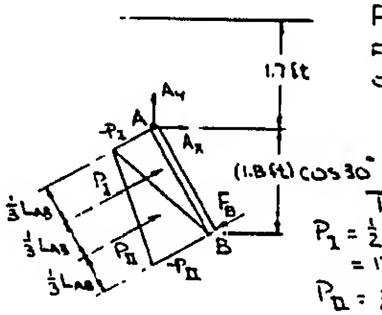
OR $\bar{x} = \frac{5}{8} h$ ◀

5.158



GIVEN: 1.8×1.8-ft GATE, d = 3.5 ft, WATER

FIND: FORCE F_B EXERTED BY PIN AT B ON GATE



FIRST CONSIDER THE FORCE OF THE WATER ON THE GATE. HAVE

$P = \frac{1}{2} A p$

$= \frac{1}{2} A (\gamma h)$

THEN ...

$P_I = \frac{1}{2} (1.8 \text{ ft})^2 (62.4 \frac{\text{lb}}{\text{ft}^3})(1.7 \text{ ft})$

$= 171.850 \text{ lb}$

$P_{II} = \frac{1}{2} (1.8 \text{ ft})^2 (62.4 \frac{\text{lb}}{\text{ft}^3})$

$\times (1.7 + 1.8 \cos 30°) \text{ ft}$

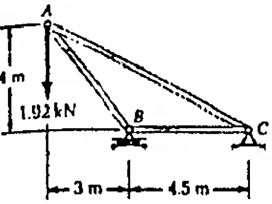
$= 329.43 \text{ lb}$

NOW.. $\Sigma M_A = 0$: $(\frac{1}{3} L_{AB}) P_I + (\frac{2}{3} L_{AB}) P_{II} - L_{AB} F_B = 0$

OR $\frac{1}{3}(171.850 \text{ lb}) + \frac{2}{3}(329.43 \text{ lb}) - F_B = 0$

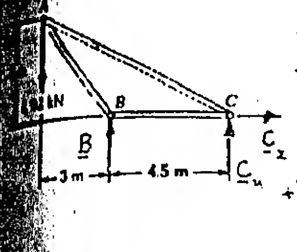
OR $F_B = 276.90 \text{ lb}$

$F_B = 277 \text{ lb} \measuredangle 30°$ ◀



GIVEN:
TRUSS AND LOADING SHOWN.
FIND:
FORCE IN EACH MEMBER

FREE BODY: ENTIRE TRUSS

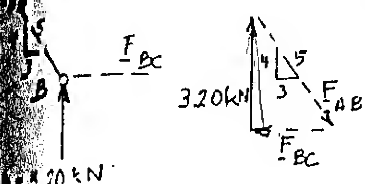


$\xrightarrow{+} \Sigma F_x = 0:\ C_x = 0 \qquad C_x = 0$

$+\circlearrowleft \Sigma M_B = 0:$
$(1.92\ kN)(3\ m) + C_y(4.5\ m) = 0$
$C_y = -1.28\ kN \qquad C_y = 1.28\ kN \downarrow$

$+\uparrow \Sigma F_y = 0:\ B - 1.92\ kN - 1.28\ kN = 0$
$B = 3.20\ kN \uparrow$

FREE BODY: JOINT B

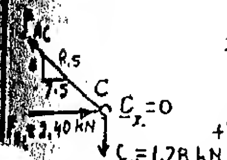


$\frac{F_{AB}}{5} = \frac{F_{BC}}{3} = \frac{3.20\ kN}{4}$

$F_{AB} = 4.00\ kN\ C$ ◀

$F_{BC} = 2.40\ kN\ C$ ◀

FREE BODY: JOINT C

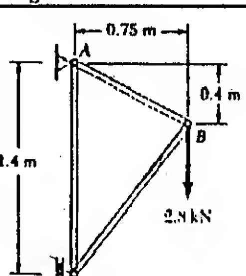


$\xrightarrow{+} \Sigma F_x = 0:\ -\frac{7.5}{8.5} F_{AC} + 2.40\ kN = 0$

$F_{AC} = +2.72\ kN \qquad F_{AC} = 2.72\ kN\ T$ ◀

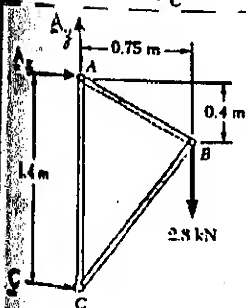
$+\uparrow \Sigma F_y = \frac{4}{8.5}(2.72\ kN) - 1.28\ kN = 0$ (CHECKS)

6.2



GIVEN:
TRUSS AND LOADING SHOWN.
FIND:
FORCE IN EACH MEMBER.

FREE BODY: ENTIRE TRUSS

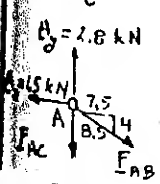


$+\circlearrowleft \Sigma M_A = 0:$
$C(1.4\ m) - (2.8\ kN)(0.75\ m) = 0$
$C = +1.500\ kN \qquad C = 1.500\ kN \rightarrow$

$\xrightarrow{+} \Sigma F_x = 0:\ A_x + 1.500\ kN = 0$
$A_x = -1.500\ kN \qquad A_x = 1.500\ kN \leftarrow$

$+\uparrow \Sigma F_y = 0:\ A_y - 2.8\ kN = 0$
$A_y = 2.8\ kN \uparrow$

FREE BODY: JOINT A



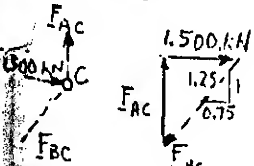
$\xrightarrow{+} \Sigma F_x = 0:\ \frac{7.5}{8.5} F_{AB} - 1.500\ kN = 0$

$F_{AB} = 1.700\ kN\ T$ ◀

$+\uparrow \Sigma F_y = 0:\ 2.8\ kN - \frac{4}{8.5}(1.700\ kN) - F_{AC} = 0$

$F_{AC} = 2.00\ kN\ T$ ◀

FREE BODY: JOINT C

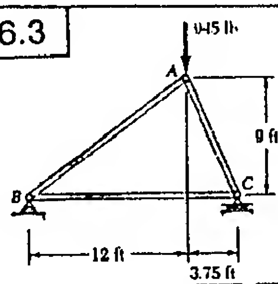


$\frac{F_{BC}}{1.25} = \frac{F_{AC}}{1} = \frac{1.500\ kN}{0.75}$

$F_{BC} = 2.50\ kN\ C$ ◀

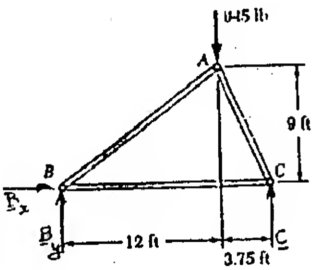
$F_{AC} = 2.00\ kN\ T$ (CHECKS)

6.3



GIVEN:
TRUSS AND LOADING SHOWN
FIND:
FORCE IN EACH MEMBER

FREE BODY: ENTIRE TRUSS

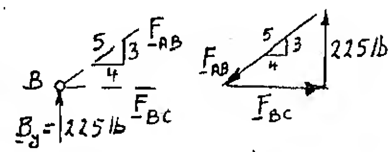


$\xrightarrow{+} \Sigma F_x = 0:\ B_x = 0$

$+\circlearrowleft \Sigma M_B = 0:$
$C(15.75\ ft) - (945\ lb)(12\ ft) = 0$
$C = 720\ lb \uparrow$

$+\uparrow \Sigma F_y = 0:\ B_y + 720\ lb - 945\ lb = 0$
$B_y = 225\ lb \uparrow$

FREE BODY: JOINT B

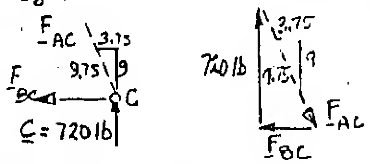


$\frac{F_{AB}}{5} = \frac{F_{BC}}{4} = \frac{225\ lb}{3}$

$F_{AB} = 375\ lb\ C$ ◀

$F_{BC} = 300\ lb\ T$ ◀

FREE BODY: JOINT C

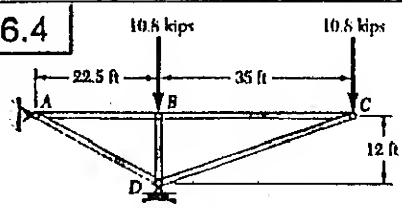


$\frac{F_{AC}}{9.75} = \frac{F_{BC}}{3.75} = \frac{720\ lb}{9}$

$F_{AC} = 780\ lb\ C$ ◀

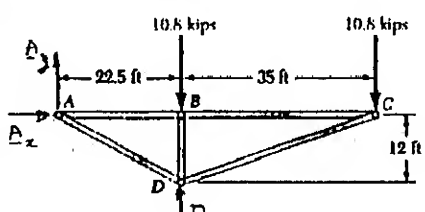
$F_{BC} = 300\ lb\ T$ (CHECKS)

6.4



GIVEN:
TRUSS AND LOADING SHOWN.
FIND:
FORCE IN EACH MEMBER

FREE BODY: TRUSS

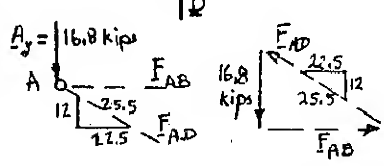


$\xrightarrow{+} \Sigma F_x = 0:\ A_x = 0$

$+\circlearrowleft \Sigma M_A = 0:$
$D(22.5) - (10.8\ kips)(22.5) - (10.8\ kips)(57.5) = 0$
$D = 38.4\ kips \uparrow$

$\Sigma F_y = 0:\ A_y = 16.8\ kips \downarrow$

FREE BODY: JOINT A

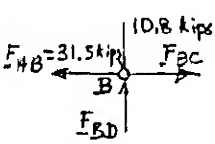


$\frac{F_{AB}}{22.5} = \frac{F_{AD}}{25.5} = \frac{16.8\ kips}{12}$

$F_{AB} = 31.5\ kips\ T$ ◀

$F_{AD} = 35.7\ kips\ C$ ◀

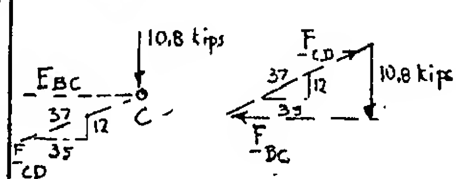
FREE BODY: JOINT B



$\Sigma F_x = 0:\ \qquad F_{BC} = 31.5\ kips\ T$ ◀

$\Sigma F_y = 0:\ \qquad F_{BD} = 10.80\ kips\ C$ ◀

FREE BODY: JOINT C

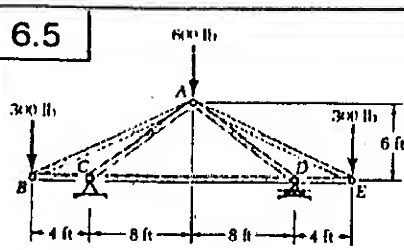


$\frac{F_{CD}}{37} = \frac{F_{BC}}{35} = \frac{10.8\ kips}{12}$

$F_{CD} = 33.3\ kips\ C$ ◀

$F_{BC} = 31.5\ kips\ T$ (CHECKS)

6.5



GIVEN: TRUSS AND LOADING SHOWN

FIND: FORCE IN EACH MEMBER.

FREE BODY: TRUSS: FROM THE SYMMETRY OF THE TRUSS AND LOADING, WE FIND $C = E = 600\ \text{lb}\uparrow$

FREE BODY: JOINT B

$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{BC}}{2} = \frac{300\ \text{lb}}{1}$$

$F_{AB} = 671\ \text{lb}\ T$, $F_{BC} = 600\ \text{lb}\ C$ ◀

FREE BODY: JOINT C

$F_{BC} = 600\ \text{lb}$, $C = 600\ \text{lb}$

$+\uparrow \Sigma F_y = 0:\ \frac{3}{5}F_{AC} + 600\ \text{lb} = 0$

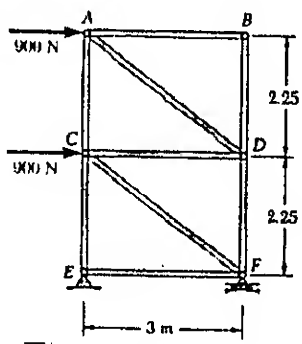
$F_{AC} = -1000\ \text{lb}$ $\quad F_{AC} = 1000\ \text{lb}\ C$ ◀

$\xrightarrow{+} \Sigma F_x = 0:\ \frac{4}{5}(-1000\ \text{lb}) + 600\ \text{lb} + F_{CD} = 0$ $\quad F_{CD} = 200\ \text{lb}\ T$ ◀

FROM SYMMETRY:

$F_{AD} = F_{AC} = 1000\ \text{lb}\ C$, $F_{AE} = F_{AB} = 671\ \text{lb}\ T$, $F_{DE} = F_{BC} = 600\ \text{lb}\ C$ ◀

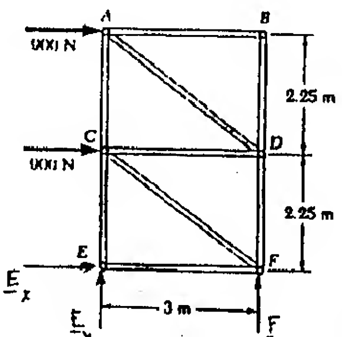
6.6



GIVEN: TRUSS AND LOADING SHOWN.

FIND: FORCE IN EACH MEMBER.

FREE BODY: TRUSS



$+\circlearrowleft \Sigma M_E = 0:$

$F(3\text{m}) - (900\text{N})(2.25\text{m}) - (900\text{N})(4.5\text{m}) = 0$

$F = 2025\ \text{N}\uparrow$

$\xrightarrow{+} \Sigma F_x = 0:\ E_x + 900\text{N} + 900\text{N} = 0$

$E_x = -1800\text{N}$ $\quad E_x = 1800\ \text{N}\leftarrow$

$+\uparrow \Sigma F_y = 0:\ E_y + 2025\ \text{N} = 0$

$E_y = -2025\text{N}$ $\quad E_y = 2025\ \text{N}\downarrow$

WE NOTE THAT AB AND BD ARE ZERO-FORCE MEMBERS: $F_{AB} = F_{BD} = 0$ ◀

FREE BODY: JOINT A

$$\frac{F_{AC}}{2.25} = \frac{F_{AD}}{3.75} = \frac{900\text{N}}{3}$$

$F_{AC} = 675\ \text{N}\ T$ ◀

$F_{AD} = 1125\ \text{N}\ C$ ◀

FREE BODY: JOINT D

$F_{AD} = 1125\ \text{N}$

$$\frac{F_{CD}}{3} = \frac{F_{DF}}{2.25} = \frac{1125\text{N}}{3.75}$$

$F_{CD} = 900\ \text{N}\ T$ ◀

$F_{DF} = 675\ \text{N}\ C$ ◀

CONTINUED

6.6 CONTINUED

FREE BODY: ...

$E_x = 1800\ \text{N}$, $E_y = 2025\ \text{N}$

$\xrightarrow{+} \Sigma F_x = 0:\ F_{EF} - 1800\text{N} = 0$

$+\uparrow \Sigma F_y = 0:\ F_{CE} - 2025\ \text{N} = 0$

FREE BODY: ...

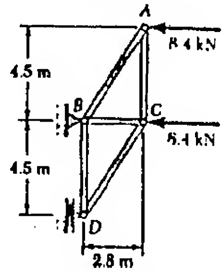
$F_{DF} = 675\ \text{N}$, $F_{EF} = 1800\ \text{N}$, $F_{CE} = 2025\ \text{N}$

$+\uparrow \Sigma F_y = 0:\ \frac{2.25}{3.75}F_{CF} + 2025\ \text{N} ...$

$F_{CF} = -2250\ \text{N}$

$\xrightarrow{+} \Sigma F_x = -\frac{3}{3.75}(-2250\text{N}) - 1800 ...$

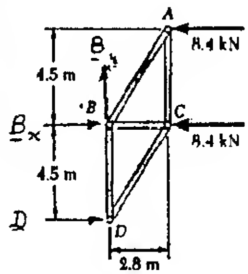
6.7



GIVEN: TRUSS ... SHOWN

FIND: FORCE IN ... MEMBER...

FREE BODY: TRU...



$+\uparrow \Sigma F_y = 0:\ ...$

$+\circlearrowleft \Sigma M_B = 0:$

$D(4.5\text{m}) + (8.4\ \text{kN}) ...$

$D = -8.4\ \text{kN}$

$\xrightarrow{+} \Sigma F_x = 0:$

$B_x - 8.4\text{kN} - 8.4\ \text{kN} ...$

$B_x = +25.2\ \text{kN}$

FREE BODY: ...

$$\frac{F_{AB}}{5.3} = ...$$

$F_{AB} = ...$

$F_{AC} = ...$

FREE BODY: JOINT C

$F_{AC} = 13.50\ \text{kN}$

$+\uparrow \Sigma F_y = 0:\ 13.50\ \text{kN} - \frac{4.5}{5.3} ...$

$F_{CD} = +15.90\ \text{kN}$ $\quad F_{CD} = ...$

$\xrightarrow{+} \Sigma F_x = 0:$

$-F_{BC} - 8.4\ \text{kN} - \frac{2.8}{5.3}(15.9 ...$

$F_{BC} = -16.80\ \text{kN}$ $\quad F_{BC} = ...$

FREE BODY: JOINT D

$F_{CD} = 15.90\ \text{kN}$

$D = 8.4\ \text{kN}$

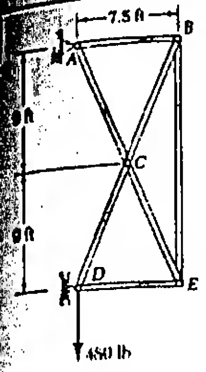
$$\frac{F_{BD}}{4.5} = ...$$

$F_{BD} = 13.5...$

WE CAN ALSO WRITE THE PROPORTION

$$\frac{F_{BD}}{4.5} = \frac{15.90\ \text{kN}}{5.3} \qquad F_{BD} = 13.50\ ...$$

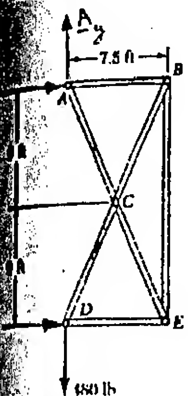
(CHEC...



GIVEN: TRUSS AND LOADING SHOWN

FIND: FORCE IN EACH MEMBER

FREE BODY: TRUSS



$+\uparrow \Sigma F_y = 0: \quad A_y - 480\text{ lb} = 0$

$A_y = +480\text{ lb} \qquad A_y = 480\text{ lb}\uparrow$

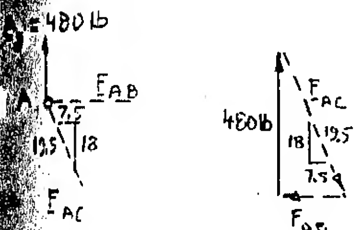
$+\circlearrowleft \Sigma M_A = 0: \quad D(18\text{ ft}) = 0$

$D = 0$

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + D = 0$

$A_x = 0$

FREE BODY: JOINT A

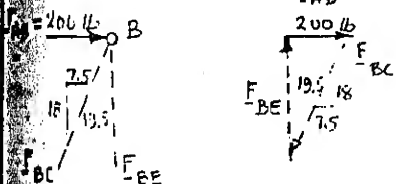


$\dfrac{F_{AB}}{7.5} = \dfrac{F_{AC}}{19.5} = \dfrac{480\text{ lb}}{18}$

$F_{AB} = 200\text{ lb } C$

$F_{AC} = 520\text{ lb } T$

FREE BODY: JOINT B

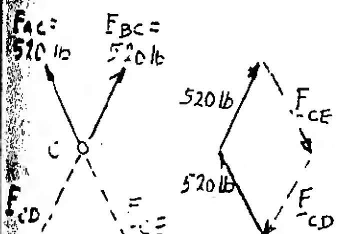


$\dfrac{F_{BC}}{19.5} = \dfrac{F_{BE}}{18} = \dfrac{200\text{ lb}}{7.5}$

$F_{BC} = 520\text{ lb } T$

$F_{BE} = 480\text{ lb } C$

FREE BODY: JOINT C

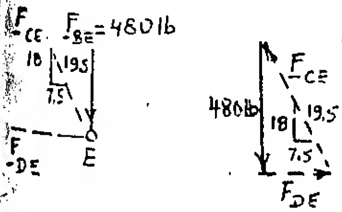


SINCE THE FORCE POLYGON IS A RHOMBUS:

$F_{CD} = 520\text{ lb } T$

$F_{CE} = 520\text{ lb } T$

FREE BODY: JOINT E

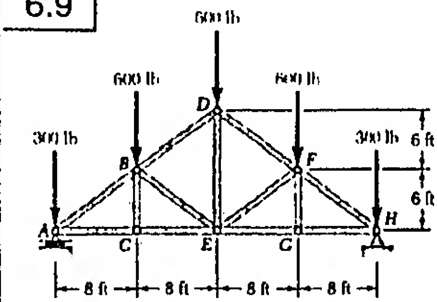


$\dfrac{F_{DE}}{7.5} = \dfrac{F_{CE}}{19.5} = \dfrac{480\text{ lb}}{18}$

$F_{DE} = 200\text{ lb } C$

$F_{CE} = 520\text{ lb } T$ (CHECKS)

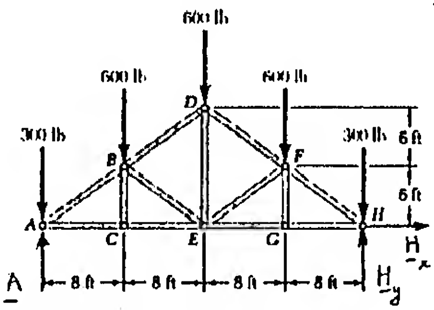
6.9



GIVEN: HOWE ROOF TRUSS LOADED AS SHOWN.

FIND: FORCE IN EACH MEMBER.

FREE BODY: TRUSS



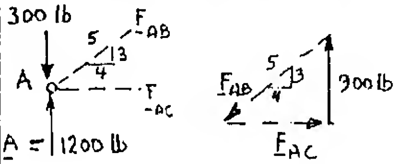
$\Sigma F_x = 0: \quad H_x = 0$

BECAUSE OF THE SYMMETRY OF THE TRUSS AND LOADING:

$A = H_y = \frac{1}{2}$ TOTAL LOAD

$A = H_y = 1200\text{ lb}\uparrow$

FREE BODY: JOINT A

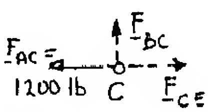


$\dfrac{F_{AB}}{5} = \dfrac{F_{AC}}{4} = \dfrac{900\text{ lb}}{3}$

$F_{AB} = 1500\text{ lb } C$

$F_{AC} = 1200\text{ lb } T$

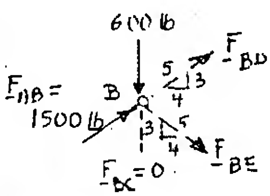
FREE BODY: JOINT C



BC IS A ZERO-FORCE MEMBER

$F_{BC} = 0 \qquad F_{CE} = 1200\text{ lb } T$

FREE BODY: JOINT B



$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{BD} + \frac{4}{5}F_{BE} + \frac{4}{5}(1500\text{ lb}) = 0$

OR: $F_{BD} + F_{BE} = -1500\text{ lb}$ (1)

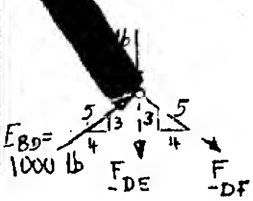
$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BD} - \frac{3}{5}F_{BE} + \frac{3}{5}(1500\text{ lb}) - 600\text{ lb} = 0$

OR: $F_{BD} - F_{BE} = -500\text{ lb}$ (2)

ADD EQS. (1) AND (2): $2F_{BD} = -2000\text{ lb}$ $\qquad F_{BD} = 1000\text{ lb } C$

SUBTRACT (2) FROM (1): $2F_{BE} = -1000\text{ lb}$ $\qquad F_{BE} = 500\text{ lb } C$

FREE BODY: JOINT D



$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}(1000\text{ lb}) + \frac{4}{5}F_{DF} = 0$

$F_{DF} = -1000\text{ lb} \qquad F_{DF} = 1000\text{ lb } C$

$+\uparrow \Sigma F_y = 0:$

$\frac{3}{5}(1000\text{ lb}) - \frac{3}{5}(-1000\text{ lb}) - 600\text{ lb} - F_{DE} = 0$

$F_{DE} = +600\text{ lb} \qquad F_{DE} = 600\text{ lb } T$

BECAUSE OF THE SYMMETRY OF THE TRUSS AND LOADING, WE DEDUCE THAT

$F_{EF} = F_{BE} \qquad F_{EF} = 500\text{ lb } C$

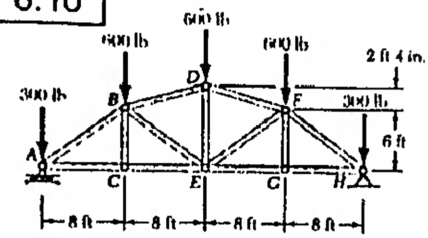
$F_{EG} = F_{CE} \qquad F_{EG} = 1200\text{ lb } T$

$F_{FG} = F_{BC} \qquad F_{FG} = 0$

$F_{FH} = F_{AB} \qquad F_{FH} = 1500\text{ lb } C$

$F_{GH} = F_{AC} \qquad F_{GH} = 1200\text{ lb } T$

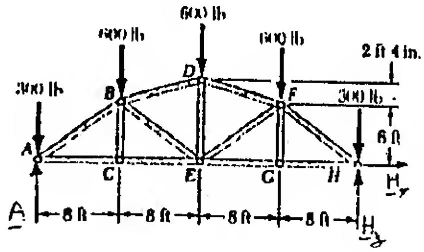
6.10



GIVEN: GAMBREL ROOF TRUSS WITH LOADING SHOWN

FIND: FORCE IN EACH MEMBER.

FREE BODY: TRUSS



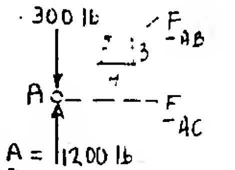
$\Sigma F_x = 0: \quad H_x = 0$

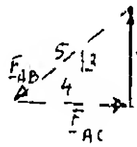
BECAUSE OF THE SYMMETRY OF THE TRUSS AND LOADING:

$A = H_y = \frac{1}{2}$ TOTAL LOAD

$A = H_y = 1200$ lb ↑

FREE BODY: JOINT A



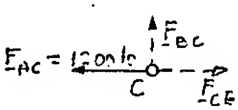


$\frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{900 \text{ lb}}{3}$

$F_{AB} = 1500$ lb C ◄

$F_{AC} = 1200$ lb T ◄

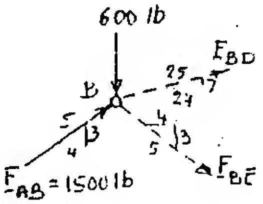
FREE BODY: JOINT C



BC IS A ZERO-FORCE MEMBER

$F_{BC} = 0 \qquad F_{CE} = 1200$ lb T ◄

FREE BODY: JOINT B



$\xrightarrow{+} \Sigma F_x = 0: \frac{24}{25} F_{BD} + \frac{4}{5} F_{BE} + \frac{4}{5}(1500 \text{ lb}) = 0$

OR $\quad 24 F_{BD} + 20 F_{BE} = -30{,}000$ lb (1)

$+\uparrow \Sigma F_y = 0: \frac{7}{25} F_{BD} - \frac{3}{5} F_{BE} + \frac{3}{5}(1500) - 600 = 0$

OR $\quad 7 F_{BD} - 15 F_{BE} = -7{,}500$ lb (2)

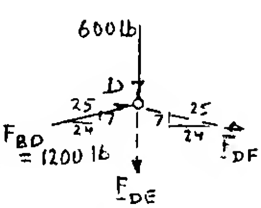
MULTIPLY (1) BY 3, (2) BY 4, AND ADD:

$100 F_{BD} = -120{,}000$ lb $\qquad F_{BD} = 1200$ lb C ◄

MULTIPLY (1) BY 7, (2) BY −24, AND ADD:

$500 F_{BE} = -30{,}000$ lb $\qquad F_{BE} = 60.0$ lb C ◄

FREE BODY: JOINT D



$\xrightarrow{+} \Sigma F_x = 0: \frac{24}{25}(1200 \text{ lb}) + \frac{24}{25} F_{DF} = 0$

$F_{DF} = -1200$ lb $\qquad F_{DF} = 1200$ lb C ◄

$+\uparrow \Sigma F_y = 0:$

$\frac{7}{25}(1200 \text{ lb}) - \frac{7}{25}(-1200 \text{ lb}) - 600 \text{ lb} - F_{DE} = 0$

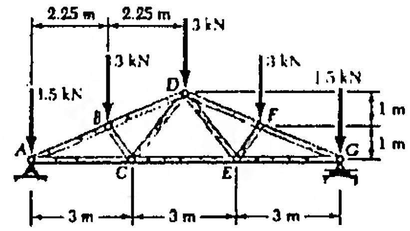
$F_{DE} = 72.0$ lb $\qquad F_{DE} = 72.0$ lb T ◄

BECAUSE OF THE SYMMETRY OF THE TRUSS AND LOADING, WE DEDUCE THAT

$F_{EF} = F_{BE}$	$F_{EF} = 60.0$ lb C	◄
$F_{EG} = F_{CE}$	$F_{EG} = 1200$ lb T	◄
$F_{FG} = F_{BC}$	$F_{FG} = 0$	◄
$F_{FH} = F_{AB}$	$F_{FH} = 1500$ lb C	◄
$F_{GH} = F_{AC}$	$F_{GH} = 1200$ lb T	◄

NOTE: COMPARE RESULTS WITH THOSE OF PROB. 6.9.

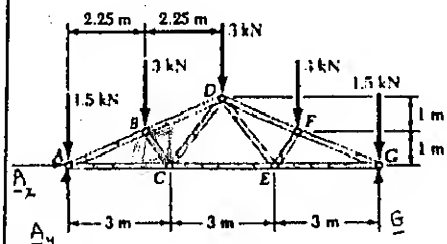
6.11



GIVEN: FINK ROOF WITH LOADING SHOWN.

FIND: FORCE IN EACH MEMBER

FREE BODY: TRUSS



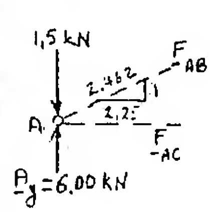
$\Sigma F_x = 0: \quad A_x = 0$

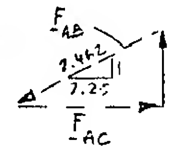
BECAUSE OF THE SYMMETRY OF THE TRUSS AND LOADING

$A_y = G = \frac{1}{2}$ TOTAL LOAD

$A_y = G = 6.00$ kN

FREE BODY: JOINT A



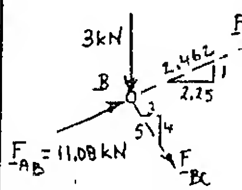


$\frac{F_{AB}}{2.462} = \frac{F_{AC}}{2.25} = \frac{4.50 \text{ kN}}{1}$

$F_{AB} = 11.08$ kN C

$F_{AC} = 10.125$ kN, $\quad F_{AC} = 10.13$ kN T

FREE BODY: JOINT B



$\xrightarrow{+} \Sigma F_x = 0:$

$\frac{3}{5} F_{BC} + \frac{2.25}{2.462} F_{BD} + \frac{2.25}{2.462}(11.08 \text{ kN}) = 0$ (1)

$+\uparrow \Sigma F_y = 0:$

$-\frac{4}{5} F_{BC} + \frac{F_{BD}}{2.462} + \frac{11.08 \text{ kN}}{2.462} - 3 \text{ kN} = 0$ (2)

MULTIPLY EQ.(2) BY −2.25 AND ADD TO EQ.(1):

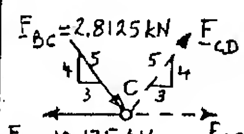
$\frac{12}{5} F_{BC} + 6.75 \text{ kN} = 0 \qquad F_{BC} = -2.8125 \qquad F_{BC} = 2.81$ kN C

MULTIPLY EQ(1) BY 4, EQ (2) BY 3, AND ADD:

$\frac{12}{2.462} F_{BD} + \frac{12}{2.462}(11.08 \text{ kN}) - 9 \text{ kN} = 0$

$F_{BD} = -9.2335$ kN $\qquad F_{BD} = 9.23$ kN C

FREE BODY: JOINT C



$+\uparrow \Sigma F_y = 0: \frac{4}{5} F_{CD} - \frac{4}{5}(2.8125 \text{ kN}) = 0$

$F_{CD} = 2.8125$ kN, $\quad F_{CD} = 2.81$ kN T

$\xrightarrow{+} \Sigma F_x = 0:$

$F_{CE} - 10.125 \text{ kN} + \frac{3}{5}(2.8125 \text{ kN}) + \frac{3}{5}(2.8125 \text{ kN}) = 0$

$F_{CE} = +6.7500$ kN $\qquad F_{CE} = 6.75$ kN T

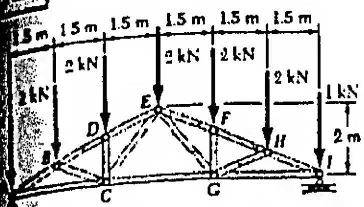
BECAUSE OF THE SYMMETRY OF THE TRUSS AND LOADING, WE DEDUCE THAT

$F_{DE} = F_{CD}$	$F_{CD} = 2.81$ kN T
$F_{DF} = F_{BD}$	$F_{DF} = 9.23$ kN C
$F_{EF} = F_{BC}$	$F_{EF} = 2.81$ kN C
$F_{EG} = F_{AC}$	$F_{EG} = 10.13$ kN T
$F_{FG} = F_{AB}$	$F_{FG} = 11.08$ kN C

6.12

GIVEN:
FAN ROOF TRUSS AND LOADING SHOWN.

FIND:
FORCE IN EACH MEMBER.



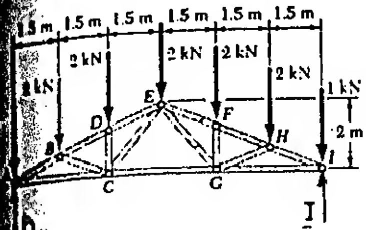
FREE BODY; TRUSS

$\Sigma F_x = 0: \quad A_x = 0$

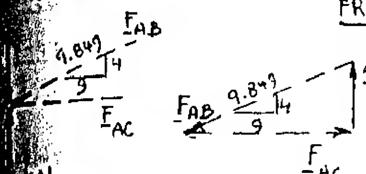
FROM SYMMETRY OF TRUSS AND LOADING:

$A_y = I = \frac{1}{2}$ TOTAL LOAD

$A_y = I = 6$ kN ↑



FREE BODY; JOINT A

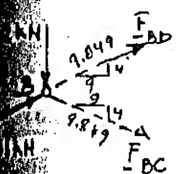


$\frac{F_{AB}}{9.849} = \frac{F_{AC}}{9} = \frac{5 \text{ kN}}{4}$

$F_{AB} = 12.31$ kN C ◀

$F_{AC} = 11.25$ kN T ◀

FREE BODY; JOINT B



$\xrightarrow{+} \Sigma F_x = \frac{9}{9.849}(12.31 \text{kN} + F_{BD} + F_{BC}) = 0$

OR: $F_{BD} + F_{BC} = -12.31$ kN (1)

$+\uparrow \Sigma F_y = \frac{4}{9.849}(12.31 \text{kN} + F_{BD} - F_{BC}) - 2 \text{kN} = 0$

OR: $F_{BD} - F_{BC} = -7.386$ kN (2)

(1) AND (2):

$F_{BD} = -19.70$ kN

SUBTRACT (2) FROM (1):

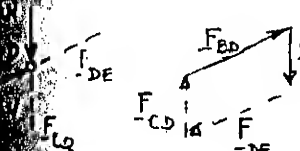
$F_{BC} = -4.924$ kN

$F_{BD} = 9.85$ kN C ◀

$F_{BC} = 2.46$ kN C ◀

FREE BODY; JOINT D

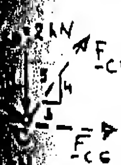
FROM FORCE POLYGON:



$F_{CD} = 2.00$ kN C ◀

$F_{DE} = 9.85$ kN C ◀

FREE BODY: JOINT C



$+\uparrow \Sigma F_y = \frac{4}{5} F_{CE} - \frac{4}{9.849}(2.46 \text{kN}) - 2 \text{kN} = 0$

$F_{CE} = 3.75$ kN T ◀

$\xrightarrow{+} \Sigma F_x = 0:$

$F_{CG} + \frac{3}{5}(3.75 \text{kN}) + \frac{9}{9.849}(2.46 \text{kN}) - 11.25 \text{kN} = 0$

$F_{CG} = +6.75$ kN $\quad F_{CG} = 6.75$ kN T ◀

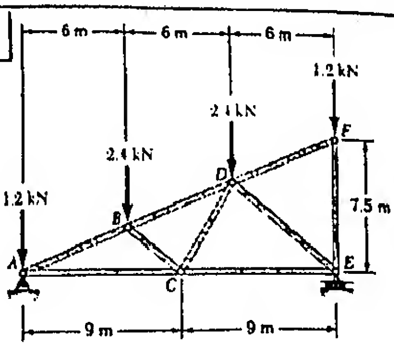
THE SYMMETRY OF THE TRUSS AND LOADING:

$F_{EF} = F_{DE}$	$F_{EF} = 9.85$ kN C ◀
$F_{EG} = F_{CE}$	$F_{EG} = 3.75$ kN T ◀
$F_{FG} = F_{CD}$	$F_{FG} = 2.00$ kN C ◀
$F_{FH} = F_{BD}$	$F_{FH} = 9.85$ kN C ◀
$F_{GH} = F_{BC}$	$F_{GH} = 2.46$ kN C ◀
$F_{GI} = F_{AC}$	$F_{GI} = 11.25$ kN T ◀
$F_{HI} = F_{AB}$	$F_{HI} = 12.31$ kN C ◀

6.13

GIVEN:
ROOF TRUSS AND LOADING SHOWN.

FIND:
FORCE IN EACH MEMBER.



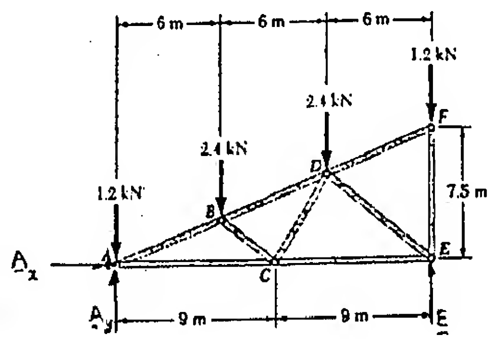
FREE BODY: TRUSS

$\Sigma F_x = 0: \quad A_x = 0$

FROM SYMMETRY OF LOADING:

$A_y = E = \frac{1}{2}$ TOTAL LOAD

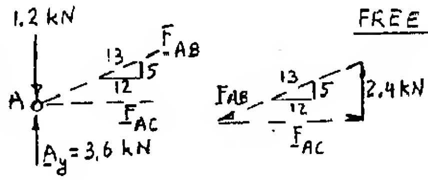
$A_y = E = 3.6$ kN ↑



WE NOTE THAT DF IS A ZERO-FORCE MEMBER AND THAT EF IS ALIGNED WITH THE LOAD. THUS $F_{DF} = 0$ ◀

$F_{EF} = 1.2$ kN C ◀

FREE BODY: JOINT A

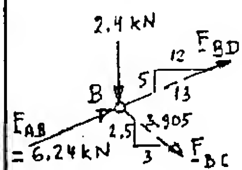


$\frac{F_{AB}}{13} = \frac{F_{AC}}{12} = \frac{2.4 \text{ kN}}{5}$

$F_{AB} = 6.24$ kN C ◀

$F_{AC} = 5.76$ kN T ◀

FREE BODY: JOINT B



$\xrightarrow{+} \Sigma F_x = 0:$

$\frac{3}{3.905} F_{BC} + \frac{12}{13} F_{BD} + \frac{12}{13}(6.24 \text{kN}) = 0$ (1)

$+\uparrow \Sigma F_y = 0:$

$-\frac{2.5}{3.905} F_{BC} + \frac{5}{13} F_{BD} + \frac{5}{13}(6.24 \text{kN}) - 2.4 \text{kN} = 0$ (2)

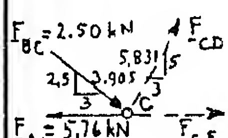
MULTIPLY (1) BY 2.5, (2) BY 3, AND ADD:

$\frac{45}{13} F_{BD} + \frac{45}{13}(6.24 \text{kN}) - 7.2 \text{kN} = 0, \quad F_{BD} = -4.16$ kN, $\quad F_{BD} = 4.16$ kN C ◀

MULTIPLY (1) BY 5, (2) BY −12, AND ADD:

$\frac{45}{3.905} F_{BC} + 28.8 \text{ kN} = 0, \quad F_{BC} = -2.50$ kN, $\quad F_{BC} = 2.50$ kN C ◀

FREE BODY: JOINT C



$+\uparrow \Sigma F_y = 0: \frac{5}{5.831} F_{CD} - \frac{2.5}{3.905}(2.50 \text{kN}) = 0$

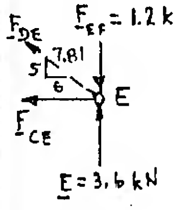
$F_{CD} = 1.867$ kN T ◀

$\xrightarrow{+} \Sigma F_x = 0:$

$F_{CE} - 5.76 \text{ kN} + \frac{3}{3.905}(2.50 \text{kN}) + \frac{3}{5.831}(1.867 \text{kN}) = 0$

$F_{CE} = 2.88$ kN T ◀

FREE BODY: JOINT E



$+\uparrow \Sigma F_y = 0: \frac{5}{7.81} F_{DE} + 3.6 \text{ kN} - 1.2 \text{ kN} = 0$

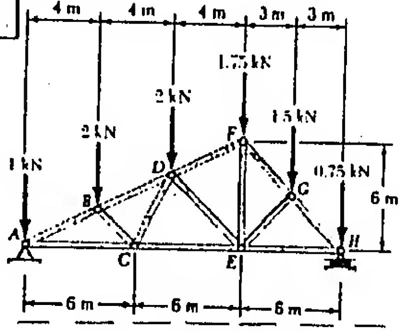
$F_{DE} = -3.75$ kN $\quad F_{DE} = 3.75$ kN C ◀

$\xrightarrow{+} \Sigma F_x = 0: -F_{CE} - \frac{6}{7.81}(-3.75 \text{ kN}) = 0$

$F_{CE} = +2.88$ kN $\quad F_{CE} = 2.88$ kN T

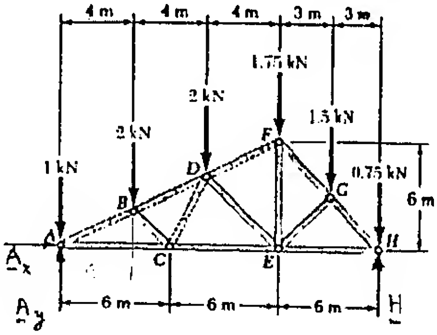
(CHECKS)

6.14



GIVEN: DOUBLE-PITCH ROOF TRUSS AND LOADING SHOWN.

FIND: FORCE IN EACH MEMBER.



FREE BODY: TRUSS

$+\circlearrowleft \Sigma M_A = 0:$

$H(18\,m) - (2\,kN)(4\,m) - (2\,kN)(8\,m) - (1.75\,kN)(12\,m) - (1.5\,kN)(15\,m) - (0.75\,kN)(18\,m) = 0$

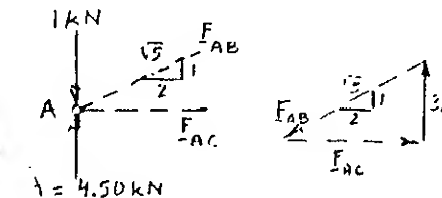
$H = 4.50\,kN \uparrow$

$\Sigma F_x = 0: \quad A_x = 0$

$\Sigma F_y = 0: \; A_y + H - 9 = 0$

$A_y = 9 - 4.50, \; A_y = 4.50\,kN \uparrow$

FREE BODY: JOINT A



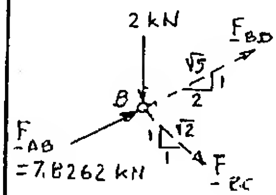
$\dfrac{F_{AB}}{\sqrt{5}} = \dfrac{F_{AC}}{2} = \dfrac{3.50\,kN}{1}$

$F_{AB} = 7.8262\,kN\ C$

$F_{AB} = 7.83\,kN\ C$ ◀

$F_{AC} = 7.00\,kN\ T$ ◀

FREE BODY: JOINT B



$\xrightarrow{+} \Sigma F_x = 0:$

$\frac{2}{\sqrt{5}} F_{BD} + \frac{2}{\sqrt{5}}(7.8262\,kN) + \frac{1}{\sqrt{2}} F_{BC} = 0$

OR: $F_{BD} + 0.79057\,F_{BC} = -7.8262\,kN$ (1)

$+\uparrow \Sigma F_y = 0:$

$\frac{1}{\sqrt{5}} F_{BD} + \frac{1}{\sqrt{5}}(7.8262\,kN) - \frac{1}{\sqrt{2}} F_{BC} - 2\,kN = 0$

OR: $F_{BD} - 1.58114\,F_{BC} = -3.3541$ (2)

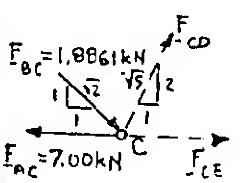
MULTIPLY (1) BY 2 AND ADD (2):

$3F_{BD} = -19.0065, \quad F_{BD} = -6.3355\,kN \qquad F_{BD} = 6.34\,kN\ C$ ◀

SUBTRACT (2) FROM (1):

$2.37171\,F_{BC} = -4.4721, \quad F_{BC} = -1.8861\,kN \qquad F_{BC} = 1.886\,kN\ C$ ◀

FREE BODY: JOINT C



$+\uparrow \Sigma F_y = 0: \; \frac{2}{\sqrt{5}} F_{CD} - \frac{1}{\sqrt{2}}(1.8861\,kN) = 0$

$F_{CD} = +1.4911\,kN \qquad F_{CD} = 1.491\,kN\ T$ ◀

$\xrightarrow{+} \Sigma F_x = 0:$

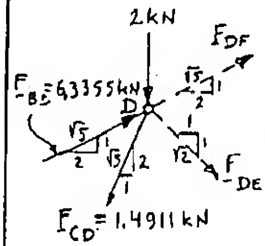
$F_{CE} - 7.00\,kN + \frac{1}{\sqrt{2}}(1.8861\,kN) + \frac{1}{\sqrt{5}}(1.4911\,kN) = 0$

$F_{CE} = 5.000\,kN \qquad F_{CE} = 5.00\,kN\ T$ ◀

CONTINUED

6.14 CONTINUED

FREE BODY: JOINT D



$\xrightarrow{+} \Sigma F_x = 0:$

$\frac{2}{\sqrt{5}} F_{DF} + \frac{1}{\sqrt{2}} F_{DE} + \frac{2}{\sqrt{5}}(6.3355\,kN) - \frac{1}{\sqrt{5}}(1.4911\,kN)$

OR: $F_{DF} + 0.79057\,F_{DE} = -5.5900$ (1)

$+\uparrow \Sigma F_y = 0:$

$\frac{1}{\sqrt{5}} F_{DF} - \frac{1}{\sqrt{2}} F_{DE} + \frac{1}{\sqrt{5}}(6.3355\,kN) - \frac{2}{\sqrt{5}}(1.4911\,kN) - 2\,kN = 0$

OR: $F_{DF} - 0.79057\,F_{DE} = -1.1188\,kN$ (2)

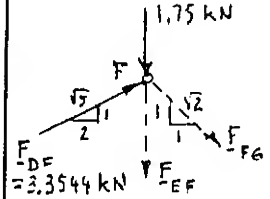
ADD (1) AND (2): $2F_{DF} = -6.7088\,kN$

$F_{DF} = -3.3544\,kN \qquad F_{DF} = 3.35\,kN\ C$ ◀

SUBTRACT (2) FROM (1): $1.58114\,F_{DE} = -4.4712\,kN$

$F_{DE} = -2.8278\,kN \qquad F_{DE} = 2.83\,kN\ C$ ◀

FREE BODY: JOINT F



$\xrightarrow{+} \Sigma F_x = 0: \frac{1}{\sqrt{2}} F_{FG} + \frac{2}{\sqrt{5}}(3.3544\,kN) = 0$

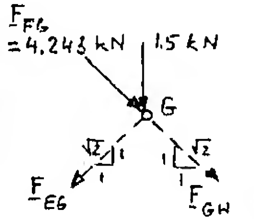
$F_{FG} = -4.243\,kN, \quad F_{FG} = 4.24\,kN\ C$ ◀

$+\uparrow \Sigma F_y = 0:$

$-F_{EF} - 1.75\,kN + \frac{1}{\sqrt{5}}(3.3544\,kN) - \frac{1}{\sqrt{2}}(-4.243\,kN) = 0$

$F_{EF} = 2.750\,kN \qquad F_{EF} = 2.75\,kN\ T$ ◀

FREE BODY: JOINT G



$\xrightarrow{+} \Sigma F_x = 0:$

$\frac{1}{\sqrt{2}} F_{GH} - \frac{1}{\sqrt{2}} F_{EG} + \frac{1}{\sqrt{2}}(4.243\,kN) = 0$

OR: $F_{GH} - F_{EG} = -4.243\,kN$ (1)

$+\uparrow \Sigma F_y = 0:$

$-\frac{1}{\sqrt{2}} F_{GH} - \frac{1}{\sqrt{2}} F_{EG} - \frac{1}{\sqrt{2}}(4.243\,kN) - 1.5\,kN = 0$

OR: $F_{GH} + F_{EG} = -6.364\,kN$ (2)

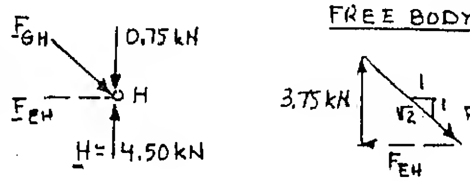
ADD (1) AND (2): $2F_{GH} = -10.607$

$F_{GH} = -5.303 \qquad F_{GH} = 5.30\,kN\ C$ ◀

SUBTRACT (1) FROM (2): $2F_{EG} = -2.121\,kN$

$F_{EG} = -1.0605\,kN \qquad F_{EG} = 1.061\,kN\ C$ ◀

FREE BODY: JOINT H

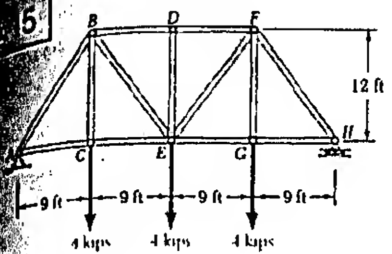


$\dfrac{F_{EH}}{1} = \dfrac{3.75\,kN}{1} \qquad F_{EH} = 3.75\,kN\ T$ ◀

WE CAN ALSO WRITE:

$\dfrac{F_{GH}}{\sqrt{2}} = \dfrac{3.75\,kN}{1} \qquad F_{GH} = 5.30\,kN\ C$ (CHECKS)

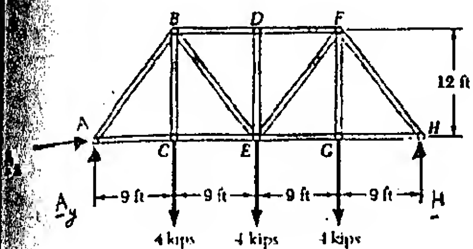
5



GIVEN: PRATT BRIDGE TRUSS AND LOADING SHOWN.

FIND: FORCE IN EACH MEMBER.

FREE BODY: TRUSS



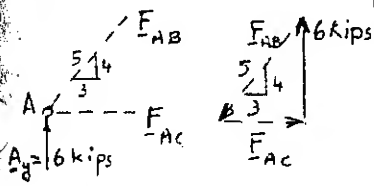
$\xrightarrow{+}\Sigma F_x = 0: \quad A_x = 0$

$+\circlearrowleft \Sigma M_A = 0: \; H(36\,ft) - (4\,kips)(9\,ft) - (4\,kips)(18\,ft) - (4\,kips)(27\,ft) = 0$

$H = 6\ kips \uparrow$

$+\uparrow \Sigma F_y = 0: \quad A_y + 6\ kips - 12\ kips = 0 \qquad A_y = 6\ kips \uparrow$

FREE BODY: JOINT A

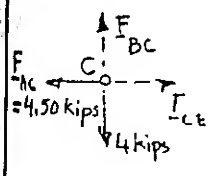


$$\frac{F_{AB}}{5} = \frac{F_{AC}}{3} = \frac{6\ kips}{4}$$

$F_{AB} = 7.50\ kips\ C$ ◀

$F_{AC} = 4.50\ kips\ T$ ◀

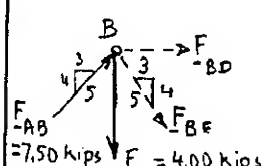
FREE BODY: JOINT C



$\Sigma F_x = 0: \qquad F_{CE} = 4.50\ kips\ T$ ◀

$\Sigma F_y = 0: \qquad F_{BC} = 4.00\ kips\ T$ ◀

FREE BODY: JOINT B



$+\uparrow \Sigma F_y = 0:$

$-\frac{4}{5}F_{BE} + \frac{4}{5}(7.50\ kips) - 4.00\ kips = 0$

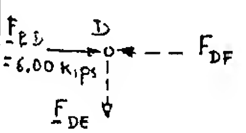
$F_{BE} = 2.50\ kips\ T$ ◀

$\xrightarrow{+}\Sigma F_x = 0:$

$\frac{3}{5}(7.50\ kips) + \frac{3}{5}(2.50\ kips) + F_{BD} = 0$

$F_{BD} = -6.00\ kips \qquad F_{BD} = 6.00\ kips\ C$ ◀

FREE BODY: JOINT D



WE NOTE THAT DE IS A ZERO-FORCE MEMBER: $F_{DE} = 0$ ◀

ALSO: $\qquad F_{DF} = 6.00\ kips\ C$ ◀

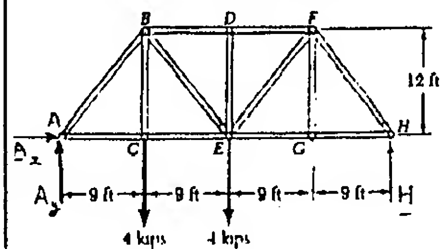
FROM SYMMETRY:

$F_{EF} = F_{BE}$	$F_{EF} = 2.50\ kips\ T$ ◀
$F_{EG} = F_{CE}$	$F_{EG} = 4.50\ kips\ T$ ◀
$F_{FG} = F_{BC}$	$F_{FG} = 4.00\ kips\ T$ ◀
$F_{FH} = F_{AB}$	$F_{FH} = 7.50\ kips\ C$ ◀
$F_{GH} = F_{AC}$	$F_{GH} = 4.50\ kips\ T$ ◀

6.16

GIVEN: TRUSS OF PROB. 6.15, ASSUMING THAT THE LOAD APPLIED AT G HAS BEEN REMOVED.

FIND: FORCE IN EACH MEMBER.



FREE BODY: TRUSS

$\Sigma F_x = 0: \quad A_x = 0$

$+\circlearrowleft \Sigma M_A = 0:$

$H(36\,ft) - (4\,kips)(9\,ft) - (4\,kips)(18\,ft) = 0$

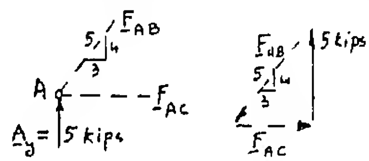
$H = 3.00\ kips \uparrow$

$+\uparrow \Sigma F_y = 0: \quad A_y = 5.00\ kips \uparrow$

WE NOTE THAT DE AND FG ARE ZERO-FORCE MEMBERS. THEREFORE: $F_{DE} = 0, \; F_{FG} = 0$ ◀

ALSO: $F_{BD} = F_{DF}$ (1) AND $F_{EG} = F_{GH}$ (2)

FREE BODY: JOINT A

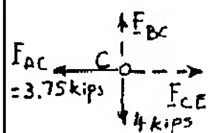


$$\frac{F_{AB}}{5} = \frac{F_{AC}}{3} = \frac{5\ kips}{4}$$

$F_{AB} = 6.25\ kips\ C$ ◀

$F_{AC} = 3.75\ kips\ T$ ◀

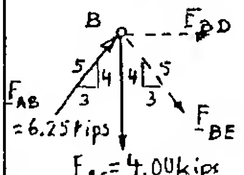
FREE BODY: JOINT C



$\Sigma F_x = 0: \qquad F_{CE} = 3.75\ kips\ T$ ◀

$\Sigma F_y = 0: \qquad F_{BC} = 4.00\ kips\ T$ ◀

FREE BODY: JOINT B



$+\uparrow \Sigma F_y = 0: \; \frac{4}{5}(6.25\ kips) - 4.00\ kips - \frac{4}{5}F_{BE} = 0$

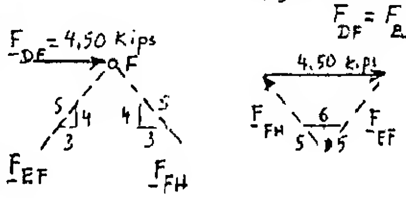
$F_{BE} = 1.250\ kips\ T$ ◀

$\xrightarrow{+}\Sigma F_x = 0: F_{BD} + \frac{3}{5}(6.25\ kips) + \frac{3}{5}(1.250\ kips) = 0$

$F_{BD} = -4.50\ kips \qquad F_{BD} = 4.50\ kips\ C$ ◀

FREE BODY: JOINT F

WE RECALL THAT $F_{FG} = 0$, AND FROM (1) THAT $F_{DF} = F_{BD}$ $\qquad F_{DF} = 4.50\ kips\ C$ ◀

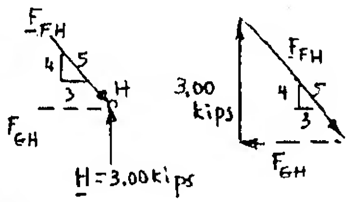


$$\frac{F_{EF}}{5} = \frac{F_{FH}}{5} = \frac{4.50\ kips}{6}$$

$F_{EF} = 3.75\ kips\ T$ ◀

$F_{FH} = 3.75\ kips\ C$ ◀

FREE BODY: JOINT H



$$\frac{F_{GH}}{3} = \frac{3.00\ kips}{4}$$

$F_{GH} = 2.25\ kips\ T$ ◀

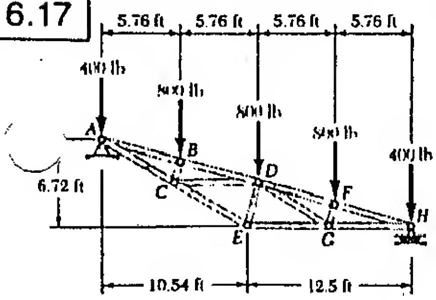
ALSO:

$$\frac{F_{FH}}{5} = \frac{3.00\ kips}{4}$$

$F_{FH} = 3.75\ kips\ C$ (CHECKS)

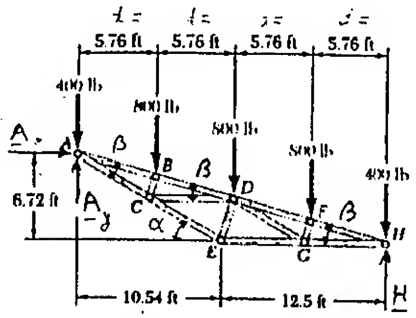
FROM EQ.(2): $\quad F_{EG} = F_{GH} \qquad F_{EG} = 2.25\ kips\ T$ ◀

6.17



GIVEN:
INVERTED HOWE ROOF TRUSS AND LOADING SHOWN.

FIND:
FORCE IN MEMBER DE AND IN MEMBERS TO THE LEFT OF DE.



FREE BODY: TRUSS

$\Sigma F_x = 0: \quad A_x = 0$

$+\circlearrowleft \Sigma M_H = 0:$

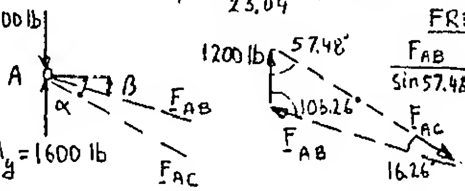
$(400\text{ lb})(4d) + (800\text{ lb})(3d) + (800\text{ lb})(2d) + (800\text{ lb})d - A_y(4d) = 0$

$A_y = 1600\text{ lb}\uparrow$

ANGLES: $\tan\alpha = \dfrac{6.72}{10.54} \quad \alpha = 32.52°$

$\tan\beta = \dfrac{6.72}{23.04} \quad \beta = 16.26°$

FREE BODY: JOINT A



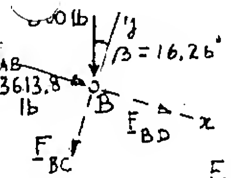
$\dfrac{F_{AB}}{\sin 57.48°} = \dfrac{F_{AC}}{\sin 106.26°} = \dfrac{1200\text{ lb}}{\sin 16.26°}$

$F_{AB} = 3613.8\text{ lb } C$

$F_{AC} = 4114.3\text{ lb } T$

$F_{AB} = 3610\text{ lb } C, \; F_{AC} = 4110\text{ lb } T$ ◀

FREE BODY: JOINT B



$+\nearrow\Sigma F_y = 0: -F_{BC} - (800\text{ lb})\cos 16.26° = 0$

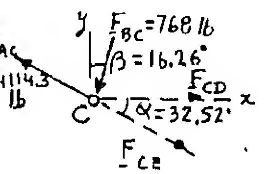
$F_{BC} = -768.0\text{ lb} \qquad F_{BC} = 768\text{ lb } C$ ◀

$\overset{+}{\rightarrow}\Sigma F_x = 0:$

$F_{BD} + 3613.8\text{ lb} + (800\text{ lb})\sin 16.26° = 0$

$F_{BD} = -3837.8\text{ lb} \qquad F_{BD} = 3840\text{ lb } C$ ◀

FREE BODY: JOINT C



$+\uparrow\Sigma F_y = 0:$

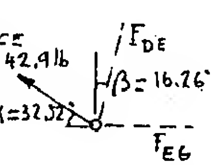
$-F_{CE}\sin 32.52° + (4114.3\text{ lb})\sin 32.52° - (768\text{ lb})\cos 16.26° = 0$

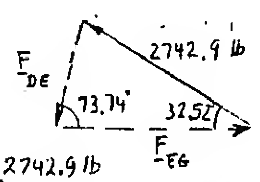
$F_{CE} = 2742.9\text{ lb} \qquad F_{CE} = 2740\text{ lb } T$ ◀

$\Sigma F_x = 0: \; F_{CD} - (4114.3\text{ lb})\cos 32.52° + (2742.9\text{ lb})\cos 32.52° - (768\text{ lb})\sin 16.26° = 0$

$F_{CD} = 1371.4\text{ lb} \qquad F_{CD} = 1371\text{ lb } T$ ◀

FREE BODY: JOINT E

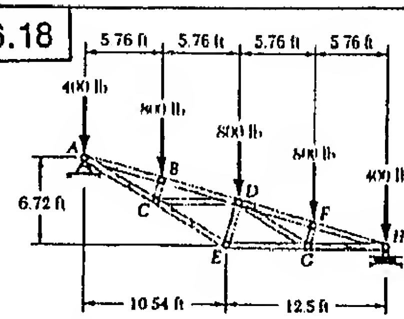




$\dfrac{F_{DE}}{\sin 32.52°} = \dfrac{2742.9\text{ lb}}{\sin 73.74°}$

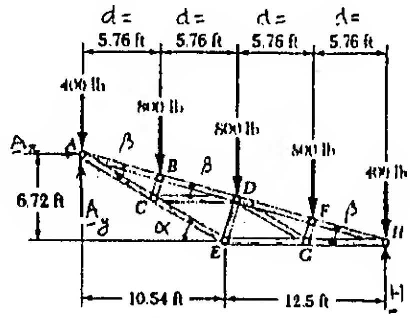
$F_{DE} = 1536\text{ lb } C$ ◀

6.18



GIVEN:
INVERTED HOWE ROOF TRUSS AND LOADING SHOWN.

FIND:
FORCE IN MEMBERS TO THE RIGHT OF DE.



FREE BODY: TRUSS

$+\circlearrowleft\Sigma M_A = 0:$

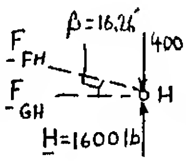
$H(4d) - (800\text{ lb})d - (800\text{ lb})(2d) - (800\text{ lb})(3d) - (400\text{ lb})(4d) = 0$

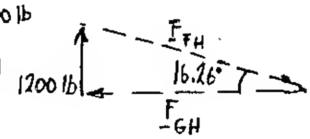
$H = 1600\text{ lb}\uparrow$

ANGLES: $\tan\alpha = \dfrac{6.72}{10.54} \qquad \alpha = 32.52°$

$\tan\beta = \dfrac{6.72}{23.04} \qquad \beta = 16.26°$

FREE BODY: JOINT H





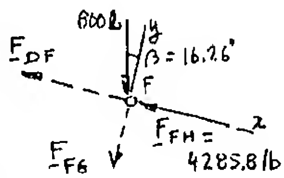
$F_{GH} = (1200\text{ lb})\cot 16.26°$

$F_{GH} = 4114.3\text{ lb } T$

$F_{GH} = 4110\text{ lb } T$ ◀

$F_{FH} = \dfrac{1200\text{ lb}}{\sin 16.26°} = 4285.8\text{ lb} \qquad F_{FH} = 4290\text{ lb } C$ ◀

FREE BODY: JOINT F



$+\nearrow\Sigma F_y = 0:$

$-F_{FG} - (800\text{ lb})\cos 16.26° = 0$

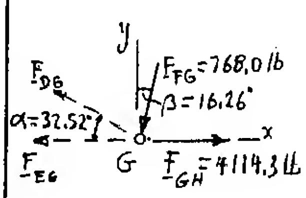
$F_{FG} = -768.0\text{ lb} \qquad F_{FG} = 768\text{ lb } C$ ◀

$\overset{+}{\rightarrow}\Sigma F_x = 0:$

$-F_{DF} - 4285.8\text{ lb} + (800\text{ lb})\sin 16.26° = 0$

$F_{DF} = -4061.8 \qquad F_{DF} = 4060\text{ lb } C$ ◀

FREE BODY: JOINT G



$+\uparrow\Sigma F_y = 0:$

$F_{DG}\sin 32.52° - (768.0\text{ lb})\cos 16.26° = 0$

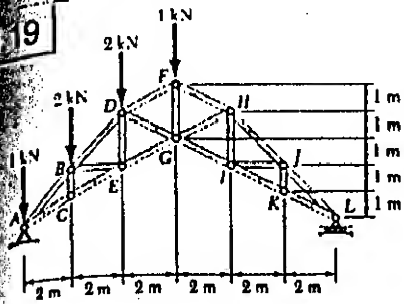
$F_{DG} = +1371.4\text{ lb}$

$F_{DG} = 1371\text{ lb } T$ ◀

$\overset{+}{\rightarrow}\Sigma F_x = 0: \; -F_{EG} + 4114.3\text{ lb} - (768.0\text{ lb})\sin 16.26° - (1371.4\text{ lb})\cos 32.52° = 0$

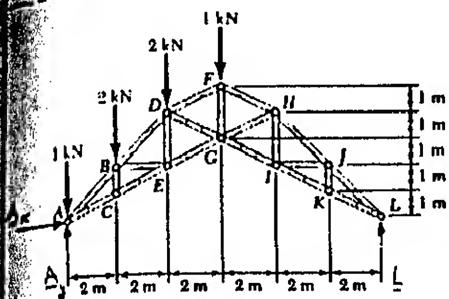
$F_{EG} = +2742.9\text{ lb} \qquad F_{EG} = 2740\text{ lb } T$ ◀

6.19



GIVEN: SCISSORS ROOF TRUSS AND LOADING SHOWN.

FIND: FORCE IN MEMBERS TO THE LEFT OF FG.



FREE BODY: TRUSS

$\Sigma F_x = 0$: $A_x = 0$

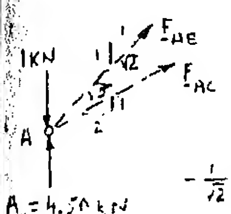
$+\circlearrowleft \Sigma M_L = 0$: $(1\text{ kN})(12\text{ m}) + (2\text{ kN})(10\text{ m}) + (2\text{ kN})(8\text{ m}) + (1\text{ kN})(6\text{ m}) - A_y(12\text{ m}) = 0$

$A_y = 4.50\text{ kN}\uparrow$

WE NOTE THAT BC IS A ZERO-FORCE MEMBER: $F_{BC} = 0$ ◀

ALSO: $F_{CE} = F_{AC}$ (1)

FREE BODY: JOINT A



$\xrightarrow{+} \Sigma F_x = 0$: $\frac{1}{\sqrt{2}} F_{AB} + \frac{2}{\sqrt{5}} F_{AC} = 0$ (2)

$+\uparrow \Sigma F_y = 0$: $\frac{1}{\sqrt{2}} F_{AB} + \frac{1}{\sqrt{5}} F_{AC} + 3.50\text{ kN} = 0$ (3)

MULTIPLY (3) BY −2 AND ADD (2):

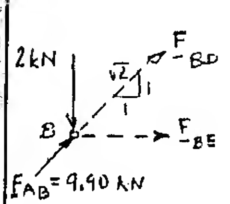
$-\frac{1}{\sqrt{2}} F_{AB} - 7\text{ kN} = 0$ $F_{AB} = 9.90\text{ kN C}$ ◀

SUBTRACT (3) FROM (2):

$\frac{1}{\sqrt{5}} F_{AC} - 3.50\text{ kN} = 0$, $F_{AC} = 7.826\text{ kN}$, $F_{AC} = 7.83\text{ kN T}$ ◀

FROM (1): $F_{CE} = F_{AC} = 7.826\text{ kN}$ $F_{CE} = 7.83\text{ kN T}$ ◀

FREE BODY: JOINT B



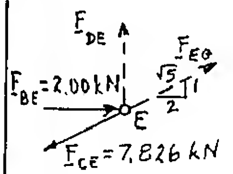
$+\uparrow \Sigma F_y = 0$: $\frac{1}{\sqrt{2}} F_{BD} + \frac{1}{\sqrt{2}}(9.90\text{ kN}) - 2\text{ kN} = 0$

$F_{BD} = -7.071\text{ kN}$ $F_{BD} = 7.07\text{ kN C}$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $F_{BE} + \frac{1}{\sqrt{2}}(9.90 - 7.071)\text{ kN} = 0$

$F_{BE} = -2.000\text{ kN}$ $F_{BE} = 2.00\text{ kN C}$ ◀

FREE BODY: JOINT E



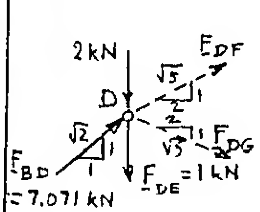
$\xrightarrow{+} \Sigma F_x = 0$: $\frac{2}{\sqrt{5}}(F_{EG} - 7.826\text{ kN}) + 2.00\text{ kN} = 0$

$F_{EG} = 5.590\text{ kN}$ $F_{EG} = 5.59\text{ kN T}$ ◀

$+\uparrow \Sigma F_y = 0$: $F_{DE} - \frac{1}{\sqrt{5}}(7.826 - 5.590)\text{ kN} = 0$

$F_{DE} = 1.000\text{ kN}$ $F_{DE} = 1.000\text{ kN T}$ ◀

FREE BODY: JOINT D



$\xrightarrow{+} \Sigma F_x = 0$: $\frac{2}{\sqrt{5}}(F_{DF} + F_{DG}) + \frac{1}{\sqrt{2}}(7.071\text{ kN}) = 0$

OR: $F_{DF} + F_{DG} = -5.590\text{ kN}$ (4)

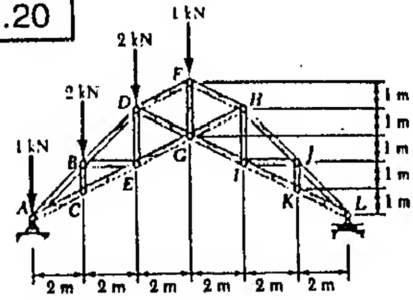
$+\uparrow \Sigma F_y = 0$: $\frac{1}{\sqrt{5}}(F_{DF} - F_{DG}) + \frac{1}{\sqrt{2}}(7.071\text{ kN}) - 2\text{ kN} - 1\text{ kN} = 0$

OR: $F_{DF} - F_{DG} = -4.472$ (5)

ADD (4) AND (5): $2F_{DF} = -10.062\text{ kN}$ $F_{DF} = 5.03\text{ kN C}$ ◀

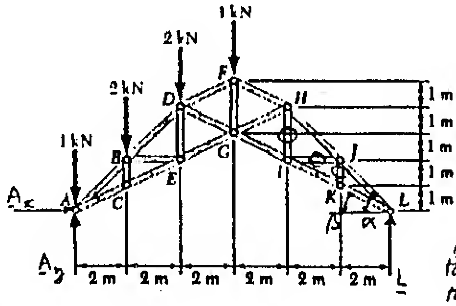
SUBTRACT (5) FROM (4): $2F_{DG} = -1.1180\text{ kN}$ $F_{DG} = 0.559\text{ kN C}$ ◀

6.20



GIVEN: SCISSORS ROOF TRUSS AND LOADING SHOWN.

FIND: FORCE IN MEMBER FG AND IN MEMBERS TO THE RIGHT OF FG.



FREE BODY: TRUSS

$+\circlearrowleft \Sigma M_A = 0$: $L(12\text{ m}) - (2\text{ kN})(2\text{ m}) - (2\text{ kN})(4\text{ m}) - (1\text{ kN})(6\text{ m}) = 0$

$L = 1.500\text{ kN}\uparrow$

ANGLES: $\tan\alpha = 1$ $\alpha = 45°$

$\tan\beta = \frac{1}{2}$ $\beta = 26.57°$

ZERO-FORCE MEMBERS:

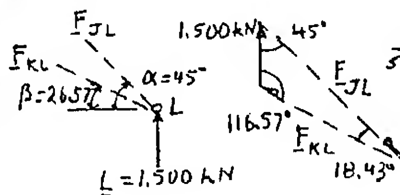
EXAMINING SUCCESSIVELY JOINTS K, J, AND I, WE NOTE THAT THE FOLLOWING MEMBERS TO THE RIGHT OF FG ARE ZERO-FORCE MEMBERS: JK, IJ, AND HI.

THUS: $F_{HI} = F_{IJ} = F_{JK} = 0$ ◀

WE ALSO NOTE THAT

$F_{GI} = F_{IK} = F_{KL}$ (1) AND $F_{HJ} = F_{JL}$ (2)

FREE BODY: JOINT L



$\frac{F_{JL}}{\sin 116.57°} = \frac{F_{KL}}{\sin 45°} = \frac{1.500\text{ kN}}{\sin 18.43°}$

$F_{JL} = 4.2436\text{ kN}$

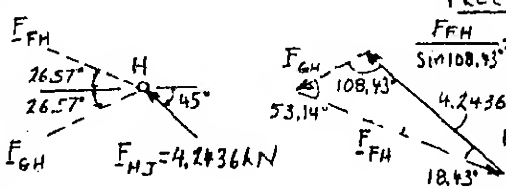
$F_{JL} = 4.24\text{ kN C}$ ◀

$F_{KL} = 3.35\text{ kN T}$ ◀

FROM EQ. (1): $F_{GI} = F_{IK} = F_{KL}$ $F_{GI} = F_{IK} = 3.35\text{ kN T}$ ◀

FROM EQ. (2): $F_{HJ} = F_{JL} = 4.2436\text{ kN}$, $F_{HJ} = 4.24\text{ kN C}$ ◀

FREE BODY: JOINT H

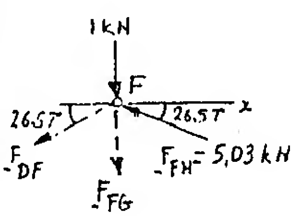


$\frac{F_{FH}}{\sin 108.43°} = \frac{F_{GH}}{\sin 18.43°} = \frac{4.2436}{\sin 53.14°}$

$F_{FH} = 5.03\text{ kN C}$ ◀

$F_{GH} = 1.677\text{ kN T}$ ◀

FREE BODY: JOINT F



$\xrightarrow{+} \Sigma F_x = 0$:

$-F_{DF}\cos 26.57° - (5.03\text{ kN})\cos 26.57° = 0$

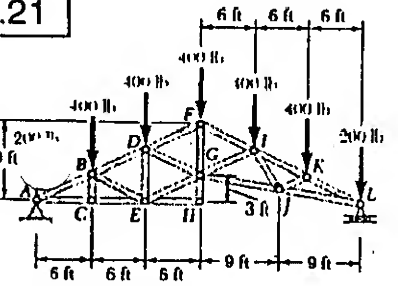
$F_{DF} = -5.03\text{ kN}$

$+\uparrow \Sigma F_y = 0$:

$-F_{FG} - 1\text{ kN} + (5.03\text{ kN})\sin 26.57° - (-5.03\text{ kN})\sin 26.57° = 0$

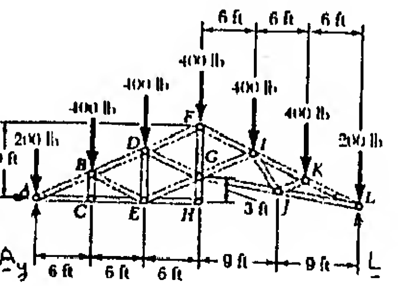
$F_{FG} = +3.500\text{ kN}$ $F_{FG} = 3.50\text{ kN T}$ ◀

6.21



GIVEN: STUDIO ROOF TRUSS AND LOADING SHOWN.

FIND: FORCE IN MEMBERS TO THE LEFT OF LINE FGH.



FREE BODY: TRUSS

$\Sigma F_x = 0$: $A_x = 0$

BECAUSE OF SYMMETRY OF LOADING:

$A_y = L = \frac{1}{2}$ TOTAL LOAD

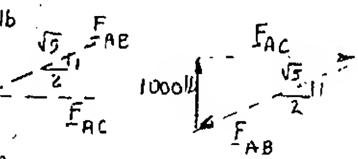
$A_y = L = 1200$ lb ↑

ZERO-FORCE MEMBERS. EXAMINING JOINTS C AND H, WE CONCLUDE THAT BC, EH, AND GH ARE ZERO-FORCE MEMBERS. THUS:

$F_{BC} = F_{EH} = 0$ ◀

ALSO: $F_{CE} = F_{AC}$ (1)

FREE BODY: JOINT A



$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{1000\text{ lb}}{1}$

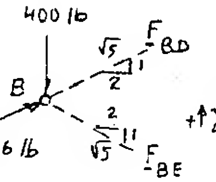
$F_{AB} = 2236$ lb C

$F_{AB} = 2240$ lb C ◀

$F_{AC} = 2000$ lb T ◀

FROM EQ.(1): $F_{CE} = 2000$ lb T ◀

FREE BODY: JOINT B



$\xrightarrow{+} \Sigma F_x = 0$: $\frac{2}{\sqrt{5}} F_{BD} + \frac{2}{\sqrt{5}} F_{BE} + \frac{2}{\sqrt{5}}(2236\text{ lb}) = 0$

OR: $F_{BD} + F_{BE} = -2236$ lb (2)

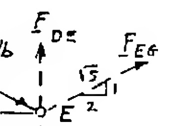
$+\uparrow \Sigma F_y = 0$: $\frac{1}{\sqrt{5}} F_{BD} - \frac{1}{\sqrt{5}} F_{BE} + \frac{1}{\sqrt{5}}(2236\text{ lb}) - 400\text{ lb} = 0$

OR: $F_{BD} - F_{BE} = -1342$ lb (3)

ADD (2) AND (3): $2F_{BD} = -3578$ lb $F_{BD} = 1789$ lb C ◀

SUBTRACT (3) FROM (2): $2F_{BE} = -894$ lb $F_{BE} = 447$ lb C ◀

FREE BODY: JOINT E



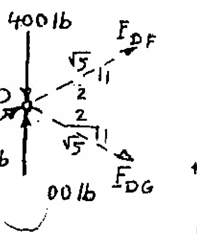
$\xrightarrow{+} \Sigma F_x = 0$: $\frac{2}{\sqrt{5}} F_{EG} + \frac{2}{\sqrt{5}}(447\text{ lb}) - 2000\text{ lb} = 0$

$F_{EG} = 1789$ lb T ◀

$+\uparrow \Sigma F_y = 0$: $F_{DE} + \frac{1}{\sqrt{5}}(1789\text{ lb}) - \frac{1}{\sqrt{5}}(447\text{ lb}) = 0$

$F_{DE} = -600$ lb $F_{DE} = 600$ lb C ◀

FREE BODY: JOINT D



$\xrightarrow{+} \Sigma F_x = 0$: $\frac{2}{\sqrt{5}} F_{DF} + \frac{2}{\sqrt{5}} F_{DG} + \frac{2}{\sqrt{5}}(1789\text{ lb}) = 0$

OR: $F_{DF} + F_{DG} = -1789$ lb (4)

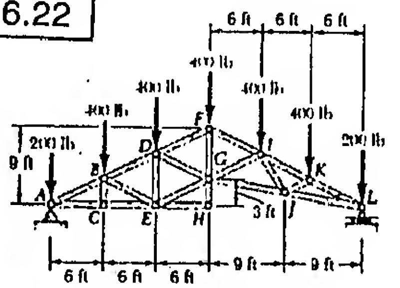
$+\uparrow \Sigma F_y = 0$: $\frac{1}{\sqrt{5}} F_{DF} - \frac{1}{\sqrt{5}} F_{DG} + \frac{1}{\sqrt{5}}(1789\text{ lb}) + 600\text{ lb} - 400\text{ lb} = 0$

OR: $F_{DF} - F_{DG} = -2236$ lb (5)

ADD (4) AND (5): $2F_{DF} = -4025$ lb $F_{DF} = 2010$ lb C ◀

SUBTRACT (5) FROM (4): $2F_{DG} = 447$ lb $F_{DG} = 224$ lb T ◀

6.22



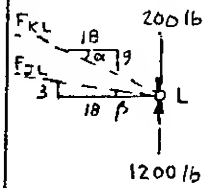
GIVEN: STUDIO ROOF TRUSS AND LOADING SHOWN.

FIND: FORCE IN FG AND IN MEMBERS TO THE RIGHT OF FG.

REACTION AT L: BECAUSE OF THE SYMMETRY OF THE LOADING, $L = \frac{1}{2}$ TOTAL LOAD, $L = 1200$ lb ↑

(SEE F.B. DIAGRAM TO THE LEFT FOR MORE DETAILS.)

FREE BODY: JOINT L



$\alpha = \tan^{-1}\frac{9}{18} = 26.57°$

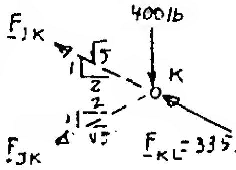
$\beta = \tan^{-1}\frac{3}{18} = 9.46°$

$\frac{F_{JL}}{\sin 63.43°} = \frac{F_{KL}}{\sin 99.46°} = \frac{1000\text{ lb}}{\sin 17.11°}$

$F_{JL} = 3040$ lb T ◀

$F_{KL} = 3352.7$ lb C $F_{KL} = 3350$ lb C ◀

FREE BODY: JOINT K



$\xrightarrow{+} \Sigma F_x = 0$: $-\frac{2}{\sqrt{5}} F_{IK} - \frac{2}{\sqrt{5}} F_{JK} - \frac{2}{\sqrt{5}}(3352.7\text{ lb}) = 0$

OR: $F_{IK} + F_{JK} = -3352.7$ lb (1)

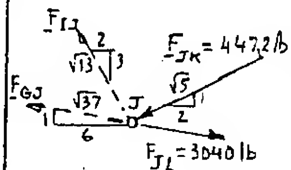
$+\uparrow \Sigma F_y = 0$: $\frac{1}{\sqrt{5}} F_{IK} - \frac{1}{\sqrt{5}} F_{JK} + \frac{1}{\sqrt{5}}(3352.7) - 400 = 0$

OR: $F_{IK} - F_{JK} = -2458.3$ lb (2)

ADD (1) AND (2): $2F_{IK} = -5811.0$, $F_{IK} = -2905.5$ lb, $F_{IK} = 2910$ lb C ◀

SUBTRACT (2) FROM (1): $2F_{JK} = -894.4$, $F_{JK} = -447.2$ lb, $F_{JK} = 447$ lb C ◀

FREE BODY: JOINT J



$\xrightarrow{+} \Sigma F_x = 0$:

$-\frac{2}{\sqrt{13}} F_{IJ} - \frac{6}{\sqrt{37}} F_{GJ} + \frac{6}{\sqrt{37}}(3040\text{ lb}) - \frac{2}{\sqrt{5}}(447.2) = 0$ (3)

$+\uparrow \Sigma F_y = 0$:

$\frac{3}{\sqrt{13}} F_{IJ} + \frac{1}{\sqrt{37}} F_{GJ} - \frac{1}{\sqrt{37}}(3040\text{ lb}) - \frac{1}{\sqrt{5}}(447.2) = 0$ (4)

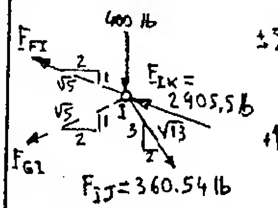
MULTIPLY (4) BY 6 AND ADD TO (3):

$\frac{16}{\sqrt{13}} F_{IJ} - \frac{8}{\sqrt{5}}(447.2) = 0$, $F_{IJ} = 360.54$ lb $F_{IJ} = 361$ lb T ◀

MULTIPLY (3) BY 3, (4) BY 2, AND ADD:

$-\frac{16}{\sqrt{37}}(F_{GJ} - 3040) - \frac{8}{\sqrt{5}}(447.2) = 0$, $F_{GJ} = 2431.7$ lb $F_{GJ} = 2430$ lb T ◀

FREE BODY: JOINT I



$\xrightarrow{+} \Sigma F_x = 0$: $-\frac{2}{\sqrt{5}} F_{FI} - \frac{2}{\sqrt{5}} F_{GI} - \frac{2}{\sqrt{5}}(2905.5) + \frac{2}{\sqrt{13}}(360.54) = 0$

OR: $F_{FI} + F_{GI} = -2681.9$ lb (5)

$+\uparrow \Sigma F_y = 0$: $\frac{1}{\sqrt{5}} F_{FI} - \frac{1}{\sqrt{5}} F_{GI} + \frac{1}{\sqrt{5}}(2905.5) - \frac{3}{\sqrt{13}}(360.54) - 400 = 0$

OR: $F_{FI} - F_{GI} = -1340.3$ lb (6)

ADD (5) AND (6): $2F_{FI} = -4022.2$, $F_{FI} = -2011.1$ lb, $F_{FI} = 2010$ lb C ◀

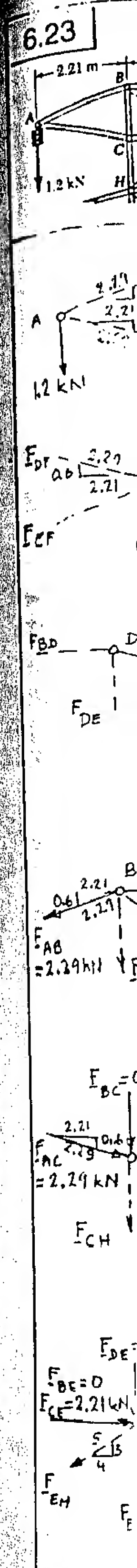
SUBTRACT (6) FROM (5): $2F_{GI} = -1341.6$ lb $F_{GI} = 671$ lb C ◀

FREE BODY: JOINT F

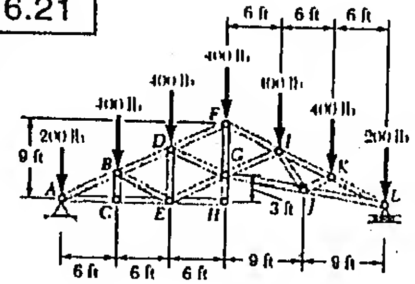
FROM $\Sigma F_x = 0$: $F_{DF} = F_{FI} = 2011.1$ lb C

$+\uparrow \Sigma F_y = 0$: $-F_{FG} - 400\text{ lb} + 2(\frac{1}{\sqrt{5}} 2011.1\text{ lb}) = 0$

$F_{FG} = +1400$ lb $F_{FG} = 1400$ lb T ◀

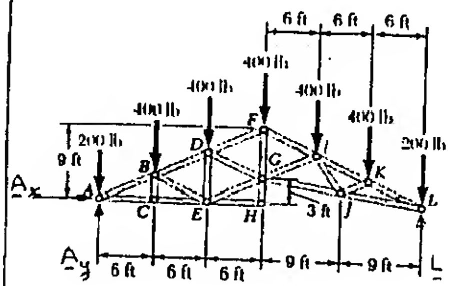


6.21



GIVEN: STUDIO ROOF TRUSS AND LOADING SHOWN.

FIND: FORCE IN MEMBERS TO THE LEFT OF LINE FGH.



FREE BODY: TRUSS

$\Sigma F_x = 0: \quad A_x = 0$

BECAUSE OF SYMMETRY OF LOADING:

$A_y = L = \frac{1}{2}$ TOTAL LOAD

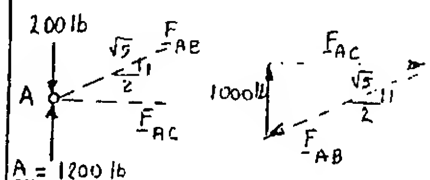
$A_y = L = 1200\ lb \uparrow$

ZERO-FORCE MEMBERS. EXAMINING JOINTS C AND H, WE CONCLUDE THAT BC, EH, AND GH ARE ZERO-FORCE MEMBERS. THUS:

$F_{BC} = F_{EH} = 0$

ALSO: $F_{CE} = F_{AC}$ (1)

FREE BODY: JOINT A



$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{1000\ lb}{1}$

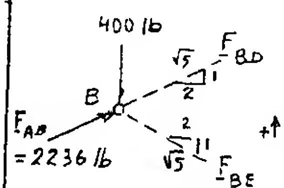
$F_{AB} = 2236\ lb\ C$

$F_{AB} = 2240\ lb\ C$

$F_{AC} = 2000\ lb\ T$

FROM EQ.(1): $F_{CE} = 2000\ lb\ T$

FREE BODY: JOINT B



$\xrightarrow{+} \Sigma F_x = 0: \frac{2}{\sqrt{5}} F_{BD} + \frac{2}{\sqrt{5}} F_{BE} + \frac{2}{\sqrt{5}}(2236\ lb) = 0$

OR: $F_{BD} + F_{BE} = -2236\ lb$ (2)

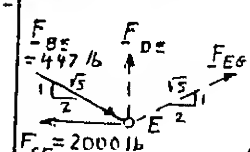
$+\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{5}} F_{BD} - \frac{1}{\sqrt{5}} F_{BE} + \frac{1}{\sqrt{5}}(2236\ lb) - 400\ lb = 0$

OR: $F_{BD} - F_{BE} = -1342\ lb$ (3)

ADD(2) AND (3): $2F_{BD} = -3578\ lb$ — $F_{BD} = 1789\ lb\ C$

SUBTRACT (3) FROM (1): $2F_{BE} = -894\ lb$ — $F_{BE} = 447\ lb\ C$

FREE BODY: JOINT E



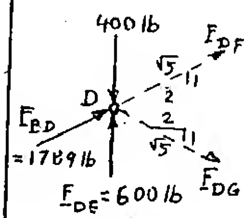
$\xrightarrow{+} \Sigma F_x = 0: \frac{2}{\sqrt{5}} F_{EG} + \frac{2}{\sqrt{5}}(447\ lb) - 2000\ lb = 0$

$F_{EG} = 1789\ lb\ T$

$+\uparrow \Sigma F_y = 0: \quad F_{DE} + \frac{1}{\sqrt{5}}(1789\ lb) - \frac{1}{\sqrt{5}}(447\ lb) = 0$

$F_{DE} = -600\ lb$ — $F_{DE} = 600\ lb\ C$

FREE BODY; JOINT D



$\xrightarrow{+} \Sigma F_x = 0: \frac{2}{\sqrt{5}} F_{DF} + \frac{2}{\sqrt{5}} F_{DG} + \frac{2}{\sqrt{5}}(1789\ lb) = 0$

OR: $F_{DF} + F_{DG} = -1789\ lb$ (4)

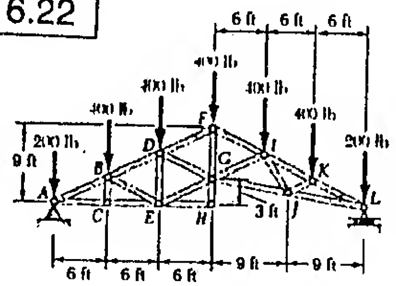
$+\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{5}} F_{DF} - \frac{1}{\sqrt{5}} F_{DG} + \frac{1}{\sqrt{5}}(1789\ lb) + 600\ lb - 400\ lb = 0$

OR: $F_{DF} - F_{DG} = -2236\ lb$ (5)

ADD (4) AND (5): $2F_{DF} = -4025\ lb$ — $F_{DF} = 2010\ lb\ C$

SUBTRACT (5) FROM (4): $2F_{DG} = 447\ lb$ — $F_{DG} = 224\ lb\ T$

6.22



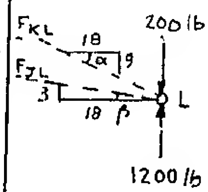
GIVEN: STUDIO ROOF T[RUSS] AND LOADING S[HOWN].

FIND: FORCE IN FG [AND] IN MEMBERS [TO THE] RIGHT OF FG[H].

REACTION AT L: BECAUSE OF THE SYMMETRY OF LOADING, $L = \frac{1}{2}$ TOTAL LOAD, $L = 1200\ lb \uparrow$

(SEE F.B. DIAGRAM TO THE LEFT FOR MORE DETAIL)

FREE BODY: JOINT [L]



$\alpha = \tan^{-1}\frac{9}{18} = 26.57°$

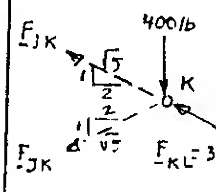
$\beta = \tan^{-1}\frac{3}{18} = 9.46°$

$\frac{F_{JL}}{\sin 63.43°} = \frac{F_{KL}}{\sin 99.46°} = \frac{1000\ lb}{\sin 17.11°}$

$F_{JL} = 3040\ lb\ T$

$F_{KL} = 3352.7\ lb\ C$ — $F_{KL} = 3350\ lb\ C$

FREE BODY: JOINT [K]



$\xrightarrow{+} \Sigma F_x = 0: -\frac{2}{\sqrt{5}} F_{IK} - \frac{2}{\sqrt{5}} F_{JK} - \frac{2}{\sqrt{5}}(3352.7\ lb)$

OR: $F_{IK} + F_{JK} = -3352.7\ lb$

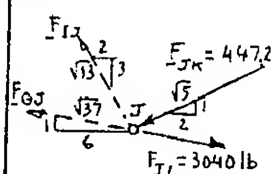
$+\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{5}} F_{IK} - \frac{1}{\sqrt{5}} F_{JK} + \frac{1}{\sqrt{5}}(3352.7) - 400$

OR: $F_{IK} - F_{JK} = -2458.3\ lb$

ADD (1) AND (2): $2F_{IK} = -5811.0$, $F_{IK} = -2905.5\ lb$, $F_{IK} = 2910\ lb\ C$

SUBTRACT (2) FROM (1): $2F_{JK} = -894.4$, $F_{JK} = -447.2\ lb$, $F_{JK} = 447\ lb\ C$

FREE BODY: JOINT J



$\xrightarrow{+} \Sigma F_x = 0:$

$-\frac{2}{\sqrt{13}} F_{IJ} - \frac{6}{\sqrt{37}} F_{GJ} + \frac{6}{\sqrt{37}}(3040\ lb) - \frac{2}{\sqrt{5}}(447.2)$

$+\uparrow \Sigma F_y = 0:$

$\frac{3}{\sqrt{13}} F_{IJ} + \frac{1}{\sqrt{37}} F_{GJ} - \frac{1}{\sqrt{37}}(3040\ lb) - \frac{1}{\sqrt{5}}(447.2)$

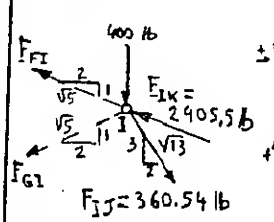
MULTIPLY (4) BY 6 AND ADD TO (3):

$\frac{16}{\sqrt{13}} F_{IJ} - \frac{8}{\sqrt{5}}(447.2) = 0$, $F_{IJ} = 360.54\ lb$ — $F_{IJ} = 361\ lb\ T$

MULTIPLY (3) BY 3, (4) BY 2, AND ADD:

$-\frac{16}{\sqrt{37}}(F_{GJ} - 3040) - \frac{8}{\sqrt{5}}(447.2) = 0$, $F_{GJ} = 2431.7\ lb$ — $F_{GJ} = 2430\ lb\ T$

FREE BODY: JOINT I



$\xrightarrow{+} \Sigma F_x = 0: -\frac{2}{\sqrt{5}} F_{FI} - \frac{2}{\sqrt{5}} F_{GI} - \frac{2}{\sqrt{5}}(2905.5) + \frac{2}{\sqrt{13}}(360.54) = 0$

OR: $F_{FI} + F_{GI} = -2681.9\ lb$ (5)

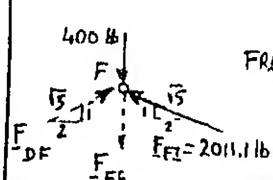
$+\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{5}} F_{FI} - \frac{1}{\sqrt{5}} F_{GI} + \frac{1}{\sqrt{5}}(2905.5) - \frac{3}{\sqrt{13}}(360.54) - 400 = 0$

OR: $F_{FI} - F_{GI} = -1390.3\ lb$ (6)

ADD (5) AND (6): $2F_{FI} = -4022.2$, $F_{FI} = -2011.1\ lb$, $F_{FI} = 2010\ lb\ C$

SUBTRACT (6) FROM (5): $2F_{GI} = -1341.6\ lb$ — $F_{GI} = 671\ lb\ C$

FREE BODY; JOINT F



FROM $\Sigma F_x = 0$: $F_{DF} = F_{FI} = 2011.1\ lb\ C$

$+\uparrow \Sigma F_y = 0: \ -F_{FG} - 400\ lb + 2\left(\frac{1}{\sqrt{5}} 2011.1\ lb\right) = 0$

$F_{FG} = +1400\ lb$ — $F_{FG} = 1400\ lb\ T$

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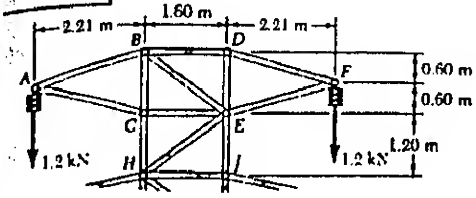
N FG AND
...E TO THE
FG.

...Y OF THE
...ETAILS.)
JOINT L
99.46°
1000 lb
...43°
...00 lb
...17.11°
...lb T
...lb C
JOINT K
...352.7 lb) = 0
...7 lb (1)
...7) - 400 = 0
(2)
...7 lb
...INT J
...(447.2) = 0 (3)
...(447.2) = 0 (4)
T
T
...NT I
...360.54) = 0 (5)
...360.54)
(6)
C
F

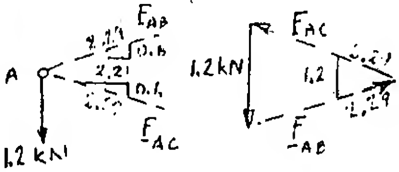
6.23



GIVEN: TOP PORTION OF PYLER TRANSMISSION LINE TOWER AND LOADING SHOWN.

FIND: FORCE IN MEMBERS LOCATED ABOVE HJ.

FREE BODY: JOINT A

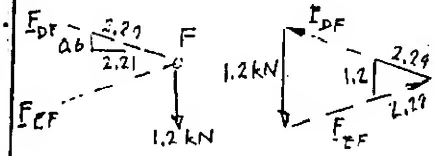


$$\frac{F_{AB}}{2.29} = \frac{F_{AC}}{2.29} = \frac{1.2\,kN}{1.2}$$

$F_{AB} = 2.29\,kN\ T$

$F_{AC} = 2.29\,kN\ C$

FREE BODY: JOINT F

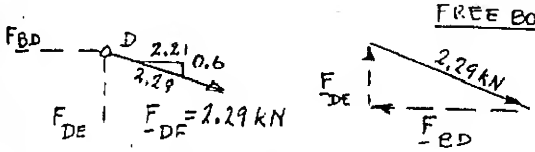


$$\frac{F_{DF}}{2.29} = \frac{F_{EF}}{2.29} = \frac{1.2\,kN}{2.1}$$

$F_{DF} = 2.29\,kN\ T$

$F_{EF} = 2.29\,kN\ C$

FREE BODY: JOINT D

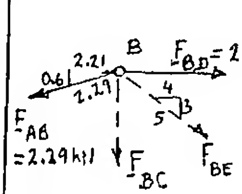


$$\frac{F_{BD}}{2.21} = \frac{F_{DE}}{0.6} = \frac{2.29\,kN}{2.29}$$

$F_{BD} = 2.21\,kN\ T$

$F_{DE} = 0.600\,kN\ C$

FREE BODY: JOINT B



$\overset{+}{\rightarrow}\Sigma F_x = 0$:

$$\frac{4}{5}F_{BE} + 2.21\,kN - \frac{2.21}{2.29}(2.29\,kN) = 0$$

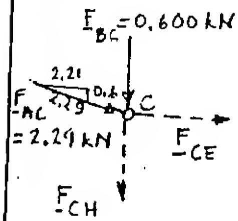
$F_{BE} = 0$

$+\uparrow \Sigma F_y = 0$:

$$-F_{BC} - \frac{3}{5}(0) - \frac{0.6}{2.29}(2.29\,kN) = 0$$

$F_{BC} = -0.600\,kN$, $F_{BC} = 0.600\,kN\,C$

FREE BODY: JOINT C



$\overset{+}{\rightarrow}\Sigma F_x = 0$:

$$F_{CE} + \frac{2.21}{2.29}(2.29\,kN) = 0$$

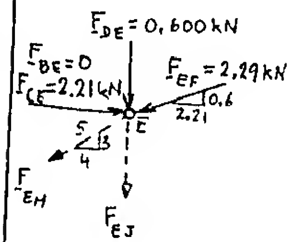
$F_{CE} = -2.21\,kN$, $F_{CE} = 2.21\,kN\ C$

$+\uparrow \Sigma F_y = 0$:

$$-F_{CH} - 0.600\,kN - \frac{0.6}{2.29}(2.29\,kN) = 0$$

$F_{CH} = -1.200\,kN$, $F_{CH} = 1.200\,kN\ C$

FREE BODY: JOINT E



$\overset{+}{\rightarrow}\Sigma F_x = 0$:

$$2.21\,kN - \frac{2.21}{2.29}(2.29\,kN) - \frac{4}{5}F_{EH} = 0$$

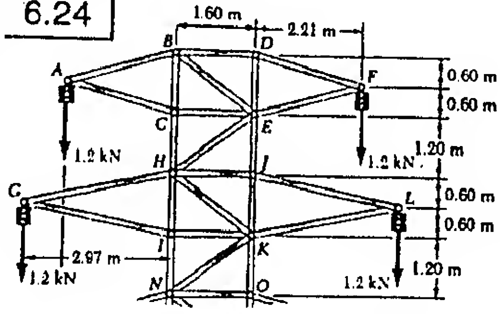
$F_{EH} = 0$

$+\uparrow \Sigma F_y = 0$:

$$-F_{EJ} - 0.600\,kN - \frac{0.6}{2.29}(2.29\,kN) - 0 = 0$$

$F_{EJ} = -1.200\,kN$, $F_{EJ} = 1.200\,kN\ C$

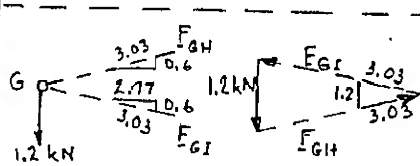
6.24



GIVEN: PORTION OF TOWER TRANSMISSION LINE TOWER LOADING SHOWN WITH F_{CH} = 1.2 kN C AND ...

FIND: FORCE IN MEMBERS BETWEEN HJ AND N...

FREE BODY: JOINT G

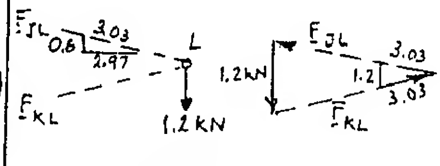


$$\frac{F_{GH}}{3.03} = \frac{F_{GI}}{3.03} = \frac{1.2...}{1.2...}$$

$F_{GH} = 3.03\,kN\ T$

$F_{GI} = 3.03\,kN\ C$

FREE BODY: JOINT L

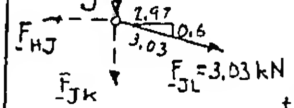


$$\frac{F_{JL}}{3.03} = \frac{F_{KL}}{3.03} = \frac{1.2...}{1...}$$

$F_{JL} = 3.03\,kN\ T$

$F_{KL} = 3.03\,kN\ C$

$F_{EJ} = 1.2\,kN$ (GIVEN)



FREE BODY: JOINT J

$\overset{+}{\rightarrow}\Sigma F_x = 0$: $-F_{HJ} + \frac{2.97}{3.03}(3.03\,kN...$

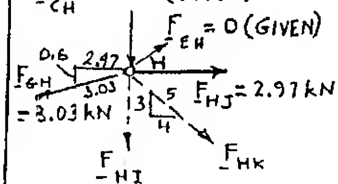
$F_{HJ} = 2.97\,kN\,T$

$+\uparrow \Sigma F_y = 0$: $-F_{JK} - 1.2\,kN - \frac{0.6}{3.03}(3.03\,k...$

$F_{JK} = -1.800\,kN$, $F_{JK} = 1.800\,kN\,C$

$F_{CH} = 1.2\,kN$ (GIVEN)

$F_{EH} = 0$ (GIVEN)



FREE BODY: JOINT H

$\overset{+}{\rightarrow}\Sigma F_x = 0$:

$$\frac{4}{5}F_{HK} + 2.97\,kN - \frac{2.97}{3.03}(3.03\,k...$$

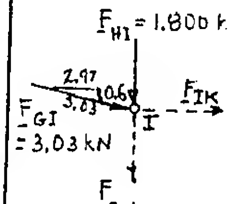
$F_{HK} = 0$

$+\uparrow \Sigma F_y = 0$:

$$-F_{HI} - 1.2\,kN - \frac{0.6}{3.03}(3.03)\,kN - \frac{3}{5}(0) = 0$$

$F_{HI} = -1.800\,kN$, $F_{HI} = 1.800\,kN\,C$

$F_{HI} = 1.800\,kN$



FREE BODY: JOINT I

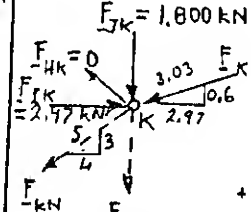
$\overset{+}{\rightarrow}\Sigma F_x = 0$: $F_{IK} + \frac{2.97}{3.03}(3.03\,kN) = 0$

$F_{IK} = -2.97\,kN$, $F_{IK} = 2.97\,kN\,C$

$+\uparrow \Sigma F_y = 0$: $-F_{IN} - 1.800\,kN - \frac{0.6}{3.03}(3.03\,kN) = ...$

$F_{IN} = -2.40\,kN$, $F_{IN} = 2.40\,kN\,C$

$F_{JK} = 1.800\,kN$



FREE BODY: JOINT K

$\overset{+}{\rightarrow}\Sigma F_x = 0$:

$$-\frac{4}{5}F_{KN} + 2.97\,kN - \frac{2.97}{3.03}(3.03\,kN) = 0$$

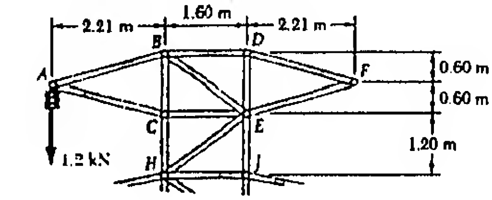
$F_{KN} = 0$

$+\uparrow \Sigma F_y = 0$:

$$-F_{KO} - \frac{0.6}{3.03}(3.03\,kN) - 1.800\,kN - \frac{3}{5}(0) = 0$$

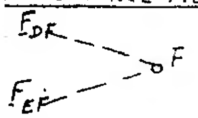
$F_{KO} = -2.40\,kN$, $F_{KO} = 2.40\,kN\,C$

6.25



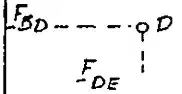
GIVEN: TOP PORTION OF TOWER TRANSMISSION LINE OF PROB. 6.23, ASSUMING THAT CABLES ON RIGHT-HAND SIDE ARE MISSING. (LOADING WILL BE AS SHOWN ABOVE.)
FIND: FORCE IN MEMBERS LOCATED ABOVE HJ.

ZERO-FORCE MEMBERS.



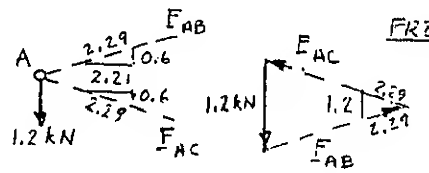
CONSIDERING JOINT F, WE NOTE THAT DF AND EF ARE ZERO-FORCE MEMBERS:

$F_{DF} = F_{EF} = 0$ ◀



CONSIDERING NEXT JOINT D, WE NOTE THAT BD AND DE ARE ZERO-FORCE MEMBERS:

$F_{BD} = F_{DE} = 0$ ◀



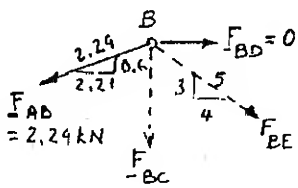
FREE BODY: JOINT A

$$\frac{F_{AB}}{2.29} = \frac{F_{AC}}{2.29} = \frac{1.2\,kN}{1.2}$$

$F_{AB} = 2.29\,kN\,T$ ◀

$F_{AC} = 2.29\,kN\,C$ ◀

FREE BODY: JOINT B



$\stackrel{+}{\rightarrow} \Sigma F_x = 0: \frac{4}{5} F_{BE} - \frac{2.21}{2.29}(2.29\,kN) = 0$

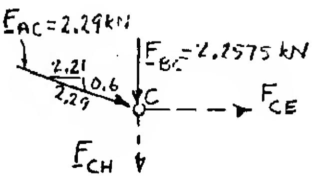
$F_{BE} = 2.7625\,kN$, $F_{BE} = 2.76\,kN\,T$ ◀

$+\uparrow \Sigma F_y = 0:$

$-F_{BC} - \frac{0.6}{2.29}(2.29\,kN) - \frac{3}{5}(2.7625\,kN) = 0$

$F_{BC} = -2.2575\,kN$ $\quad F_{BC} = 2.26\,kN\,C$ ◀

FREE BODY: JOINT C



$\stackrel{+}{\rightarrow} \Sigma F_x = 0: F_{CE} + \frac{2.21}{2.29}(2.29\,kN) = 0$

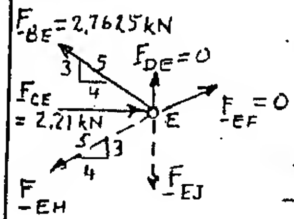
$F_{CE} = 2.21\,kN\,C$ ◀

$+\uparrow \Sigma F_y = 0:$

$-F_{CH} - 2.2575\,kN - \frac{0.6}{2.29}(2.29\,kN) = 0$

$F_{CH} = -2.8575\,kN$ $\quad F_{CH} = 2.86\,kN\,C$ ◀

FREE BODY: JOINT E



$\stackrel{+}{\rightarrow} \Sigma F_x = 0:$

$-\frac{4}{5} F_{EH} - \frac{4}{5}(2.7625\,kN) + 2.21\,kN = 0$

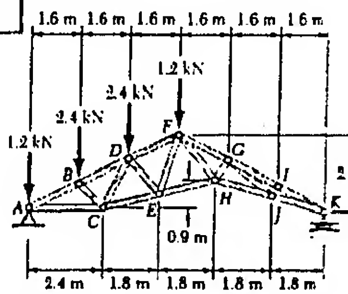
$F_{EH} = 0$ ◀

$+\uparrow \Sigma F_y = 0:$

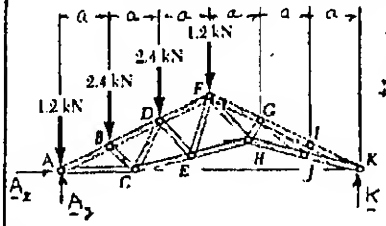
$-F_{EJ} + \frac{3}{5}(2.7625\,kN) - \frac{3}{5}(0) = 0$

$F_{EJ} = +1.6575\,kN$ $\quad F_{EJ} = 1.658\,kN\,T$ ◀

6.26



GIVEN: [illegible] TRUSS AND LOADING SHOWN.
FIND: FORCE IN MEMBERS CONNECTING [illegible] A THROUGH F.



FREE BODY: TRUSS

$\Sigma F_x = 0:$ $\quad A_x = 0$

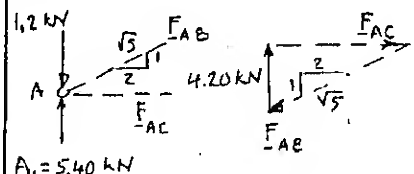
$\circlearrowleft \Sigma M_K = 0:$

$(1.2\,kN)6a - (2.4\,kN)5a$

$+(2.4\,kN)4a - (1.2\,kN)3a$

$-A_y(6a) = 0$ $\quad A_y = 5.40\,kN\uparrow$

FREE BODY: JOINT A



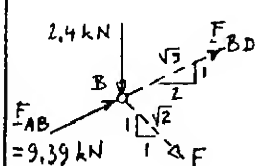
$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{4.20\,kN}{1}$$

$F_{AB} = [illegible]$

$F_{AB} = 9.39\,kN\,C$ ◀

$F_{AC} = 8.40\,kN\,T$ ◀

FREE BODY: JOINT B



$\stackrel{+}{\rightarrow} \Sigma F_x = 0: \frac{2}{\sqrt{5}} F_{BD} + \frac{1}{\sqrt{2}} F_{BC} - \frac{2}{\sqrt{5}}([illegible]) = 0$ (1)

$+\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{5}} F_{BD} - \frac{1}{\sqrt{2}} F_{BC} - \frac{1}{\sqrt{5}}([illegible]) - 2.4 = 0$ (2)

ADD (1) AND (2):

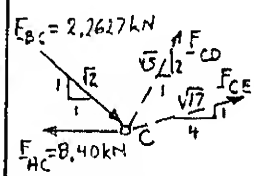
$\frac{3}{\sqrt{5}} F_{BD} + \frac{3}{\sqrt{5}}(9.39[illegible]\,kN) - [illegible] = 0$

$F_{BD} = -7.6026\,kN$ $\quad F_{BD} = 7.60\,kN\,C$ ◀

MULTIPLY (2) BY −2 AND ADD (1): $\frac{3}{\sqrt{2}} F_{BC} - [illegible] = 0$

$F_{BC} = -2.2627\,kN$ $\quad F_{BC} = 2.26\,kN\,C$ ◀

FREE BODY: JOINT C



$\stackrel{+}{\rightarrow} \Sigma F_x = 0:$

$\frac{1}{\sqrt{5}} F_{CD} + \frac{4}{\sqrt{17}} F_{CE} - \frac{1}{\sqrt{2}}([illegible]) - [illegible] = 0$ (3)

$+\uparrow \Sigma F_y = 0: \frac{2}{\sqrt{5}} F_{CD} - \frac{1}{\sqrt{17}} F_{CE} - \frac{1}{\sqrt{2}}([illegible]) = 0$ (4)

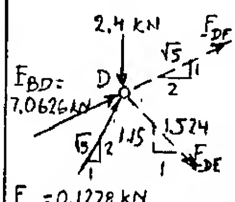
MULTIPLY (4) BY −4 AND ADD (3): $-\frac{7}{\sqrt{5}} F_{CD} + \frac{5}{\sqrt{2}}([illegible]) - [illegible] = 0$

$F_{CD} = -0.1278\,kN$ $\quad F_{CD} = 0.128\,kN\,C$ ◀

MULTIPLY (3) BY 2 AND SUBTRACT (4): $\frac{9}{\sqrt{17}} F_{CE} + \frac{1}{\sqrt{2}}([illegible]) - [illegible] = 0$

$F_{CE} = 7.068\,kN$ $\quad F_{CE} = 7.07\,kN\,T$ ◀

FREE BODY: JOINT D



$\stackrel{+}{\rightarrow} \Sigma F_x = 0: \frac{2}{\sqrt{5}} F_{DF} + \frac{1}{1.524} F_{DE} [illegible] = 0$ (5)

$+\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{5}} F_{DF} - \frac{1.15}{1.524} F_{DE} [illegible] = 0$ (6)

MULTIPLY (5) BY 1.15 AND ADD (6):

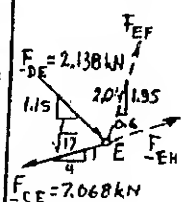
$\frac{3.30}{\sqrt{5}} F_{DF} + \frac{3.30}{\sqrt{5}}(7.6026) + \frac{3.15}{\sqrt{5}}([illegible]) [illegible] = 0$

$F_{DF} = -6.098\,kN$ $\quad F_{DF} = 6.10\,kN\,C$ ◀

MULTIPLY (6) BY −2 AND ADD (5): $\frac{3.30}{1.524} F_{DE} - \frac{3}{\sqrt{5}}([illegible]) [illegible] = 0$

$F_{DE} = -2.138\,kN$ $\quad F_{DE} = 2.14\,kN\,C$ ◀

FREE BODY: JOINT E



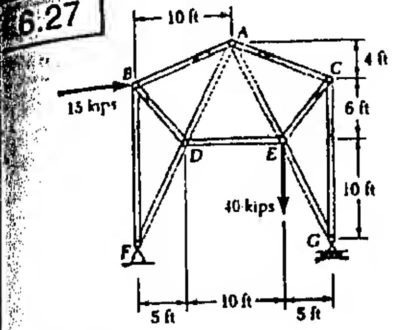
$\stackrel{+}{\rightarrow} \Sigma F_x = 0: \frac{0.6}{2.04} F_{EF} + \frac{4}{\sqrt{17}}(F_{EH} - F_{CE}) - \frac{1}{1.524}([illegible]) = 0$ (7)

$+\uparrow \Sigma F_y = 0: \frac{1.95}{2.04} F_{EF} + \frac{1}{\sqrt{17}}(F_{EH} - F_{CE}) - \frac{1.15}{1.524}([illegible]) = 0$ (8)

MULTIPLY (8) BY 4 AND SUBTRACT (7):

$\frac{7.2}{2.04} F_{EF} - 7.856\,kN = 0$ $\quad F_{EF} = [illegible]$ ◀

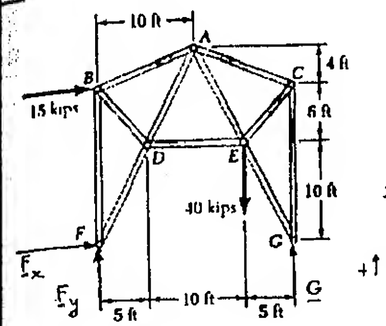
6.27



GIVEN: TRUSS AND LOADING SHOWN.

FIND: FORCE IN EACH MEMBER OF THE TRUSS.

FREE BODY: TRUSS



$\circlearrowleft \Sigma M_F = 0$: $G(20\text{ ft}) - (15\text{ kips})(16\text{ ft}) - (40\text{ kips})(15\text{ ft}) = 0$

$G = 42\text{ kips}\uparrow$

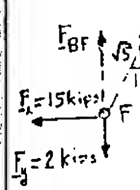
$\xrightarrow{+} \Sigma F_x = 0$: $F_x + 15\text{ kips} = 0$

$F_x = 15\text{ kips} \leftarrow$

$+\uparrow \Sigma F_y = 0$: $F_y - 40\text{ kips} + 42\text{ kips} = 0$

$F_y = 2\text{ kips}\downarrow$

FREE BODY: JOINT F



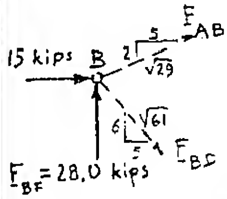
$\xrightarrow{+} \Sigma F_x = 0$: $\frac{1}{\sqrt{5}}F_{DF} - 15\text{ kips} = 0$

$F_{DF} = 33.54\text{ kips}$, $F_{DF} = 33.5\text{ kips } T$

$+\uparrow \Sigma F_y = 0$: $F_{BF} - 2\text{ kips} + \frac{2}{\sqrt{5}}(33.54\text{ kips}) = 0$

$F_{BF} = -28.00\text{ kips}$, $F_{BF} = 28.0\text{ kips } C$

FREE BODY: JOINT B



$\xrightarrow{+} \Sigma F_x = 0$: $\frac{5}{\sqrt{29}}F_{AB} + \frac{5}{\sqrt{61}}F_{BD} + 15\text{ kips} = 0$ (1)

$+\uparrow \Sigma F_y = 0$: $\frac{2}{\sqrt{29}}F_{AB} - \frac{6}{\sqrt{61}}F_{BD} + 28\text{ kips} = 0$ (2)

MULTIPLY (1) BY 6, (2) BY 5, AND ADD:

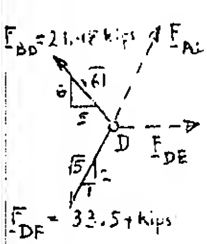
$\frac{40}{\sqrt{29}}F_{AB} + 230\text{ kips} = 0$ $F_{AB} = -30.96\text{ kips}$

$F_{AB} = 31.0\text{ kips } C$

MULTIPLY (1) BY 2, (2) BY −5, AND ADD:

$\frac{40}{\sqrt{61}}F_{BD} - 110\text{ kips} = 0$, $F_{BD} = 21.48\text{ kips}$, $F_{BD} = 21.5\text{ kips } T$

FREE BODY: JOINT D



$+\uparrow \Sigma F_y = 0$: $\frac{2}{\sqrt{5}}F_{AD} - \frac{2}{\sqrt{5}}(33.54) + \frac{6}{\sqrt{61}}(21.48) = 0$

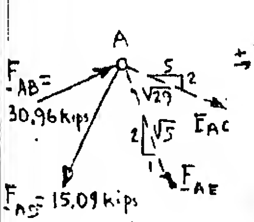
$F_{AD} = 15.09\text{ kips } T$

$\xrightarrow{+} \Sigma F_x = 0$:

$F_{DE} + \frac{1}{\sqrt{5}}(15.09 - 33.54) - \frac{5}{\sqrt{61}}(21.48) = 0$

$F_{DE} = 22.0\text{ kips } T$

FREE BODY: JOINT A



$\xrightarrow{+} \Sigma F_x = 0$: $\frac{5}{\sqrt{29}}F_{AC} + \frac{1}{\sqrt{5}}F_{AE} + \frac{5}{\sqrt{29}}(30.96) - \frac{1}{\sqrt{5}}(15.09) = 0$ (3)

$+\uparrow \Sigma F_y = 0$: $-\frac{2}{\sqrt{29}}F_{AC} - \frac{2}{\sqrt{5}}F_{AE} + \frac{2}{\sqrt{29}}(30.96) - \frac{2}{\sqrt{5}}(15.09) = 0$ (4)

MULTIPLY (3) BY 2 AND ADD (4):

$\frac{8}{\sqrt{29}}F_{AC} + \frac{12}{\sqrt{29}}(30.96) - \frac{4}{\sqrt{5}}(15.09) = 0$

$F_{AC} = -28.27\text{ kips}$, $F_{AC} = 28.3\text{ kips } C$

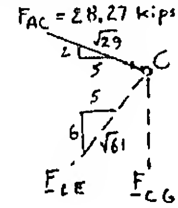
MULTIPLY (3) BY 2, (4) BY 5 AND ADD:

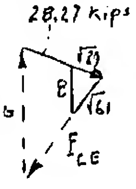
$-\frac{8}{\sqrt{5}}F_{AE} + \frac{20}{\sqrt{29}}(30.96) - \frac{12}{\sqrt{5}}(15.09) = 0$

$F_{AE} = 9.50\text{ kips } T$

(CONTINUED)

6.27 CONTINUED





FREE BODY: JOINT C

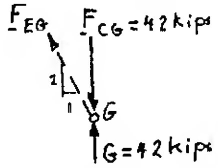
FROM FORCE TRIANGLE:

$\frac{F_{CE}}{\sqrt{61}} = \frac{F_{CG}}{8} = \frac{28.27\text{ kips}}{\sqrt{29}}$

$F_{CE} = 41.0\text{ kips } T$

$F_{CG} = 42.0\text{ kips } C$

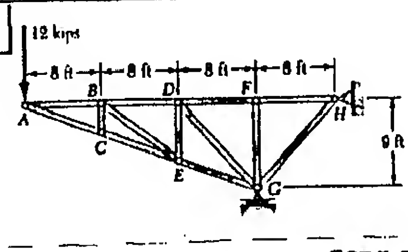
FREE BODY: JOINT G



$\xrightarrow{+} \Sigma F_x = 0$: $F_{EG} = 0$

$+\uparrow \Sigma F_y = 0$: $42\text{ kips} - 42\text{ kips} = 0$ (CHECKS)

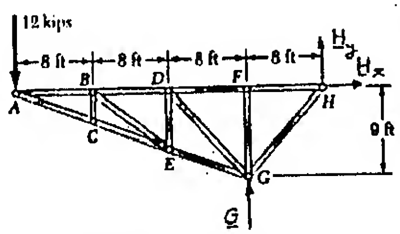
6.28



GIVEN: TRUSS AND LOADING SHOWN.

FIND: FORCE IN EACH MEMBER

FREE BODY: TRUSS



$\xrightarrow{+} \Sigma F_x = 0$: $H_x = 0$

$+\circlearrowleft \Sigma M_G = 0$: $(12\text{ kips})(24\text{ ft}) + H_y(8\text{ ft}) = 0$

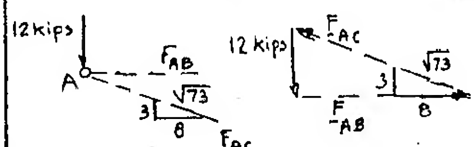
$H_y = -36\text{ kips}$ $H_y = 36\text{ kips}\downarrow$

$\Sigma F_y = 0$: $G = 48\text{ kips}\uparrow$

ZERO-FORCE MEMBERS

JOINT F:	$F_{DF} = F_{FH}$ (1)	AND	$F_{FG} = 0$
JOINT C:	$F_{AC} = F_{CE}$ (2)	AND	$F_{BC} = 0$
JOINT B:	$F_{AB} = F_{BD}$ (3)	AND	$F_{BE} = 0$
JOINT E:	$F_{CE} = F_{EG}$ (4)	AND	$F_{DE} = 0$
JOINT D:	$F_{BD} = F_{DF}$ (5)	AND	$F_{DG} = 0$

FREE BODY: JOINT A



$\frac{F_{AB}}{8} = \frac{F_{AC}}{\sqrt{73}} = \frac{12\text{ kips}}{3}$

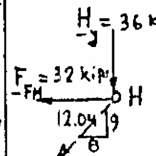
$F_{AB} = 32.0\text{ kips } T$

$F_{AC} = 34.2\text{ kips } C$

FROM EQS. (3), (5), AND (1): $F_{BD} = F_{DF} = F_{FH} = 32.0\text{ kips } T$

FROM EQS. (2) AND (4): $F_{CE} = F_{EG} = 34.2\text{ kips } C$

FREE BODY: JOINT H

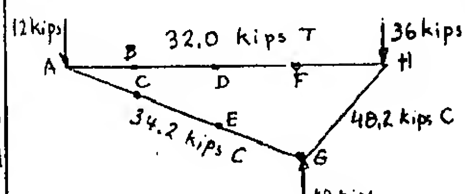


$+\uparrow \Sigma F_y = 0$: $-\frac{9}{12.04}F_{GH} - 36\text{ kips} = 0$

$F_{GH} = -48.16\text{ kips}$ $F_{GH} = 48.2\text{ kips } C$

$\xrightarrow{+} \Sigma F_x = -32\text{ kips} - \frac{8}{12.04}(-48.16\text{ kips}) = 0$ (CHECKS)

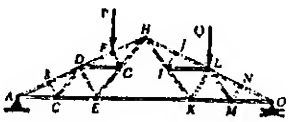
SUMMARY OF FORCES



(MEMBERS NOT SHOWN ARE ZERO-FORCE MEMBERS)

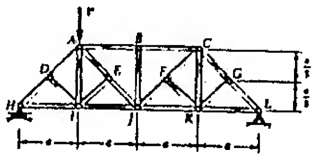
6.29 DETERMINE WHETHER THE TRUSSES OF PROBS. 6.31a, 6.32a, AND 6.33a ARE SIMPLE TRUSSES.

TRUSS OF PROB. 6.31a



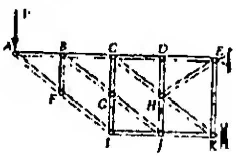
STARTING WITH TRIANGLE ABC AND ADDING TWO MEMBERS AT A TIME, WE OBTAIN JOINTS D, E, G, F, AND H, BUT CANNOT GO FURTHER. THUS, THIS TRUSS IS NOT A SIMPLE TRUSS ◀

TRUSS OF PROB. 6.32a



STARTING WITH TRIANGLE HDI AND ADDING TWO MEMBERS AT A TIME, WE OBTAIN SUCCESSIVELY JOINTS A, E, J, AND B, BUT CANNOT GO FURTHER. THUS, THIS TRUSS IS NOT A SIMPLE TRUSS ◀

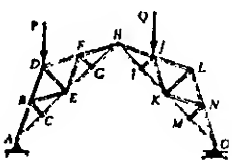
TRUSS OF PROB. 6.33a



STARTING WITH TRIANGLE EHK AND ADDING TWO MEMBERS AT A TIME, WE OBTAIN SUCCESSIVELY JOINTS D, J, C, G, I, B, F, AND A, THUS COMPLETING THE TRUSS. THEREFORE, THIS TRUSS IS A SIMPLE TRUSS ◀

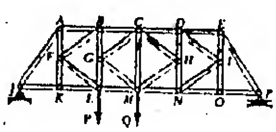
6.30 DETERMINE WHETHER THE TRUSSES OF PROBLEMS 6.31b, 6.32b, AND 6.33b ARE SIMPLE TRUSSES.

TRUSS OF PROB. 6.31b.



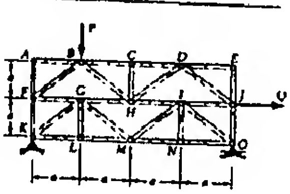
STARTING WITH TRIANGLE ABC AND ADDING TWO MEMBERS AT A TIME, WE OBTAIN SUCCESSIVELY JOINTS E, D, F, G, AND H, BUT CANNOT GO FURTHER. THUS, THIS TRUSS IS NOT A SIMPLE TRUSS ◀

TRUSS OF PROB. 6.32b.



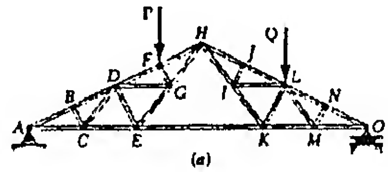
STARTING WITH TRIANGLE CGM AND ADDING TWO MEMBERS AT A TIME, WE OBTAIN SUCCESSIVELY JOINTS B, L, F, A, K, J, THEN H, D, N, I, E, O, AND P, THUS COMPLETING THE TRUSS. THEREFORE, THIS TRUSS IS A SIMPLE TRUSS ◀

TRUSS OF PROB. 6.33b.



STARTING WITH TRIANGLE GLM AND ADDING TWO MEMBERS AT A TIME, WE OBTAIN JOINTS K AND I BUT CANNOT CONTINUE. STARTING INSTEAD WITH TRIANGLE BCH, WE OBTAIN JOINT D BUT CANNOT CONTINUE. THUS, THIS TRUSS IS NOT A SIMPLE TRUSS ◀

6.31 DETERMINE THE ZERO-FORCE MEMBERS IN EACH OF THE TRUSSES SHOWN FOR THE GIVEN LOADING



(a)

TRUSS (a)

FB: JOINT B: $F_{BC}=0$

FB: JOINT C: $F_{CD}=0$

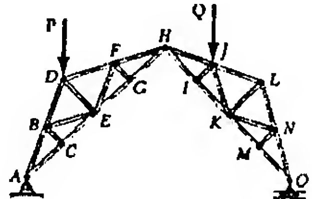
FB: JOINT J: $F_{IJ}=0$

FB: JOINT I: $F_{IL}=0$

FB: JOINT N: $F_{MN}=0$

FB: JOINT M: $F_{LM}=0$

THE ZERO-FORCE MEMBERS, THEREFORE, ARE BC, CD, IJ, IL, LM, MN ◀



(b)

TRUSS (b)

FB: JOINT C: $F_{BC}=0$

FB: JOINT B: $F_{BE}=0$

FB: JOINT G: $F_{FG}=0$

FB: JOINT F: $F_{EF}=0$

FB: JOINT E: $F_{DE}=0$

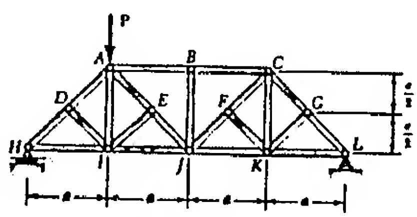
FB: JOINT I: $F_{IJ}=0$

FB: JOINT M: $F_{MN}=0$

FB: JOINT N: $F_{KN}=0$

THE ZERO-FORCE MEMBERS, THEREFORE, ARE BC, BE, DE, EF, FG, IJ, KN, MN ◀

6.32 DETERMINE THE ZERO-FORCE MEMBERS IN EACH OF THE TRUSSES SHOWN FOR THE GIVEN LOADING



(a)

TRUSS (a)

FB: JOINT B: $F_{BJ}=0$

FB: JOINT D: $F_{DI}=0$

FB: JOINT E: $F_{EI}=0$

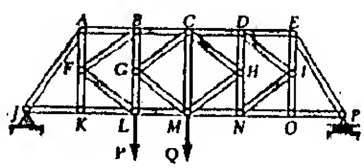
FB: JOINT I: $F_{AI}=0$

FB: JOINT F: $F_{FK}=0$

FB: JOINT G: $F_{GK}=0$

FB: JOINT K: $F_{CK}=0$

THE ZERO-FORCE MEMBERS, THEREFORE, ARE AI, BJ, CK, DI, EI, FK, GK ◀



(b)

TRUSS (b)

FB: JOINT K: $F_{FK}=0$

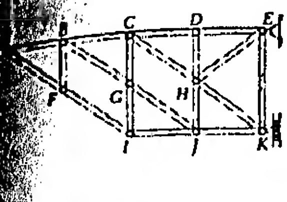
FB: JOINT O: $F_{IO}=0$

THE ZERO-FORCE MEMBERS, THEREFORE, ARE FK AND IO ◀

ALL OTHER MEMBERS ARE EITHER IN TENSION OR COMPRESSION.

6.33

DETERMINE THE ZERO-FORCE MEMBERS IN EACH OF THE TRUSSES SHOWN FOR THE GIVEN LOADING.



(a)

TRUSS (a)

FB: JOINT F: $F_{BF}=0$

FB: JOINT B: $F_{BG}=0$

FB: JOINT G: $F_{GJ}=0$

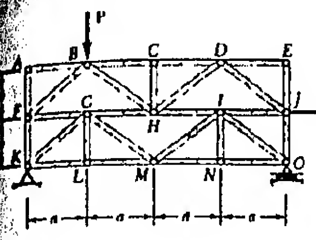
FB: JOINT D: $F_{DH}=0$

FB: JOINT J: $F_{HJ}=0$

FB: JOINT H: $F_{EH}=0$

THE ZERO-FORCE MEMBERS, THEREFORE, ARE

BF, BG, DH, EH, GJ, HJ ◄



(b)

TRUSS (b)

FB: JOINT A: $F_{AB}=F_{AF}=0$

FB: JOINT C: $F_{CH}=0$

FB: JOINT E: $F_{DE}=F_{EJ}=0$

FB: JOINT L: $F_{GL}=0$

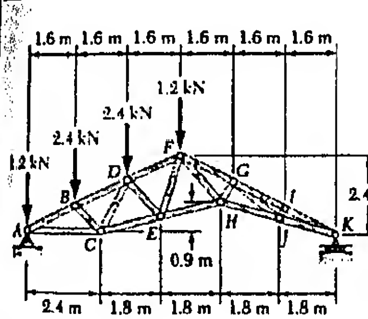
FB: JOINT N: $F_{IN}=0$

THE ZERO-FORCE MEMBERS, THEREFORE, ARE

AB, AF, CH, DE, EJ, GL, IN ◄

6.34

DETERMINE THE ZERO-FORCE MEMBERS IN THE TRUSS OF (a) PROB. 6.26, (b) PROB. 6.28



(a) TRUSS OF PROB. 6.26

FB: JOINT I: $F_{IJ}=0$

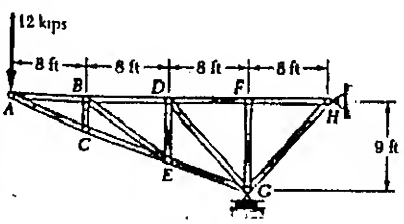
FB: JOINT J: $F_{GJ}=0$

FB: JOINT G: $F_{GH}=0$

THE ZERO-FORCE MEMBERS, THEREFORE, ARE

GH, GJ, IJ ◄

(b) TRUSS OF PROB. 6.28



FB: JOINT C: $F_{BC}=0$

FB: JOINT B: $F_{BE}=0$

FB: JOINT E: $F_{DE}=0$

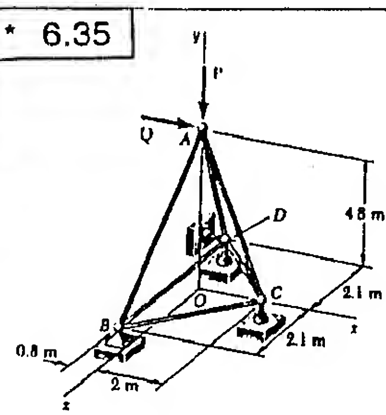
FB: JOINT D: $F_{DG}=0$

FB: JOINT F: $F_{FG}=0$

THE ZERO-FORCE MEMBERS, THEREFORE, ARE

BC, BE, DE, DG, FG ◄

* 6.35



GIVEN:

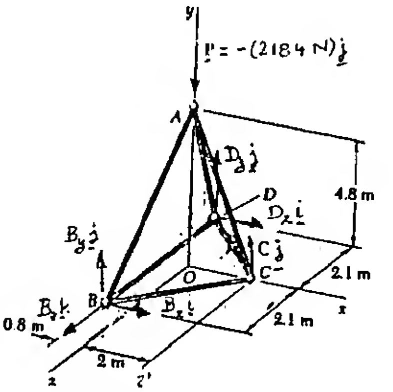
TRUSS SHOWN, WITH

$\underline{P}=(-2184\text{ N})\underline{j}$

$\underline{Q}=0$

FIND:

FORCE IN EACH MEMBER



FREE BODY: TRUSS

FROM SYMMETRY:

$D_x=B_x$ AND $D_y=B_y$

$\Sigma F_x=0:\ 2B_x=0$

$B_x=D_x=0$

$\Sigma F_z=0:\ B_z=0$

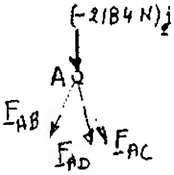
$\Sigma M_{CZ'}=0:$

$-2B_y(2.8\text{ m})+(2184\text{ N})(2\text{ m})=0$

$B_y=780\text{ N}$

THUS: $\underline{B}=(780\text{ N})\underline{j}$ ◄

FREE BODY: A



$\underline{F}_{AB}=F_{AB}\frac{\overrightarrow{AB}}{AB}=\frac{F_{AB}}{5.30}(-0.8\underline{i}-4.8\underline{j}+2.1\underline{k})$

$\underline{F}_{AC}=F_{AC}\frac{\overrightarrow{AC}}{AC}=\frac{F_{AC}}{5.20}(2\underline{i}-4.8\underline{j})$

$\underline{F}_{AD}=F_{AD}\frac{\overrightarrow{AD}}{AD}=\frac{F_{AD}}{5.30}(-0.8\underline{i}-4.8\underline{j}-2.1\underline{k})$

$\Sigma\underline{F}=0:\ \underline{F}_{AB}+\underline{F}_{AC}+\underline{F}_{AD}-(2184\text{ N})\underline{j}=0$

SUBSTITUTING FOR $\underline{F}_{AB}, \underline{F}_{AC}, \underline{F}_{AD}$, AND EQUATING TO ZERO THE COEFFICIENTS OF $\underline{i}, \underline{j}, \underline{k}$:

(i) $-\frac{0.8}{5.30}(F_{AB}+F_{AD})+\frac{2}{5.20}F_{AC}=0$ (1)

(j) $-\frac{4.8}{5.30}(F_{AB}+F_{AD})-\frac{4.8}{5.20}F_{AC}-2184\text{ N}=0$ (2)

(k) $\frac{2.1}{5.30}(F_{AB}-F_{AD})=0$ $F_{AD}=F_{AB}$

MULTIPLY (1) BY -6 AND ADD (2):

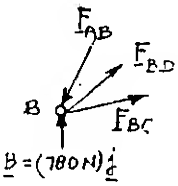
$-(16.8/5.20)F_{AC}-2184\text{ N}=0,\ F_{AC}=-676\text{ N},\ F_{AC}=676\text{ N C}$ ◄

SUBSTITUTE FOR F_{AC} AND F_{AD} IN (1):

$-(0.8/5.30)2F_{AB}+(2/5.20)(-676\text{ N})=0,\ F_{AB}=-861.25\text{ N}$

$F_{AB}=F_{AD}=861\text{ N C}$ ◄

FREE BODY: B



$\underline{F}_{AB}=(861.25\text{ N})\frac{\overrightarrow{AB}}{AB}=-(130\text{ N})\underline{i}-(780\text{ N})\underline{j}+(341.25\text{ N})\underline{k}$

$\underline{F}_{BC}=F_{BC}\left(\frac{2.8\underline{i}-2.1\underline{k}}{3.5}\right)=F_{BC}(0.8\underline{i}-0.6\underline{k})$

$\underline{F}_{BD}=-F_{BD}\underline{k}$

$\Sigma\underline{F}=0:\ \underline{F}_{AB}+\underline{F}_{BC}+\underline{F}_{BD}+(780\text{ N})\underline{j}=0$

SUBSTITUTING FOR $\underline{F}_{AB}, \underline{F}_{BC}, \underline{F}_{BD}$ AND EQUATING TO ZERO THE COEFFICIENTS OF $\underline{i}$ AND $\underline{k}$:

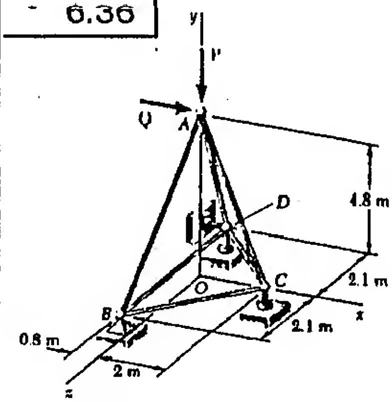
(i) $-130\text{ N}+0.8F_{BC}=0$ $F_{BC}=+162.5\text{ N},\ F_{BC}=162.5\text{ N T}$ ◄

(k) $341.25\text{ N}-0.6F_{BC}-F_{BD}=0$

$F_{BD}=341.25-0.6(162.5)=+243.75\text{ N}$ $F_{BD}=244\text{ N T}$ ◄

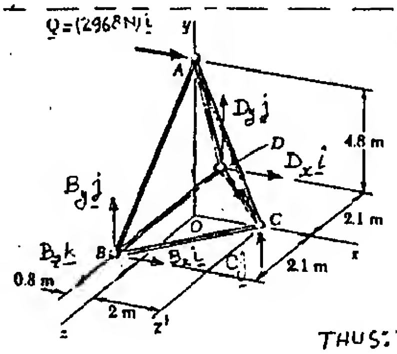
FROM SYMMETRY: $F_{CD}=F_{BC}$ $F_{CD}=162.5\text{ N T}$ ◄

6.36



GIVEN:
TRUSS SHOWN, WITH
$P = 0$
$\underline{Q} = (2968\ N)\underline{i}$

FIND:
FORCE IN EACH MEMBER



FREE BODY: TRUSS

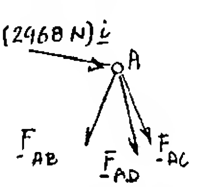
FROM SYMMETRY:
$D_x = B_x$ AND $D_y = B_y$

$\Sigma F_x = 0: 2B_x + 2968N = 0$
$B_x = D_x = -1484\ N$

$\Sigma M_{CZ'} = 0:$
$-2B_y(2.8m) - (2968N)(4.8m) = 0$
$B_y = -2544\ N$

THUS: $\underline{B} = -(1484N)\underline{i} - (2544N)\underline{j}$

FREE BODY: A



$\underline{F}_{AB} = F_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{F_{AB}}{5.30}(-0.8\underline{i} - 4.8\underline{j} + 2.1\underline{k})$

$\underline{F}_{AC} = F_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{F_{AC}}{5.20}(2\underline{i} - 4.8\underline{j})$

$\underline{F}_{AD} = F_{AD}\frac{\overrightarrow{AD}}{AD} = \frac{F_{AD}}{5.30}(-0.8\underline{i} - 4.8\underline{j} - 2.1\underline{k})$

$\Sigma \underline{F} = 0: \underline{F}_{AB} + \underline{F}_{AC} + \underline{F}_{AD} + (2968N)\underline{i} = 0$

SUBSTITUTING FOR F_{AB}, F_{AC}, F_{AD}, AND EQUATING TO ZERO THE COEFFICIENTS OF $\underline{i}, \underline{j}, \underline{k}$:

(i) $-\frac{0.8}{5.30}(F_{AB} + F_{AD}) + \frac{2}{5.20}F_{AC} + 2968\ N = 0$ (1)

(j) $-\frac{4.8}{5.30}(F_{AB} + F_{AD}) - \frac{4.8}{5.20}F_{AC} = 0$ (2)

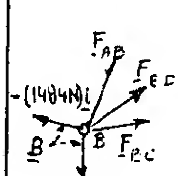
(k) $\frac{2.1}{5.30}(F_{AB} - F_{AD}) = 0$ $F_{AD} = F_{AB}$

MULTIPLY (1) BY -6 AND ADD (2):
$-(16.8/5.20)F_{AC} - 6(2968N) = 0$, $F_{AC} = -5512\ N$, $F_{AC} = 5510\ N\ C$

SUBSTITUTE FOR F_{AC} AND F_{AD} IN (2):
$-(4.8/5.30)2F_{AB} - (4.8/5.20)(-5512N) = 0$, $F_{AB} = +2809\ N$

$F_{AB} = F_{AD} = 2810\ N\ T$

FREE BODY: B



$\underline{F}_{BA} = (2809\ N)\frac{\overrightarrow{BA}}{BA} = (424N)\underline{i} + (2544N)\underline{j} - (1113N)\underline{k}$

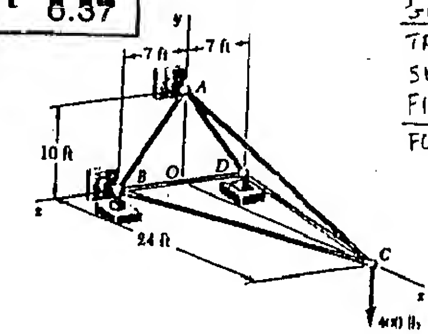
$\underline{F}_{BC} = F_{BC}\left(\frac{2.8\underline{i} - 2.1\underline{k}}{3.5}\right) = F_{BC}(0.8\underline{i} - 0.6\underline{k})$

$\underline{F}_{BD} = -F_{BD}\underline{k}$

$\Sigma \underline{F} = 0: \underline{F}_{AB} + \underline{F}_{BC} + \underline{F}_{BD} - (1484N)\underline{i} - (2544N)\underline{j} = 0$

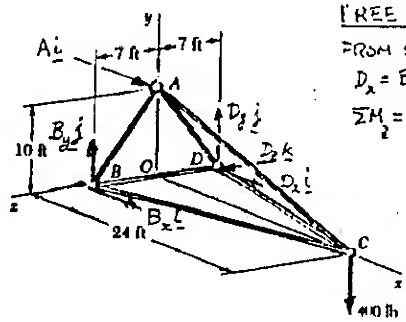
SUBSTITUTING FOR F_{AB}, F_{BC}, F_{BD} AND EQUATING TO ZERO THE COEFFICIENTS OF $\underline{i}$ AND $\underline{k}$:

6.37



GIVEN:
TRUSS AND LOADING SHOWN

FIND:
FORCE IN EACH MEMBER



FREE BODY: TRUSS

FROM SYMMETRY:
$D_x = B_x$ AND $D_y = B_y$

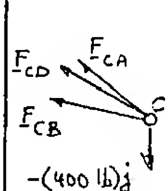
$\Sigma M_z = 0: -A(10ft) - (400lb)(24ft) = 0$
$A = -960\ lb$

$\Sigma F_x = 0: B_x + D_x + A = 0$
$2B_x - 960\ lb = 0$, $B_x = +480\ lb$

$\Sigma F_y = 0: B_y + D_y - 400\ lb = 0$
$2B_y = 400\ lb$
$B_y = +200\ lb$

THUS: $\underline{B} = (480\ lb)\underline{i} + (200\ lb)\underline{j}$

FREE BODY: C



$\underline{F}_{CA} = F_{AC}\frac{\overrightarrow{CA}}{CA} = \frac{F_{AC}}{26}(-24\underline{i} + 10\underline{j})$

$\underline{F}_{CB} = F_{BC}\frac{\overrightarrow{CB}}{CB} = \frac{F_{BC}}{25}(-24\underline{i} + 7\underline{k})$

$\underline{F}_{CD} = F_{CD}\frac{\overrightarrow{CD}}{CD} = \frac{F_{CD}}{25}(-24\underline{i} - 7\underline{k})$

$\Sigma \underline{F} = 0: \underline{F}_{CA} + \underline{F}_{CB} + \underline{F}_{CD} - (400\ lb)\underline{j} = 0$

SUBSTITUTING FOR F_{CA}, F_{CB}, F_{CD}, AND EQUATING TO ZERO THE COEFFICIENTS OF $\underline{i}, \underline{j}, \underline{k}$:

(i) $-\frac{24}{26}F_{AC} - \frac{24}{25}(F_{BC} + F_{CD}) = 0$ (1)

(j) $\frac{10}{26}F_{AC} - 400\ lb = 0$ $F_{AC} = 1040\ lb\ T$

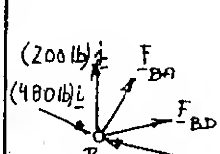
(k) $\frac{7}{25}(F_{BC} - F_{CD}) = 0$ $F_{CD} = F_{BC}$

SUBSTITUTE FOR F_{AC} AND F_{CD} IN EQ.(1):

$-\frac{24}{26}(1040\ lb) - \frac{24}{25}(2F_{BC}) = 0$ $F_{BC} = -500\ lb$

$F_{BC} = F_{CD} = 500\ lb\ C$

FREE BODY: B



$\underline{F}_{BC} = (500\ lb)\frac{\overrightarrow{CB}}{CB} = -(480\ lb)\underline{i} + (140\ lb)\underline{k}$

$\underline{F}_{BA} = F_{AB}\frac{\overrightarrow{BA}}{BA} = \frac{F_{AB}}{12.21}(10\underline{j} - 7\underline{k})$

$\underline{F}_{BD} = -F_{BD}\underline{k}$

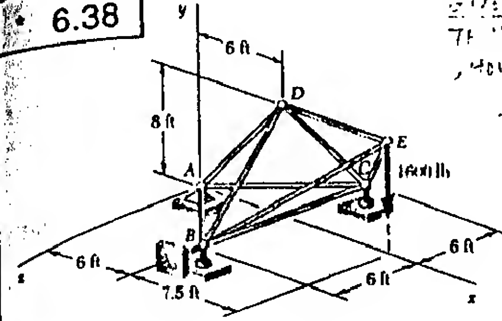
$\Sigma \underline{F} = 0: \underline{F}_{BA} + \underline{F}_{BD} + \underline{F}_{BC} + (480\ lb)\underline{i} + (200\ lb)\underline{j} = 0$

SUBSTITUTING FOR F_{BA}, F_{BD}, F_{BC} AND EQUATING TO ZERO THE COEFFICIENTS OF $\underline{j}$ AND $\underline{k}$:

*6.38

GIVEN: TRUSS AND LOADING SHOWN.

FIND: FORCE IN EACH MEMBER.



FREE BODY: TRUSS

FROM SYMMETRY: $A_z = B_z = 0$

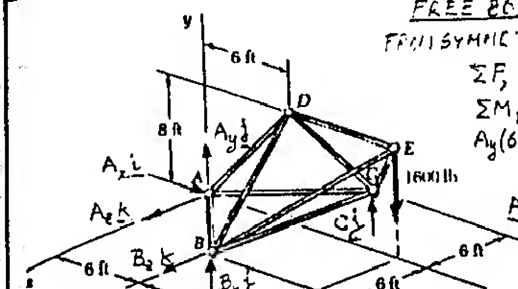
$\Sigma F_x = 0: \quad A_x = 0$

$\Sigma M_{BC} = 0:$

$A_y(6\,ft) + (1800\,lb)(7.5\,ft) = 0$

$A_y = -2000\ lb$

$\underline{A} = -(2000\,lb)\underline{j}$ ◁



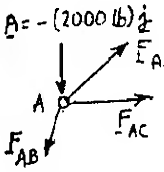
FROM SYMMETRY: $B_y = C$

$\Sigma F_y = 0:$

$2B_y - 2000\,lb - 1600\,lb = 0$

$B_y = 1800\,lb \quad \underline{B} = (1800\,lb)\underline{j}$ ◁

FREE BODY: A



$\Sigma \underline{F} = 0: \ \underline{F}_{AB} + \underline{F}_{AC} + \underline{F}_{AD} - (2000\,lb)\underline{j} = 0$

$F_{AB}\frac{\underline{i}+\underline{k}}{\sqrt{2}} + F_{AC}\frac{\underline{i}-\underline{k}}{\sqrt{2}} + F_{AD}(0.6\underline{i} + 0.8\underline{j}) - (2000\,lb)\underline{j} = 0$

FACTORING $\underline{i}, \underline{j}, \underline{k}$ AND EQUATING THEIR COEFFICIENTS TO ZERO:

$\frac{1}{\sqrt{2}}F_{AB} + \frac{1}{\sqrt{2}}F_{AC} + 0.6\,F_{AD} = 0 \qquad (1)$

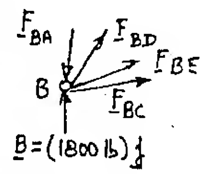
$0.8\,F_{AD} - 2000\,lb = 0 \qquad F_{AD} = 2500\,lb\ T$ ◀

$\frac{1}{\sqrt{2}}F_{AB} - \frac{1}{\sqrt{2}}F_{AC} = 0 \qquad F_{AC} = F_{AB}$

SUBSTITUTE FOR F_{AD} AND F_{AC} INTO (1):

$\frac{2}{\sqrt{2}}F_{AB} + 0.6(2500\,lb) = 0, \ F_{AB} = -1060.7\,lb, \ F_{AB} = F_{AC} = 1061\,lb\,C$ ◀

FREE BODY: B



$\underline{F}_{BA} = F_{AB}\frac{\overrightarrow{BA}}{BA} = +(1060.7\,lb)\frac{\underline{i}+\underline{k}}{\sqrt{2}} = (750\,lb)(\underline{i}+\underline{k})$

$\underline{F}_{BC} = -F_{BC}\underline{k}, \quad \underline{F}_{BD} = F_{BD}(0.8\underline{j} - 0.6\underline{k})$

$\underline{F}_{BE} = F_{BE}\frac{\overrightarrow{BE}}{BE} = \frac{F_{BE}}{12.5}(7.5\underline{i} + 8\underline{j} - 6\underline{k})$

$\Sigma \underline{F} = 0: \ \underline{F}_{BA} + \underline{F}_{BC} + \underline{F}_{BD} + \underline{F}_{BE} + (1800\,lb)\underline{j} = 0$

SUBSTITUTE FOR $\underline{F}_{BA}, \underline{F}_{BC}, \underline{F}_{BD}, \underline{F}_{BE}$ AND EQUATE TO ZERO THE COEFFICIENTS OF $\underline{i}, \underline{j}, \underline{k}$:

(i) $750\,lb + (7.5/12.5)F_{BE} = 0, \ F_{BE} = -1250\,lb, \ F_{BE} = 1250\,lb\,C$ ◀

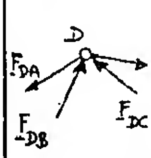
(j) $0.8\,F_{BD} + (8/12.5)(-1250\,lb) + 1800\,lb = 0, \ F_{BD} = 1250\,lb\,C$ ◀

(k) $750\,lb - F_{BC} - 0.6(-1250\,lb) - \frac{6}{12.5}(-1250\,lb) = 0$

$F_{BC} = 2100\,lb\ T$ ◀

FROM SYMMETRY: $F_{CE} = F_{CD} = 1250\,lb\ C$ ◀

FREE BODY: D



$\Sigma \underline{F} = 0: \ \underline{F}_{DA} + \underline{F}_{DB} + \underline{F}_{DC} + F_{DE}\underline{i} = 0$

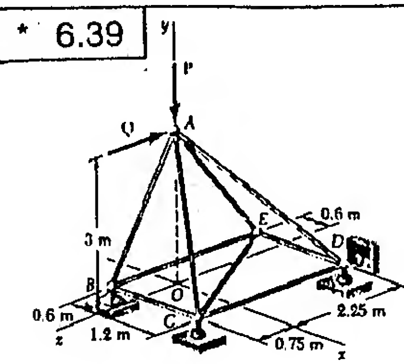
WE NOW SUBSTITUTE FOR $\underline{F}_{DA}, \underline{F}_{DB}, \underline{F}_{DC}$ AND EQUATE TO ZERO THE COEFFICIENT OF $\underline{i}$. ONLY $\underline{F}_{DA}$ CONTAINS $\underline{i}$ AND ITS COEFFICIENT IS $-0.6\,F_{AD} = -0.6(2500\,lb) = -1500\,lb$

(i) $-1500\,lb + F_{DE} = 0 \qquad F_{DE} = 1500\,lb\ T$ ◀

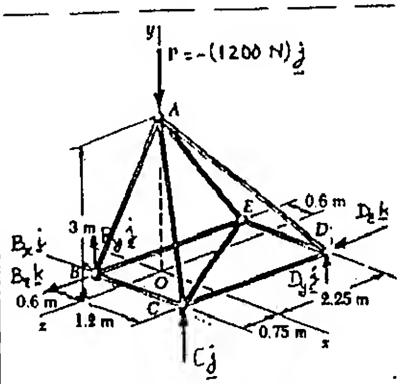
*6.39

GIVEN: TRUSS SHOWN WITH $\underline{P} = -(1200\,N)\underline{j}$, $\underline{Q} = 0$

FIND: FORCE IN EACH MEMBER



FREE BODY: TRUSS



$\Sigma \underline{M}_B = 0:$

$1.8\underline{i} \times C\underline{j}$

$+ (1.8\underline{i} - 3\underline{k}) \times (D_y\underline{j} + D_z\underline{k})$

$+ (0.6\underline{i} - 0.75\underline{k}) \times (-1200\underline{j}) = 0$

$1.8C\underline{k} + 1.8D_y\underline{k} - 1.8D_z\underline{j}$

$+ 3D_y\underline{i} - 720\underline{k} - 900\underline{i} = 0$

EQUATE TO ZERO THE COEFFICIENTS OF $\underline{i}, \underline{j}, \underline{k}$:

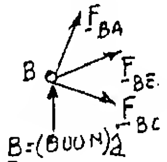
(i) $3D_y - 900 = 0, \ D_y = 300\,N$

(j) $D_z = 0, \ \underline{D} = (300\,N)\underline{j}$ ◁

(k) $1.8C + 1.8(300) - 720 = 0, \ \underline{C} = (100\,N)\underline{j}$ ◁

$\Sigma \underline{F} = 0: \ \underline{B} + 300\underline{j} + 100\underline{j} - 1200\underline{j} = 0 \qquad \underline{B} = (800\,N)\underline{j}$ ◁

FREE BODY: B



$\Sigma \underline{F} = 0: \ \underline{F}_{BA} + \underline{F}_{BC} + \underline{F}_{BE} + (800\,N)\underline{j} = 0$, WITH

$\underline{F}_{BA} = F_{AB}\frac{\overrightarrow{BA}}{BA} = \frac{F_{AB}}{3.15}(0.6\underline{i} + 3\underline{j} - 0.75\underline{k})$

$\underline{F}_{BC} = F_{BC}\underline{i} \qquad \underline{F}_{BE} = -F_{BE}\underline{k}$

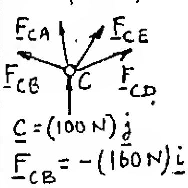
SUBSTITUTE AND EQUATE TO ZERO THE COEFF. OF $\underline{j}, \underline{i}, \underline{k}$:

(j) $(3/3.15)F_{AB} + 800\,N = 0, \ F_{AB} = -840\,N, \quad F_{AB} = 840\,N\,C$ ◀

(i) $(0.6/3.15)(-840\,N) + F_{BC} = 0 \qquad F_{BC} = 160.0\,N\,T$ ◀

(k) $(-0.75/3.15)(-840\,N) - F_{BE} = 0 \qquad F_{BE} = 200\,N\,T$ ◀

FREE BODY: C



$\Sigma \underline{F} = 0: \underline{F}_{CA} + \underline{F}_{CB} + \underline{F}_{CD} + \underline{F}_{CE} + (100\,N)\underline{j} = 0$, WITH

$\underline{F}_{CA} = F_{AC}\frac{\overrightarrow{CA}}{CA} = \frac{F_{AC}}{3.317}(-1.2\underline{i} + 3\underline{j} - 0.75\underline{k})$

$\underline{F}_{CB} = -(160\,N)\underline{i} \qquad \underline{F}_{CD} = -F_{CD}\underline{k} \qquad \underline{F}_{CE} = F_{CE}\frac{\overrightarrow{CE}}{CE} = \frac{F_{CE}}{3.499}(-1.8\underline{i} - 3\underline{k})$

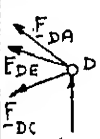
SUBSTITUTE AND EQUATE TO ZERO THE COEFF. OF $\underline{j}, \underline{i}, \underline{k}$:

(j) $(3/3.317)F_{AC} + 100\,N = 0, \ F_{AC} = -110.57\,N, \ F_{AC} = 110.6\,N\,C$ ◀

(i) $-\frac{1.2}{3.317}(-110.57) - 160 - \frac{1.8}{3.499}F_{CE} = 0, \ F_{CE} = -233.3, \ F_{CE} = 233\,N\,C$ ◀

(k) $-\frac{0.75}{3.317}(-110.57) - F_{CD} - \frac{3}{3.499}(-233.3) = 0 \qquad F_{CD} = 225\,N\,T$ ◀

FREE BODY: D



$\Sigma \underline{F} = 0: \underline{F}_{DA} + \underline{F}_{DC} + \underline{F}_{DE} + (300\,N)\underline{j} = 0$, WITH

$\underline{F}_{DA} = F_{AD}\frac{\overrightarrow{DA}}{DA} = \frac{F_{AD}}{3.937}(-1.2\underline{i} + 3\underline{j} + 2.25\underline{k})$

$\underline{F}_{DC} = F_{CD}\underline{k} = (225\,N)\underline{k} \qquad \underline{F}_{DE} = -F_{DE}\underline{i}$

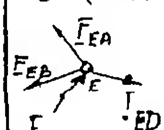
SUBSTITUTE AND EQUATE TO ZERO THE COEFF. OF $\underline{j}, \underline{i}, \underline{k}$:

(j) $(3/3.937)F_{AD} + 300\,N = 0, \ F_{AD} = -393.7\,N, \ F_{AD} = 394\,N\,C$ ◀

(i) $(-1.2/3.937)(-393.7\,N) - F_{DE} = 0 \qquad F_{DE} = 120.0\,N\,T$ ◀

(k) $(2.25/3.937)(-393.7\,N) + 225\,N = 0$ (CHECKS)

FREE BODY: E

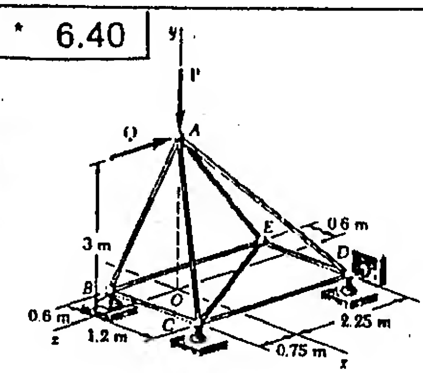


MEMBER AE IS THE ONLY MEMBER AT E WHICH DOES NOT LIE IN THE XZ PLANE. THEREFORE, IT IS A ZERO-FORCE MEMBER.

$F_{AE} = 0$

6.43 | 1.2 kN 1.2 kN 1.2 kN 1.2 kN 1.2 kN | GIVEN: MANSARD ROOF TRUSS AND

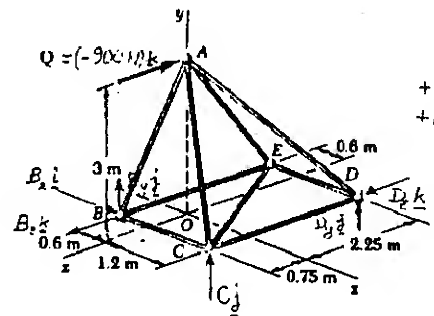
* 6.40



GIVEN:
TRUSS SHOWN WITH
$\underline{P}=0$
$\underline{Q}=(-900\,N)\underline{k}$

FIND:
FORCE IN EACH MEMBER.

FREE BODY: TRUSS



$\Sigma \underline{M}_B=0$: $1.8\underline{i}\times C\underline{j}$
$+(1.8\underline{i}-3\underline{k})\times(D_y\underline{j}+D_z\underline{k})$
$+(0.6\underline{i}+3\underline{j}-0.75\underline{k})\times(-900N)\underline{k}$
$=0$

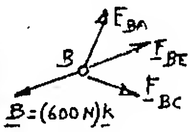
$1.8C\underline{k}+1.8D_y\underline{k}-1.8D_z\underline{j}$
$+3D_y\underline{i}+540\underline{j}-2700\underline{i}=0$

EQUATE TO ZERO THE COEFF. OF $\underline{i},\underline{j},\underline{k}$:
$3D_y-2700=0$ $D_y=900\,N$
$-1.8D_z+540=0$ $D_z=300\,N$
$1.8C+1.8D_y=0$, $C=-D_y=-900\,N$

THUS: $\underline{C}=-(900N)\underline{j}$; $\underline{D}=(900N)\underline{j}+(300N)\underline{k}$ ◁

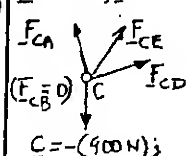
$\Sigma\underline{F}=0$: $\underline{B}-900\underline{j}+900\underline{j}+300\underline{k}-900\underline{k}=0$ $\underline{B}=(600N)\underline{k}$ ◁

FREE BODY: B



SINCE $\underline{B}$ IS ALIGNED WITH MEMBER BE:
$F_{AB}=F_{BC}=0$, $F_{BE}=600\,N\ T$ ◀

FREE BODY: C



$\Sigma\underline{F}=0$: $\underline{F}_{CA}+\underline{F}_{CD}+\underline{F}_{CE}-(900N)\underline{j}=0$, WITH

$\underline{F}_{CA}=F_{AC}\dfrac{\vec{CA}}{CA}=\dfrac{F_{AC}}{3.317}(-1.2\underline{i}+3\underline{j}-0.75\underline{k})$

$\underline{F}_{CD}=-F_{CD}\underline{k}$ $\underline{F}_{CE}=F_{CE}\dfrac{\vec{CE}}{CE}=\dfrac{F_{CE}}{3.499}(-1.8\underline{i}-3\underline{k})$

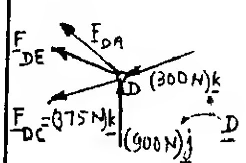
SUBSTITUTE AND EQUATE TO ZERO THE COEFF. OF $\underline{j},\underline{i},\underline{k}$:

($\underline{j}$) $(3/3.317)F_{AC}-900\,N=0$, $F_{AC}=995.1\,N$, $F_{AC}=995\,N\ T$ ◀

($\underline{i}$) $-\dfrac{1.2}{3.317}(995.1)-\dfrac{1.8}{3.499}F_{CE}=0$, $F_{CE}=-699.8\,N$, $F_{CE}=700\,N\ C$ ◀

($\underline{k}$) $-\dfrac{0.75}{3.317}(995.1)-F_{CD}-\dfrac{3}{3.499}(-699.8)=0$ $F_{CD}=375\,N\ T$ ◀

FREE BODY: D



$\Sigma\underline{F}=0$: $\underline{F}_{DA}+\underline{F}_{DE}+(375N)\underline{k}+(900N)\underline{j}+(300N)\underline{k}=0$

WITH $\underline{F}_{DA}=F_{AD}\dfrac{\vec{DA}}{DA}=\dfrac{F_{AD}}{3.937}(-1.2\underline{i}+3\underline{j}+2.25\underline{k})$

AND $\underline{F}_{DE}=-F_{DE}\underline{i}$

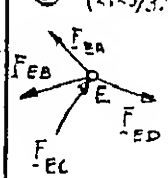
SUBSTITUTE AND EQUATE TO ZERO THE COEFF. OF $\underline{j},\underline{i},\underline{k}$:

($\underline{j}$) $(3/3.937)F_{AD}+900N=0$, $F_{AD}=-1181.1\,N$, $F_{AD}=1181\,N\ C$ ◀

($\underline{i}$) $-(1.2/3.937)(-1181.1N)-F_{DE}=0$ $F_{DE}=360\,N\ T$ ◀

($\underline{k}$) $(2.25/3.937)(-1181.1N+375N+300N=0$ (CHECKS)

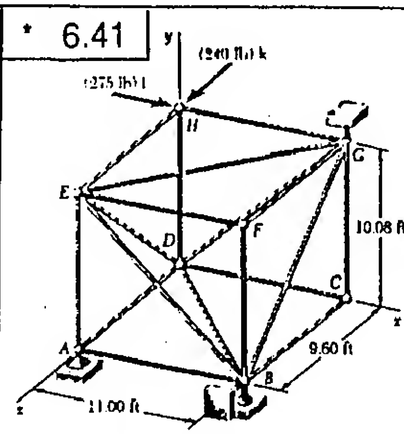
FREE BODY: E



MEMBER AE IS THE ONLY MEMBER AT E WHICH DOES NOT LIE IN THE XZ PLANE. THEREFORE, IT IS A ZERO-FORCE MEMBER.

$F_{AE}=0$ ◀

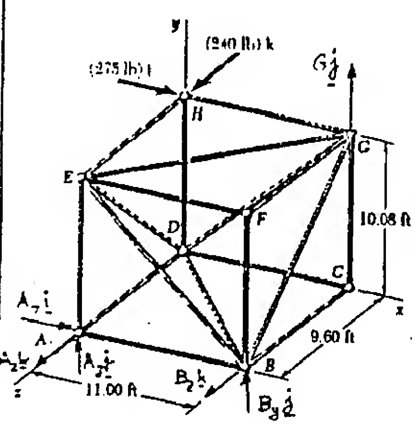
* 6.41



GIVEN:
TRUSS AND LOADING SHOWN.
(a) CHECK THAT TRUSS IS A SIMPLE TRUSS, COMPLETELY CONSTRAINED, AND REACTIONS STATICALLY DETERMINATE.
(b) FIND:
FORCE IN EACH OF THE SIX MEMBERS JOINED AT E.

(a) CHECK SIMPLE TRUSS. (1) START WITH TETRAHEDRON BEFG
(2) ADD MEMBERS BD, ED, GD JOINING AT D.
(3) ADD MEMBERS BA, DA, EA JOINING AT A.
(4) ADD MEMBERS DH, EH, GH JOINING AT H.
(5) ADD MEMBERS BC, DC, GC JOINING AT C
TRUSS HAS BEEN COMPLETED: IT IS A SIMPLE TRUSS

FREE BODY: TRUSS



CHECK CONSTRAINTS AND REACTIONS:
SIX UNKNOWN REACTIONS - OK - MOREOVER SUPPORTS AT A AND B CONSTRAIN TRUSS TO ROTATE ABOUT AB AND SUPPORT AT G PREVENTS SUCH A ROTATION. THUS, TRUSS IS COMPLETELY CONSTRAINED AND REACTIONS ARE STATICALLY DETERMINATE

DETERMINATION OF REACTIONS:

$\Sigma\underline{M}_A=0$: $11\underline{i}\times(B_y\underline{j}+B_z\underline{k})+(11\underline{i}-9.6\underline{k})\times G\underline{j}+(10.08\underline{j}-9.6\underline{k})\times(275\underline{i}+240\underline{k})=0$

$11B_y\underline{k}-11B_z\underline{j}+11G\underline{k}+9.6G\underline{i}-(10.08)(275)\underline{k}+(10.08)(240)\underline{i}-(9.6)(275)\underline{j}=0$

EQUATE TO ZERO THE COEFF. OF $\underline{i},\underline{j},\underline{k}$:

($\underline{i}$) $9.6G+(10.08)(240)=0$ $G=-252\,lb$ $\underline{G}=(-252\,lb)\underline{j}$ ◁

($\underline{j}$) $-11B_z-(9.6)(275)=0$ $B_z=-240\,lb$

($\underline{k}$) $11B_y+11(-252)-(10.08)(275)=0$, $B_y=504\,lb$

$\underline{B}=(504\,lb)\underline{j}-(240\,lb)\underline{k}$ ◁

$\Sigma\underline{F}=0$: $\underline{A}+(504\,lb)\underline{j}-(240\,lb)\underline{k}-(252\,lb)\underline{j}+(275\,lb)\underline{i}+(240\,lb)\underline{k}=0$

$\underline{A}=-(275\,lb)\underline{i}-(252\,lb)\underline{j}$ ◁

ZERO-FORCE MEMBERS

THE DETERMINATION OF THESE MEMBERS WILL FACILITATE OUR SOLUTION

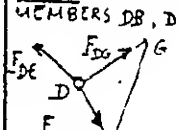
FB: C. WRITING $\Sigma F_x=0, \Sigma F_y=0, \Sigma F_z=0$ YIELDS $F_{BC}=F_{CD}=F_{CG}=0$ ◁

FB: F. WRITING $\Sigma F_x=0, \Sigma F_y=0, \Sigma F_z=0$ YIELDS $F_{BF}=F_{EF}=F_{FG}=0$ ◁

FB: A: SINCE $A_z=0$, WRITING $\Sigma F_z=0$ YIELDS $F_{AD}=0$ ◁

FB: H: WRITING $\Sigma F_y=0$ YIELDS $F_{DH}=0$ ◁

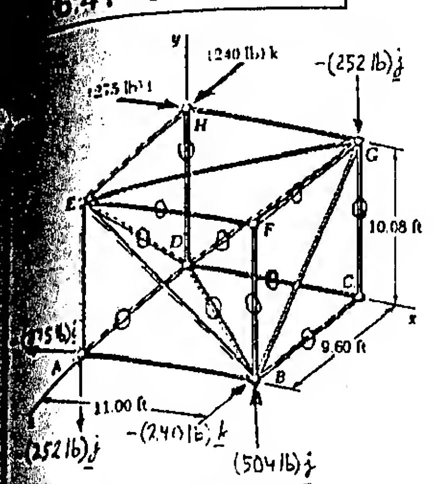
FB: D: SINCE $F_{AD}=F_{CD}=F_{DH}=0$, WE NEED CONSIDER ONLY MEMBERS DB, DE, AND DG.



SINCE F_{DE} IS THE ONLY FORCE NOT CONTAINED IN PLANE BDG, IT MUST BE ZERO. SIMILAR REASONINGS SHOW THAT THE OTHER TWO FORCES ARE ALSO ZERO $F_{BD}=F_{DE}=F_{DG}=0$ ◁

(CONTINUED)

6.41 CONTINUED



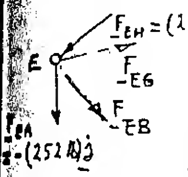
THE RESULTS OBTAINED FOR THE REACTIONS AT THE SUPPORTS AND FOR THE ZERO-FORCE MEMBERS ARE SHOWN ON THE ADJACENT FIGURE. ZERO-FORCE MEMBERS ARE INDICATED BY A ZERO ("0").

(b) FORCE IN EACH OF THE MEMBERS JOINED AT E

WE ALREADY FOUND THAT $F_{DE} = F_{EF} = 0$

FREE BODY: A $\quad \Sigma F_y = 0$ YIELDS $F_{AE} = 252$ lb T

FREE BODY: H $\quad \Sigma F_z = 0$ YIELDS $F_{EH} = 240$ lb C



FREE BODY: E

$\Sigma F = 0$: $F_{EB} + F_{EG} + (240\text{ lb})k - (252\text{ lb})j = 0$

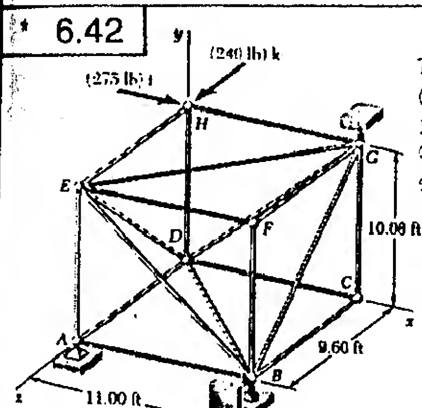
$\frac{F_{BE}}{14.92}(11i - 10.08j) + \frac{F_{EG}}{14.6}(11i - 9.6k) + 240k - 252j = 0$

EQUATE TO ZERO THE COEFF. OF j AND k:

(j) $\quad -(10.08/14.92)F_{BE} - 252 = 0 \qquad F_{BE} = 373$ lb C

(k) $\quad -(9.6/14.6)F_{EG} + 240 = 0 \qquad F_{EG} = 365$ lb T

* 6.42



GIVEN: TRUSS AND LOADING SHOWN.

(a) CHECK THAT TRUSS IS SIMPLE TRUSS, COMPLETELY CONSTRAINED, AND REACTIONS STATICALLY DETERMINATE

(b) FIND: FORCE IN EACH OF THE SIX MEMBERS JOINED AT G.

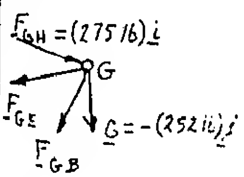
SEE SOLUTION OF PROB. 6.41 FOR PART (a) AND FOR REACTIONS AND ZERO-FORCE MEMBERS

(b) FORCE IN EACH OF THE MEMBERS JOINED AT G.

WE ALREADY KNOW (SEE FIG. AT TOP OF PAGE) THAT

$F_{CG} = F_{DG} = F_{FG} = 0$

FREE BODY: H $\quad \Sigma F_x = 0$ YIELDS $F_{GH} = 275$ lb C



FREE BODY: G

$\Sigma F = 0$: $F_{GB} + F_{GE} + (275\text{ lb})i - (252\text{ lb})j = 0$

$\frac{F_{BG}}{13.92}(-10.08j + 9.6k) + \frac{F_{EG}}{14.6}(-11i + 9.6k) + 275i - 252j = 0$

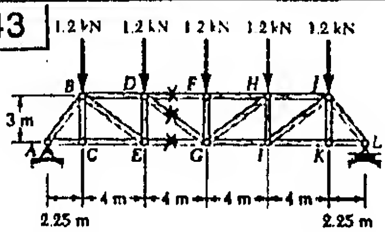
EQUATE TO ZERO THE COEFF. OF i, j, k:

(i) $\quad -(11/14.6)F_{EG} + 275 = 0 \qquad F_{EG} = 365$ lb T

(j) $\quad -(10.08/13.92)F_{BG} - 252 = 0 \qquad F_{BG} = 348$ lb C

(k) $\quad (9.6/13.92)(-348) + (9.6/14.6)(365) = 0 \qquad$ (CHECKS)

6.43



GIVEN: MANSARD ROOF TRUSS AND LOADING SHOWN.

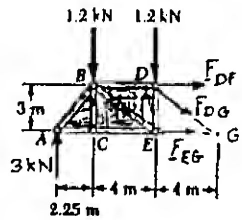
FIND: FORCE IN MEMBERS DF, DG, AND EG.

REACTIONS AT SUPPORTS

BECAUSE OF THE SYMMETRY OF THE TRUSS AND LOADING,

$A_x = 0$, $A_y = L = \frac{1}{2}$(TOTAL LOAD) $= \frac{1}{2}(6\text{ kN}) \qquad A = L = 3$ kN ↑

WE PASS A SECTION THROUGH DF, DG, AND EG AND USE THE FREE BODY SHOWN:



$+\circlearrowleft \Sigma M_G = 0$:

$(1.2\text{ kN})(8\text{ m}) + (1.2\text{ kN})(4\text{ m}) - (3\text{ kN})(10.25\text{ m}) - F_{DF}(3\text{ m}) = 0$

$F_{DF} = -5.45$ kN, $\quad F_{DF} = 5.45$ kN C

$+\uparrow \Sigma F_y = 0$:

$3\text{ kN} - 1.2\text{ kN} - 1.2\text{ kN} - \frac{3}{5}F_{DG} = 0$

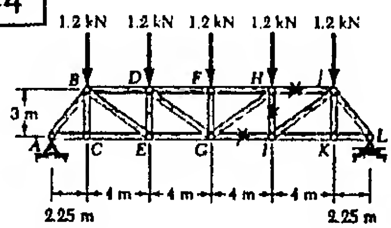
$F_{DG} = +1.00$ kN, $\quad F_{DG} = 1.00$ kN T

$+\circlearrowleft \Sigma M_D = 0$:

$(1.2\text{ kN})(4\text{ m}) - (3\text{ kN})(6.25\text{ m}) + F_{EG}(3\text{ m}) = 0$

$F_{EG} = +4.65$ kN, $\quad F_{EG} = 4.65$ kN T

6.44



GIVEN: MANSARD ROOF TRUSS AND LOADING SHOWN

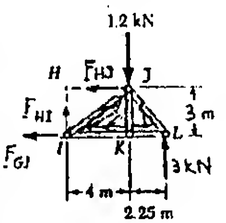
FIND: FORCE IN MEMBERS GI, HI, AND HJ

REACTIONS AT SUPPORTS

BECAUSE OF THE SYMMETRY OF THE TRUSS AND LOADING,

$A_x = 0$, $A_y = L = \frac{1}{2}$(TOTAL LOAD) $= \frac{1}{2}(6\text{ kN}) \qquad A = L = 3$ kN ↑

WE PASS A SECTION THROUGH GI, HI, AND HJ AND USE THE FREE BODY SHOWN



$+\circlearrowleft \Sigma M_H = 0$:

$(3\text{ kN})(6.25\text{ m}) - (1.2\text{ kN})(4\text{ m}) - F_{GI}(3\text{ m}) = 0$

$F_{GI} = +4.65$ kN, $\quad F_{GI} = 4.65$ kN T

$+\uparrow \Sigma F_y = 0$:

$F_{HI} - 1.2\text{ kN} + 3\text{ kN} = 0$

$F_{HI} = -1.80$ kN $\qquad F_{HI} = 1.80$ kN C

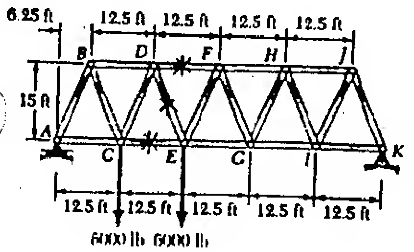
$+\circlearrowleft \Sigma M_I = 0$:

$F_{HJ}(3\text{ m}) - (1.2\text{ kN})(4\text{ m}) + (3\text{ kN})(6.25\text{ m}) = 0$

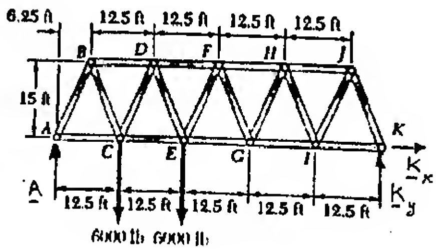
$F_{HJ} = -4.65$ kN $\qquad F_{HJ} = 4.65$ kN C

CHECK: $\xrightarrow{+} \Sigma F_x = 4.65\text{ kN} - 4.65\text{ kN} = 0$

6.45



GIVEN: WARREN BRIDGE TRUSS AND LOADING SHOWN.
FIND: FORCE IN MEMBERS CE, DE, AND DF.



FREE BODY: TRUSS

$\xrightarrow{+} \Sigma F_x = 0: \quad K_x = 0$

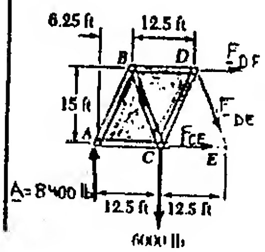
$+\circlearrowleft \Sigma M_A = 0: \quad K_y(62.5\,ft) - (6000\,lb)(12.5\,ft) - (6000\,lb)(25\,ft) = 0$

$K = K_y = 3600\,lb\uparrow$ ◁

$+\uparrow \Sigma F_y = 0: \quad A + 3600\,lb - 6000\,lb - 6000\,lb = 0$

$A = 8400\,lb\uparrow$ ◁

WE PASS A SECTION THROUGH MEMBERS CE, DE, AND DF. AND USE THE FREE BODY SHOWN.



$+\circlearrowleft \Sigma M_D = 0:$

$F_{CE}(15\,ft) - (8400\,lb)(18.75\,ft) + (6000\,lb)(6.25\,ft) = 0$

$F_{CE} = +8000\,lb \qquad F_{CE} = 8000\,lb\ T$ ◀

$+\uparrow \Sigma F_y = 0:$

$8400\,lb - 6000\,lb - \frac{15}{16.25} F_{DE} = 0$

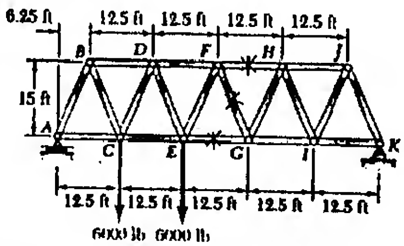
$F_{DE} = +2600\,lb \qquad F_{DE} = 2600\,lb\ T$ ◀

$+\circlearrowleft \Sigma M_E = 0:$

$6000\,lb(12.5\,ft) - (8400\,lb)(25\,ft) - F_{DF}(15\,ft) = 0$

$F_{DF} = -9000\,lb \qquad F_{DF} = 9000\,lb\ C$ ◀

6.46

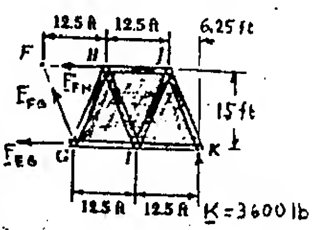


GIVEN: WARREN BRIDGE TRUSS AND LOADING SHOWN.
FIND: FORCE IN MEMBERS EG, FG, AND FH.

SEE SOLUTION OF PROB. 6.45 FOR FREE-BODY DIAGRAM OF TRUSS AND DETERMINATION OF REACTIONS.

$A = 8400\,lb, \; K = 3600\,lb$ ◁

WE PASS A SECTION THROUGH MEMBERS EG, FG, AND FH. AND USE THE FREE BODY SHOWN.



$+\circlearrowleft \Sigma M_F = 0:$

$(3600\,lb)(31.25\,ft) - F_{EG}(15\,ft) = 0$

$F_{EG} = +7500\,lb, \qquad F_{EG} = 7500\,lb\ T$ ◀

$+\uparrow \Sigma F_y = 0: \; \frac{15}{16.25} F_{FG} + 3600\,lb = 0$

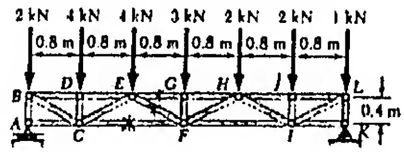
$F_{FG} = -3900\,lb, \qquad F_{FG} = 3900\,lb\ C$ ◀

$+\circlearrowleft \Sigma M_G = 0:$

$F_{FH}(15\,ft) + (3600\,lb)(25\,ft) = 0$

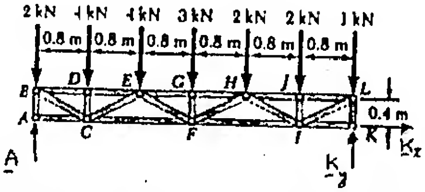
$F_{FH} = -6000\,lb, \qquad F_{FH} = 6000\,lb\ C$ ◀

6.47



GIVEN: FLOOR TRUSS WITH LOADING SHOWN.
FIND: FORCE IN MEMBERS CF, EF, AND EG.

FREE BODY: TRUSS



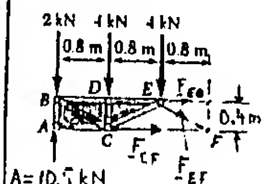
$\xrightarrow{+} \Sigma F_x = 0: K_x = 0$

$+\circlearrowleft \Sigma M_A = 0: \; K_y(4.8\,m) - (4\,kN)(0.8\,m) - (4\,kN)(1.6\,m) - (3\,kN)(2.4\,m) - (2\,kN)(3.2\,m) - (2\,kN)(4\,m) - (1\,kN)(4.8\,m) = 0$

$K_y = 7.5\,kN \qquad$ THUS: $K = 7.5\,kN\uparrow$ ◁

$+\uparrow \Sigma F_y = 0: \; A + 7.5\,kN - 18\,kN = 0 \qquad A = 10.5\,kN \qquad A = 10.5\,kN\uparrow$ ◁

WE PASS A SECTION THROUGH MEMBERS CF, EF, AND EG AND USE THE FREE BODY SHOWN.



$+\circlearrowleft \Sigma M_E = 0:$

$F_{CF}(0.4\,m) - (10.5\,kN)(1.6\,m) + (2\,kN)(1.6\,m) + (4\,kN)(0.8\,m) = 0$

$F_{CF} = +26.0\,kN \qquad F_{CF} = 26.0\,kN\ T$ ◀

$+\uparrow \Sigma F_y = 0:$

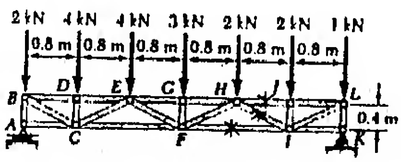
$10.5\,kN - 2\,kN - 4\,kN - 4\,kN - \frac{1}{\sqrt{5}} F_{EF} = 0$

$F_{EF} = +1.118\,kN \qquad F_{EF} = 1.118\,kN\ T$ ◀

$+\circlearrowleft \Sigma M_F = 0: \; (2\,kN)(2.4\,m) + (4\,kN)(1.6\,m) + (4\,kN)(0.8\,m) - (10.5\,kN)(2.4\,m) - F_{EG}(0.4\,m) = 0$

$F_{EG} = -27.0\,kN \qquad F_{EG} = 27.0\,kN\ C$ ◀

6.48

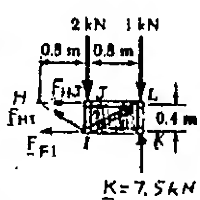


GIVEN: FLOOR TRUSS WITH LOADING SHOWN.
FIND: FORCE IN MEMBERS FI, HI, AND HJ.

SEE SOLUTION OF PROB. 6.47 FOR FREE-BODY DIAGRAM OF TRUSS AND DETERMINATION OF REACTIONS.

$A = 10.5\,kN\uparrow, \; K = 7.5\,kN\uparrow$ ◁

WE PASS A SECTION THROUGH MEMBERS FI, HI, AND HJ. AND USE THE FREE BODY SHOWN.



$+\circlearrowleft \Sigma M_H = 0:$

$(7.5\,kN)(1.6\,m) - (2\,kN)(0.8\,m) - (1\,kN)(1.6\,m) - F_{FI}(0.4\,m) = 0$

$F_{FI} = +22.0\,kN \qquad F_{FI} = 22.0\,kN\ T$ ◀

$+\uparrow \Sigma F_y = 0:$

$\frac{1}{\sqrt{5}} F_{HI} - 2\,kN - 1\,kN + 7.5\,kN = 0$

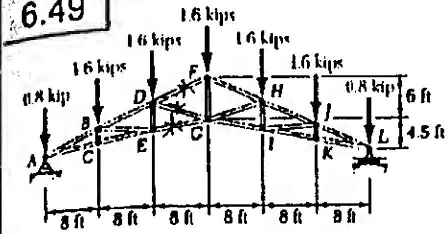
$F_{HI} = -10.06\,kN \qquad F_{HI} = 10.06\,kN\ C$ ◀

$+\circlearrowleft \Sigma M_I = 0:$

$F_{HJ}(0.4\,m) + (7.5\,kN)(0.8\,m) - (1\,kN)(0.8\,m) = 0$

$F_{HJ} = -13.00\,kN \qquad F_{HJ} = 13.00\,kN\ C$ ◀

6.49



GIVEN: HOWE SCISSORS ROOF TRUSS WITH LOADING SHOWN

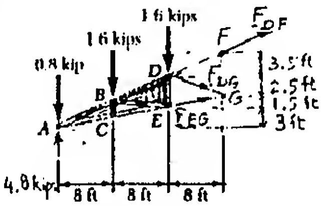
FIND: FORCE IN MEMBERS DF, DG, AND EG.

REACTIONS AT SUPPORTS:
BECAUSE OF SYMMETRY OF LOADING,

$A_x=0,\ A_y=L=\frac{1}{2}(\text{TOTAL LOAD})=\frac{1}{2}(9.60\text{ kips})=4.80\text{ kips}$

$\underline{A}=\underline{L}=4.80\text{ kips}\uparrow$ ◁

WE PASS A SECTION THROUGH MEMBERS DF, DG, AND EG, AND USE THE FREE BODY SHOWN.



WE SLIDE F_{DF} TO APPLY IT AT F.

$+\circlearrowleft \Sigma M_G=0:\ (0.8\text{ kip})(24\text{ ft})+(1.6\text{ kips})(16\text{ ft})+(1.6\text{ kips})(8\text{ ft})-(4.8\text{ kips})(24\text{ ft})-\frac{8F_{DF}}{\sqrt{8^2+3.5^2}}(6\text{ ft})=0$

$F_{DF}=-10.48$ kips, $F_{DF}=10.48$ kips C ◀

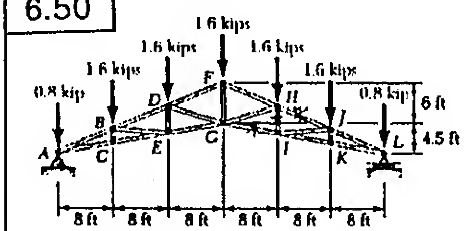
$+\circlearrowleft \Sigma M_A=0:\ -(1.6\text{ kips})(8\text{ ft})-(1.6\text{ kips})(16\text{ ft})-\frac{2.5F_{DG}}{\sqrt{8^2+2.5^2}}(16\text{ ft})-\frac{8F_{DG}}{\sqrt{8^2+2.5^2}}(7\text{ ft})=0$

$F_{DG}=-3.35$ kips, $F_{DG}=3.35$ kips C ◀

$+\circlearrowleft \Sigma M_D=0:\ (0.8\text{ kips})(16\text{ ft})+(1.6\text{ kips})(8\text{ ft})-(4.8\text{ kips})(16\text{ ft})+\frac{8F_{EG}}{\sqrt{8^2+1.5^2}}(4\text{ ft})=0$

$F_{EG}=+13.02$ kips, $F_{EG}=13.02$ kips T ◀

6.50



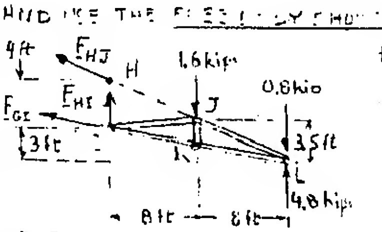
GIVEN: HOWE SCISSORS ROOF TRUSS WITH LOADING SHOWN.

FIND: FORCE IN MEMBERS GI, HI, AND HJ.

REACTIONS AT SUPPORTS: BECAUSE OF SYMMETRY OF LOADING:

$A_x=0,\ A_y=L=\frac{1}{2}(\text{TOTAL LOAD})=\frac{1}{2}(9.60\text{ kips})=4.80\text{ kips}$, $\underline{A}=\underline{L}=4.80\text{ kips}\uparrow$ ◁

WE PASS A SECTION THROUGH MEMBERS GI, HI, AND HJ, AND USE THE FREE BODY SHOWN.



$+\circlearrowleft \Sigma M_H=0:\ -\frac{16F_{GI}}{\sqrt{16^2+3^2}}(4\text{ ft})+(4.8\text{ kips})(16\text{ ft})-(0.8\text{ kip})(16\text{ ft})-(1.6\text{ kips})(8\text{ ft})=0$

$F_{GI}=+13.02$ kips, $F_{GI}=13.02$ kips T ◀

$+\circlearrowleft \Sigma M_L=0:$
$(1.6\text{ kips})(8\text{ ft})-F_{HI}(16\text{ ft})=0,\ F_{HI}=+0.800$ kips, $F_{HI}=0.800$ kips T ◀

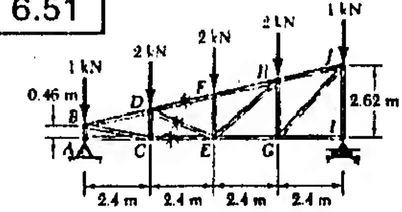
WE SLIDE F_{HJ} TO APPLY IT AT H.

$+\circlearrowleft \Sigma M_I=0:$

$\frac{8F_{HJ}}{\sqrt{8^2+3.5^2}}(4\text{ ft})+(4.8\text{ kips})(16\text{ ft})-(1.6\text{ kips})(8\text{ ft})-(0.8\text{ kip})(16\text{ ft})=0$

$F_{HJ}=-13.17$ kips, $F_{HJ}=13.17$ kips C ◀

6.51

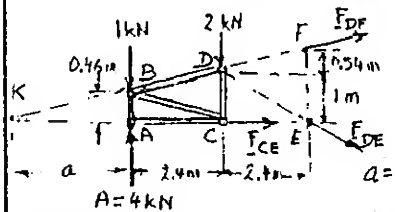


GIVEN: PITCHED FLAT ROOF TRUSS WITH LOADING SHOWN.

FIND: FORCE IN MEMBERS CE, DE, AND DF.

REACTIONS AT SUPPORTS: BECAUSE OF THE SYMMETRY OF THE LOADING, $A_x=0,\ A_y=I=\frac{1}{2}(\text{TOTAL LOAD})=\frac{1}{2}(8\text{ kN})$ $\underline{A}=\underline{I}=4\text{ kN}\uparrow$ ◁

WE PASS A SECTION THROUGH MEMBERS CE, DE, AND DF, AND USE THE FREE BODY SHOWN. (WE MOVED F_{DE} TO E AND F_{DF} TO F.)



SLOPE BJ $=\frac{2.16\text{ m}}{9.6\text{ m}}=\frac{9}{40}$

SLOPE DE $=\frac{-1\text{ m}}{2.4\text{ m}}=\frac{-5}{12}$

$a=\frac{0.46\text{ m}}{\text{SLOPE BJ}}=\frac{0.46\text{ m}}{9/40}=2.0444$ m

$+\circlearrowleft \Sigma M_D=0:\ F_{CE}(1\text{ m})+(1\text{ kN})(2.4\text{ m})-(4\text{ kN})(2.4\text{ m})=0$

$F_{CE}=+7.20$ kN $\qquad F_{CE}=7.20$ kN T ◀

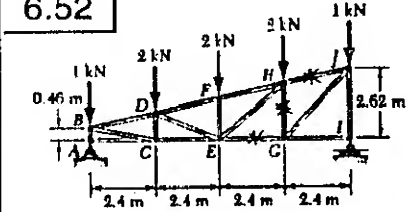
$+\circlearrowleft \Sigma M_K=0:\ (4\text{ kN})(2.0444\text{ m})-(1\text{ kN})(2.0444\text{ m})-(2\text{ kN})(4.4444\text{ m})-\left(\frac{5}{13}F_{DE}\right)(6.8444\text{ m})=0$

$F_{DE}=-1.047$ kN $\qquad F_{DE}=1.047$ kN C ◀

$+\circlearrowleft \Sigma M_E=0:\ (1\text{ kN})(4.8\text{ m})+(2\text{ kN})(2.4\text{ m})-(4\text{ kN})(4.8\text{ m})-\left(\frac{40}{41}F_{DF}\right)(1.54\text{ m})=0$

$F_{DF}=-6.39$ kN $\qquad F_{DF}=6.39$ kN C ◀

6.52

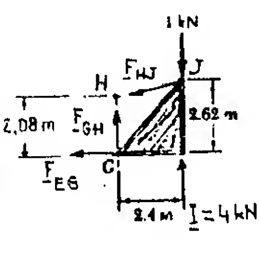


GIVEN: PITCHED FLAT ROOF TRUSS WITH LOADING SHOWN.

FIND: FORCE IN MEMBERS EG, GH, AND HJ.

REACTIONS AT SUPPORTS: BECAUSE OF THE SYMMETRY OF THE LOADING, $A_x=0,\ A_y=I=\frac{1}{2}(\text{TOTAL LOAD})=\frac{1}{2}(8\text{ kN})$ $\underline{A}=\underline{I}=4\text{ kN}\uparrow$ ◁

WE PASS A SECTION THROUGH MEMBERS EG, GH, AND HJ, AND USE THE FREE BODY SHOWN.



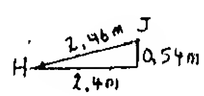
$+\circlearrowleft \Sigma M_H=0:$
$(4\text{ kN})(2.4\text{ m})-(1\text{ kN})(2.4\text{ m})-F_{EG}(2.08\text{ m})=0$

$F_{EG}=+3.4615$ kN $\qquad F_{EG}=3.46$ kN T ◀

$+\circlearrowleft \Sigma M_J=0:$
$-F_{GH}(2.4\text{ m})-F_{EG}(2.62\text{ m})=0$

$F_{GH}=-\frac{2.62}{2.4}(3.4615\text{ kN})$

$F_{GH}=-3.7788$ kN $\qquad F_{GH}=3.78$ kN C ◀



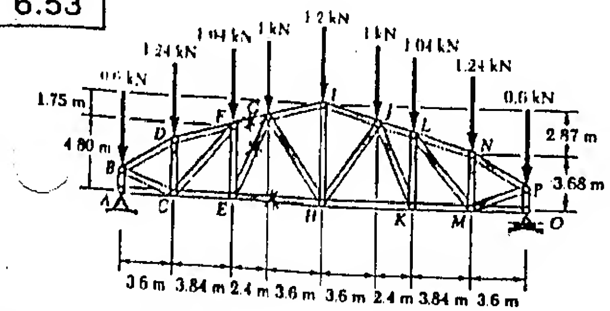
$\xrightarrow{+}\Sigma F_x=0:$

$-F_{EG}-\frac{2.4}{2.46}F_{HJ}=0$

$F_{HJ}=-\frac{2.46}{2.4}F_{EG}=-\frac{2.46}{2.4}(3.4615\text{ kN})$

$F_{HJ}=-3.548$ kN $\qquad F_{HJ}=3.55$ kN C ◀

6.53

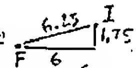


GIVEN: MARKET ROOF TRUSS WITH LOADING SHOWN.
FIND: FORCE IN MEMBERS FG, EG, AND EH.

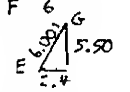
REACTIONS AT SUPPORTS. BECAUSE OF THE SYMMETRY OF THE LOADING, $A_x = 0$, $A_y = O = \frac{1}{2}$ LOAD. $A = O = 4.48\ \text{kN}\uparrow$

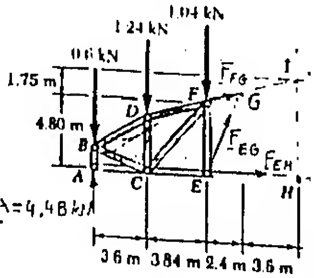
WE PASS A SECTION THROUGH MEMBERS FG, EG, AND EH, AND USE THE FREE BODY SHOWN.

SLOPE FG = SLOPE FI = $\frac{1.75\ \text{m}}{6\ \text{m}}$



SLOPE EG = $\frac{5.50\ \text{m}}{2.4\ \text{m}}$





$+\circlearrowleft \Sigma M_F = 0: (0.6\ \text{kN})(7.44\ \text{m}) + (1.24\ \text{kN})(3.84\ \text{m}) - (4.48\ \text{kN})(7.44\ \text{m}) - \left(\frac{6}{6.25} F_{FG}\right)(4.80\ \text{m}) = 0$

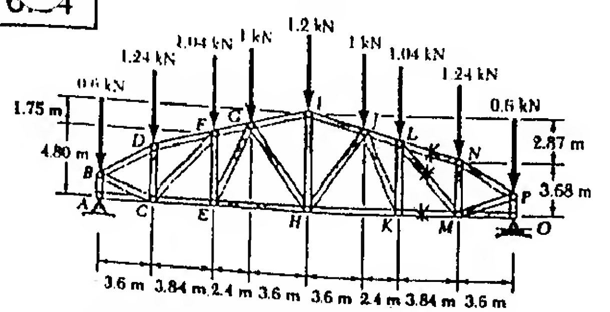
$F_{FG} = -5.231\ \text{kN}$, $\quad F_{FG} = 5.23\ \text{kN}\ C$

$+\circlearrowleft \Sigma M_G = 0: F_{EH}(5.50\ \text{m}) + (0.6\ \text{kN})(9.84\ \text{m}) + (1.24\ \text{kN})(6.24\ \text{m}) + (1.04\ \text{kN})(2.4\ \text{m}) - (4.48\ \text{kN})(9.84\ \text{m}) = 0$, $\quad F_{EH} = 5.08\ \text{kN}\ T$

$+\uparrow \Sigma F_y = 0: \frac{5.50}{6.001} F_{EG} + \frac{1.75}{6.25}(-5.231\ \text{kN}) + 4.48\ \text{kN} - 0.6\ \text{kN} - 1.24\ \text{kN} - 1.04\ \text{kN} = 0$

$F_{EG} = -0.1476\ \text{kN}$ $\quad F_{EG} = 0.1476\ \text{kN}\ C$

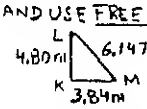
6.54

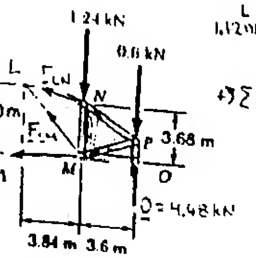


GIVEN: MARKET ROOF TRUSS WITH LOADING SHOWN
FIND: FORCE IN MEMBERS KM, LM, AND LN.

BECAUSE OF SYMMETRY OF LOADING, $O = \frac{1}{2}$(LOAD), $O = 4.48\ \text{kN}\uparrow$

WE PASS A SECTION THROUGH KM, LM, LN, AND USE FREE BODY SHOWN





$+\circlearrowleft \Sigma M_M = 0: \left(\frac{3.84}{4} F_{LN}\right)(3.68\ \text{m}) + (4.48\ \text{kN} - 0.6\ \text{kN})(3.6\ \text{m}) = 0$

$F_{LN} = -3.954\ \text{kN}$, $\quad F_{LN} = 3.95\ \text{kN}\ C$

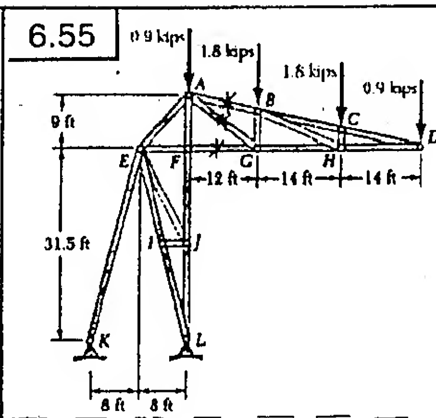
$+\circlearrowleft \Sigma M_L = 0: -F_{KM}(4.80\ \text{m}) - (1.24\ \text{kN})(3.84\ \text{m}) + (4.48\ \text{kN} - 0.6\ \text{kN})(7.44\ \text{m}) = 0$

$F_{KM} = +5.02\ \text{kN}$, $\quad F_{KM} = 5.02\ \text{kN}\ T$

$+\uparrow \Sigma F_y = 0: \frac{4.80}{6.147} F_{LM} + \frac{1.12}{4}(-3.954\ \text{kN}) - 1.24\ \text{kN} - 0.6\ \text{kN} + 4.48\ \text{kN} = 0$

$F_{LM} = -1.963\ \text{kN}$ $\quad F_{LM} = 1.963\ \text{kN}\ C$

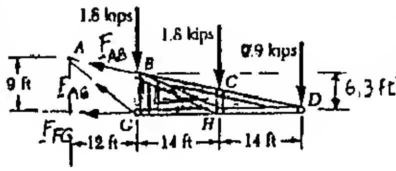
6.55

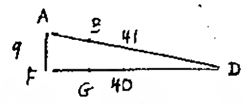


GIVEN: STADIUM ROOF TRUSS WITH LOADING SHOWN

FIND: FORCE IN MEMBERS AB, AG, AND FG.

WE PASS A SECTION THROUGH MEMBERS AB, AG, AND FG. AND USE THE FREE BODY SHOWN.





$+\circlearrowleft \Sigma M_G = 0: \left(\frac{40}{41} F_{AB}\right)(6.3\ \text{ft}) - (1.8\ \text{kips})(14\ \text{ft}) - (0.9\ \text{kips})(28\ \text{ft}) = 0$

$F_{AB} = +8.20\ \text{kips}$ $\quad F_{AB} = 8.20\ \text{kips}\ T$

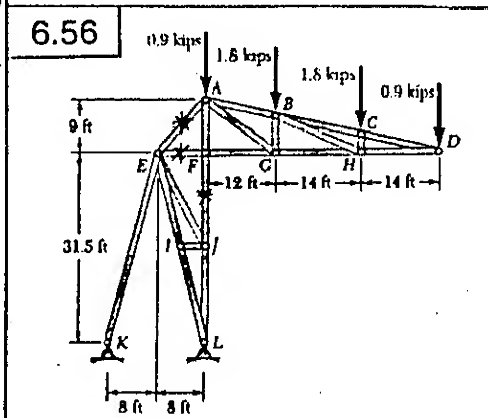
$+\circlearrowleft \Sigma M_D = 0: -\left(\frac{3}{5} F_{AG}\right)(28\ \text{ft}) + (1.8\ \text{kips})(28\ \text{ft}) + (1.8\ \text{kips})(14\ \text{ft}) = 0$

$F_{AG} = +4.50\ \text{kips}$ $\quad F_{AG} = 4.50\ \text{kips}\ T$

$+\circlearrowleft M_A = 0: -F_{FG}(9\ \text{ft}) - (1.8\ \text{kips})(12\ \text{ft}) - (1.8\ \text{kips})(26\ \text{ft}) - (0.9\ \text{kips})(40\ \text{ft}) = 0$

$F_{FG} = -11.60\ \text{kips}$ $\quad F_{FG} = 11.60\ \text{kips}\ C$

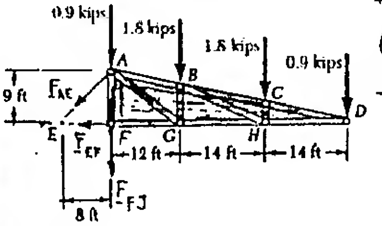
6.56



GIVEN: STADIUM ROOF TRUSS WITH LOADING SHOWN.

FIND: FORCE IN MEMBERS AE, EF, AND FJ.

WE PASS A SECTION THROUGH MEMBERS AE, EF, AND FJ, AND USE THE FREE BODY SHOWN.



$+\circlearrowleft \Sigma M_F = 0:$

$\left(\frac{8}{\sqrt{8^2+9^2}} F_{AE}\right)(9\ \text{ft}) - (1.8\ \text{kips})(12\ \text{ft}) - (1.8\ \text{kips})(26\ \text{ft}) - (0.9\ \text{kips})(40\ \text{ft}) = 0$

$F_{AE} = +17.46\ \text{kips}$

$F_{AE} = 17.46\ \text{kips}\ T$

$+\circlearrowleft \Sigma M_A = 0: -F_{EF}(9\ \text{ft}) - (1.8\ \text{kips})(12\ \text{ft}) - (1.8\ \text{kips})(26\ \text{ft}) - (0.9\ \text{kips})(40\ \text{ft}) = 0$

$F_{EF} = -11.60\ \text{kips}$ $\quad F_{EF} = 11.60\ \text{kips}\ C$

$+\circlearrowleft \Sigma M_E = 0: -F_{FJ}(8\ \text{ft}) - (0.9\ \text{kips})(8\ \text{ft}) - (1.8\ \text{kips})(20\ \text{ft}) - (1.8\ \text{kips})(34\ \text{ft}) - (0.9\ \text{kips})(48\ \text{ft}) = 0$

$F_{FJ} = -18.45\ \text{kips}$ $\quad F_{FJ} = 18.45\ \text{kips}\ C$

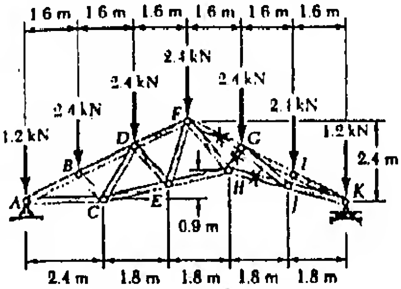
6.57

GIVEN:
FIND:

6.58

BECAUS
WE NE

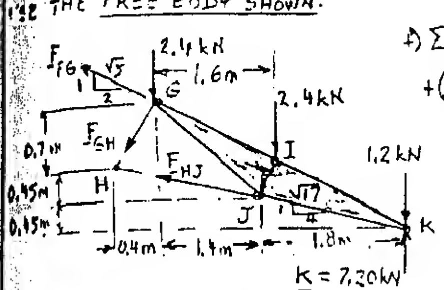
6.57



GIVEN: VAULTED ROOF TRUSS WITH LOADING SHOWN.
FIND: FORCE IN MEMBERS FG, GH, AND HJ.

BECAUSE OF THE SYMMETRY OF THE LOADING, $A = K = 7.20\,kN\uparrow$

WE PASS A SECTION THROUGH MEMBERS FG, GH, AND HJ, AND USE THE FREE BODY SHOWN.



$+\circlearrowleft \Sigma M_H = 0$: $\left(\frac{2}{\sqrt{5}}F_{FG}\right)(0.7\,m) + \left(\frac{1}{\sqrt{5}}F_{FG}\right)(0.4\,m) - (2.4\,kN)(0.4\,m) - (2.4\,kN)(2\,m) - (1.2\,kN)(3.6\,m) + (7.2\,kN)(3.6\,m) = 0$

$\frac{1.8}{\sqrt{5}}F_{FG} = -15.84$

$F_{FG} = 19.68\,kN\,C$

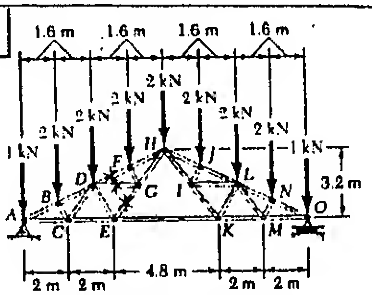
$+\circlearrowleft \Sigma M_K = 0$: $\left(\frac{4}{\sqrt{65}}F_{GH}\right)(1.6\,m) + \left(\frac{7}{\sqrt{65}}F_{GH}\right)(3.2\,m) + (2.4\,kN)(3.2\,m) + (2.4\,kN)(1.6\,m) = 0$

$(28.8/\sqrt{65})F_{GH} = -11.52$ $\qquad F_{GH} = 3.22\,kN\,C$

$+\circlearrowleft \Sigma M_G = 0$: $-\left(\frac{4}{\sqrt{17}}F_{HJ}\right)(1.15\,m) + \left(\frac{1}{\sqrt{17}}F_{HJ}\right)(1.4\,m) - (2.4\,kN)(1.6\,m) + (6\,kN)(3.2\,m) = 0$

$-(3.2/\sqrt{17})F_{HJ} = -15.36$ $\qquad F_{HJ} = 19.79\,kN\,T$

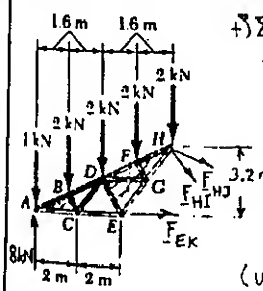
6.58



GIVEN: FINK ROOF TRUSS WITH LOADING SHOWN.
FIND: FORCE IN MEMBERS DF, DG, AND EG.

BECAUSE OF THE SYMMETRY OF THE LOADING: $A = 8\,kN\uparrow$

WE NEXT DETERMINE F_{EK} FROM THE F.B. DIAGRAM OF PANEL AEH:

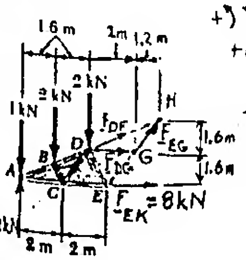


$+\circlearrowleft \Sigma M_H = 0$: $(1\,kN)(6.4\,m) + (2\,kN)(4.8\,m) + (2\,kN)(3.2\,m) + (2\,kN)(1.6\,m) + F_{EK}(3.2\,m) - (8\,kN)(6.4\,m) = 0$

$F_{EK} = 8.00\,kN\,T$

FREE BODY: PANEL ADE

(WE SLIDE F_{EG} TO APPLY IT AT G)



$+\circlearrowleft \Sigma M_G = 0$: $(8\,kN)(1.6\,m) - (8\,kN)(5.2\,m) + (1\,kN)(5.2\,m) + (2\,kN)(3.6\,m) + (2\,kN)(2\,m) - \left(\frac{1}{\sqrt{3}}F_{DF}\right)(2\,m) = 0$

$F_{DF} = -13.86\,kN$ $\qquad F_{DF} = 13.86\,kN\,C$

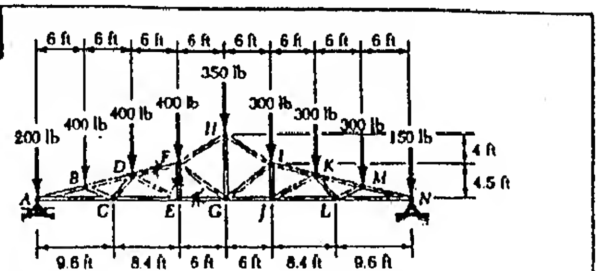
$+\circlearrowleft \Sigma M_H = 0$: $F_{DG}(1.6\,m) + (8\,kN)(3.2\,m) - (8\,kN)(6.4\,m) + (1\,kN)(6.4\,m) + (2\,kN)(4.8\,m) + (2\,kN)(3.2\,m) = 0$

$F_{DG} = +2.00\,kN$ $\qquad F_{DG} = 2.00\,kN\,T$

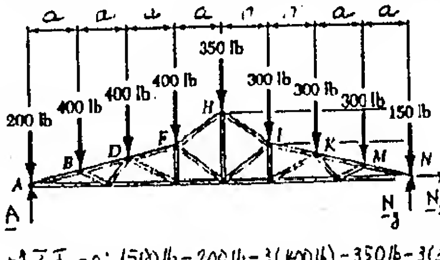
$+\circlearrowleft \Sigma M_D = 0$: $\left(\frac{4}{5}F_{EG}\right)(2\,m) + (8\,kN)(1.6\,m) - (8\,kN)(3.2\,m) + (1\,kN)(3.2\,m) + (2\,kN)(1.6\,m) = 0$

$F_{EG} = +4.00\,kN$ $\qquad F_{EG} = 4.00\,kN\,T$

6.59



GIVEN: DUOPITCH ROOF TRUSS WITH LOADING SHOWN.
FIND: FORCE IN MEMBERS DF, EF, AND EG.



FREE BODY: TRUSS

$\Sigma F_x = 0$: $N_x = 0$

$+\circlearrowleft \Sigma M_N = 0$: $(200\,lb)(8a) + (400\,lb)(7a + 6a + 5a) + (350\,lb)(4a) + (300\,lb)(3a + 2a + a) - A(8a) = 0$

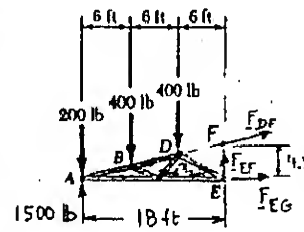
$A = 1500\,lb\uparrow$

$+\uparrow \Sigma F_y = 0$: $1500\,lb - 200\,lb - 3(400\,lb) - 350\,lb - 3(300\,lb) - 150\,lb + N_y = 0$

$N_y = 1300\,lb$, $N = 1300\,lb\uparrow$

WE PASS A SECTION THROUGH DF, EF, AND EG, AND USE THE FREE BODY SHOWN.

(WE APPLY F_{DF} AT F)



$+\circlearrowleft \Sigma M_E = 0$:

$(200\,lb)(18\,ft) + (400\,lb)(12\,ft) + (400\,lb)(6\,ft) - (1500\,lb)(18\,ft) - \left(\frac{18}{\sqrt{18^2+4.5^2}}F_{DF}\right)(4.5\,ft) = 0$

$F_{DF} = -3711\,lb$, $\quad F_{DF} = 3710\,lb\,C$

$+\circlearrowleft \Sigma M_A = 0$:

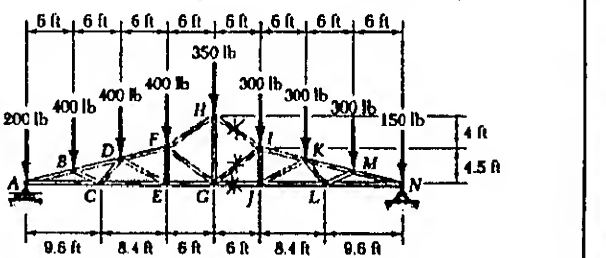
$F_{EF}(18\,ft) - (400\,lb)(6\,ft) - (400\,lb)(12\,ft) = 0$

$F_{EF} = +400\,lb$, $\quad F_{EF} = 400\,lb\,T$

$+\circlearrowleft \Sigma M_F = 0$: $F_{EG}(4.5\,ft) - (1500\,lb)(18\,ft) + (200\,lb)(18\,ft) + (400\,lb)(12\,ft) + (400\,lb)(6\,ft) = 0$

$F_{EG} = +3600\,lb$, $\quad F_{EG} = 3600\,lb\,T$

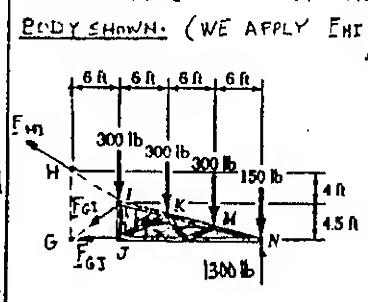
6.60



GIVEN: DUOPITCH ROOF TRUSS WITH LOADING SHOWN.
FIND: FORCE IN MEMBERS HI, GI, AND GJ.

SEE SOLUTION OF PROB. 6.59 FOR REACTIONS: $A = 1500\,lb\uparrow$, $N = 1300\,lb\uparrow$

WE PASS A SECTION THROUGH HI, GI, AND GJ, AND USE THE FREE BODY SHOWN. (WE APPLY F_{HI} AT H.)



$+\circlearrowleft \Sigma M_G = 0$: $\left(\frac{6}{\sqrt{6^2+4^2}}F_{HI}\right)(8.5\,ft) + (1300\,lb)(24\,ft) - (300\,lb)(6\,ft) - (300\,lb)(12\,ft) - (300\,lb)(18\,ft) - (150\,lb)(24\,ft) = 0$

$F_{HI} = -2375.4\,lb$, $\quad F_{HI} = 2375\,lb\,C$

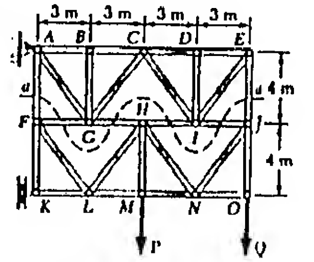
$+\circlearrowleft \Sigma M_I = 0$: $(1300\,lb)(18\,ft) - (300\,lb)(6\,ft) - (300\,lb)(12\,ft) - (150\,lb)(18\,ft) - F_{GJ}(4.5\,ft) = 0$

$F_{GJ} = +3400\,lb$, $\quad F_{GJ} = 3400\,lb\,T$

$\xrightarrow{+} \Sigma F_x = 0$: $-\frac{4}{5}F_{GI} - \frac{6}{\sqrt{6^2+4^2}}(-2375.4\,lb) - 3400\,lb = 0$

$F_{GI} = -1179.4\,lb$, $\quad F_{GI} = 1179\,lb\,C$

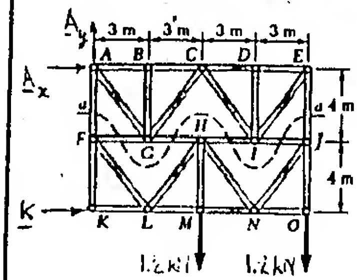
6.61



GIVEN: TRUSS SHOWN WITH $P = Q = 1.2\ \text{kN}$

FIND: FORCE IN MEMBERS AF AND EJ (USE SECTION aa)

FREE BODY: ENTIRE TRUSS

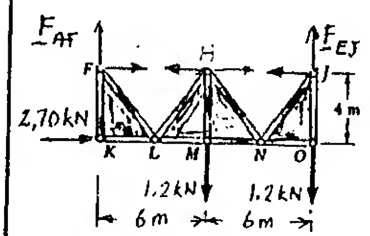


$+\circlearrowleft \Sigma M_A = 0$: $K(8\ \text{m}) - (1.2\ \text{kN})(6\ \text{m}) - (1.2\ \text{kN})(12\ \text{m}) = 0$

$K = +2.70\ \text{kN}$

$K = 2.70\ \text{kN} \rightarrow$

FREE BODY: LOWER PORTION



$+\circlearrowleft \Sigma M_F = 0$: $F_{EJ}(12\ \text{m}) + (2.70\ \text{kN})(4\ \text{m}) - (1.2\ \text{kN})(6\ \text{m}) - (1.2\ \text{kN})(12\ \text{m}) = 0$

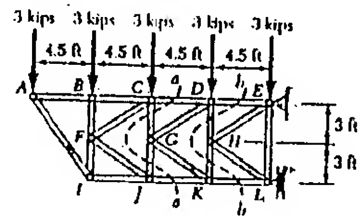
$F_{EJ} = +0.900\ \text{kN}$

$F_{EJ} = 0.900\ \text{kN T}$

$+\uparrow \Sigma F_y = 0$: $F_{AF} + 0.9\ \text{kN} - 1.2\ \text{kN} - 1.2\ \text{kN} = 0$

$F_{AF} = +1.500\ \text{kN}$ $F_{AF} = 1.500\ \text{kN T}$

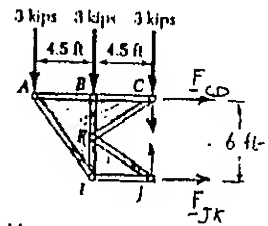
6.63



GIVEN: TRUSS AND LOADING SHOWN

FIND: FORCE IN MEMBERS CD AND JK (USE SECTION aa)

FREE BODY: PORTION OF TRUSS SHOWN



$+\circlearrowleft \Sigma M_C = 0$: $F_{JK}(6\ \text{ft}) + (3\ \text{kips})(9\ \text{ft}) + (3\ \text{kips})(4.5\ \text{ft}) = 0$

$F_{JK} = -6.75\ \text{kips}$

$F_{JK} = 6.75\ \text{kips C}$

$\xrightarrow{+} \Sigma F_x = 0$:

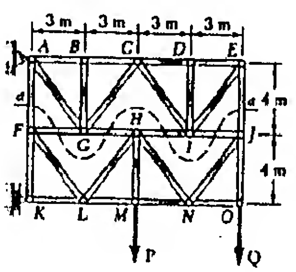
$F_{CD} + F_{JK} = 0$

$F_{CD} - 6.75\ \text{kips} = 0$

$F_{CD} = +6.75\ \text{kips}$

$F_{CD} = 6.75\ \text{kips T}$

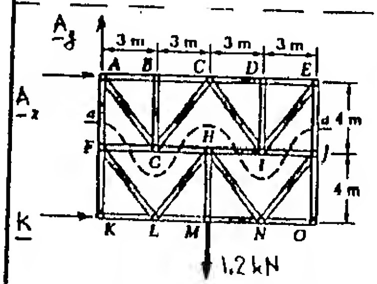
6.62



GIVEN: TRUSS SHOWN WITH $P = 1.2\ \text{kN}$, $Q = 0$

FIND: FORCE IN MEMBERS AF AND EJ (USE SECTION aa)

FREE BODY: ENTIRE TRUSS

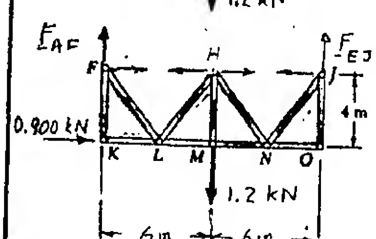


$+\circlearrowleft \Sigma M_A = 0$: $K(8\ \text{m}) - (1.2\ \text{kN})(6\ \text{m}) = 0$

$K = +0.900\ \text{kN}$

$K = 0.900\ \text{kN} \rightarrow$

FREE BODY: LOWER PORTION



$+\circlearrowleft \Sigma M_F = 0$: $F_{EJ}(12\ \text{m}) + (0.900\ \text{kN})(4\ \text{m}) - (1.2\ \text{kN})(6\ \text{m}) = 0$

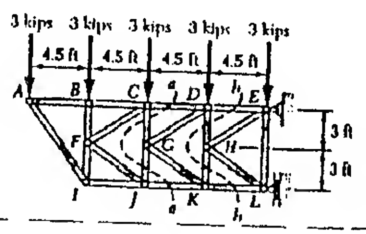
$F_{EJ} = +0.300\ \text{kN}$

$F_{EJ} = 0.300\ \text{kN T}$

$+\uparrow \Sigma F_y = 0$: $F_{AF} + 0.300\ \text{kN} - 1.2\ \text{kN} = 0$

$F_{AF} = +0.900\ \text{kN}$, $F_{AF} = 0.900\ \text{kN T}$

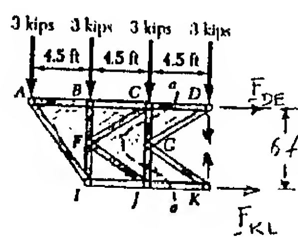
6.64



GIVEN: TRUSS AND LOADING SHOWN.

FIND: FORCE IN MEMBERS DE AND KL. (USE SECTION b-b)

FREE BODY: PORTION OF TRUSS SHOWN



$+\circlearrowleft \Sigma M_D = 0$: $F_{KL}(6\ \text{ft}) + (3\ \text{kips})(13.5\ \text{ft}) + (3\ \text{kips})(9\ \text{ft}) + (3\ \text{kips})(4.5\ \text{ft}) = 0$

$F_{KL} = -13.50\ \text{kips}$

$F_{KL} = 13.50\ \text{kips C}$

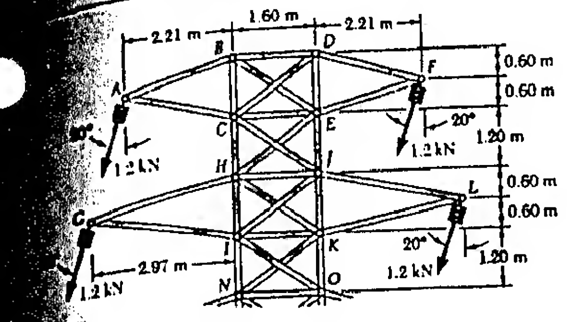
$\xrightarrow{+} \Sigma F_x = 0$:

$F_{DE} + F_{KL} = 0$

$F_{DE} - 13.50\ \text{kips} = 0$

$F_{DE} = +13.50\ \text{kips}$, $F_{DE} = 13.50\ \text{kips T}$

6.65 AND 6.66



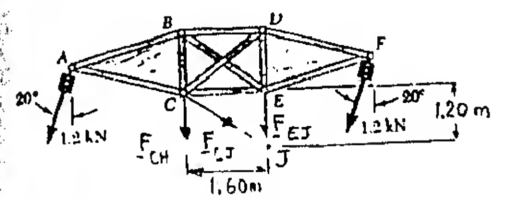
GIVEN: POWER TRANSMISSION LINE TOWER AND LOADING SHOWN.

6.65

FIND: (a) WHICH OF THE COUNTERS CJ AND HE IS ACTING
(b) THE FORCE IN THAT COUNTER

FREE BODY: PORTION ABDFEC OF TOWER

WE ASSUME THAT COUNTER CJ IS ACTING AND SHOW THE FORCES EXERTED BY THAT COUNTER AND BY MEMBERS CH AND EJ



$\xrightarrow{+}\Sigma F_x = 0: \frac{4}{5}F_{CJ} - 2(1.2\text{ kN})\sin 20^\circ = 0 \qquad F_{CJ} = +1.026\text{ kN}$

SINCE CJ IS FOUND TO BE IN TENSION, OUR ASSUMPTION WAS CORRECT. THUS, THE ANSWERS ARE

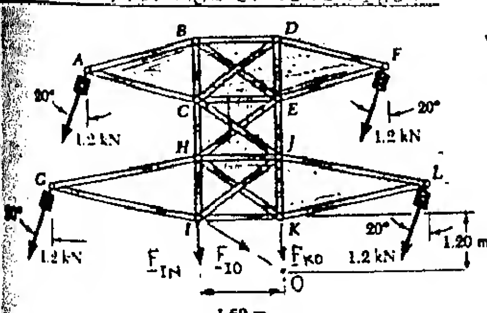
(a) CJ

(b) 1.026 kN T

6.66

FIND: (a) WHICH OF THE COUNTERS IO AND KN IS ACTING
(b) THE FORCE IN THAT COUNTER.

FREE BODY: PORTION OF TOWER SHOWN



WE ASSUME THAT COUNTER IO IS ACTING AND SHOW THE FORCES EXERTED BY THAT COUNTER AND BY MEMBERS IN AND KO.

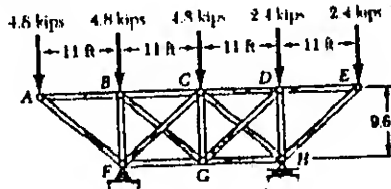
$\xrightarrow{+}\Sigma F_x = 0: \frac{4}{5}F_{IO} - 4(1.2\text{ kN})\sin 20^\circ = 0 \qquad F_{IO} = +2.05\text{ kN}$

SINCE IO IS FOUND TO BE IN TENSION, OUR ASSUMPTION WAS CORRECT. THUS, THE ANSWERS ARE

(a) IO

(b) 2.05 kN T

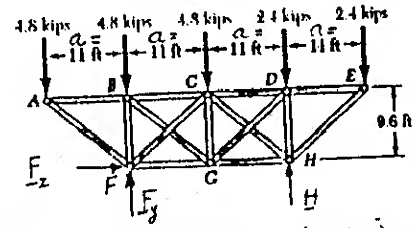
6.67



GIVEN: TRUSS AND LOADING SHOWN.

FIND: FORCES IN THE COUNTERS ACTING UNDER THIS LOADING

FREE BODY: TRUSS



$\Sigma F_x = 0: \quad F_x = 0$

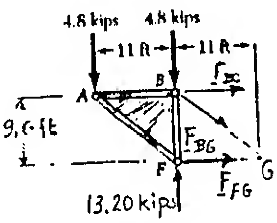
$+\circlearrowleft \Sigma M_H = 0:$
$4.8(3a) + 4.8(2a) + 4.8a - 2.4a - F_y(2a) = 0$

$F_y = +13.20$ kips $\qquad F = 13.20\text{ kips}\uparrow$

$+\uparrow\Sigma F_y = 0: H + 13.20\text{ kips} - 3(4.8\text{ kips}) - 2(2.4\text{ kips}) = 0$

$H = +6.00$ kips $\qquad H = 6.00\text{ kips}\uparrow$

FREE BODY: ABF



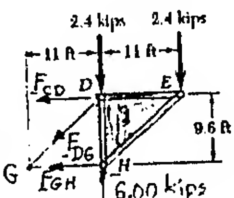
WE ASSUME THAT COUNTER BG IS ACTING

$+\uparrow\Sigma F_y = 0: -\frac{9.6}{14.6}F_{BG} + 13.20 - 2(4.8) = 0$

$F_{BG} = +5.475 \qquad F_{BG} = 5.48\text{ kips T}$

SINCE BG IS IN TENSION, OUR ASSUMPTION WAS CORRECT

FREE BODY: DEH



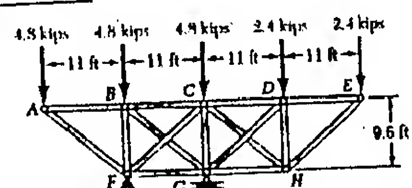
WE ASSUME THAT COUNTER DG IS ACTING.

$+\uparrow\Sigma F_y = 0: -\frac{9.6}{14.6}F_{DG} + 6.00 - 2(2.4) = 0$

$F_{DG} = +1.825 \qquad F_{DG} = 1.825\text{ kips T}$

SINCE DG IS IN TENSION, O.K.

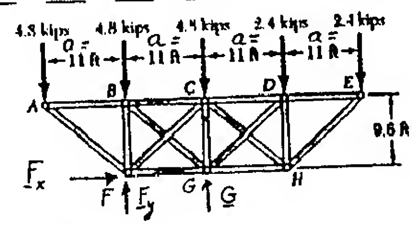
6.68



GIVEN: TRUSS AND LOADING SHOWN.

FIND: FORCES IN THE COUNTERS ACTING UNDER THIS LOADING

FREE BODY: TRUSS

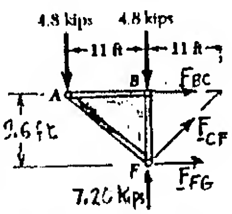


$\Sigma F_x = 0: \quad F_x = 0$

$+\circlearrowleft \Sigma M_G = 0: -F_y a + 4.8(2a) + 4.8a - 2.4a - 2.4(2a) = 0$

$F_y = 7.20, \quad F = 7.20\text{ kips}\uparrow$

FREE BODY: ABF



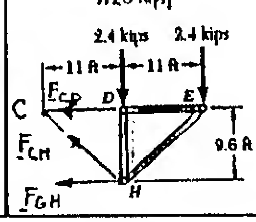
WE ASSUME THAT COUNTER CF IS ACTING.

$+\uparrow\Sigma F_y = 0: \frac{9.6}{14.6}F_{CF} + 7.20 - 2(4.8) = 0$

$F_{CF} = +3.65 \qquad F_{CF} = 3.65\text{ kips T}$

SINCE CF IS IN TENSION, O.K.

FREE BODY: DEH



WE ASSUME THAT COUNTER CH IS ACTING

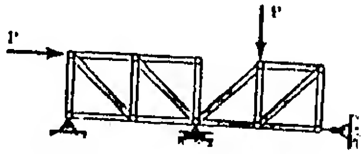
$+\Sigma F_y = 0: \frac{9.6}{14.6}F_{CH} - 2(2.4\text{ kips}) = 0$

$F_{CH} = +7.30 \qquad F_{CH} = 7.30\text{ kips T}$

SINCE CH IS IN TENSION, O.K.

CLASSIFY EACH STRUCTURE AS COMPLETELY, PARTIALLY OR IMPROPERLY CONSTRAINED. IF COMPLETELY CONSTRAINED, FURTHER CLASSIFY AS DETERMINATE OR INDETERMINATE.

STRUCTURE (a)



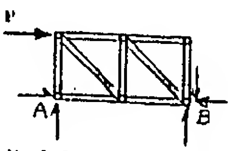
(a)

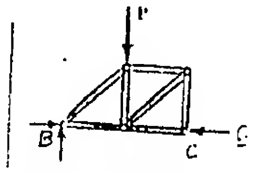
NUMBER OF MEMBERS:
$m = 16$
NUMBER OF JOINTS:
$n = 10$
REACTION COMPONENTS:
$r = 4$

$m + r = 20 \qquad 2n = 20$

THUS: $m + r = 2n$ ◁

TO DETERMINE WHETHER THE STRUCTURE IS ACTUALLY COMPLETELY CONSTRAINED AND DETERMINATE, WE MUST TRY TO FIND THE REACTIONS AT THE SUPPORTS. WE DIVIDE THE STRUCTURE INTO TWO SIMPLE TRUSSES AND DRAW THE FREE-BODY DIAGRAM OF EACH TRUSS.



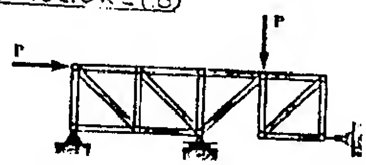


THIS IS A PROPERLY SUPPORTED SIMPLE TRUSS — O.K.

THIS IS AN IMPROPERLY SUPPORTED SIMPLE TRUSS. (REACTION AT C PASSES THROUGH B, THUS, EQ. $\Sigma M_B = 0$ CANNOT BE SATISFIED.)

STRUCTURE IS IMPROPERLY CONSTRAINED ◀

STRUCTURE (b)



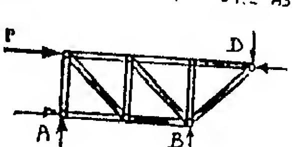
(b)

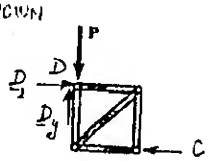
$m = 16$
$n = 10$
$r = 4$

$m + r = 20 \qquad 2n = 20$

THUS: $m + r = 2n$ ◁

WE MUST AGAIN TRY TO FIND THE REACTIONS AT THE SUPPORTS, DIVIDING THE STRUCTURE AS SHOWN

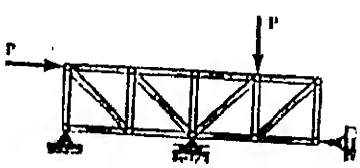




BOTH PORTIONS ARE SIMPLY SUPPORTED SIMPLE TRUSSES

STRUCTURE IS COMPLETELY CONSTRAINED AND DETERMINATE ◀

STRUCTURE (c)



(c)

$m = 17$
$n = 10$
$r = 4$

$m + r = 21 \qquad 2n = 20$

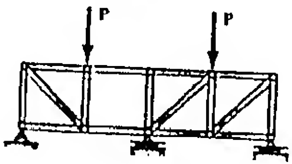
THUS: $m + r > 2n$ ◁

THIS IS A SIMPLE TRUSS WITH AN EXTRA SUPPORT WHICH CAUSES REACTIONS (AND FORCES IN MEMBERS) TO BE INDETERMINATE.

STRUCTURE IS COMPLETELY CONSTRAINED AND INDETERMINATE ◀

6.70 GIVEN: THE THREE STRUCTURES SHOWN. CLASSIFY EACH STRUCTURE AS COMPLETELY, PARTIALLY OR IMPROPERLY CONSTRAINED. IF COMPLETELY CONSTRAINED, FURTHER CLASSIFY AS DETERMINATE OR INDETERMINATE.

STRUCTURE (a)



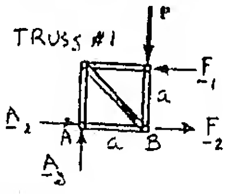
(a)

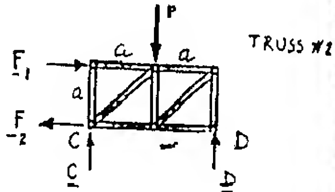
NUMBER OF MEMBERS
$m = 16$
NUMBER OF JOINTS:
$n = 10$
REACTION COMPONENTS
$r = 4$

$m + r = 20 \qquad 2n = 20$

THUS: $m + r = 2n$

TO DETERMINE WHETHER THE STRUCTURE IS ACTUALLY COMPLETELY CONSTRAINED AND DETERMINATE, WE MUST TRY TO FIND THE REACTIONS AT THE SUPPORTS. WE DIVIDE THE STRUCTURE INTO TWO SIMPLE TRUSSES AND DRAW THE FREE-BODY DIAGRAM OF EACH TRUSS





FREE BODY: TRUSS #1

$+\circlearrowleft \Sigma M_A = 0: \quad F_1 a - Pa = 0 \qquad F_1 = P$

$+\uparrow \Sigma F_y = 0: \quad A_y - P = 0 \qquad A_y = P$

FREE BODY: TRUSS #2

$+\circlearrowleft \Sigma M_C = 0: \quad D(2a) - Pa - Pa = 0 \qquad D = P$

$\xrightarrow{+} \Sigma F_x = 0: \quad F_1 - F_2 = 0 \qquad F_2 = F_1 \qquad F_2 = P$

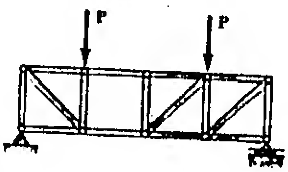
$+\uparrow \Sigma F_y = 0: \quad C - P + P = 0 \qquad C = 0$

FREE BODY: TRUSS #1

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - F_1 + F_2 = 0 \qquad A_x = 0$

SINCE ALL UNKNOWNS HAVE BEEN FOUND AND ALL EQUATIONS SATISFIED,

STRUCTURE IS COMPLETELY CONSTRAINED AND DETERMINATE ◀



(b)

STRUCTURE (b)

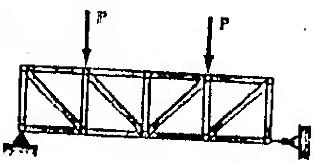
$m = 16, \ n = 10, \ r = 3$

$m + r = 19 \qquad 2n = 20$

THUS: $m + r < 2n$ ◁

STRUCTURE IS PARTIALLY CONSTRAINED ◀

STRUCTURE (c)

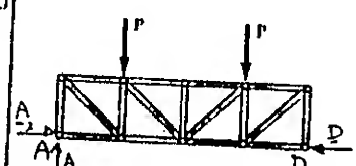


(c)

$m = 17, n = 10, r = 3$

$m + r = 20, \qquad 2n = 20$

THUS: $m + r = 2n$ ◁



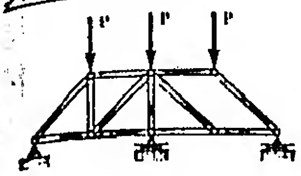
HOWEVER, WE NOTE THAT THE STRUCTURE IS A SIMPLE TRUSS WHICH IS IMPROPERLY CONSTRAINED, SINCE THE REACTION AT D PASSES THROUGH A, RESULTING IN $\Sigma M_A \neq 0$.

STRUCTURE IS IMPROPERLY CONSTRAINED ◀

6.71

GIVEN: THE THREE STRUCTURES SHOWN.
CLASSIFY EACH STRUCTURE AS COMPLETELY, PARTIALLY, OR IMPROPERLY CONSTRAINED, IF COMPLETELY CONSTRAINED, FURTHER CLASSIFY AS DETERMINATE OR INDETERMINATE.

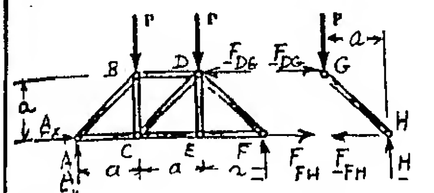
STRUCTURE (a)



(a)

NUMBER OF MEMBERS:
$m = 12$
NUMBER OF JOINTS:
$n = 6$
REACTION COMPONENTS:
$r = 4$
$m + r = 16 \qquad 2n = 16$
THUS: $m + r = 2n$

TO DETERMINE WHETHER THE STRUCTURE IS ACTUALLY COMPLETELY CONSTRAINED AND DETERMINATE, WE MUST TRY TO FIND THE REACTIONS AT THE SUPPORTS. WE PASS A SECTION AND OBTAIN THE SIMPLE TRUSS ABCDEF AND MEMBER GH.

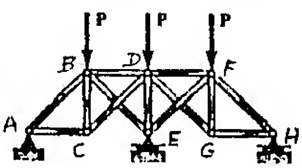


FREE BODY: GH
$+\circlearrowleft \Sigma M_H = 0:$
$Pa - F_{DG}\, a = 0$
$F_{DG} = P$
$\Sigma F_x = 0: \quad F_{FH} = F_{DG} = P$
$\Sigma F_y = 0: \quad H = P$

FREE BODY: TRUSS ABCDEF
$\xrightarrow{+} \Sigma F_x = 0: \; A_x + F_{FH} - F_{DG} = 0 \qquad A_x + P - P = 0 \qquad A_x = 0$
$+\circlearrowleft \Sigma M_A = 0: \; F(3a) + Pa - Pa - P(2a) = 0 \qquad F = \frac{2}{3}P$
$+\uparrow \Sigma F_y = 0: \; A_y - P - P + \frac{2}{3}P = 0 \qquad A_y = \frac{4}{3}P$

SINCE ALL UNKNOWNS HAVE BEEN FOUND AND ALL EQUATIONS SATISFIED.

STRUCTURE IS COMPLETELY CONSTRAINED AND DETERMINATE



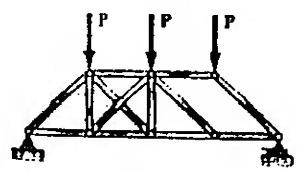
(b)

STRUCTURE (b)

$m = 13, \; n = 8, \; r = 4$
$m + r = 17 \qquad 2n = 16$
THUS: $m + r > 2n$

MOREOVER, WE NOTE THAT STRUCTURE IS A SIMPLE TRUSS (FOLLOW LETTERING TO CONSTRUCT)

STRUCTURE IS COMPLETELY CONSTRAINED AND INDETERMINATE

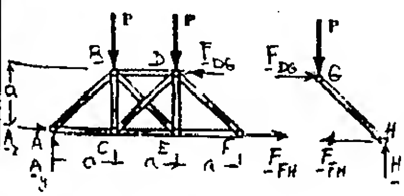


(c)

STRUCTURE (c)

$m = 13, \; n = 8, \; r = 3$
$m + r = 16 \qquad 2n = 16$
THUS: $m + r = 2n$

WE PASS A SECTION AND OBTAIN THE TWO FREE BODIES SHOWN.



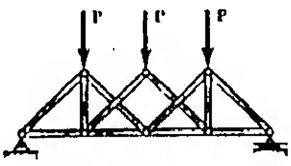
FREE BODY: FG
WE RECALL FROM PART (a) THAT
$F_{DG} = F_{FH} = H = P$

FREE BODY: ABCDEF
$+\circlearrowleft \Sigma M_A = F_{DG}\, a - Pa - P(2a) = Pa - 3Pa = 2Pa \neq 0$
THIS EQUILIBRIUM EQUATION IS NOT SATISFIED, THEREFORE
STRUCTURE IS IMPROPERLY CONSTRAINED

6.72

GIVEN: THE THREE STRUCTURES SHOWN
CLASSIFY EACH STRUCTURE AS COMPLETELY, PARTIALLY, OR IMPROPERLY CONSTRAINED, IF COMPLETELY CONSTRAINED, FURTHER CLASSIFY AS DETERMINATE OR INDETERMINATE.

STRUCTURE (a)

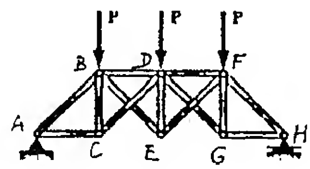


(a)

NUMBER OF MEMBERS:
$m = 12$
NUMBER OF JOINTS:
$n = 8$
REACTION COMPONENTS:
$r = 3$
$m + r = 15 \qquad 2n = 16$
THUS: $m + r < 2n$

STRUCTURE IS PARTIALLY CONSTRAINED

STRUCTURE (b)



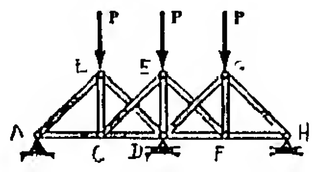
(b)

$m = 13, \quad n = 8$
$r = 3$
$m + r = 16 \qquad 2n = 16$
THUS: $m + r = 2n$

TO VERIFY THAT THE STRUCTURE IS ACTUALLY COMPLETELY CONSTRAINED AND DETERMINATE, WE OBSERVE THAT IT IS A SIMPLE TRUSS (FOLLOW LETTERING TO CHECK THIS) AND THAT IT IS SIMPLY SUPPORTED BY A PIN-AND-BRACKET AND A ROLLER. THUS:

STRUCTURE IS COMPLETELY CONSTRAINED AND DETERMINATE.

STRUCTURE (c)



(c)

$m = 13, \quad n = 8$
$r = 4$
$m + r = 17 \qquad 2n = 16$
THUS: $m + r > 2n$

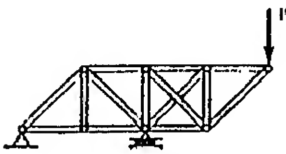
STRUCTURE IS COMPLETELY CONSTRAINED AND INDETERMINATE

THIS RESULT CAN BE VERIFIED BY OBSERVING THAT THE STRUCTURE IS A SIMPLE TRUSS (FOLLOW LETTERING TO CHECK THIS), THEREFORE RIGID, AND THAT ITS SUPPORTS INVOLVE 4 UNKNOWNS.

6.73

GIVEN: THE THREE STRUCTURES SHOWN. CLASSIFY EACH STRUCTURE AS COMPLETELY, PARTIALLY, OR IMPROPERLY CONSTRAINED. IF COMPLETELY CONSTRAINED, FURTHER CLASSIFY AS DETERMINATE OR INDETERMINATE.

STRUCTURE (a)



(a)

NUMBER OF MEMBERS:

$m = 14$

NUMBER OF JOINTS:

$n = 8$

REACTION COMPONENTS:

$r = 3$

$m + r = 17 \qquad 2n = 16$

THUS: $m + r > 2n$

STRUCTURE IS COMPLETELY CONSTRAINED AND INDETERMINATE

THIS RESULT CAN BE VERIFIED BY OBSERVING THAT THE STRUCTURE IS AN OVERRIGID TRUSS (ONE EXTRA MEMBER).

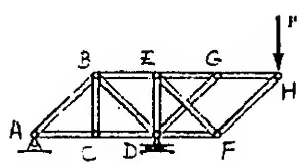
STRUCTURE (b)

$m = 13, \quad n = 8$

$r = 3$

$m + r = 16 \qquad 2n = 16$

THUS: $m + r = 2n$



WE OBSERVE THAT THE STRUCTURE IS A SIMPLE TRUSS (FOLLOW LETTERING TO CHECK THIS) AND THAT IT IS SIMPLY SUPPORTED BY A PIN-AND-BRACKET AND A ROLLER. THUS:

STRUCTURE IS COMPLETELY CONSTRAINED AND DETERMINATE

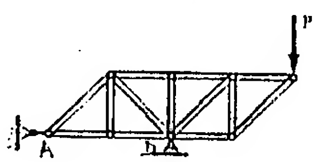
STRUCTURE (c)

$m = 13, \quad n = 8$

$r = 3$

$m + r = 16 \qquad 2n = 16$

THUS: $m + r = 2n$



(c)

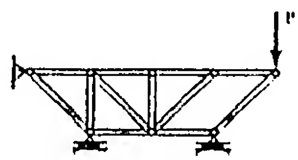
WE OBSERVE THAT THE STRUCTURE IS A SIMPLE TRUSS, BUT THAT IT IS IMPROPERLY CONSTRAINED, SINCE THE REACTION AT A PASSES THROUGH THE SUPPORT D. THE EQUATION $\Sigma M_D = 0$, THEREFORE, IS NOT SATISFIED.

THUS: STRUCTURE IS IMPROPERLY CONSTRAINED

6.74

GIVEN: THE THREE STRUCTURES SHOWN. CLASSIFY EACH STRUCTURE AS COMPLETELY, PARTIALLY, OR IMPROPERLY CONSTRAINED. IF COMPLETELY CONSTRAINED, FURTHER CLASSIFY AS DETERMINATE OR INDETERMINATE.

STRUCTURE (a)



(a)

NUMBER OF MEMBERS:

$m = 12$

NUMBER OF JOINTS:

$n = 8$

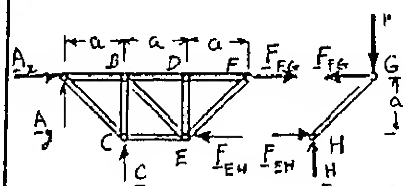
REACTION COMPONENTS:

$r = 4$

$m + r = 16 \qquad 2n = 16$

THUS: $m + r = 2n$

TO VERIFY WHETHER OR NOT THE STRUCTURE IS COMPLETELY CONSTRAINED AND DETERMINATE, WE PASS A SECTION AND CONSIDER THE FREE BODIES ABCDEF (A SIMPLE TRUSS) AND GH



FREE BODY: GH

$+\circlearrowleft \Sigma M_H = 0: \; F_{FG}a - Pa = 0$

$F_{FG} = P$

$\xrightarrow{+} \Sigma F_x = 0: \; F_{EH} - F_{FG} = 0$

$F_{EH} = F_{FG} = P$

$+\uparrow \Sigma F_y = 0: \; H - P = 0 \qquad H = P$

FREE BODY: TRUSS ABCDEF

$+\circlearrowleft \Sigma M_A = 0: \; Ca - F_{EH}a = 0 \qquad C = F_{EH} = P$

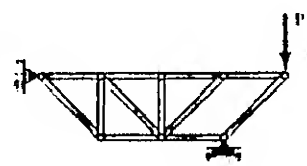
$\xrightarrow{+} \Sigma F_x = 0: \; A_x + F_{FG} - F_{EH} = 0 \qquad A_x = 0$

$+\uparrow \Sigma F_y = 0: \; A_y + C = 0 \qquad A_y = -C = -P$

SINCE ALL UNKNOWNS HAVE BEEN FOUND AND ALL EQUATIONS SATISFIED,

STRUCTURE IS COMPLETELY CONSTRAINED AND DETERMINATE

STRUCTURE (b)



(b)

$m = 12, \quad n = 8$

$r = 3$

$m + r = 15 \qquad 2n = 16$

THUS: $m + r < 2n$

STRUCTURE IS PARTIALLY CONSTRAINED

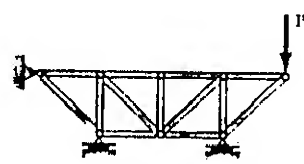
STRUCTURE (c)

$m = 13, \quad n = 8$

$r = 4$

$m + r = 17 \qquad 2n = 16$

THUS: $m + r > 2n$

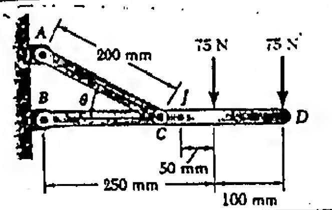


(c)

WE OBSERVE THAT THE STRUCTURE IS A SIMPLE TRUSS AND THAT ITS SUPPORTS INVOLVE 4 UNKNOWNS (INSTEAD OF 3 FOR A SIMPLY SUPPORTED TRUSS), THUS

STRUCTURE IS COMPLETELY CONSTRAINED AND INDETERMINATE

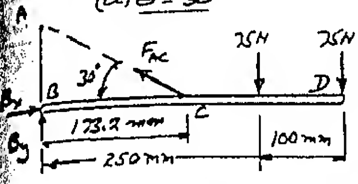
6.75



FIND: FORCE IN AC AND REACTION AT B WHEN (a) $\theta = 30°$, (b) $\theta = 60°$.

(a) $\theta = 30°$

SINCE $AC = 200$ mm
$BC = (200)\cos 30° = 173.2$ mm



$+\circlearrowleft \Sigma M_B = 0$:
$-75N(250\,mm) - 75N(350) + F_{AC}\sin 30°(173.2\,mm) = 0$

$F_{AC} = 519.6N$; $F_{AC} = 520\,N\ T$ ◀

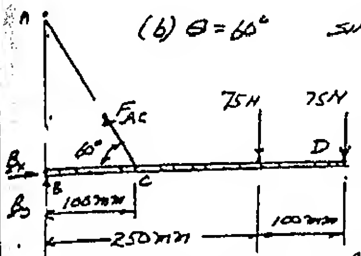
$\overset{+}{\rightarrow} \Sigma F_x = 0$: $B_x - (519.6N)\cos 30° = 0$ $\quad B_x = 450 \rightarrow$

$+\uparrow \Sigma F_y = 0$: $B_y + (519.6N)\sin 30° - 75N - 75N = 0$

$B_y = -109.8\,N$ $\quad B_y = 109.8N \downarrow$

109.8N, 450N, B $\quad B = 463N \measuredangle 13.7°$ ◀

(b) $\theta = 60°$ $\quad$ SINCE $AC = 200$ mm
$BC = (200)\cos 60° = 100$ mm



$+\circlearrowleft \Sigma M_B = 0$:
$-75N(250\,mm) - 75N(350\,mm) + F_{AC}\sin 60°(100\,mm) = 0$

$F_{AC} = 519.6N$; $F_{AC} = 520N\ T$ ◀

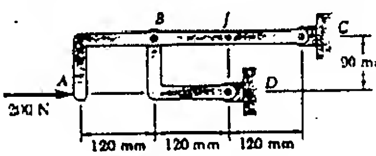
$\overset{+}{\rightarrow} \Sigma F_x = 0$: $B_x - (519.6N)\cos 60° = 0$ $\quad B_x = 259.8N \rightarrow$

$+\uparrow \Sigma F_y = 0$: $B_y + (519.6N)\sin 60° - 75N - 75N = 0$

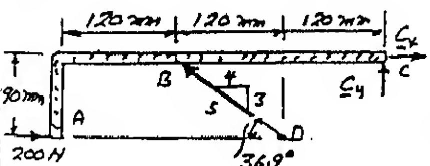
$B_y = -300N$ $\quad B_y = 300N \downarrow$

259.8N, 300N, B $\quad B = 397N \measuredangle 49.1°$ ◀

6.76



FIND: FORCE ACTING ON MEMBER ABC (a) AT B, (b) AT C.



NOTE THAT BD IS A TWO-FORCE MEMBER. FORCE B IS DIRECTED ALONG DB

(a)

$+\circlearrowleft \Sigma M_C = 0$: $(200N)(90\,mm) - \frac{3}{5}B(240\,mm) = 0$

$B = 125N$ $\quad \underline{B} = 125N \measuredangle 36.9°$ ◀

(b)

$\overset{+}{\rightarrow} \Sigma F_x = 0$: $C_x + 200N - \frac{4}{5}(125N) = 0$

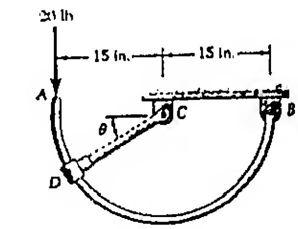
$C_x = -100N$ $\quad C_x = 100N \leftarrow$

$+\uparrow \Sigma F_y = 0$: $C_y + \frac{3}{5}(125N) = 0$

$C_y = -75N$ $\quad C_y = 75N \downarrow$

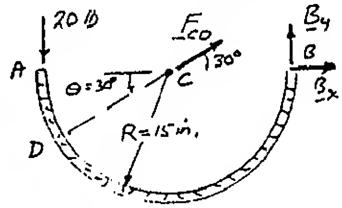
100N, 75N, $\underline{C}$ $\quad \underline{C} = 125N \measuredangle 36.9°$ ◀

6.77



GIVEN: $\theta = 30°$

FIND: (a) FORCE IN CD, (b) REACTION AT B.



NOTE THAT DC IS A TWO-FORCE MEMBER. F_{CD} MUST BE DIRECTED ALONG DC.

(a) $+\circlearrowleft \Sigma M_B = 0$: $(20lb)(2R) - (F_{CD}\sin 30°)R = 0$

$F_{CD} = 80\,lb$ $\quad F_{CD} = 80\,lb\ T$ ◀

(b) $+\circlearrowleft \Sigma M_C = 0$: $(20lb)R + (B_y)R = 0$

$B_y = -20\,lb$ $\quad B_y = 20\,lb \downarrow$

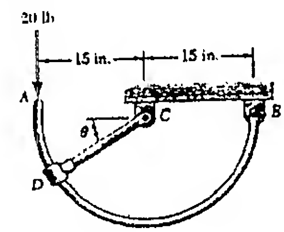
$\overset{+}{\rightarrow} \Sigma F_x = 0$: $F_{CD}\cos 30° + B_x = 0$

$(80\,lb)(\cos 30°) + B_x = 0$

$B_x = -69.28\,lb$ $\quad B_x = 69.28\,lb \leftarrow$

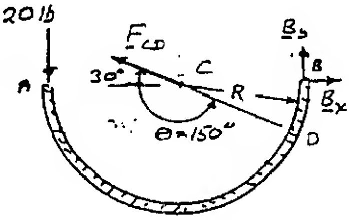
69.28 lb, 20 lb, $\underline{B}$ $\quad \underline{B} = 72.1\,lb \measuredangle 16.1°$ ◀

6.78



GIVEN: $\theta = 150°$

FIND: (a) FORCE IN CD, (b) REACTION AT B.



NOTE THAT CD IS A TWO-FORCE MEMBER. F_{CD} MUST BE DIRECTED ALONG DC.

(a) $+\circlearrowleft \Sigma M_B = 0$: $(20lb)(2R) - (F_{CD}\sin 30)R = 0$

$F_{CD} = 80\,lb$ $\quad F_{CD} = 80\,lb\ T$ ◀

(b) $+\circlearrowleft \Sigma M_C = 0$: $(20lb)R + (B_y)R = 0$

$B_y = -20\,lb$ $\quad B_y = 20\,lb \downarrow$

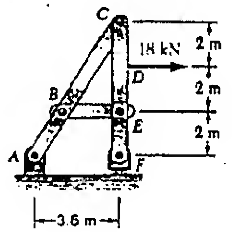
$\overset{+}{\rightarrow} \Sigma F_x = 0$: $-F_{CD}\cos 30° + B_x = 0$

$-(20\,lb)\cos 30° + B_x = 0$

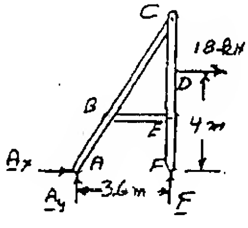
$B_x = 69.28\,lb$ $\quad B_x = 69.28\,lb \rightarrow$

69.28 lb, 20 lb, $\underline{B}$ $\quad \underline{B} = 72.1\,lb \measuredangle 16.1°$ ◀

6.79



FIND: COMPONENTS OF FORCES ACTING ON MEMBER ABC.



FREE BODY: ENTIRE FRAME

$\xrightarrow{+}\Sigma F_x=0$: $A_x+18\text{ kN}=0$

$A_x=-18\text{ kN}$ $\quad A_x=18\text{ kN}\leftarrow$

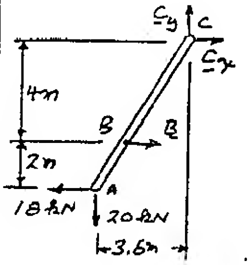
$+\circlearrowleft\Sigma M_F=0$:

$-(18\text{ kN})(4\text{ m})-A_y(3.6\text{ m})=0$

$A_y=-20\text{ kN}$ $\quad A_y=20\text{ kN}\downarrow$

$+\uparrow\Sigma F_y=0$: $-20\text{ kN}+F=0$

$F=+20\text{ kN}$ $\quad F=20\text{ kN}\uparrow$



FREE BODY: MEMBER ABC

NOTE: BE IS A TWO-FORCE MEMBER, THUS $\underline{B}$ IS DIRECTED ALONG LINE BE.

$+\circlearrowleft\Sigma M_C=0$:

$B(4\text{ m})-(18\text{ kN})(6\text{ m})+(20\text{ kN})(3.6\text{ m})=0$

$B=9\text{ kN}$ $\quad \underline{B}=9\text{ kN}\rightarrow$

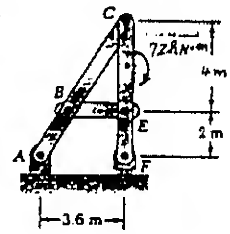
$\xrightarrow{+}\Sigma F_x=0$: $C_x-18\text{ kN}+9\text{ kN}=0$

$C_x=9\text{ kN}$ $\quad \underline{C}_x=9\text{ kN}\rightarrow$

$+\uparrow\Sigma F_y=0$: $C_y-20\text{ kN}=0$

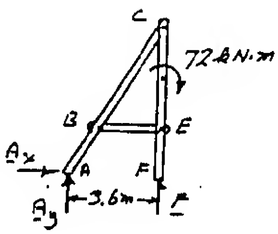
$C_y=20\text{ kN}$ $\quad \underline{C}_y=20\text{ kN}\uparrow$

6.80



GIVEN: $M=72\text{ kN}\cdot\text{m}$.

FIND: COMPONENTS OF FORCES ACTING ON MEMBER ABC.



FREE BODY: ENTIRE FRAME

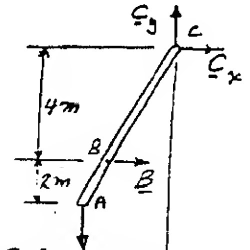
$\xrightarrow{+}\Sigma F_x=0$: $A_x=0$

$+\circlearrowleft\Sigma M_F=0$:

$-72\text{ kN}\cdot\text{m}-A_y(3.6\text{ m})=0$

$A_y=-20\text{ kN}$ $\quad A_y=20\text{ kN}\downarrow$

$\underline{A}=20\text{ kN}\downarrow$



FREE BODY: MEMBER ABC

NOTE: $\underline{BE}$ IS A TWO-FORCE MEMBER. THUS $\underline{B}$ IS DIRECTED ALONG LINE BE.

$+\circlearrowleft\Sigma M_C=0$

$B(4\text{ m})+(20\text{ kN})(3.6\text{ m})=0$

$B=-18\text{ kN}$ $\quad \underline{B}=18\text{ kN}\leftarrow$

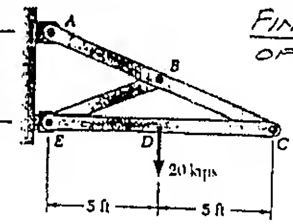
$\xrightarrow{+}F_x=0$: $-18\text{ kN}+C_x=0$

$C_x=18\text{ kN}$ $\quad \underline{C}_x=18\text{ kN}\rightarrow$

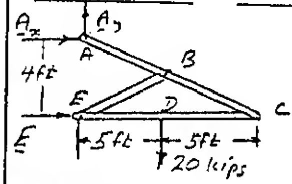
$+\uparrow\Sigma F_y=0$: $C_y-20\text{ kN}=0$

$C_y=20\text{ kN}$ $\quad \underline{C}_y=20\text{ kN}\uparrow$

6.81



FIND: COMPONENTS OF FORCES ACTING ON MEMBER ABC



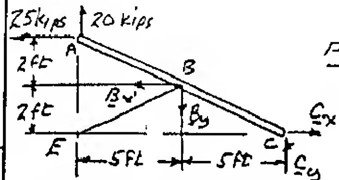
FREE BODY: ENTIRE FRAME

$+\circlearrowleft\Sigma M_E=0$: $-A_x(4)-(20\text{ kips})(5)=0$

$A_x=-25\text{ kips}$, $\quad A_x=25\text{ kips}\leftarrow$

$+\uparrow\Sigma F_y=0$: $A_y-20\text{ kips}=0$

$A_y=20\text{ kips}$ $\quad A_y=20\text{ kips}\uparrow$



FREE BODY: MEMBER ABC

NOTE: BE IS A TWO-FORCE MEMBER, THUS $\underline{B}$ IS DIRECTED ALONG LINE BE. AND $B_y=\frac{2}{5}B_x$

$+\circlearrowleft\Sigma M_C=0$: $(25\text{ kips})(4\text{ ft})-(20\text{ kips})(10\text{ ft})+B_x(2\text{ ft})+B_y(5\text{ ft})=0$

$-100\text{ kip}\cdot\text{ft}+B_x(2\text{ ft})+\frac{2}{5}B_x(5\text{ ft})=0$

$B_x=25\text{ kips}$ $\quad B_x=25\text{ kips}\rightarrow$

$B_y=\frac{2}{5}(B_x)=\frac{2}{5}(25)=10\text{ kips}$ $\quad B_y=10\text{ kips}\downarrow$

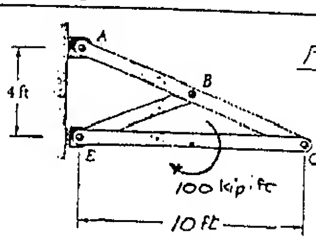
$\xrightarrow{+}\Sigma F_x=0$: $C_x-25\text{ kips}-25\text{ kips}=0$

$C_x=50\text{ kips}$ $\quad \underline{C}_x=50\text{ kips}\rightarrow$

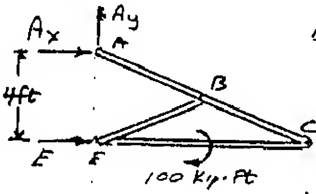
$+\uparrow\Sigma F_y=0$: $C_y+20\text{ kips}-10\text{ kips}=0$

$C_y=-10\text{ kips}$ $\quad C_y=10\text{ kips}\downarrow$

6.82



FIND: COMPONENTS OF FORCES ACTING ON MEMBER ABC



FREE BODY: ENTIRE FRAME

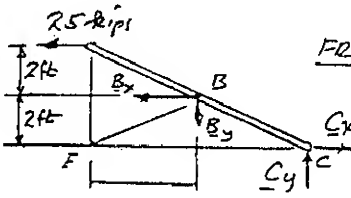
$+\uparrow\Sigma F_y=0$: $\quad A_y=0$

$+\circlearrowleft\Sigma M_E=0$:

$-A_x(4\text{ ft})-100\text{ kip}\cdot\text{ft}=0$

$A_x=-25\text{ kips}$ $\quad A_x=25\text{ kips}\leftarrow$

$\underline{A}=25\text{ kips}\leftarrow$



FREE BODY: MEMBER ABC

NOTE: BE IS A TWO-FORCE MEMBER, THUS $\underline{B}$ IS DIRECTED ALONG LINE BE AND

$B_y=\frac{2}{5}B_x$

$+\circlearrowleft\Sigma M_C=0$: $(25\text{ kips})(4\text{ ft})+B_x(2\text{ ft})+B_y(5\text{ ft})=0$

$100\text{ kip}\cdot\text{ft}+B_x(2\text{ ft})+\frac{2}{5}B_x(5\text{ ft})=0$

$B_x=-25\text{ kips}$ $\quad B_x=25\text{ kips}\rightarrow$

$B_y=\frac{2}{5}B_x=\frac{2}{5}(-25)=-10\text{ kips}$; $\quad B_y=10\text{ kips}\uparrow$

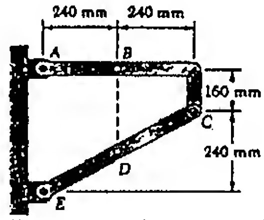
$\xrightarrow{+}\Sigma F_x=0$: $-25\text{ kips}+25\text{ kips}+C_x=0$ $\quad C_x=0$

$+\uparrow\Sigma F_y=0$: $+10\text{ kips}+C_y=0$

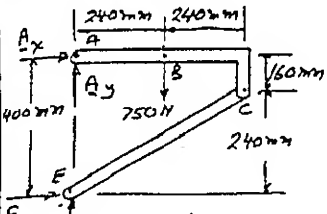
$C_y=-10\text{ kips}$ $\quad C_y=10\text{ kips}\downarrow$

$\underline{C}=10\text{ kips}\downarrow$

6.83



FIND: COMPONENTS OF REACTIONS AT A AND E IF A 750 N ↓ FORCE IS APPLIED (a) AT B, (b) AT D.



FREE-BODY: ENTIRE FRAME

THE FOLLOWING ANALYSIS IS VALID FOR BOTH (a) AND (b) SINCE POSITION OF LOAD ON ITS LINE OF ACTION IS INMATERIAL.

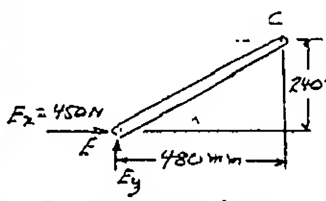
$+\circlearrowleft \Sigma M_E = 0$: $-(750\,N)(240\,mm) - A_x(400\,mm) = 0$

$A_x = -450\,N$ $\qquad A_x = 450\,N \leftarrow$

$\xrightarrow{+} \Sigma F_x = 0$: $E_x - 450\,N = 0$; $E_x = 450\,N$ $\qquad E_x = 450\,N \rightarrow$

$+\uparrow \Sigma F_y = 0$: $A_y + E_y - 750\,N = 0$ $\qquad (1)$

(a) LOAD APPLIED AT B. FREE BODY: MEMBER CE



CE IS A TWO-FORCE MEMBER. THUS, THE REACTION AT E MUST BE DIRECTED ALONG CE.

$\dfrac{E_y}{450\,N} = \dfrac{240\,mm}{480\,mm}$; $\quad E_y = 225\,N \uparrow$

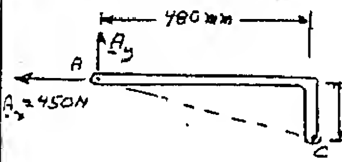
FROM EQ.(1): $A_y + 225 - 750 = 0$; $\quad A_y = 525\,lb \uparrow$

THUS, REACTIONS ARE:

$A_x = 450\,N \leftarrow$, $A_y = 525\,lb \uparrow$ ◀

$E_x = 450\,N \rightarrow$, $E_y = 225\,lb \uparrow$ ◀

(b) LOAD APPLIED AT D. FREE BODY: MEMBER AC



AC IS A TWO-FORCE MEMBER. THUS, THE REACTION AT A MUST BE DIRECTED ALONG AC.

$\dfrac{A_y}{450\,N} = \dfrac{160\,mm}{480\,mm}$ $\qquad A_y = 150\,N \uparrow$

FROM EQ.(1): $A_y + E_y - 750\,N = 0$

$150\,N + E_y - 750\,N = 0$

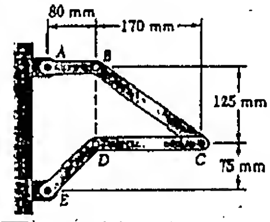
$E_y = 600\,N$ $\qquad E_y = 600\,N \uparrow$

THUS, REACTIONS ARE:

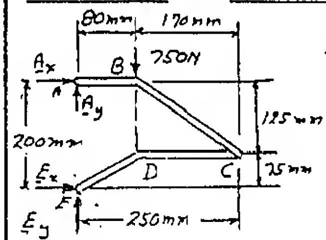
$A_x = 450\,N \leftarrow$, $A_y = 150\,N \uparrow$ ◀

$E_x = 450\,N \rightarrow$, $E_y = 600\,N \uparrow$ ◀

6.84



FIND: COMPONENTS OF REACTIONS AT A AND E IF A 750 N ↓ FORCE IS APPLIED (a) AT B, (b) AT D.



FREE BODY: ENTIRE FRAME

THE FOLLOWING ANALYSIS IS VALID FOR BOTH (a) AND (b) SINCE POSITION OF LOAD ON ITS LINE OF ACTION IS INMATERIAL.

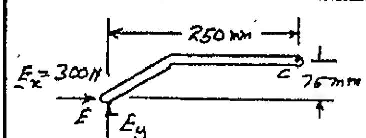
$+\circlearrowleft \Sigma M_E = 0$: $-(750\,N)(80\,mm) - A_x(200\,mm) = 0$

$A_x = -300\,N$ $\qquad A_x = 300\,N \leftarrow$

$\xrightarrow{+} \Sigma F_x = 0$: $E_x - 300\,N = 0$; $E_x = 300\,N$ $\qquad E_x = 300\,N \rightarrow$

$+\uparrow \Sigma F_y = 0$: $A_y + E_y - 750\,N = 0$ $\qquad (1)$

(a) LOAD APPLIED AT B. FREE BODY: MEMBER CE



CE IS A TWO-FORCE MEMBER. THUS, THE REACTION AT E MUST BE DIRECTED ALONG CE.

$\dfrac{E_y}{300\,N} = \dfrac{75\,mm}{250\,mm}$; $\quad E_y = 90\,N \uparrow$

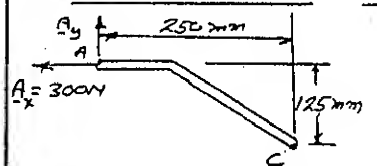
FROM EQ.(1): $A_y + 90\,N - 750\,N = 0$; $\quad A_y = 660\,N \uparrow$

THUS REACTIONS ARE:

$A_x = 300\,N \leftarrow$, $A_y = 660\,N \uparrow$ ◀

$E_x = 300\,N \rightarrow$, $E_y = 90\,N \uparrow$ ◀

(b) LOAD APPLIED AT D. FREE BODY: MEMBER AC



AC IS A TWO-FORCE MEMBER. THUS, THE REACTION AT A MUST BE DIRECTED ALONG AC.

$\dfrac{A_y}{300\,N} = \dfrac{125\,mm}{250\,mm}$ $\qquad A_y = 150\,N \uparrow$

FROM EQ.(1): $A_y + E_y - 750\,N = 0$

$150\,N + E_y - 750\,N = 0$

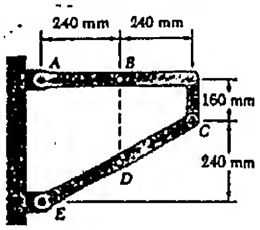
$E_y = 600\,N$ $\qquad E_y = 600\,N \uparrow$

THUS, REACTIONS ARE:

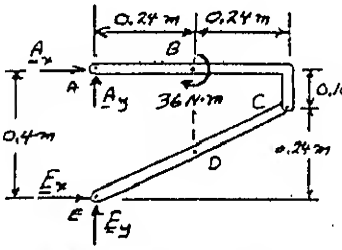
$A_x = 300\,N \leftarrow$, $A_y = 150\,N \uparrow$ ◀

$E_x = 300\,N \rightarrow$, $E_y = 600\,N \uparrow$ ◀

6.85



FIND: COMPONENTS OF REACTIONS AT A AND E IF A 36 N·m ↻ COUPLE IS APPLIED (a) AT B, (b) AT D.



FREE BODY: ENTIRE FRAME

THE FOLLOWING ANALYSIS IS VALID FOR BOTH (a) AND (b) SINCE THE POINT OF APPLICATION OF THE COUPLE IS IMMATERIAL.

$+\circlearrowleft \Sigma M_E = 0: \quad -36\,N\cdot m - A_x(0.4\,m) = 0$

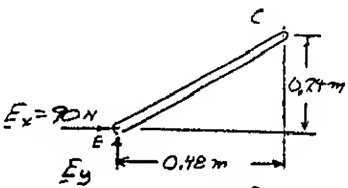
$A_x = -90\,N \qquad A_x = 90\,N \leftarrow$

$\xrightarrow{+} \Sigma F_x = 0: \quad -90 + E_x = 0$

$E_x = 90\,N \qquad E_x = 90\,N \rightarrow$

$+\uparrow \Sigma F_y = 0: \quad A_y + E_y = 0 \qquad (1)$

(a) COUPLE APPLIED AT B. FREE BODY: MEMBER CE



AC IS A TWO-FORCE MEMBER. THUS, THE REACTION AT E MUST BE DIRECTED ALONG EC.

$\frac{E_y}{90\,N} = \frac{0.24\,m}{0.48\,m}; \quad E_y = 45\,N \uparrow$

FROM EQ(1): $A_y + 45\,N = 0$

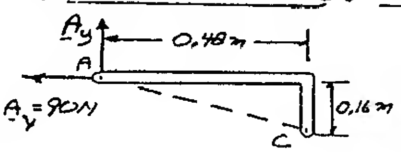
$A_y = -45\,N \qquad A_y = 45\,N \downarrow$

THUS, REACTIONS ARE

$A_x = 90\,N \leftarrow, \; A_y = 45\,N \downarrow$ ◀

$E_x = 90\,N \rightarrow, \; E_y = 45\,N \uparrow$ ◀

(b) COUPLE APPLIED AT D. FREE BODY: MEMBER AC



AC IS A TWO-FORCE MEMBER. THUS, THE REACTION AT A MUST BE DIRECTED ALONG AC.

$\frac{A_y}{90\,N} = \frac{0.16\,m}{0.48\,m}; \quad A_y = 30\,N \uparrow$

FROM EQ(1): $A_y + E_y = 0$

$30\,N + E_y = 0$

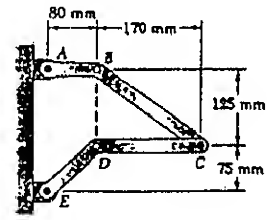
$E_y = -30\,N \qquad E_y = 30\,N \downarrow$

THUS, REACTIONS ARE:

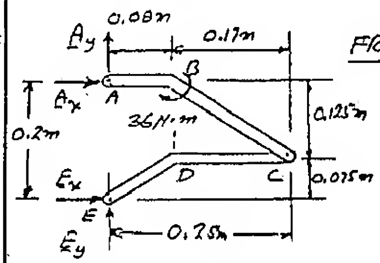
$A_x = 90\,N \leftarrow, \quad A_y = 30\,N \uparrow$ ◀

$E_x = 90\,N \rightarrow, \quad E_y = 30\,N \downarrow$ ◀

6.86



FIND: COMPONENTS OF REACTIONS AT A AND E IF A 36 N·m ↻ COUPLE IS APPLIED (a) AT B, (b) AT D.



FREE BODY: ENTIRE FRAME

THE FOLLOWING ANALYSIS IS VALID FOR BOTH (a) AND (b) SINCE THE POINT OF APPLICATION OF THE COUPLE IS IMMATERIAL.

$+\circlearrowleft \Sigma M_E = 0: \quad -36\,N\cdot m - A_x(0.2\,m) = 0$

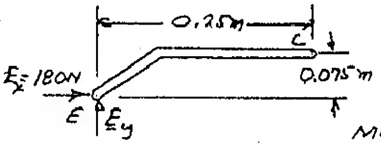
$A_x = -180\,N \qquad A_x = 180\,N \leftarrow$

$\xrightarrow{+} \Sigma F_x = 0: \quad -180\,N + E_x = 0$

$E_x = 180\,N \qquad E_x = 180\,N \rightarrow$

$+\uparrow \Sigma F_y = 0: \quad A_y + E_y = 0 \qquad (1)$

(a) COUPLE APPLIED AT B. FREE BODY: MEMBER CE



AC IS A TWO-FORCE MEMBER. THUS, THE REACTION AT E MUST BE DIRECTED ALONG EC.

$\frac{E_y}{180\,N} = \frac{0.075\,m}{0.25\,m} \qquad E_y = 54\,N \uparrow$

FROM EQ.(1): $A_y + 54\,N = 0$

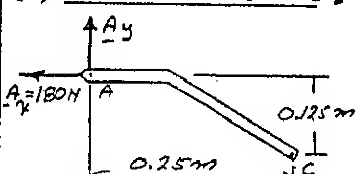
$A_y = -54\,N \qquad A_y = 54\,N \downarrow$

THUS, REACTIONS ARE

$A_x = 180\,N \leftarrow, \; A_y = 54\,N \downarrow$ ◀

$E_x = 180\,N \rightarrow, \; E_y = 54\,N \uparrow$ ◀

(b) COUPLE APPLIED AT D. FREE BODY: MEMBER AC



AC IS A TWO-FORCE MEMBER. THUS, THE REACTION AT A MUST BE DIRECTED ALONG EC.

$\frac{A_y}{180\,N} = \frac{0.125\,m}{0.25\,m} \qquad A_y = 90\,N \uparrow$

FROM EQ.(1): $A_y + E_y = 0$

$90\,N + E_y = 0$

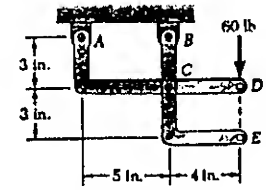
$E_y = -90\,N \qquad E_y = 90\,N \downarrow$

THUS, REACTIONS ARE

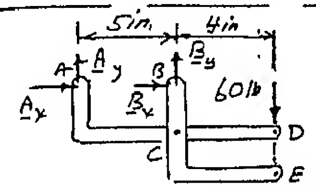
$A_y = 180\,N \leftarrow, \; A_y = 90\,N \uparrow$ ◀

$E_x = 180\,N \rightarrow, \; E_y = 90\,N \downarrow$ ◀

6.87



FIND: THE COMPONENTS OF THE REACTIONS AT A AND B WHEN THE 60-lb LOAD IS (a) APPLIED AT D, (b) APPLIED AT E.



FREE BODY: ENTIRE FRAME

THE FOLLOWING ANALYSIS IS VALID FOR BOTH (a) AND (b) SINCE THE POSITION OF THE LOAD ALONG ITS LINE OF ACTION IS IMMATERIAL.

$+\circlearrowleft \Sigma M_B = 0: \; -A_y(5\text{ in.}) - (60\text{ lb})(4\text{ in.}) = 0$

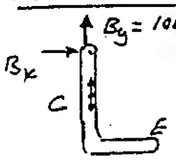
$A_y = -48\text{ lb} \qquad A_y = 48\text{ lb}\downarrow$

$+\uparrow \Sigma F_y = 0: \; -60\text{ lb} + B_y - 48\text{ lb} = 0$

$B_y = 108\text{ lb} \qquad B_y = 108\text{ lb}\uparrow$

$\xrightarrow{+} \Sigma F_x = 0: \; A_x + B_x = 0 \qquad (1)$

(a) LOAD APPLIED AT D. FREE BODY: MEMBER BCE



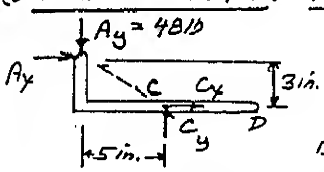
BCE IS A TWO-FORCE MEMBER. THUS, REACTION AT B IS: $B = 108\text{ lb}\uparrow$ AND $B_x = 0$

FROM EQ(1): $A_x + B_x = 0; \; A_x + 0 = 0 \qquad A_x = 0$

THUS, REACTIONS ARE:

$A = 48\text{ lb}\downarrow, \; B = 108\text{ lb}\uparrow$ ◀

(b) LOAD APPLIED AT E. FREE BODY: MEMBER ACD



ACD IS A TWO-FORCE MEMBER. THUS, THE REACTION AT A MUST BE DIRECTED ALONG AC

$\frac{48\text{ lb}}{A_x} = \frac{3\text{ in}}{5\text{ in.}} \qquad A_x = 80\text{ lb}\rightarrow$

FROM EQ(1): $A_x + B_x = 0$

$80\text{ lb} + B_x = 0$

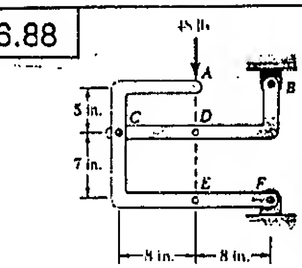
$B_x = -80\text{ lb} \qquad B_x = 80\text{ lb}\leftarrow$

THUS, REACTIONS ARE:

$A_x = 80\text{ lb}\rightarrow, \; A_y = 48\text{ lb}\downarrow$ ◀

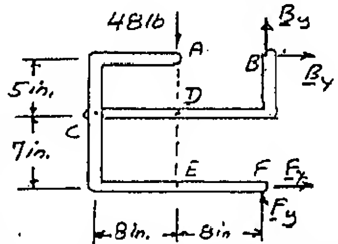
$B_x = 80\text{ lb}\leftarrow, \; B_y = 108\text{ lb}\uparrow$ ◀

6.88



FIND: THE COMPONENTS OF THE REACTIONS AT B AND F WHEN THE 48-lb LOAD IS APPLIED

(a) AT A.
(b) AT D.
(c) AT E.



FREE BODY: ENTIRE FRAME

THE FOLLOWING ANALYSIS IS VALID FOR (a), (b), AND (c), SINCE THE POSITION OF THE LOAD ALONG ITS LINE OF ACTION IS IMMATERIAL

$+\circlearrowleft \Sigma M_F = 0: \; (48\text{ lb})(8\text{ in.}) - B_x(12\text{ in.}) = 0$

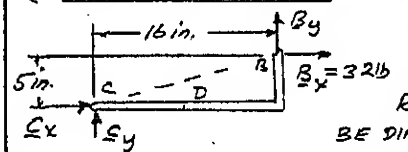
$B_x = 32\text{ lb} \qquad B_x = 32\text{ lb}\rightarrow$

$\xrightarrow{+} \Sigma F_x = 0: \; 32\text{ lb} + F_x = 0$

$F_x = -32\text{ lb} \qquad F_x = 32\text{ lb}\leftarrow$

$+\uparrow \Sigma F_y = 0: \; B_y + F_y - 48\text{ lb} = 0 \qquad (1)$

(a) LOAD APPLIED AT A. FREE BODY: MEMBER CDB



CDB IS A TWO-FORCE MEMBER. THUS, THE REACTION AT B MUST BE DIRECTED ALONG BC.

$\frac{B_y}{32\text{ lb}} = \frac{5\text{ in.}}{16\text{ in.}} \qquad B_y = 10\text{ lb}\uparrow$

FROM EQ(1): $10\text{ lb} + F_y - 48\text{ lb} = 0$

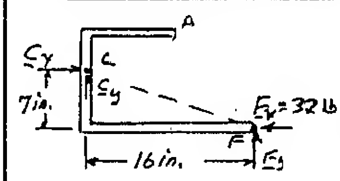
$F_y = 38\text{ lb} \qquad F_y = 38\text{ lb}\uparrow$

THUS, REACTIONS ARE:

$B_x = 32\text{ lb}\rightarrow, \; B_y = 10\text{ lb}\uparrow$ ◀

$F_x = 32\text{ lb}\leftarrow, \; F_y = 38\text{ lb}\uparrow$ ◀

(b) LOAD APPLIED AT D. FREE BODY: MEMBER ACF



ACF IS A TWO-FORCE MEMBER. THUS, THE REACTION AT F MUST BE DIRECTED ALONG CF.

$\frac{F_y}{32\text{ lb}} = \frac{7\text{ in.}}{16\text{ in.}} \qquad F_y = 14\text{ lb}\uparrow$

FROM EQ(1): $B_y + 14\text{ lb} - 48\text{ lb} = 0$

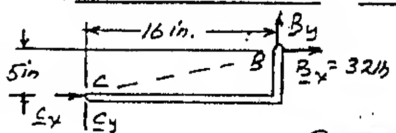
$B_y = 34\text{ lb} \qquad B_y = 34\text{ lb}\uparrow$

THUS REACTIONS ARE:

$B_x = 32\text{ lb}\leftarrow, \; B_y = 34\text{ lb}\uparrow$ ◀

$F_x = 32\text{ lb}\rightarrow, \; F_y = 14\text{ lb}\uparrow$ ◀

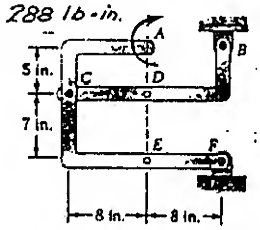
(c) LOAD APPLIED AT E: FREE BODY: MEMBER CDB



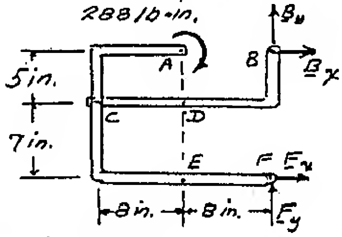
THIS IS THE SAME FREE BODY AS IN PART (a).

REACTIONS ARE SAME AS (a) ◀

6.89



FIND: THE COMPONENTS OF THE REACTIONS AT B AND F WHEN THE 288-lb·in. COUPLE IS APPLIED
(a) AT A.
(b) AT D.
(c) AT E.



FREE BODY: ENTIRE FRAME

THE FOLLOWING ANALYSIS IS VALID FOR (a), (b), AND (c), SINCE THE POINT OF APPLICATION OF THE COUPLE IS IMMATERIAL.

$+\circlearrowleft \Sigma M_F = 0: \quad -288 \text{ lb·in.} - B_x(12 \text{ in.}) = 0$

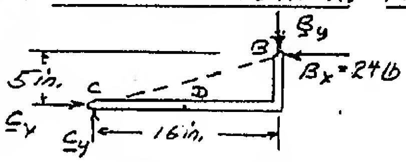
$B_x = -24 \text{ lb} \qquad B_x = 24 \text{ lb} \leftarrow$

$\xrightarrow{+} \Sigma F_x = 0: \quad -24 \text{ lb} + F_x = 0$

$F_x = 24 \text{ lb} \qquad F_x = 24 \text{ lb} \rightarrow$

$+\uparrow \Sigma F_y = 0: \quad B_y + F_y = 0 \qquad (1)$

(a) COUPLE APPLIED AT A. FREE BODY: MEMBER CDB



CDB IS A TWO-FORCE MEMBER, THUS REACTION AT B MUST BE DIRECTED ALONG BC.

$\dfrac{B_y}{24 \text{ lb}} = \dfrac{5 \text{ in.}}{16 \text{ in.}} \qquad B_y = 7.5 \text{ lb} \downarrow$

FROM EQ.(1): $-7.5 \text{ lb} + F_y = 0$

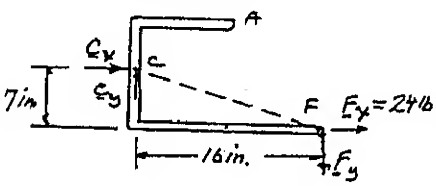
$F_y = 7.5 \text{ lb} \qquad F_y = 7.5 \text{ lb} \uparrow$

THUS, REACTIONS ARE:

$B_x = 24 \text{ lb} \leftarrow, \quad B_y = 7.5 \text{ lb} \downarrow$ ◀

$F_x = 24 \text{ lb} \rightarrow, \quad F_y = 7.5 \text{ lb} \uparrow$ ◀

(b) COUPLE APPLIED AT D. FREE BODY: MEMBER ACF



ACF IS A TWO-FORCE MEMBER. THUS, THE REACTION AT F MUST BE DIRECTED ALONG CF.

$\dfrac{F_y}{24 \text{ lb}} = \dfrac{7 \text{ in.}}{16 \text{ in.}} \qquad F_y = 10.5 \text{ lb} \downarrow$

FROM EQ.(1): $B_y - 10.5 \text{ lb} = 0$

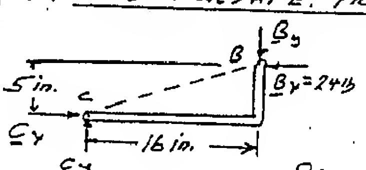
$B_y = +10.5 \text{ lb} \qquad B_y = 10.5 \text{ lb} \uparrow$

THUS, REACTIONS ARE:

$B_x = 24 \text{ lb} \leftarrow, \quad B_y = 10.5 \text{ lb} \uparrow$ ◀

$F_x = 24 \text{ lb} \rightarrow, \quad F_y = 10.5 \text{ lb} \downarrow$ ◀

(c) COUPLE APPLIED AT E. FREE BODY: MEMBER CDB

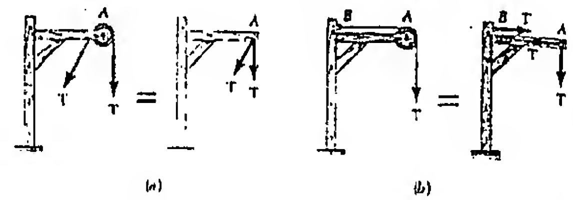


THIS IS THE SAME FREE BODY AS IN PART (a).

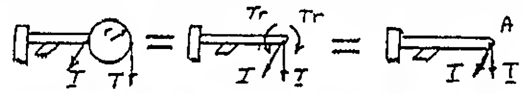
REACTION ARE SAME AS IN (a) ◀

6.90

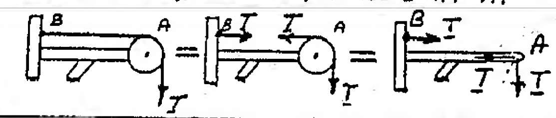
SHOW THAT THE LOADINGS SHOWN ARE EQUIVALENT IN (a) AND THEN IN (b).



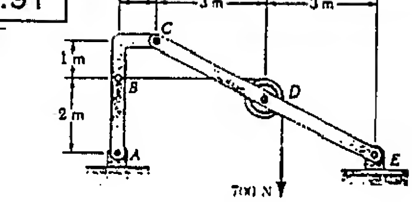
(a) REPLACE EACH FORCE BY A FORCE-COUPLE SYSTEM



(b) CUT CABLE AS SHOWN AND REPLACE FORCES ON PULLEY BY EQUIVALENT FORCES AT A.

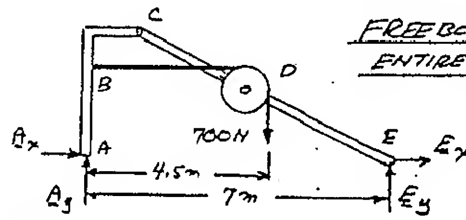


6.91



GIVEN: RADIUS OF PULLEY = 0.5 m

FIND: COMPONENTS OF REACTIONS AT A AND E.



FREE BODY: ENTIRE ASSEMBLY

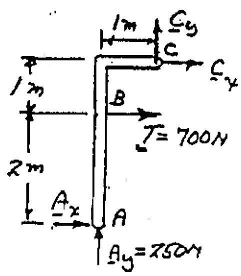
$+\circlearrowleft \Sigma M_A = 0: \quad E_y(7 \text{ m}) - (700 \text{ N})(4.5 \text{ m}) = 0$

$E_y = 450 \text{ N} \qquad E_y = 450 \text{ N} \uparrow$

$+\uparrow \Sigma F_y = 0: \quad A_y + 450 \text{ N} - 700 \text{ N} = 0$

$A_y = 250 \text{ N} \qquad A_y = 250 \text{ N} \uparrow$

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + E_x = 0 \qquad (1)$



FREE BODY: MEMBER ABC

$+\circlearrowleft \Sigma M_C = 0:$

$(700 \text{ N})(1 \text{ m}) + A_x(3 \text{ m}) - (250 \text{ N})(1 \text{ m}) = 0$

$A_x = -150 \text{ N} \qquad A_x = 150 \text{ N} \leftarrow$

FROM EQ.(1):

$A_x + E_x = 0$

$-150 \text{ N} + E_x = 0$

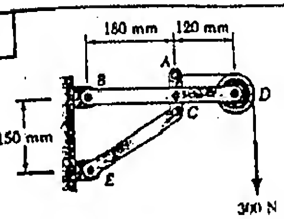
$E_x = 150 \text{ N} \qquad E_x = 150 \text{ N} \rightarrow$

THUS, REACTIONS ARE:

$A_x = 150 \text{ N} \leftarrow, \quad A_y = 250 \text{ N} \uparrow$ ◀

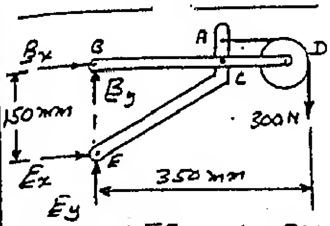
$E_x = 150 \text{ N} \rightarrow, \quad E_y = 450 \text{ N} \uparrow$ ◀

6.92



GIVEN: RADIUS OF PULLEY = 50 mm.

FIND: COMPONENTS OF REACTIONS AT B AND E.



FREE BODY: ENTIRE ASSEMBLY

$+\circlearrowleft \Sigma M_E = 0:$

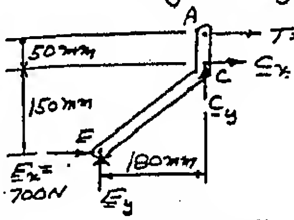
$-(300\,N)(350\,mm) - B_x(150\,mm) = 0$

$B_x = -700\,N \qquad B_x = 700\,N \leftarrow$

$\xrightarrow{+} \Sigma F_x = 0: \quad -700\,N + E_x = 0$

$E_x = 700\,N \qquad E_x = 700\,N \rightarrow$

$+\uparrow \Sigma F_y = 0: \quad B_y + E_y - 300\,N = 0 \qquad (1)$



FREE BODY: MEMBER ACE

$+\circlearrowleft \Sigma M_C = 0: \ (700\,N)(150\,mm)$

$-(300\,N)(50\,mm) - E_y(180\,mm) = 0$

$E_y = 500\,N \qquad E_y = 500\,N \uparrow$

FROM EQ.(1): $B_y + 500\,N - 300\,N = 0$

$B_y = -200\,N \qquad B_y = 200\,N \downarrow$

THUS, REACTIONS ARE:

$B_x = 700\,N \leftarrow, \quad B_y = 200\,N \downarrow$ ◀

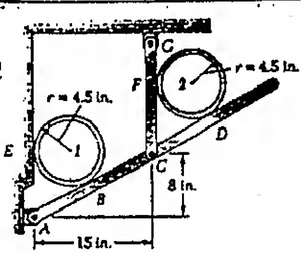
$E_x = 700\,N \rightarrow, \quad E_y = 500\,N \uparrow$ ◀

6.93 and 6.94

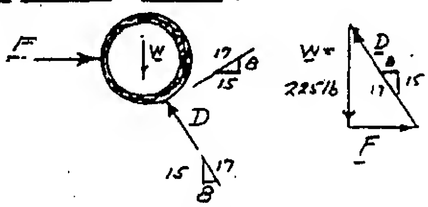
GIVEN: PIPES WEIGH 30 lb/ft

FRAMES SPACED AT 7.5 ft.

FIND: COMPONENTS OF REACTIONS AT A AND G.



FREE BODY: PIPE 2 $\qquad W = (30\,lb/ft)(7.5\,ft) = 225\,lb$

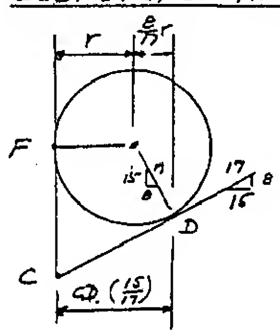


$\frac{F}{8} = \frac{D}{17} = \frac{225\,lb}{15}$

$F = 120\,lb \rightarrow$

$D = 255\,lb \nwarrow$

GEOMETRY OF PIPE 2 $\qquad r = 4.5\,in.$



BY SYMMETRY: $CF = CD \qquad (1)$

EQUATE HORIZONTAL DISTANCES

$r + \frac{8}{17}r = CD\left(\frac{15}{17}\right)$

$\frac{25}{17}r = CD\left(\frac{15}{17}\right)$

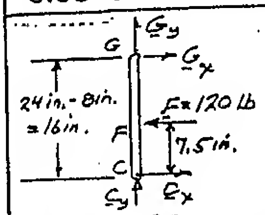
$CD = \frac{25}{15}r = \frac{5}{3}r$

FROM EQ.(1) $CF = \frac{5}{3}r = \frac{5}{3}(4.5\,in.)$

$CF = 7.5\,in.$

(CONTINUED)

6.93 and 6.94 CONTINUED



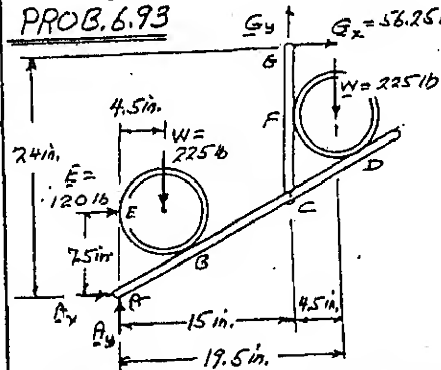
FREE BODY: MEMBER CFG

$+\circlearrowleft \Sigma M_C = 0$

$(120\,lb)(7.5\,in.) - G_x(16\,in.) = 0$

$G_x = 56.25\,lb \qquad G_x = 56.3\,lb \rightarrow$ ◀

PROB. 6.93



FREE BODY: FRAME AND PIPES

NOTE: PIPE 2 IS SIMILAR TO PIPE 1.

$AE = CF = 7.5\,in.$

$E = F = 120\,lb$

$+\circlearrowleft \Sigma M_A = 0: \ G_y(15\,in.) - (56.25\,lb)(24\,in.) - (225\,lb)(4.5\,in.)$

$-(225\,lb)(19.5\,in.) - (120\,lb)(7.5\,in.) = 0$

$G_y = 510\,lb \qquad G_y = 510\,lb \uparrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \ A_x + 120\,lb + 56.25\,lb = 0$

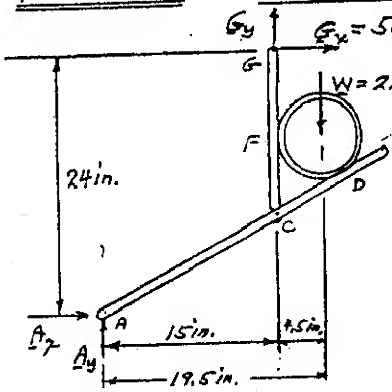
$A_x = 176.25\,lb \qquad A_x = 176.3\,lb \leftarrow$ ◀

$+\uparrow \Sigma F_y = 0: \ A_y + 510\,lb - 225\,lb - 225\,lb = 0$

$A_y = -60\,lb \qquad A_y = 60\,lb \downarrow$ ◀

PROB. 6.94 $\qquad$ FREE BODY: FRAME AND PIPE 2

$G_x = 56.3\,lb \rightarrow$ ◁



$+\circlearrowleft \Sigma M_A = 0:$

$G_y(15\,in.) - (56.25\,lb)(24\,in.) - (225\,lb)(19.5\,in.) = 0$

$G_y = 382.5\,lb \qquad G_y = 383\,lb \uparrow$ ◀

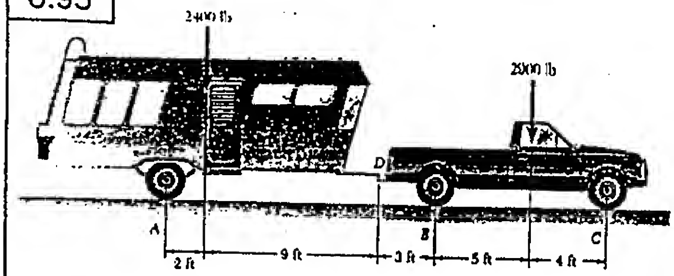
$\xrightarrow{+} \Sigma F_x = 0: \ A_x + 56.25\,lb$

$A_x = -56.25\,lb \qquad A_x = 56.3\,lb \leftarrow$ ◀

$+\uparrow \Sigma F_y = 0: \ A_y + 382.5\,lb - 225\,lb = 0$

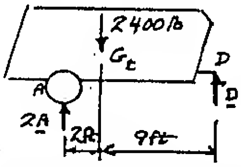
$A_y = -157.5\,lb \qquad A_y = 157.5 \downarrow$ ◀

6.95



FIND: (a) REACTIONS AT EACH OF THE SIX WHEELS. (b) ADDITIONAL LOAD ON EACH WHEEL DUE TO THE TRAILER

(a)



FREE BODY: TRAILER

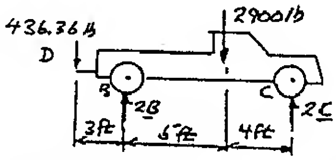
(WE SHALL DENOTE BY A, B, C THE REACTION AT ONE WHEEL)

$+\circlearrowleft \Sigma M_A = 0: \; -(2400\,lb)(2\,ft) + D(11\,ft) = 0$

$D = 436.36\,lb$

$+\uparrow \Sigma F_y = 0: \; 2A - 2400\,lb + 436.36\,lb = 0$

$A = 981.82\,lb$ $\qquad \underline{A} = 982\,lb \uparrow$ ◀



FREE BODY: TRUCK

$+\circlearrowleft \Sigma M_B = 0:$

$(436.36\,lb)(3\,ft)$

$-(2900\,lb)(5\,ft) + 2C(9\,ft) = 0$

$C = 732.83\,lb$ $\qquad \underline{C} = 733\,lb \uparrow$ ◀

$+\uparrow \Sigma F_y = 0: \; 2B - 436.36\,lb - 2900\,lb + 2(732.83\,lb) = 0$

$B = 935.35\,lb$ $\qquad \underline{B} = 935\,lb \uparrow$ ◀

(b) ADDITIONAL LOAD ON TRUCK WHEELS

USE FREE BODY DIAGRAM OF TRUCK WITHOUT 2900 lb.

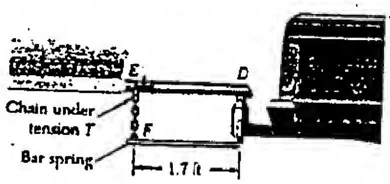
$+\circlearrowleft \Sigma M_B = 0: \; (436.36\,lb)(3\,ft) + 2C(9\,ft) = 0$

$C = -72.73\,lb$ $\qquad \Delta C = -72.7\,lb$ ◀

$+\uparrow \Sigma F_y = 0: \; 2B - 436.36\,lb - 2(72.73\,lb) = 0$

$B = 290.9\,lb$ $\qquad \Delta B = +291\,lb$ ◀

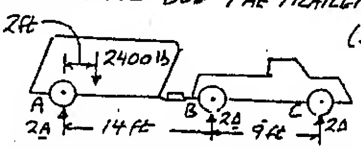
6.96



FIND: (a) TENSION IN EACH CHAIN FOR EQUAL ADDITIONAL LOAD ON TRUCK WHEELS. (b) REACTION AT EACH WHEEL.

(a) WE SHALL FIRST FIND THE ADDITIONAL REACTION "Δ" AT EACH WHEEL DUE THE TRAILER. FREE BODY DIAGRAM

(SAME Δ AT EACH TRUCK WHEEL)



$+\circlearrowleft \Sigma M_A = 0: -(2400\,lb)(2\,ft)$

$+2\Delta(14\,ft) + 2\Delta(23\,ft) = 0$

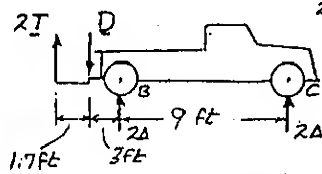
$\Delta = 64.86\,lb$

$+\uparrow \Sigma F_y = 0: \; 2A - 2400\,lb + 4(64.86\,lb) = 0; \; A = 1070\,lb; \; \underline{A} = 1077\,lb \uparrow$

(CONTINUED

6.96 CONTINUED

FREE BODY: TRUCK (TRAILER LOADING ONLY)



$+\circlearrowleft \Sigma M_D = 0:$

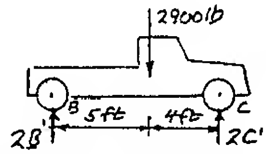
$2\Delta(12\,ft) + 2\Delta(3\,ft) - 2T(1.7\,ft)$

$T = 8.824\,\Delta$

$= 8.824(64.86\,lb)$

$T = 572.3\,lb$

$T = 572\,lb$ ◀



FREE BODY: TRUCK (TRUCK WEIGHT ONLY)

$+\circlearrowleft \Sigma M_B = 0:$

$-(2900\,lb)(5\,ft) + 2C'(9\,ft) = 0$

$C' = 805.6\,lb$ $\qquad \underline{C}' = 805.6\,lb \uparrow$

$+\uparrow \Sigma F_y = 0: \; 2B' - 2900\,lb + 2(805.6\,lb) = 0$

$B' = 644.4\,lb$ $\qquad \underline{B}' = 644.4\,lb \uparrow$

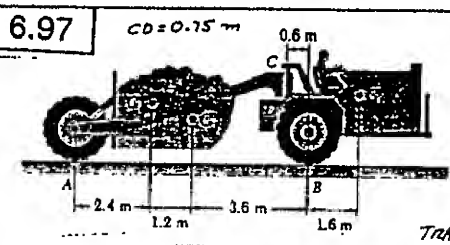
ACTUAL REACTIONS

$B = B' + \Delta = 644.4\,lb + 64.86 = 709.2\,lb$ $\qquad \underline{B} = 709\,lb \uparrow$ ◀

$C = C' + \Delta = 805.6\,lb + 64.86 = 870.46\,lb$ $\qquad \underline{C} = 870\,lb \uparrow$ ◀

(FROM PART a): $\qquad \underline{A} = 1077\,lb \uparrow$ ◀

6.97



GIVEN: $m_\ell = 45\,Mg$, $m_s = 8\,Mg$, $m_t = 10\,Mg$

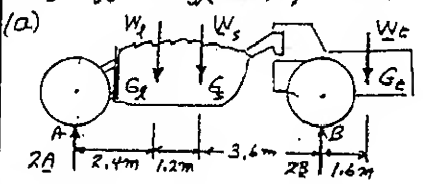
FIND: (a) REACTIONS AT EACH OF 4 WHEELS, (b) FORCES ACTING ON TRACTOR AT C AND D.

$W_\ell = m_\ell g = (45\,Mg)(9.81\,m/s^2) = 441.45\,kN$

$W_s = m_s g = (8\,Mg)(9.81\,m/s^2) = 78.48\,kN$

$W_t = m_t g = (10\,Mg)(9.81\,m/s^2) = 98.1\,kN$

(a)



FREE BODY: ENTIRE MACHINE

$+\circlearrowleft \Sigma M_A = 0: \; 2B(7.2\,m) - W_\ell(2.4\,m) - W_s(3.6\,m) - W_t(8.8\,m) = 0$

$2B(7.2\,m) - (441.45\,kN)(2.4\,m) - (78.48\,kN)(3.6\,m) - (98.1\,kN)(8.8\,m) = 0$

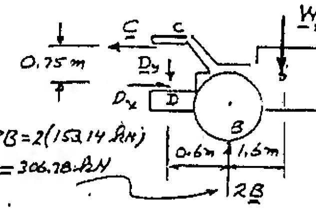
$B = 153.14\,kN$ $\qquad \underline{B} = 153.1\,kN \uparrow$ ◀

$+\uparrow \Sigma F_y = 0: \; 2A + 2(153.14\,kN) - 441.45\,kN - 78.48\,kN - 98.1\,kN = 0$

$A = 155.87\,kN$ $\qquad \underline{A} = 155.9\,kN \uparrow$ ◀

(b)

FREE BODY: TRACTOR



$2B = 2(153.14\,kN)$

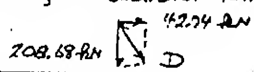
$= 306.28\,kN$

$+\circlearrowleft \Sigma M_D = 0: \; C(0.75\,m) + (306.28\,kN)(0.6\,m) - (98.1\,kN)(2.2\,m) = 0$

$C = 42.74\,kN$ $\qquad \underline{C} = 42.7\,kN \leftarrow$ ◀

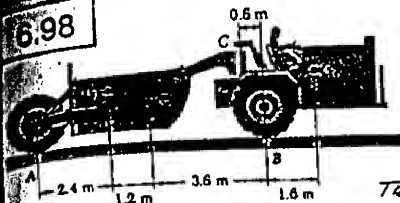
$\xrightarrow{+} \Sigma F_x = 0: -42.74\,kN + D_x = 0;$ $\qquad \underline{D}_x = 42.74\,kN \rightarrow$

$+\uparrow \Sigma F_y = 0: \; 306.28\,kN - 98.1\,kN - D_y = 0;$ $\qquad \underline{D}_y = 208.18\,kN \downarrow$



$\underline{D} = 213\,kN \measuredangle 78.4°$ ◀

6.98



GIVEN: $m_s = 8\ Mg$

$m_t = 10\ Mg$

(LOAD REMOVED)

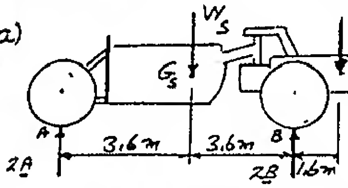
FIND: (a) REACTIONS AT EACH OF 4 WHEELS.

(b) FORCES ACTING ON TRACTOR AT C AND D.

$W_s = m_s g = (8\ Mg)(9.81\ m/s^2) = 78.48\ kN$

$W_t = m_t g = (10\ Mg)(9.81\ m/s^2) = 98.1\ kN$

(a)



FREE BODY: ENTIRE MACHINE

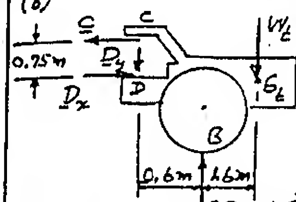
$+\circlearrowleft \Sigma M_B = 0: \ -2A(7.2\ m) + (78.48\ kN)(3.6\ m) - (98.1\ kN)(1.6\ m) = 0$

$A = 8.72\ kN \qquad \underline{A} = 8.72\ kN \uparrow$ ◀

$+\uparrow \Sigma F_y = 0: \ 2(8.72\ kN) + 2B - 78.48\ kN - 98.1\ kN = 0$

$B = 79.57\ kN \qquad \underline{B} = 79.6\ kN \uparrow$ ◀

(b)



FREE BODY: TRACTOR

$2B = 2(79.57\ kN)$

$2B = 159.14\ kN$

$+\circlearrowleft \Sigma M_D = 0: \ C(0.75\ m) + (159.14\ kN)(0.6\ m) - (98.1\ kN)(2.2\ m) = 0$

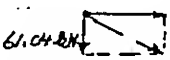
$C = 160.4\ kN \qquad \underline{C} = 160.4\ kN \leftarrow$ ◀

$+\uparrow \Sigma F_y = 0: \ 159.14\ kN - 98.1\ kN - D_y = 0$

$D_y = 61.04\ kN \qquad D_y = 61.04\ kN \downarrow$

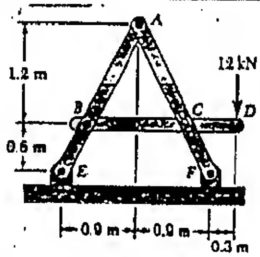
$\xrightarrow{+} \Sigma F_x = 0: \ -160.4\ kN + D_x = 0$

$D_x = 160.4\ kN \qquad D_x = 160.4\ kN \rightarrow$

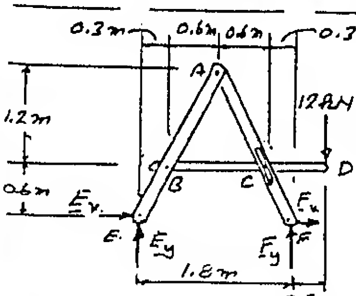


$\underline{D} = 171.6\ kN \ \searrow 20.8^\circ$ ◀

6.99



FIND: COMPONENTS OF ALL FORCES ACTING ON MEMBER ABE.



FREE BODY: ENTIRE FRAME

$+\circlearrowleft \Sigma M_E = 0:$

$F_y(1.8\ m) - (12\ kN)(2.1\ m) = 0$

$F_y = 14\ kN$

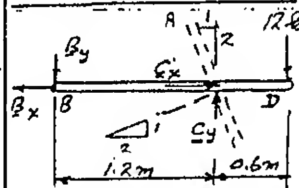
$\underline{F}_y = 14\ kN \uparrow$

$+\uparrow \Sigma F_y = 0: \ E_y + 14\ kN - 12\ kN = 0$

$E_y = -2\ kN \qquad \underline{E}_y = 2\ kN \downarrow$

(CONTINUED)

6.99 CONTINUED



FREE BODY: MEMBER BCD

$+\circlearrowleft \Sigma M_B = 0:$

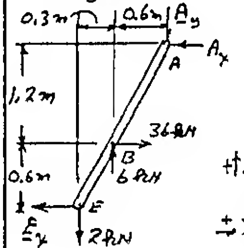
$C_y(1.2\ m) - (12\ kN)(1.8\ m) = 0$

$C_y = 18\ kN$

$\frac{C_y}{C_x} = \frac{1}{2}; \ \frac{18\ kN}{C_x} = \frac{1}{2}; \ C_x = 36\ kN$

$\xrightarrow{+} \Sigma F_x = 0: \ -B_x + 36\ kN = 0 \qquad B_x = 36\ kN$

$+\uparrow \Sigma F_y = 0: \ -B_y + 18\ kN - 12\ kN = 0 \qquad B_y = 6\ kN$



FREE BODY: MEMBER ABE

$+\circlearrowleft \Sigma M_A = 0: \ -E_x(1.8\ m) + (2\ kN)(0.9\ m)$

$+ (36\ kN)(1.2\ m) - (6\ kN)(0.6\ m) = 0$

$E_x = 23\ kN \qquad \underline{E}_x = 23\ kN \leftarrow$

$+\uparrow \Sigma F_y = 0: \ -A_y + 6\ kN - 2\ kN = 0$

$A_y = 4\ kN \qquad \underline{A}_y = 4\ kN \downarrow$

$\xrightarrow{+} \Sigma F_x = 0: \ -A_x + 36\ kN - 23\ kN = 0$

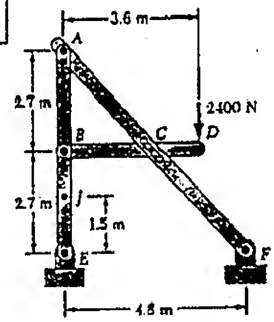
$A_x = 13\ kN \qquad \underline{A}_x = 13\ kN \leftarrow$

FORCES ACTING ON ABE:

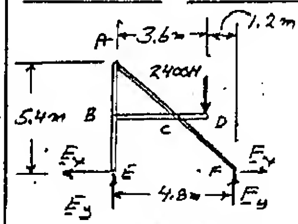
$\underline{A}_x = 13\ kN \leftarrow, \ \underline{A}_y = 4\ kN \downarrow; \ \underline{B}_x = 36\ kN \rightarrow, \ \underline{B}_y = 6\ kN \uparrow$ ◀

$\underline{E}_x = 13\ kN \leftarrow, \ \underline{E}_y = 2\ kN \downarrow.$ ◀

6.100



FIND: COMPONENTS OF ALL FORCES ACTING ON MEMBER ABE.



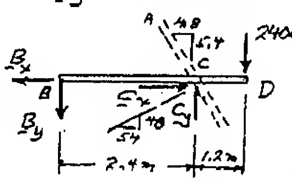
FREE BODY: ENTIRE FRAME

$+\circlearrowleft \Sigma M_F = 0$

$(2400\ N)(1.2\ m) - E_y(4.8\ m) = 0$

$E_y = 600\ N$

$\underline{E}_y = 600\ N \uparrow$



FREE BODY: MEMBER BCD

$+\circlearrowleft \Sigma M_B = 0:$

$C_y(2.4\ m) - (2400\ N)(3.6\ m) = 0$

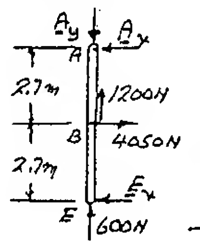
$C_y = +3600\ N$

$\frac{C_x}{C_y} = \frac{54}{48}; \ \frac{C_x}{3600\ N} = \frac{54}{48}$

$C_x = +4050\ N$

$\xrightarrow{+} \Sigma F_x = 0: \ -B_x + 4050\ N = 0 \qquad B_x = +4050\ N$

$+\uparrow \Sigma F_y = 0: \ -B_y + 3600\ N - 2400\ N = 0 \qquad B_y = +1200\ N$



FREE BODY: MEMBER ABE

FROM ABOVE: $\underline{E}_y = 600\ N \uparrow$ ◀

$\underline{B}_x = 4050\ N \rightarrow, \ \underline{B}_y = 1200\ N \uparrow$ ◀

$+\circlearrowleft \Sigma M_A = 0: \ (4050\ N)(2.7\ m) - E_x(5.4\ m) = 0$

$E_x = +2025\ N \qquad \underline{E}_x = 2025\ N \leftarrow$ ◀

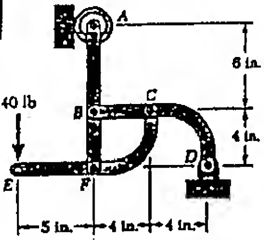
$+\uparrow \Sigma F_y = 0: \ -A_y + 1200\ N + 600\ N = 0$

$A_y = +1800\ N \qquad \underline{A}_y = 1800\ N \downarrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \ -A_x + 4050\ N - 2025\ N = 0$

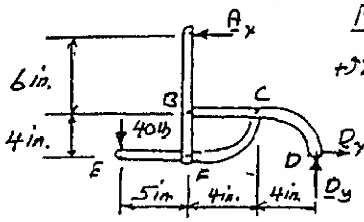
$A_x = +2025\ N \qquad \underline{A}_x = 2025\ N \leftarrow$ ◀

6.101



FIND: COMPONENTS OF FORCES ACTING ON MEMBER CFE AT C AND F.

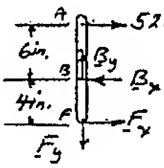
FREE BODY: ENTIRE FRAME



$+\circlearrowleft \Sigma M_D = 0: (40\,lb)(13\,in.) + A_x(10\,in.) = 0$

$A_x = -52\,lb, \quad \underline{A}_x = 52\,lb \rightarrow$

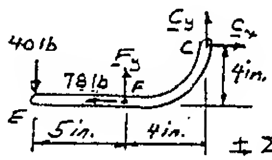
FREE BODY: MEMBER ABF



$+\circlearrowleft \Sigma M_B = 0: -(52\,lb)(6\,in.) + F_x(4\,in.) = 0$

$F_x = +78\,lb$

FREE BODY: MEMBER CFE



FROM ABOVE: $\underline{F}_x = 78\,lb \leftarrow$ ◀

$+\circlearrowleft \Sigma M_C = 0: (40\,lb)(9\,in.)$

$-(78\,lb)(4\,in.) - F_y(4\,in.) = 0$

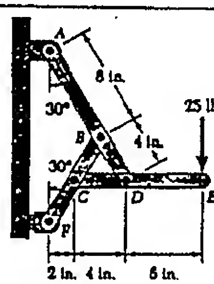
$F_y = +12\,lb \quad \underline{F}_y = 12\,lb \uparrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: C_x - 78\,lb = 0$

$C_x = +78\,lb \quad \underline{C}_x = 78\,lb \rightarrow$ ◀

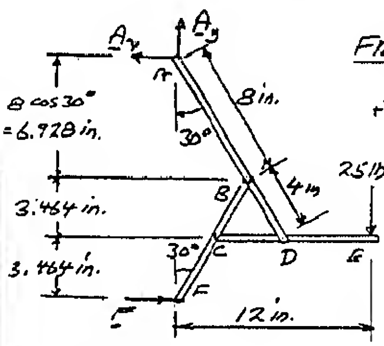
$+\uparrow \Sigma F_y = 0: -40\,lb + 12\,lb + C_y = 0;\ C_y = +28\,lb \quad \underline{C}_y = 28\,lb \uparrow$ ◀

6.102



FIND: COMPONENTS OF FORCES ACTING ON MEMBER CDE AT C AND D.

FREE BODY: ENTIRE FRAME



$+\uparrow \Sigma F_y = 0: A_y - 25\,lb = 0$

$A_y = 25\,lb \quad \underline{A}_y = 25\,lb \uparrow$

$+\circlearrowleft \Sigma M_F = 0: A_x(6.928 + 2\times 3.464) - (25\,lb)(12\,in.) = 0$

$A_x = 21.651\,lb \quad \underline{A}_x = 21.65\,lb \leftarrow$

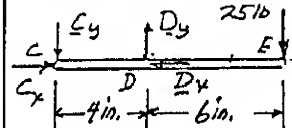
$\xrightarrow{+} \Sigma F_x = 0: \quad F - 21.651\,lb = 0$

$F = 21.651\,lb \qquad \underline{F} = 21.65\,lb \rightarrow$

(CONTINUED)

6.102 CONTINUED

FREE BODY: MEMBER CDE



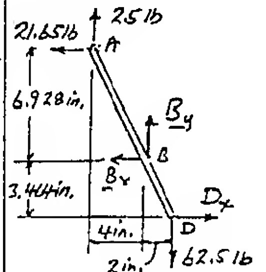
$+\circlearrowleft \Sigma M_C = 0:$

$D_y(4\,in.) - (25\,lb)(10\,in.) = 0$

$D_y = +62.5\,lb \qquad \underline{D}_y = 62.5\,lb \uparrow$ ◀

$+\uparrow \Sigma F_y = 0: -C_y + 62.5\,lb - 25\,lb = 0$

$C_y = +37.5\,lb \quad \underline{C}_y = 37.5\,lb \downarrow$ ◀



FREE BODY: MEMBER ABD

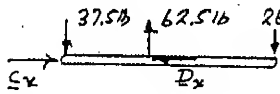
$+\circlearrowleft \Sigma M_B = 0:$

$D_x(3.464\,in.) + (21.65\,lb)(6.928\,in.)$

$-(25\,lb)(4\,in.) - (62.5\,lb)(2\,in.)$

$D_x = +21.65\,lb$

RETURN TO FREE BODY: MEMBER CDE



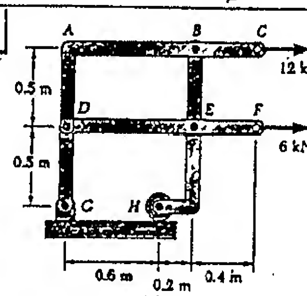
FROM ABOVE

$D_x = +21.65\,lb \quad \underline{D}_x = 21.7\,lb \leftarrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: C_x - 21.65\,lb$

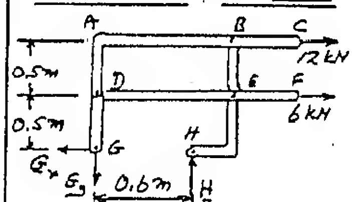
$C_x = +21.65\,lb \quad \underline{C}_x = 21.7\,lb \rightarrow$ ◀

6.103



FIND: COMPONENTS OF FORCES ACTING ON MEMBER DABC AT B AND D.

FREE BODY: ENTIRE FRAME



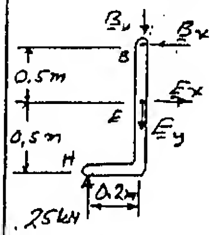
$+\circlearrowleft \Sigma M_G = 0$

$H(0.6\,m) - (12\,kN)(1\,m)$

$-(6\,kN)(0.5\,m) = 0$

$H = 25\,kN \qquad \underline{H} = 25\,kN \uparrow$

FREE BODY: MEMBER BEH

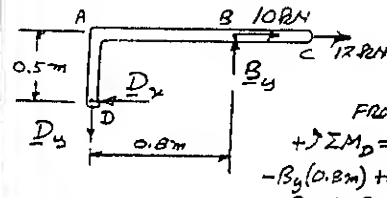


$+\circlearrowleft \Sigma M_E = 0:$

$B_x(0.5\,m) - (25\,kN)(0.2\,m) = 0$

$B_x = +10\,kN$

FREE BODY: MEMBER DABC



FROM ABOVE: $\underline{B}_x = 10\,kN \rightarrow$ ◀

$+\circlearrowleft \Sigma M_D = 0$

$-B_y(0.8\,m) + (10\,kN + 12\,kN)(0.5\,m) = 0$

$B_y = +13.75\,kN \quad \underline{B}_y = 13.75\,kN \uparrow$ ◀

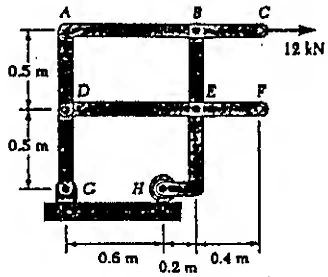
$\xrightarrow{+} \Sigma F_x = 0: -D_x + 10\,kN + 12\,kN = 0$

$D_x = +22\,kN \qquad \underline{D}_x = 22\,kN \leftarrow$ ◀

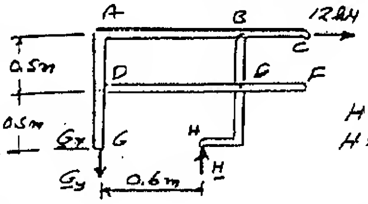
$+\uparrow \Sigma F_y = 0: -D_y + 13.75\,kN = 0$

$D_y = +13.75\,kN \quad \underline{D}_y = 13.75\,kN \downarrow$ ◀

6.104



FIND: COMPONENTS OF FORCES ACTING ON MEMBER DABC AT B AND D.

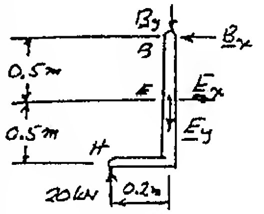


FREE BODY: ENTIRE FRAME

$+\circlearrowleft \Sigma M_G = 0$

$H(0.6\ m) - (12\ kN)(1\ m) = 0$

$H = 20\ kN \qquad H = 20\ kN \uparrow$

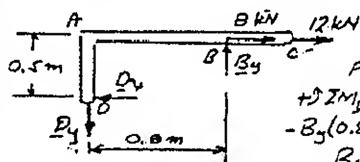


FREE BODY: MEMBER BEH

$+\circlearrowleft \Sigma M_E = 0$

$B_x(0.5\ m) - (20\ kN)(0.2\ m) = 0$

$B_x = +8\ kN$



FREE BODY: MEMBER DABC

FROM ABOVE $B_x = 8\ kN \rightarrow$ ◀

$+\circlearrowleft \Sigma M_D = 0:$

$-B_y(0.8\ m) + (8\ kN + 12\ kN)(0.5\ m) = 0$

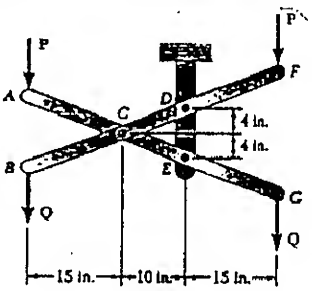
$B_y = +12.5\ kN \qquad B_y = 12.5\ kN \uparrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: -D_x + 8\ kN + 12\ kN = 0$

$D_x = +20\ kN \qquad D_x = 20\ kN \leftarrow$ ◀

$+\uparrow \Sigma F_y = 0: \ -D_y + 12.5\ kN = 0; \ D_y = +12.5\ kN \qquad D_y = 12.5\ kN \downarrow$ ◀

6.105 and 6.106



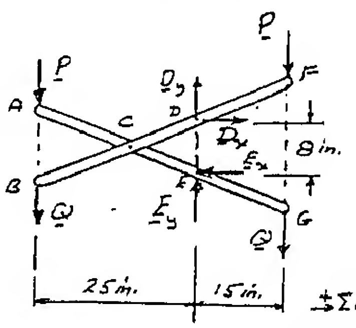
FIND: COMPONENTS OF FORCES ACTING ON

(a) MEMBER BCDF AT C AND D,

(b) MEMBER ACEG AT E,

PROB. 6.105: WHEN $P = 15\ lb$ AND $Q = 65\ lb$.

PROB. 6.106: WHEN $P = 25\ lb$ AND $Q = 55\ lb$.



FREE BODY: ENTIRE FRAME

$+\circlearrowleft \Sigma M_E = 0: (P+Q)(25\ in) - (P+Q)(15\ in) - D_x(8\ in.) = 0$

$D_x = (P+Q)\frac{10}{8}$

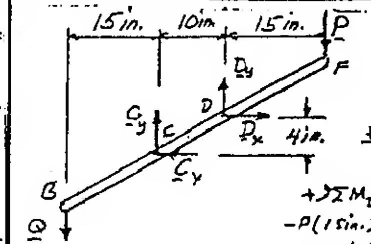
$D_x = (P+Q)\frac{10}{8} \rightarrow$

$\xrightarrow{+} \Sigma F_x = 0: \ -E_x + (P+Q)\frac{10}{8} = 0$

$E_x = (P+Q)\frac{10}{8}; \quad E_x = (P+Q)\frac{10}{8} \leftarrow$

(CONTINUED)

6.105 and 6.106 CONTINUED



FREE BODY: MEMBER BCDF

FROM ABOVE:

$D_x = (P+Q)\frac{10}{8} \rightarrow$ ◁

$\xrightarrow{+} \Sigma F_x = 0: -C_x + D_x = 0$

$C_x = (P+Q)\frac{10}{8} \leftarrow$ ◁

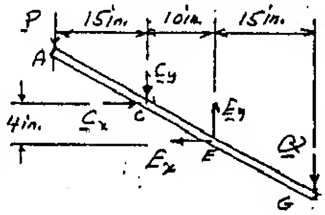
$+\circlearrowleft \Sigma M_D = 0: \ -(P+Q)\frac{10}{8}(4\ in.) - P(15\ in.) + Q(25\ in.) + C_y(10\ in.)$

$C_y = \frac{1}{10}(-20P + 20Q)$

$C_y = 2Q - 2P \qquad C_y = 2Q - 2P \uparrow$ ◁

$+\uparrow \Sigma F_y = 0: D_y + (2Q - 2P) - P - Q = 0$

$D_y = -Q + 3P \qquad D_y = -Q + 3P \uparrow$ ◁



FREE BODY: MEMBER ACEG

FROM ABOVE

$E_x = (P+Q)\frac{10}{8} \leftarrow$ ◁

AND $C_y = 2Q - P$

$+\uparrow \Sigma F_y = 0: E_y - C_y - P - Q = 0$

$E_y - (2Q - 2P) - P - Q = 0$

$E_y = 3Q - P; \quad E_y = 3Q - P \uparrow$ ◁

PROB. 6.105: $P = 15\ lb$ AND $Q = 65\ lb$

$C_x = (P+Q)\frac{10}{8} = (15+65)\frac{10}{8} = +100\ lb \qquad C_x = 100\ lb \leftarrow$ ◀

$C_y = 2Q - 2P = 2(65) - 2(15) = +100\ lb \qquad C_y = 100\ lb \uparrow$ ◀

$D_x = (P+Q)\frac{10}{8} = (15+65)\frac{10}{8} = +100\ lb \qquad D_x = 100\ lb \rightarrow$ ◀

$D_y = -Q + 3P = -65 + 3(15) = -20\ lb \qquad D_y = 20\ lb \downarrow$ ◀

$E_x = (P+Q)\frac{10}{8} = (15+65)\frac{10}{8} = +100\ lb \qquad E_x = 100\ lb \leftarrow$ ◀

$E_y = 3Q - P = 3(65) - 15 = +180\ lb \qquad E_y = 180\ lb \uparrow$ ◀

PROB. 6.106: $P = 25\ lb$ AND $Q = 55\ lb$

$C_x = (P+Q)\frac{10}{8} = (25+55)\frac{10}{8} = +100\ lb \qquad C_x = 100\ lb \leftarrow$ ◀

$C_y = 2Q - 2P = 2(55) - 2(25) = +60\ lb \qquad C_y = 60\ lb \uparrow$ ◀

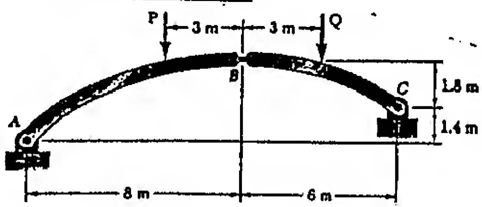
$D_x = (P+Q)\frac{10}{8} = (25+55)\frac{10}{8} = +100\ lb \qquad D_x = 100\ lb \rightarrow$ ◀

$D_y = -Q + 3P = -55 + 3(25) = +20\ lb \qquad D_y = 20\ lb \uparrow$ ◀

$E_x = (P+Q)\frac{10}{8} = (25+55)\frac{10}{8} = +100\ lb \qquad E_x = 100\ lb \leftarrow$ ◀

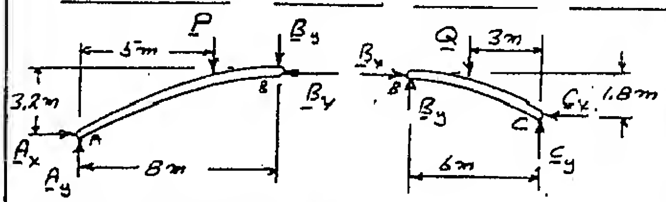
$E_y = 3Q - P = 3(55) - 25 = +140\ lb \qquad E_y = 140\ lb \uparrow$ ◀

6.107 and 6.108



FIND: THE COMPONENTS OF (a) THE REACTION AT A,
(b) THE FORCE EXERTED AT B ON SEGMENT AB

PROB. 6.107: GIVEN THAT $P = 112$ kN AND $Q = 140$ kN.
PROB. 6.108: GIVEN THAT $P = 140$ kN AND $Q = 112$ kN.



FREE BODY: SEGMENT AB:

$+\circlearrowleft \Sigma M_A = 0: \quad B_x(3.2\,m) - B_y(8\,m) - P(5\,m) = 0 \qquad (1)$

$0.75(EQ.1) \quad B_x(2.4\,m) - B_y(6\,m) - P(3.75\,m) = 0 \qquad (2)$

FREE BODY: SEGMENT BC:

$+\circlearrowleft \Sigma M_C = 0: \quad B_x(1.8\,m) + B_y(6\,m) - Q(3\,m) = 0 \qquad (3)$

ADD (2) AND (3): $\quad 4.2 B_x - 3.75P - 3Q = 0$

$B_x = (3.75P + 3Q)/4.2 \qquad (4)$

EQ(1): $\quad (3.75P + 3Q)\frac{3.2}{4.2} - 8B_y - 5P = 0$

$B_y = (-9P + 9.6Q)/33.6 \qquad (5)$

PROB. 6.107: GIVEN THAT $P = 112$ kN AND $Q = 140$ kN

(b) FORCE EXERTED AT B ON AB

EQ(4): $B_x = (3.75 \times 112 + 3 \times 140)/4.2 = 200$ kN

$B_x = 200$ kN ←

EQ(5): $B_y = (-9 \times 112 + 9.6 \times 140)/33.6 = +10$ kN

$B_y = 10$ kN ↓

(a) REACTION AT A.

CONSIDERING AGAIN AB AS A FREE BODY

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B_x = 0; \quad A_x = B_x = 200$ kN

$A_x = 200$ kN →

$+\uparrow \Sigma F_y = 0: \quad A_y - P - B_y = 0$

$A_y - 112\text{ kN} - 10\text{ kN} = 0$

$A_y = +122$ kN $\qquad A_y = 122$ kN ↑

PROB. 6.108 GIVEN THAT $P = 140$ kN AND $Q = 112$ kN.

(b) FORCE EXERTED AT B ON AB.

EQ.(4): $B_x = (3.75 \times 140 + 3 \times 112)/4.2 = 205$ kN

$B_x = 205$ kN ←

EQ(5): $B_y = (-9 \times 140 + 9.6 \times 112)/33.6 = -5.5$ kN

$B_y = 5.5$ kN ↑

(a) REACTION AT A:

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B_x = 0; \quad A_x = B_x = 205$ kN

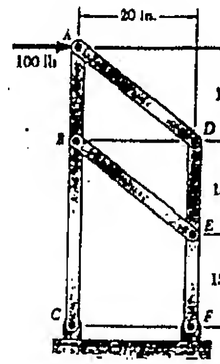
$A_x = 205$ kN →

$+\uparrow \Sigma F_y = 0: \quad A_y - P - B_y = 0$

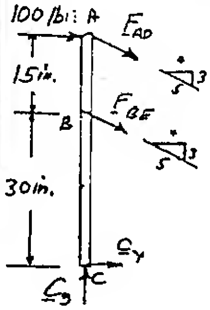
$A_y - 140\text{ kN} - (-5.5\text{ kN}) = 0$

$A_y = 134.5$ kN $\qquad A_y = 134.5$ kN ↑

6.109



FIND:
(a) REACTION AT C
(b) FORCE IN MEMBER AD

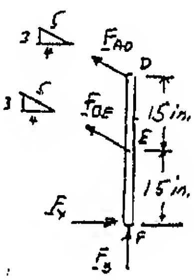


FREE BODY: MEMBER ABC

$+\circlearrowleft \Sigma M_C = 0: \; +(100\,lb)(45\,in.)$

$+ \frac{4}{5}F_{AD}(45\,in.) + \frac{4}{5}F_{BE}(30\,in.) = 0$

$3F_{AD} + 2F_{BE} = -375 \qquad (1)$



FREE BODY: MEMBER DEF

$+\circlearrowleft \Sigma M_F = 0$

$\frac{4}{5}F_{AD}(30\,in.) + \frac{4}{5}F_{BE}(15\,in.) = 0$

$F_{BE} = -2F_{AD} \qquad (2)$

(a) SUBSTITUTE FROM (2) INTO (1)

$3F_{AD} + 2(-2F_{AD}) = -375\,lb$

$F_{AD} = +375\,lb \qquad F_{AD} = 375$ lb ten.

(2) $\quad F_{BE} = -2F_{AD} = -2(375\,lb)$

$F_{BE} = -750\,lb \qquad F_{BE} = 750$ lb comp.

(b) RETURN TO FREE BODY OF MEMBER ABC

$\xrightarrow{+} \Sigma F_x = 0:$

$C_x + 100\,lb + \frac{4}{5}F_{AD} + \frac{4}{5}F_{BE} = 0$

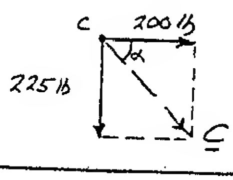
$C_x + 100 + \frac{4}{5}(375) + \frac{4}{5}(-750) = 0$

$C_x = +200\,lb \qquad C_x = 200$ lb →

$+\uparrow \Sigma F_y = 0: \quad C_y - \frac{3}{5}F_{AD} - \frac{3}{5}F_{BE} = 0$

$C_y - \frac{3}{5}(375) - \frac{3}{5}(-750) = 0$

$C_y = -225\,lb \qquad C_y = 225$ lb ↓

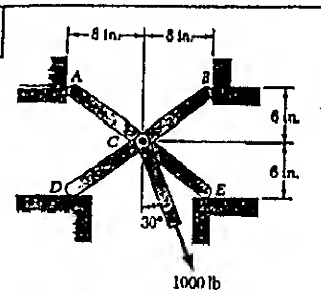


$\alpha = 48.37°$

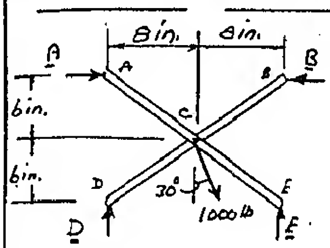
$C = 301.0\,lb$

$C = 301\,lb \; \measuredangle 48.4°$

6.110



FIND: REACTIONS AT A, B, D, AND E.



FREE BODY: ENTIRE FRAME

$\xrightarrow{+} \Sigma F_x = 0$:

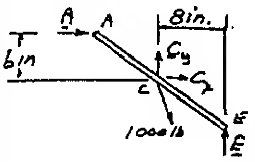
$A - B + (1000\text{ lb})\sin 30° = 0$

$A - B + 500 = 0 \quad (1)$

$+\uparrow \Sigma F_y = 0$:

$D + E - (1000\text{ lb})\cos 30° = 0$

$D + E - 866.03 = 0 \quad (2)$

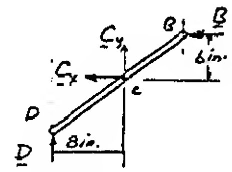


FREE BODY: MEMBER ACE

$+\circlearrowleft \Sigma M_C = 0$

$-A(6\text{ in.}) + E(8\text{ in.}) = 0$

$E = \frac{3}{4}A \quad (3)$



FREE BODY: MEMBER BCD

$+\circlearrowleft \Sigma M_C = 0$

$-D(8\text{ in.}) + B(6\text{ in.}) = 0$

$D = \frac{3}{4}B \quad (4)$

SUBSTITUTE E AND D FROM (3) AND (4) INTO (2):

$\frac{3}{4}A + \frac{3}{4}B - 866.06 = 0$

$A + B - 1154.71 = 0 \quad (5)$

(1) $A - B + 500 = 0 \quad (6)$

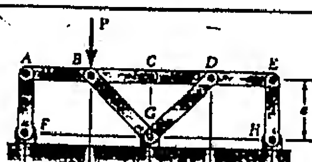
(5)+(6) $2A - 654.71 = 0$; $A = 327.4\text{ lb}$ — $A = 327\text{ lb} \rightarrow$ ◀

(5)−(6) $2B - 1654.71 = 0$; $B = 827.4\text{ lb}$ — $B = 827\text{ lb} \leftarrow$ ◀

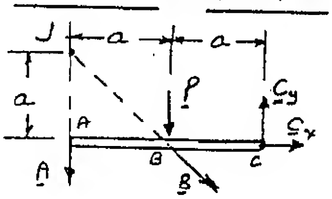
(4) $D = \frac{3}{4}(827.4)$ — $D = 620.5\text{ lb}$ — $D = 620.5\text{ lb} \uparrow$ ◀

(3) $E = \frac{3}{4}(327.4)$ — $E = 245.5\text{ lb}$ — $E = 245.5\text{ lb} \uparrow$ ◀

6.111



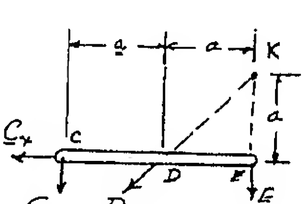
FIND: THE FORCE IN EACH LINK



FREE BODY: MEMBER ABC

$+\circlearrowleft \Sigma M_J = 0$

$-Pa + C_y(2a) + C_x(a) = 0 \quad (1)$



FREE BODY: MEMBER CDE

$+\circlearrowleft \Sigma M_K = 0$

$C_y(2a) - C_x(a) = 0$

$C_x = 2C_y \quad (2)$

6.111 CONTINUED

(2)→(1): $-Pa + C_y(2a) + 2C_y(a) = 0$; $C_y = +\frac{1}{4}P$ ◁

(2): $C_x = 2C_y = 2(\frac{1}{4}P)$; $C_x = +\frac{1}{2}P$ ◁

RETURN TO FREE BODY OF ABC:

$\xrightarrow{+} \Sigma F_x = 0$: $C_x + \frac{1}{\sqrt{2}}B = 0$;

$\frac{1}{2}P + \frac{1}{\sqrt{2}}B = 0$; $B = -P/\sqrt{2}$ — $F_{BG} = \frac{P}{\sqrt{2}}$ comp. ◀

$+\circlearrowleft \Sigma M_B = 0$: $C_y(a) + A(a)$

$\frac{1}{4}Pa + Aa = 0$; $A = -\frac{P}{4}$ — $F_{AF} = \frac{P}{4}$ comp. ◀

RETURN TO FREE BODY OF CDE:

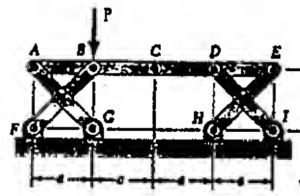
$\xrightarrow{+} \Sigma F_x = 0$: $-C_x - \frac{1}{\sqrt{2}}D = 0$

$-\frac{P}{2} - \frac{1}{\sqrt{2}}D = 0$; $D = -\frac{P}{\sqrt{2}}$; — $F_{DG} = \frac{P}{\sqrt{2}}$ comp. ◀

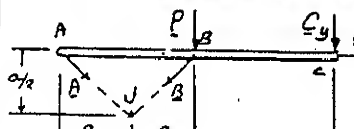
$+\circlearrowleft \Sigma M_D = 0$: $C_y(a) - E(a) = 0$

$\frac{1}{4}Pa - Ea = 0$; $E = \frac{P}{4}$; — $F_{EH} = \frac{P}{4}$ ten. ◀

6.112



FIND: THE FORCE IN EACH LINK.

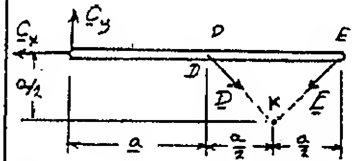


FREE BODY: MEMBER ABC

$+\circlearrowleft \Sigma M_J = 0$

$C_x(\frac{1}{2}a) + C_y(\frac{3}{2}a) + P(\frac{a}{2}) = 0$

$C_x + 3C_y + P = 0 \quad (1)$



FREE BODY: MEMBER CDE

$+\circlearrowleft \Sigma M_K = 0$

$C_x(\frac{a}{2}) - C_y(\frac{3a}{2}) = 0$

$C_x = +3C_y \quad (2)$

(2)→(1): $+3C_y + 3C_y + P = 0$:

$6C_y + P = 0$ — $C_y = -\frac{P}{6}$ ◁

(2): $C_x = 3C_y = 3(-\frac{P}{6})$ — $C_x = -\frac{P}{2}$ ◁

RETURN TO FREE BODY OF ABC:

$+\circlearrowleft \Sigma M_B = 0$: $-\frac{A}{\sqrt{2}}(a) + C_y(a) = 0$

$-\frac{A}{\sqrt{2}}a - \frac{P}{6}a = 0$; $A = -\frac{\sqrt{2}}{6}P$ — $F_{AG} = \frac{\sqrt{2}}{6}P$ comp. ◀

$+\circlearrowleft \Sigma M_A = 0$: $\frac{B}{\sqrt{2}}(a) + C_y(2a) + P(a) = 0$

$\frac{B}{\sqrt{2}}(a) - \frac{P}{6}(2a) + P(a) = 0$

$B = -\frac{2\sqrt{2}}{3}P$ — $F_{BF} = \frac{2\sqrt{2}}{3}P$ comp. ◀

RETURN TO FREE BODY OF CDE

$+\circlearrowleft \Sigma M_E = 0$: $\frac{D}{\sqrt{2}}(a) - C_y(2a) = 0$

$\frac{D}{\sqrt{2}}(a) - (-\frac{P}{6})(2a) = 0$

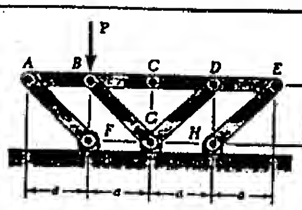
$D = -\frac{\sqrt{2}}{3}P$ — $F_{DI} = \frac{\sqrt{2}}{3}P$ comp. ◀

$+\circlearrowleft \Sigma M_D = 0$: $\frac{E}{\sqrt{2}}(a) + C_y(a) = 0$

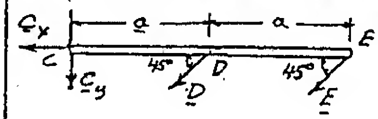
$\frac{E}{\sqrt{2}}(a) - \frac{P}{6}(a) = 0$

$E = \frac{\sqrt{2}}{6}P$ — $F_{EH} = \frac{\sqrt{2}}{6}P$ ten. ◀

6.113



FIND: THE FORCE IN EACH LINK



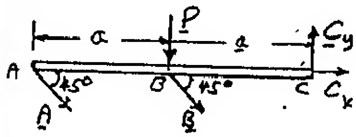
FREE BODY: MEMBER CDE

$\xrightarrow{+}\Sigma F_x=0:\quad C_x+\frac{D}{\sqrt{2}}+\frac{E}{\sqrt{2}}=0$

$C_x-\frac{2E}{\sqrt{2}}+\frac{E}{\sqrt{2}}=0 \qquad C_x=\frac{E}{\sqrt{2}}$

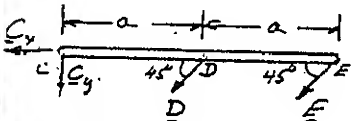
$+\uparrow\Sigma F_y=0:\quad C_y+\frac{D}{\sqrt{2}}+\frac{E}{\sqrt{2}}=0$

$C_y-\frac{2E}{\sqrt{2}}+\frac{E}{\sqrt{2}}=0 \qquad C_y=\frac{E}{\sqrt{2}}$



FREE BODY: MEMBER ABC

$\measuredangle 45^\circ\ \Sigma F=0:\quad -\frac{P}{\sqrt{2}}+\frac{C_x}{\sqrt{2}}+\frac{C_y}{\sqrt{2}}=0: \qquad C_x+C_y=+P \qquad (1)$



FREE BODY: MEMBER CDE

$45^\circ\ \Sigma F=0:\quad +\frac{C_x}{\sqrt{2}}-\frac{C_y}{\sqrt{2}}=0 \qquad C_x=C_y \qquad (2)$

$(2)\rightarrow(1) \quad C_y+C_y=P; \quad C_y=\frac{P}{2} \triangleleft; \ C_x=\frac{P}{2} \triangleleft$

$+\circlearrowleft\Sigma M_D=0:\quad C_y(a)-\frac{E}{\sqrt{2}}(a)=0$

$E=\sqrt{2}\,C_y=\frac{\sqrt{2}}{2}P \qquad F_{EH}=\frac{\sqrt{2}}{2}P$ ten. ◀

$+\circlearrowleft\Sigma M_E=0:\quad \frac{D}{\sqrt{2}}(a)+C_y(2a)=0$

$D=-2\sqrt{2}\,C_y=-2\sqrt{2}\,\frac{P}{2} \qquad F_{DG}=\sqrt{2}P$ comp. ◀

RETURN TO FREE BODY OF ABC

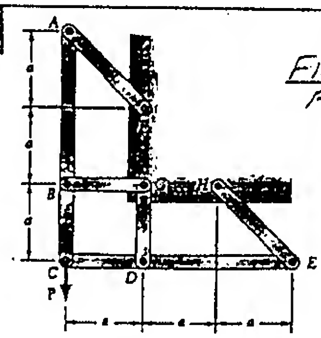
$+\circlearrowleft\Sigma M_B=0:\quad \frac{A}{\sqrt{2}}(a)+C_y(a)$

$A=\sqrt{2}\,C_y=\frac{\sqrt{2}}{2}P \qquad F_{AF}=\frac{\sqrt{2}}{2}P$ ten. ◀

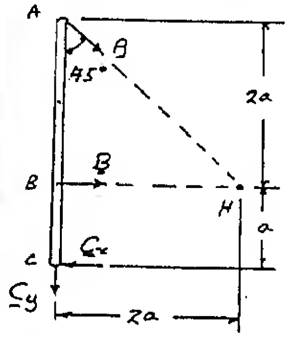
$+\circlearrowleft\Sigma M_A=0:\quad \frac{B}{\sqrt{2}}a+Pa-C_y(2a)=0$

$B=\sqrt{2}\left(P-\frac{P}{2}2\right)=0 \qquad F_{BG}=0$ ◀

6.114



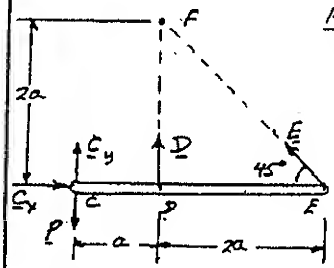
FIND: THE FORCE IN EACH LINK



FREE BODY: MEMBER ABC

$+\circlearrowleft\Sigma M_H=0:\quad C_x(a)-C_y(2a)=0$

$C_x=2C_y$



FREE BODY: MEMBER CDE

$+\circlearrowleft\Sigma M_F=0$

$C_x(2a)-C_y(a)+P(a)=0$

$2C_x-C_y+P=0$

$2(2C_y)-C_y+P=0$

$C_y=-\frac{1}{3}P \quad \triangleleft$

$C_x=2C_y: \quad C_x=-\frac{2}{3}P \quad \triangleleft$

$\xrightarrow{+}\Sigma F=0:\quad C_x-\frac{E}{\sqrt{2}}=0; \quad -\frac{2}{3}P-\frac{E}{\sqrt{2}}=0$

$E=-\frac{2\sqrt{2}}{3}P \qquad F_{EH}=\frac{2\sqrt{2}}{3}P$ comp. ◀

$+\circlearrowleft\Sigma M_E=0:\quad D(2a)+C_y(3a)-P(3a)=0$

$D(2a)-\frac{P}{3}(3a)-P(3a)=0$

$D=+2P \qquad F_{DG}=2P$ ten. ◀

RETURN TO FREE BODY OF ABC

$+\uparrow\Sigma F_y=0:\quad \frac{A}{\sqrt{2}}+C_y=0$

$\frac{A}{\sqrt{2}}-\frac{P}{3}=0$

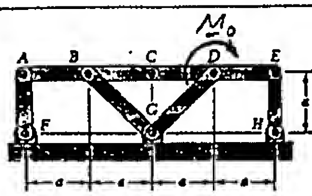
$A=+\frac{\sqrt{2}}{3}P \qquad F_{AF}=\frac{\sqrt{2}}{3}P$ ten. ◀

$+\circlearrowleft\Sigma M_A=0:\quad B(2a)-C_x(3a)=0$

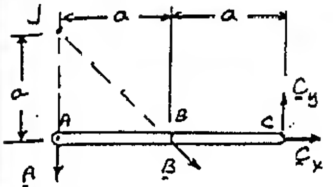
$B(2a)+\frac{2}{3}P(3a)=0$

$B=-P \qquad F_{BG}=P$ comp. ◀

6.115



FIND: THE FORCE IN EACH LINK

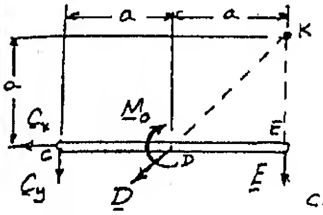


FREE BODY: MEMBER ABC

$+\circlearrowleft \Sigma M_J = 0$

$C_y(2a) + C_x(a) = 0$

$C_x = -2C_y$



FREE BODY: MEMBER CDE

$+\circlearrowleft \Sigma M_K = 0$:

$C_y(2a) - C_x(a) - M_0 = 0$

$C_y(2a) - (-2C_y)(a) - M_0 = 0$

$C_y = M_0/4a$ ◁

$C_x = -2C_y$: $C_x = -M_0/2a$ ◁

$\pm \Sigma F_x = 0$: $\frac{D}{\sqrt{2}} + C_x = 0$; $\frac{D}{\sqrt{2}} - \frac{M_0}{2a} = 0$

$D = \frac{M_0}{\sqrt{2}a}$ $\qquad F_{DG} = \frac{M_0}{\sqrt{2}a}$ ten. ◀

$+\circlearrowleft \Sigma M_D = 0$: $E(a) - C_y(a) + M_0 = 0$

$E(a) - (\frac{M_0}{4a})(a) + M_0 = 0$

$E = -\frac{3}{4}\frac{M_0}{a}$ $\qquad F_{EH} = \frac{3}{4}\frac{M_0}{a}$ comp. ◀

RETURN TO FREE BODY OF ABC

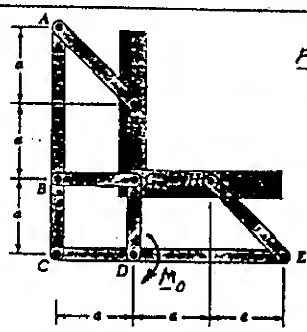
$\pm \Sigma F_x = 0$: $\frac{B}{\sqrt{2}} + C_x = 0$; $\frac{B}{\sqrt{2}} - \frac{M_0}{2a} = 0$

$B = \frac{M_0}{\sqrt{2}a}$ $\qquad F_{BG} = \frac{M_0}{\sqrt{2}a}$ ten. ◀

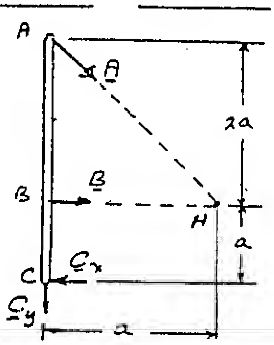
$+\circlearrowleft \Sigma M_B = 0$: $A(a) + C_y(a)$; $A(a) + \frac{M_0}{4a}(a) = 0$

$A = -\frac{M_0}{4a}$ $\qquad F_{AF} = \frac{M_0}{4a}$ comp. ◀

6.116



FIND: THE FORCE IN EACH LINK



FREE BODY: MEMBER ABC

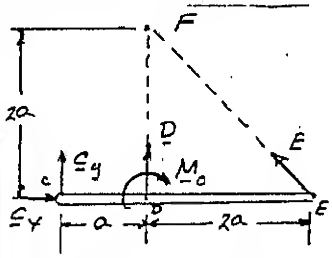
$+\circlearrowleft \Sigma M_H = 0$:

$C_x(a) - C_y(2a) = 0$

$C_x = 2C_y$

(CONTINUED)

6.116 CONTINUED



FREE BODY: MEMBER CDE

$+\circlearrowleft \Sigma M_F = 0$:

$C_x(2a) - C_y(a) - M_0 = 0$

$(2C_y)(2a) - C_y(a) - M_0 = 0$

$C_y = \frac{M_0}{3a}$ ◁

$C_x = 2C_y$: $C_x = \frac{2M_0}{3a}$ ◁

$\pm \Sigma F_x = 0$: $C_x - \frac{E}{\sqrt{2}} = 0$; $\frac{2M_0}{3a} - \frac{E}{\sqrt{2}} = 0$

$E = \frac{2\sqrt{2}}{3}\frac{M_0}{a}$ $\qquad F_{EH} = \frac{2\sqrt{2}}{3}\frac{M_0}{a}$ ◀

$+\uparrow \Sigma F_y = 0$: $D + \frac{E}{\sqrt{2}} + C_y = 0$

$D + \frac{2\sqrt{2}}{3}\frac{M_0}{a}\frac{1}{\sqrt{2}} + \frac{M_0}{3a} = 0$

$D = -\frac{M_0}{a}$ $\qquad F_{DG} = \frac{M_0}{a}$ comp. ◀

RETURN TO FREE BODY OF ABC

$+\uparrow \Sigma F_y = 0$: $\frac{A}{\sqrt{2}} + C_y = 0$; $\frac{A}{\sqrt{2}} + \frac{M_0}{3a} = 0$

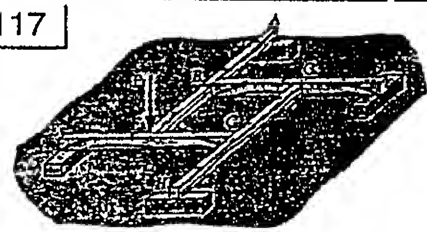
$A = -\frac{\sqrt{2}}{3}\frac{M_0}{a}$ $\qquad F_{AF} = \frac{\sqrt{2}}{3}\frac{M_0}{a}$ comp. ◀

$+\circlearrowleft \Sigma M_A = 0$: $B(2a) - C_x(3a) = 0$

$B(2a) - (\frac{2}{3}\frac{M_0}{a})(3a) = 0$

$B = +\frac{M_0}{a}$ $\qquad F_{BG} = \frac{M_0}{a}$ ten. ◀

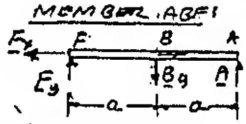
6.117



FIND: THE VERTICAL REACTIONS AT A, D, E, AND H

WE SHALL DRAW A FREE BODY OF EACH MEMBER. FORCE P WILL BE APPLIED TO MEMBER EFG. STARTING WITH MEMBER ABF, WE SHALL EXPRESS ALL FORCES IN TERMS OF REACTION A.

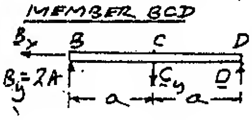
MEMBER ABF:



$+\circlearrowleft \Sigma M_F = 0$: $A(2a) - B_y(a) = 0$; $B_y = 2A$

$+\circlearrowleft \Sigma M_B = 0$: $-F_y(a) + A(a) = 0$; $F_y = A$

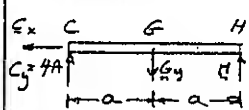
MEMBER BCD



$+\circlearrowleft \Sigma M_C = 0$: $-(2A)(a) + D(a) = 0$; $D = 2A$ (1)

$+\circlearrowleft \Sigma M_D = 0$: $-(2A)(2a) + C_y(a) = 0$; $C_y = 4A$

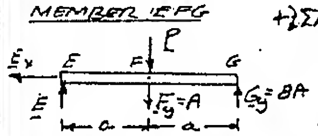
MEMBER CGH



$+\circlearrowleft \Sigma M_G = 0$: $-(4A)(a) + H(a) = 0$; $H = 4A$ (2)

$+\circlearrowleft \Sigma M_H = 0$: $-(4A)(2a) + G_y(a) = 0$; $G_y = 8A$

MEMBER EFG



$+\circlearrowleft \Sigma M_F = 0$: $-(8A)(a) + E(a) = 0$

$E = 8A$ (3)

$+\uparrow \Sigma F_y = 0$:

$E - A + 8A - P = 0$

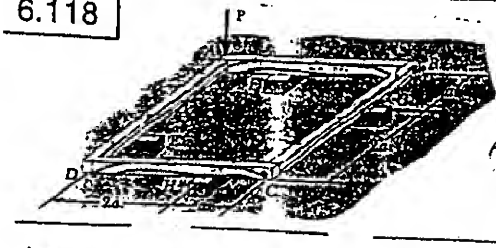
$8A - A + 8A - P = 0$; $A = \frac{P}{15}$

SUBSTITUTE $A = P/15$ INTO EQS. (1), (2), AND (3):

ANSWERS:

$A = \frac{1}{15}P\uparrow$; $D = \frac{2}{15}P\uparrow$; $H = \frac{4}{15}P\uparrow$; $E = \frac{8}{15}P\uparrow$ ◀

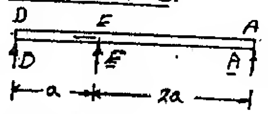
6.118



FIND: THE VERTICAL REACTIONS AT A, D, E, AND H.

WE SHALL DRAW THE FREE BODY OF EACH MEMBER. FORCE P WILL BE APPLIED TO MEMBER AFB. STARTING WITH MEMBER AED, WE SHALL EXPRESS ALL FORCES IN TERMS OF REACTION E.

MEMBER AEB:



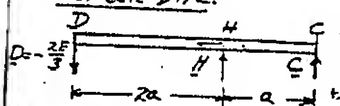
$+\circlearrowleft \Sigma M_D = 0: \; A(3a) + E(a) = 0$

$A = -E/3$

$+\circlearrowleft \Sigma M_A = 0: \; -D(3a) - E(2a) = 0$

$D = -2E/3$

MEMBER DHC:



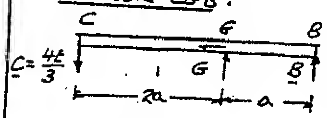
$+\circlearrowleft \Sigma M_C = 0: \; (-\frac{2E}{3})(3a) - H(a) = 0$

$H = -2E \qquad (1)$

$+\circlearrowleft \Sigma M_H = 0: \; (-\frac{2E}{3})(2a) + C(a) = 0$

$C = +4E/3$

MEMBER CGB:



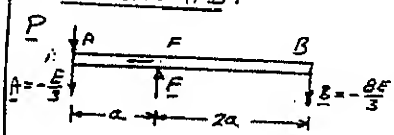
$+\circlearrowleft \Sigma M_B = 0: \; +(\frac{4E}{3})(3a) - G(a) = 0$

$G = +4E \qquad (2)$

$+\circlearrowleft \Sigma M_G = 0: \; +(\frac{4E}{3})(2a) + B(a) = 0$

$B = -8E/3$

MEMBER AFB:



$+\uparrow \Sigma F_y = 0$

$F - A - B - P = 0$

$F - (-\frac{E}{3}) - (-\frac{8E}{3}) - P = 0$

$F = P - 3E \qquad (3)$

$+\circlearrowleft \Sigma M_A = 0: \quad F(a) - B(3a) = 0$

$(P-3E)(a) - (-\frac{8E}{3})(3a) = 0$

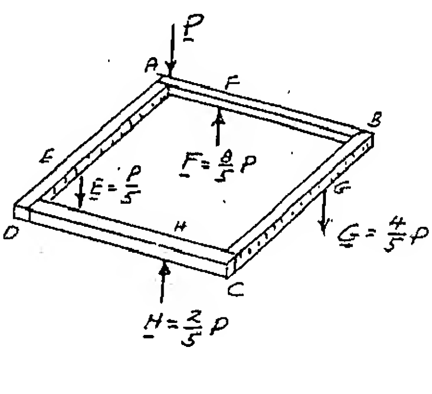
$P - 3E + 8E = 0; \quad E = -\frac{P}{5} \qquad \underline{E} = \frac{P}{5}\downarrow$ ◀

SUBSTITUTE $E = -P/5$ INTO EQS. (1), (2), AND (3).

$H = -2E = -2(-\frac{P}{5}); \quad H = +\frac{2P}{5} \qquad \underline{H} = \frac{2P}{5}\uparrow$ ◀

$G = +4E = 4(-\frac{P}{5}): \quad G = -\frac{4P}{5} \qquad \underline{G} = \frac{4P}{5}\downarrow$ ◀

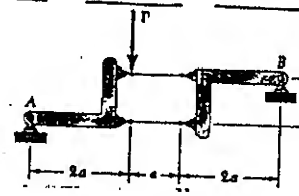
$F = P - 3E = P - 3(-\frac{P}{5}); \quad F = +\frac{8P}{5} \qquad \underline{F} = \frac{8P}{5}\uparrow$ ◀

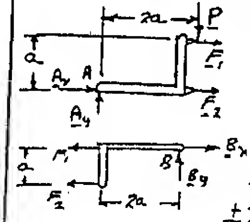


6.119

FOR EACH FRAME SHOWN FIND THE REACTIONS AND WHETHER FRAME IS RIGID

(a)





FREE BODY: LEFT PORTION

$+\uparrow \Sigma F_y = 0: \; A_y - P = 0; \; A_y = +P; \quad \underline{A_y} = P\uparrow$ ◁

$+\circlearrowleft \Sigma M_A = 0: \; -P(2a) - F_1(a) = 0 \quad F_1 = -2P$

$\xrightarrow{+} \Sigma F_x = 0: \; A_x + F_1 + F_2 = 0; \; A_x - 2P + F_2 = 0 \qquad (1)$

FREE BODY: RIGHT PORTION

$+\uparrow \Sigma F_y = 0: \; B_y = 0 \qquad B_y = 0$ ◁

$+\circlearrowleft \Sigma M_B = 0: \; F_2(a) = 0 \qquad F_2 = 0$

$\xrightarrow{+} \Sigma F_x = 0: \; B_x - F_1 - F_2 = 0: \; B_x - (-2P) = 0$

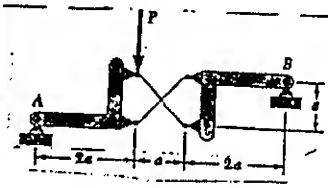
$B_x = -2P \qquad B_x = 2P \leftarrow$ ◁

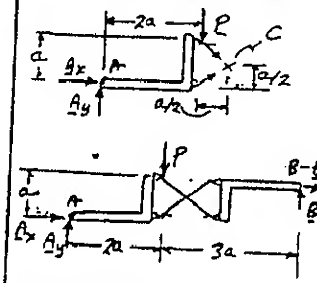
FROM (1) WITH $F_2 = 0$, $A_x = 2P \qquad A_x = 2P \rightarrow$ ◁

$\underline{A} = 2.24P \angle 26.6°; \; \underline{B} = 2P \rightarrow$ ◀

FRAME IS RIGID ◀

(b)





FREE BODY: LEFT PORTION

$+\circlearrowleft \Sigma M_C = 0 \quad A_x(\frac{a}{2}) - A_y(\frac{5}{2}a) + P(\frac{a}{2}) = 0$

$P - A_x + 5A_y \qquad (1)$

FREE BODY: ENTIRE FRAME

$+\circlearrowleft \Sigma M_B = 0$

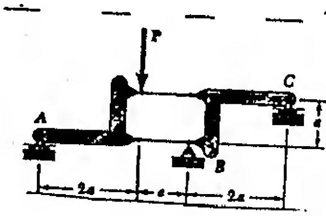
$A_x(a) - A_y(5a) + P(3a) = 0$

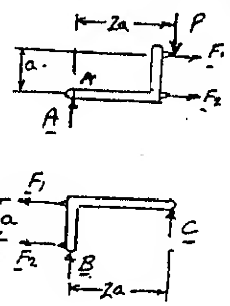
$3P - A_x + 5A_y = 0 \qquad (2)$

EQ(2) - EQ(1): $\; 3P - P = 0; \; 2P = 0 \qquad P = 0$

FOR $P \neq 0$, EQUILIBRIUM IS NOT MAINTAINED AND FRAME IS NOT RIGID ◀

(c)





FREE BODY: LEFT PORTION

$+\uparrow \Sigma F_y = 0: \; A - P = 0; \; A = P; \quad \underline{A} = P\uparrow$ ◀

$+\circlearrowleft \Sigma M_A = 0: \; -P(2a) - F_1(a) = 0$

$F_1 = -2P$

FREE BODY: RIGHT PORTION

$+\circlearrowleft \Sigma M_B = 0: \; F_1(a) + C(2a) = 0$

$C = -\frac{1}{2}F_1 = -\frac{1}{2}(-2P)$

$C = P \qquad \underline{C} = P\uparrow$ ◀

$+\uparrow \Sigma F_y = 0: \; B + C = 0$

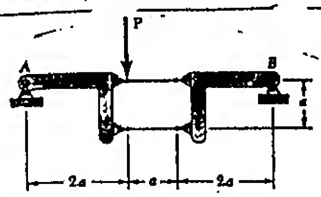
$B + P = 0$

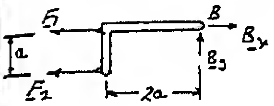
$B = -P \qquad \underline{B} = P\downarrow$ ◀

FRAME IS RIGID ◀

6.120 FOR EACH FRAME FIND THE REACTIONS AND WHETHER THE FRAME IS RIGID.

(a)

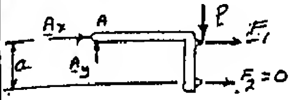




FREE BODY: RIGHT PORTION

$+\circlearrowleft \Sigma M_B = 0: \quad F_2(a) = 0$

$F_2 = 0$

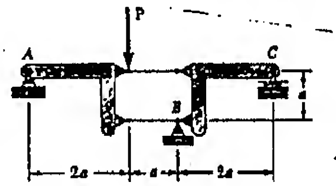


FREE BODY: LEFT PORTION

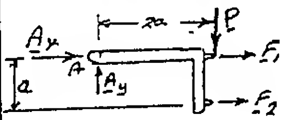
$+\circlearrowleft \Sigma M_A = 0: \quad P(2a) = 0$

$P = 0$

FOR $P \neq 0$, EQUILIBRIUM IS NOT MAINTAINED. FRAME IS NOT RIGID ◀



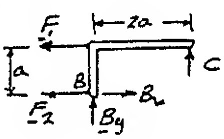
(b) THIS FRAME IS INDETERMINATE, 2 FREE BODIES = 6 EQS; 5 REACTION COMPONENTS PLUS 2 LINK FORCES = 7 UNKS



FREE BODY: LEFT PORTION

$+\circlearrowleft \Sigma M_A = 0: \quad F_2(a) - P(2a) = 0$

$F_2 = 2P$



FREE BODY: RIGHT PORTION

$+\circlearrowleft \Sigma M_C = 0: \ -F_2(a) + B_x(a) - B_y(2a) = 0$

$B_y = \frac{1}{2}(B_x - P)$ ◁

$+\uparrow \Sigma F_y = 0: \ C + B_y = 0; \quad C = -\frac{1}{2}(B_x - P)$ ◁

$\xrightarrow{+} \Sigma F_x = 0: \ -F_1 - F_2 + B_x = 0$

$F_1 = B_x - F_2 = B_x - 2P$

RETURN TO FREE BODY OF LEFT PORTION

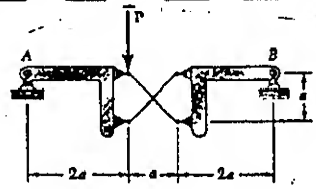
$+\uparrow \Sigma F_y = 0: \ A_y - P = 0 \qquad A_y = P$ ◁

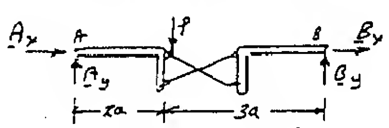
$\xrightarrow{+} \Sigma F_x = 0: \ A_x + F_1 + F_2 = 0; \quad A_x = -F_1 - F_2$

$A_x = -(B_x - 2P) - 2P: \qquad A_x = -B_x$ ◁

WE NOTE THAT REACTIONS CAN BE FOUND FOR AN ARBITRARY VALUE OF B_x. FRAME IS RIGID ◀

(c)



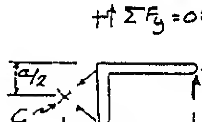


FREE BODY: ENTIRE FRAME

$+\circlearrowleft \Sigma M_A = 0: B_y(5a) - P(2a) = 0$

$B_y = \frac{2}{5}P \qquad B_y = \frac{2}{5}P\uparrow$ ◁

$+\uparrow \Sigma F_y = 0: A_y - P + \frac{2}{5}P = 0; \quad A_y = \frac{3}{5}P \qquad A_y = \frac{3}{5}P\uparrow$ ◁



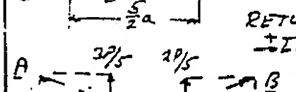
FREE BODY: RIGHT PORTION

$+\circlearrowleft \Sigma M_C = 0: \ \frac{2}{5}P(\frac{5}{2}a) - B_x(\frac{a}{2}) = 0$

$B_x = 2P \qquad B_x = 2P \rightarrow$ ◁

RETURN TO FREE BODY OF ENTIRE FRAME

$\xrightarrow{+} \Sigma F_x = 0 \ \ A_x + 2P = 0, \ A_x = -2P; \ A_x = 2P \leftarrow$ ◁

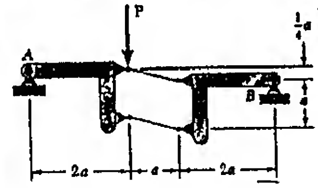


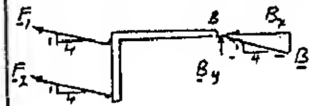
$A = 2.09P \measuredangle 16.7°$ ◀

$B = 2.04P \measuredangle 11.3°$ ◀

6.121 FOR EACH FRAME FIND THE REACTIONS AND WHETHER THE FRAME IS RIGID.

(a)

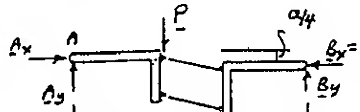




FREE BODY: RIGHT PORTION

FOR EQUILIBRIUM, B MUST BE PARALLEL TO LINKS, THAT IS

$B_x = 4B_y$



FREE BODY: ENTIRE FRAME

$+\circlearrowleft \Sigma M_A = 0:$

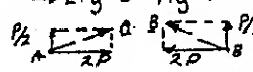
$B_y(5a) - (4B_y)\frac{a}{4} - P(2a) = 0$

$B_y = P/2 \qquad B_y = P/2\uparrow$ ◁

$B_x = 4B_y \qquad B_x = 2P \leftarrow$ ◁

$\xrightarrow{+} \Sigma F_x = 0: A_x - B_x = 0; \quad A_x = 2P \qquad A_x = 2P \rightarrow$ ◁

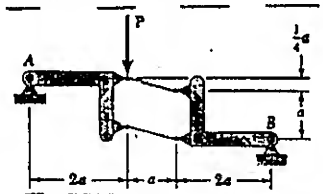
$+\uparrow \Sigma F_y = 0: A_y - P + P/2 = 0; \quad A_y = P/2 \qquad A_y = P/2\uparrow$ ◁

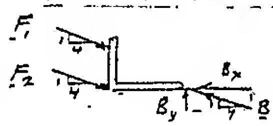


$A = 2.06P \measuredangle 14.0°$ ◀

$B = 2.06P \measuredangle 14.0°$ ◀

(b)

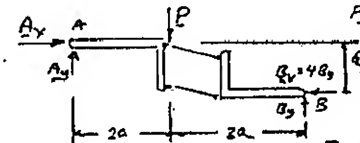




FREE BODY: RIGHT PORTION

FOR EQUILIBRIUM, B MUST BE PARALLEL TO LINKS, THAT IS

$B_x = 4B_y$



FREE BODY: ENTIRE FRAME

$+\circlearrowleft \Sigma M_A = 0:$

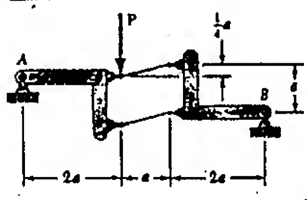
$B_y(5a) - 4B_y(\frac{5a}{4}) - P(2a) = 0$

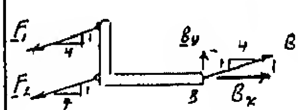
$5B_y - 5B_y - 2P = 0$

$P = 0$

FOR $P \neq 0$, EQUILIBRIUM IS NOT MAINTAINED. FRAME IS NOT RIGID. ◀

(c)

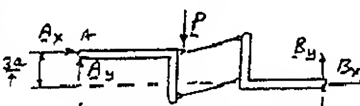




FREE BODY: RIGHT PORTION

FOR EQUILIBRIUM, B MUST BE PARALLEL TO LINKS, THAT IS

$B_x = 4B_y$



FREE BODY: ENTIRE FRAME

$+\circlearrowleft \Sigma M_A = 0$

$B_y(5a) + (4B_y)\frac{3a}{4} - P(2a) = 0$

$B_y = +P/4 \qquad B_y = P/4\uparrow$ ◁

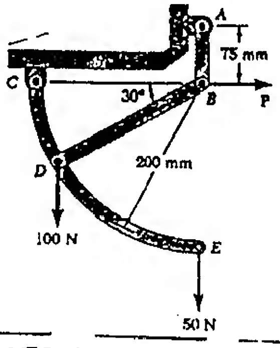
$B_x = 4(P/4) \qquad B_x = P \rightarrow$ ◁

$\xrightarrow{+} \Sigma F_x = 0: A_x + B_x = 0 \quad A_x + P = 0; \quad A_x = -P \qquad A_x = P \leftarrow$ ◁

$+\uparrow \Sigma F_y = 0: A_y - P + P/4 = 0; \quad A_y = +3P/4 \qquad A_y = 3P/4\uparrow$ ◁

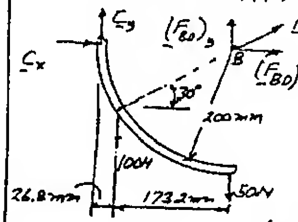
FRAME IS RIGID: $A = 1.25P \measuredangle 36.9°$; $B = 1.031P \measuredangle 14.0°$

6.122



FIND:
(a) FORCE P FOR EQUILIBRIUM,
(b) FORCE IN BD,
(c) REACTION AT C.

WE NOTE THAT BD IS A TWO-FORCE MEMBER



FREE BODY: MEMBER CDE

$+\circlearrowleft \Sigma M_C = 0$:

$(F_{BD})_y(200) - (50N)(300) - (100N)(26.8) = 0$

$(F_{BD})_y = 63.4N$

$F_{BD} = (F_{BD})_y / \sin 30° = (63.4N)/\sin 30°$

$F_{BD} = +126.8N$. $F_{BD} = 126.8N$ ten. ◀

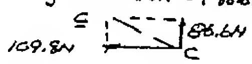
$(F_{BD})_x = (F_{BD})_y / \tan 30° = (63.4N)/\tan 30°$

$(F_{BD})_x = 109.8N$

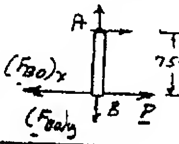
$\xrightarrow{+} \Sigma F_x = 0$: $+C_x + (F_{BD})_x = 0$; $C_x = -109.8N$ $\quad C_x = 109.8N \leftarrow$ ◁

$+\uparrow \Sigma F_y = 0$: $+C_y - 100N - 50N + (F_{BD})_y = 0$

$C_y = 150 - 63.4 = +86.6N$ $\quad C_y = 86.6N\uparrow$ ◁



$C = 139.8N \measuredangle 38.3°$ ◀

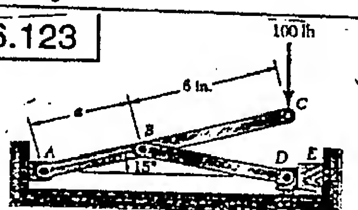


FREE BODY: MEMBER AB

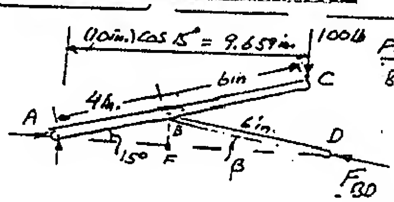
$+\circlearrowleft \Sigma M_A = 0$ $\quad P(75) - (F_{BD})_x(75) = 0$

$P = (F_{BD})_x = 109.8N$ $\quad P = 109.8N \rightarrow$ ◀

6.123



GIVEN: $a = 4in.$
$BD = 6in.$
FIND: HORIZONTAL FORCE EXERTED ON BLOCK E.



FREE BODY: ENTIRE FRAME

$BF = 4 \sin 15° = 1.0353$ in.

IN Δ BDF:

$\sin\beta = \frac{1.0353 in}{6 in}$

$\beta = 9.936°$

$AD = AF + FD = 4 \cos 15° + 6 \cos 9.936°$; $AD = 9.774$ in

$+\circlearrowleft \Sigma M_A = 0$: $(100 lb)(9.659 in.) - F_{BD} \sin\beta (9.774 in.) = 0$

$965.9 - F_{BD}(\sin 9.936°)(9.774 in.) = 0$

$F_{BD} = 573$ lb

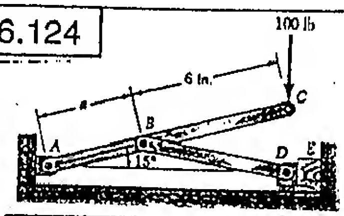
HORIZONTAL COMPONENT:

$(F_{BD})_x = F_{BD} \cos\beta = (573 lb)\cos 9.936°$

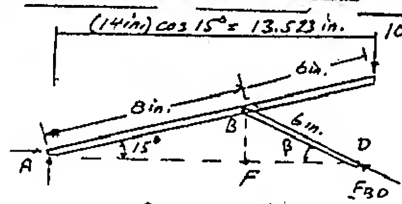
$(F_{BD})_x = 564 lb \leftarrow$

FORCE EXERTED ON BLOCK E IS DIRECTED TO THE RIGHT AND IS EQUAL TO $564 lb \rightarrow$ ◀

6.124



GIVEN: $a = 8in.$
$BD = 6in.$
FIND: HORIZONTAL FORCE EXERTED ON BLOCK E



FREE BODY: ENTIRE FRAME

$BF = (8in.) \sin 15°$
$BF = 2.0706 in.$
IN Δ BDF:
$\sin\beta = \frac{2.0706 in.}{6 in.}$
$\beta = 20.187°$

$AF = 8 \cos 15° = 7.7274 in.$

$FD = 6 \cos\beta = 5.6314 in.$

$AD = AF + FD = 13.359$ in.

$+\circlearrowleft \Sigma M_A = 0$: $(100 lb)(13.523 in.) - F_{BD} \sin\beta (AD) = 0$

$1352.3 - F_{BD} \sin 20.187° (13.359 in.) = 0$

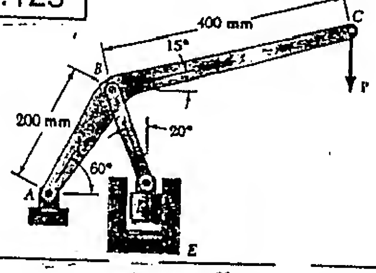
$F_{BD} = 293.3 lb$

HORIZONTAL COMPONENT: $(F_{BD})_x = F_{BD} \cos\beta$

$(F_{BD})_x = (293.3 lb)\cos 20.187°$; $\quad F_{BD} = 275 lb \leftarrow$

FORCE EXERTED ON BLOCK E IS DIRECTED TO THE RIGHT AND IS EQUAL TO $275 lb \rightarrow$ ◀

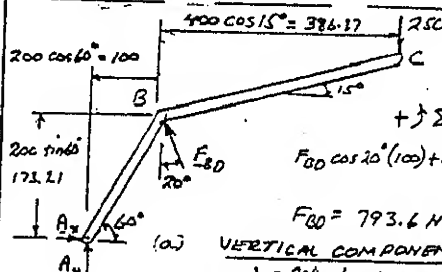
6.125



GIVEN: $P = 250 N$

FIND: (a) VERTICAL COMPONENT OF FORCE EXERTED ON SEAL.
(b) REACTION AT A.

DIMENSIONS IN mm



FREE BODY: MEMBER AC

$+\circlearrowleft \Sigma M_A = 0$:

$F_{BD} \cos 20° (100) + F_{BD} \sin 20° (173.21)$
$-(250N)(486.37) = 0$

$F_{BD} = 793.6 N$

(a) VERTICAL COMPONENT ON SEAL AT E:

$(F_{BD})_y = (793.6N)\cos 20°$

$(F_{BD})_y = 746 lb \downarrow$ ◀

(b) REACTION AT A:

$\xrightarrow{+} \Sigma F_x = 0$: $A_x - F_{BD} \sin 20° = 0$

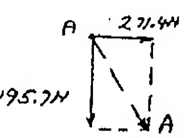
$A_x - (793.6N)\sin 20° = 0$

$A_x = 271.4N$ $\quad A_x = 271.4N \rightarrow$

$+\uparrow \Sigma F_y = 0$: $A_y + F_{BD} \cos 20° - 250N = 0$

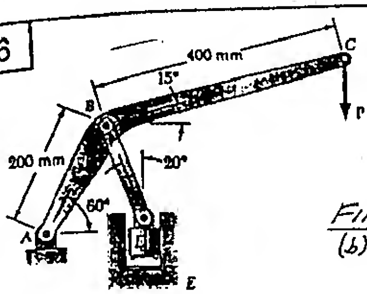
$A_y + (793.6N)\cos 20° - 250 = 0$

$A_y = -495.7N$ $\quad A_y = 495.7N\downarrow$



$A = 565N \measuredangle 61.3°$ ◀

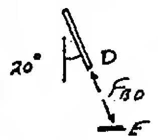
6.126



GIVEN: VERTICAL COMPONENT OF FORCE EXERTED ON SEAL E IS 900 N↓

FIND: (a) FORCE P. (b) REACTION AT A

WE NOTE THAT BD IS A TWO-FORCE MEMBER

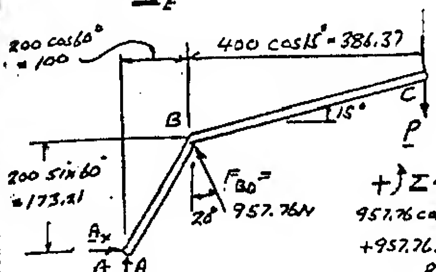


$(F_{BD})_y = F_{BD} \cos 20° = 900\ N$

$F_{BD} = 957.76\ N$ comp.

FREE BODY: MEMBER ABC

DIMENSIONS IN mm



$+\circlearrowleft \Sigma M_A = 0:$

$957.76 \cos 20° (100)$

$+ 957.76 \sin 20° (173.21)$

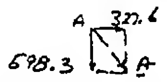
$- P(486.37) = 0$

$P = 301.7\ N$ $\quad P = 302\ N \downarrow$

$\xrightarrow{+} \Sigma F_x = 0:\ A_x - (957.76\ N)\sin 20° = 0;\ A_x = +327.6\ N;\ A_x = 327.6\ N \rightarrow$

$+\uparrow \Sigma F_y = 0:\ A_y + (957.76\ N)\cos 20° - 301.7\ N = 0$

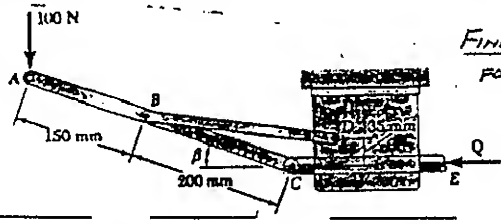
$A_y = -598.3\ N$ $\quad A_y = 598.3\ N \downarrow$



$A = 682\ N \measuredangle 61.3°$

6.127

GIVEN: $BD = 250$ mm, $\beta = 20°$.

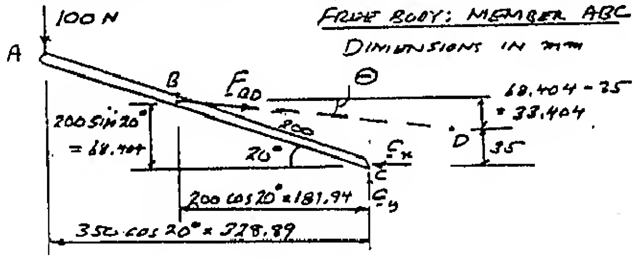


FIND: FORCE Q FOR EQUILIBRIUM

WE NOTE THAT BD IS A TWO-FORCE MEMBER

FREE BODY: MEMBER ABC

DIMENSIONS IN mm



SINCE $BD = 250$, $\theta = \sin^{-1}\frac{33.404}{250}$; $\theta = 7.679°$

$+\circlearrowleft \Sigma M_A = 0: (F_{BD}\sin\theta)187.94 - (F_{BD}\cos\theta)68.404 - (100\ N)328.89 = 0$

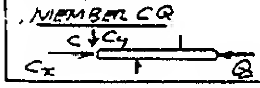
$F_{BD}[187.94 \sin 7.679° - 68.404 \cos 7.679°] = 32\,889$

$F_{BD} = 770.6\ N$

$\xrightarrow{+} \Sigma F_x = 0: (770.6\ N)\cos 7.679° - C_x = 0$

$C_x = +763.7\ N$

MEMBER CQ

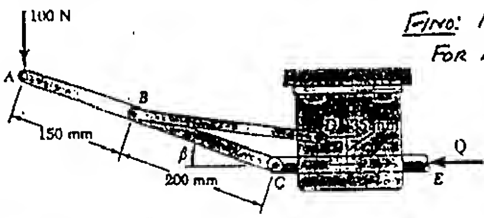


$\Sigma F_x = 0:\ Q = C_x = 763.7\ N$

$Q = 764\ N \leftarrow$

6.128

GIVEN: $BD = 250$ mm



FIND: FORCE Q FOR EQUILIBRIUM WHEN

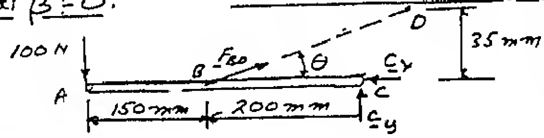
(a) $\beta = 0$

(b) $\beta = 6°$

WE NOTE THAT BD IS A TWO-FORCE MEMBER

(a) $\beta = 0$:

FREE BODY: MEMBER ABC



SINCE $BD = 250$ mm, $\sin\theta = \frac{35\ mm}{250\ mm}$; $\theta = 8.048°$

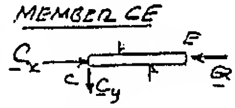
$+\circlearrowleft \Sigma M_C = 0:\ (100\ N)(350\ mm) - F_{BD}\sin\theta\,(200\ mm) = 0$

$F_{BD} = 1250\ N$

$\xrightarrow{+} \Sigma F_x = 0:\ F_{BD}\cos\theta - C_x = 0$

$(1250\ N)(\cos 8.048°) - C_x = 0$ $\quad C_x = 1237.7\ N$

MEMBER CE



$\xrightarrow{+} \Sigma F_x = 0:\ (1237.7\ N) - Q = 0$

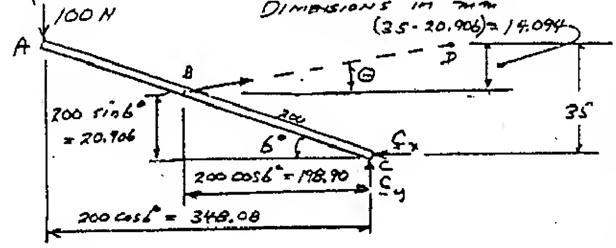
$Q = 1237.7\ N$

$Q = 1238\ N \leftarrow$

(b) $\beta = 6°$

FREE BODY: MEMBER ABC

DIMENSIONS IN mm



SINCE $BD = 250$ mm, $\theta = \sin^{-1}\frac{14.094\ mm}{250\ mm}$

$\theta = 3.232°$

$+\circlearrowleft \Sigma M_A = 0:\ (F_{BD}\sin\theta)198.90 + (F_{BD}\cos\theta)20.906 - (100\ N)348.08 = 0$

$F_{BD}[198.90 \sin 3.232° + 20.906 \cos 3.232°] = 34\,808$

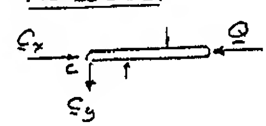
$F_{BD} = 1084.8\ N$

$\xrightarrow{+} \Sigma F_x = 0:\ F_{BD}\cos\theta - C_x = 0$

$(1084.8\ N)\cos 3.232° - C_x = 0$

$C_x = +1083.1\ N$

MEMBER DE:



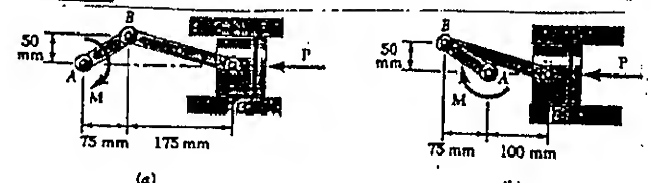
$\Sigma F_x = 0 \quad Q = C_x$

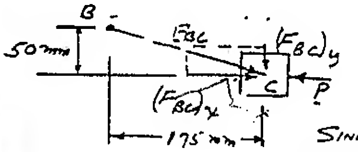
$Q = 1083.1\ N$

$Q = 1083\ N \leftarrow$

6.129

GIVEN: $\underline{M} = 1.5\ kN\cdot m$ ↻. FIND: FORCE $\underline{P}$





FREE BODY: PISTON

$\xrightarrow{+} \Sigma F_x = 0$: $(F_{BC})_x - P = 0$

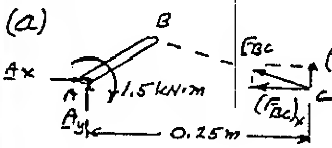
$P = (F_{BC})_x$ (1)

SINCE BC IS A TWO-FORCE MEMBER

$\frac{(F_{BC})_y}{50} = \frac{(F_{BC})_x}{175}$; $(F_{BC})_y = \frac{50}{175}(F_{BC})_x$

USE EQ(1): $(F_{BC})_y = \frac{50}{175}P$; $(F_{BC})_y = \frac{2}{7}P$

(a)



FREE BODY: CRANK AB

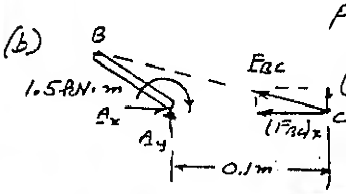
$+↺ \Sigma M_A = 0$:

$(F_{BC})_y(0.25\ m) - 1.5\ kN\cdot m = 0$

$\frac{2}{7}P(0.25) = 1.5$

$P = 21\ kN$ $\underline{P} = 21\ kN \leftarrow$ ◀

(b)



FREE BODY: CRANK AB

$+↺ \Sigma M_A = 0$:

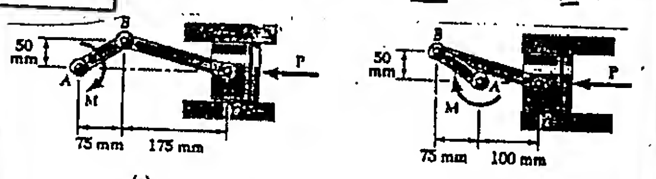
$(F_{BC})_y(0.1\ m) - 1.5\ kN\cdot m = 0$

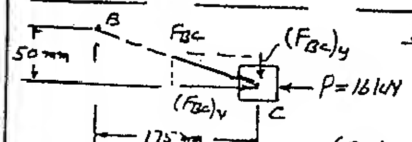
$\frac{2}{7}P(0.1) = 1.5$

$P = 52.5\ kN$ $\underline{P} = 52.5\ kN \leftarrow$ ◀

6.130

GIVEN: $\underline{P} = 16\ kN \leftarrow$. FIND: COUPLE $\underline{M}$.





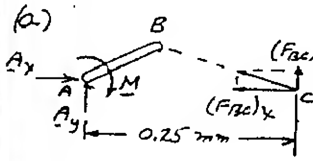
FREE BODY: PISTON

$\Sigma F_x = 0$: $(F_{BC})_x = 16\ kN$

SINCE BC IS A TWO-FORCE MEMBER

$\frac{(F_{BC})_y}{50} = \frac{16\ kN}{175}$; $(F_{BC})_y = 4.571\ kN$

(a)



FREE BODY: CRANK AB

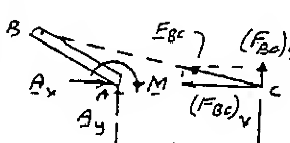
$+↺ \Sigma M_A = 0$

$M - (F_{BC})_y(0.25\ m)$

$M = (4.571\ kN)(0.25\ m)$

$M = 1.143\ kN\cdot m$ $\underline{M} = 1143\ N\cdot m$ ↺ ◀

(b)



FREE BODY: CRANK AB

$+↺ \Sigma M_A = 0$

$M - (F_{BC})_y(0.1\ m) = 0$

$M = (4.571\ kN)(0.1\ m)$

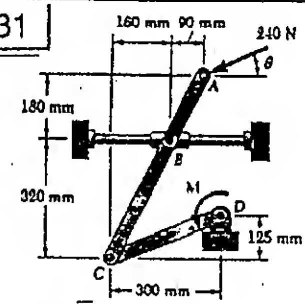
$M = 0.4571\ kN\cdot m$

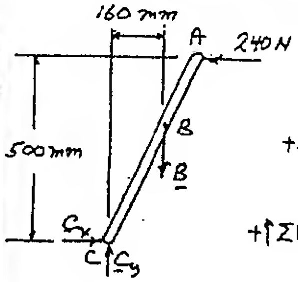
$\underline{M} = 457\ N\cdot m$ ↺ ◀

6.131

GIVEN: $\theta = 0$

FIND: COUPLE $\underline{M}$ FOR EQUILIBRIUM





FREE BODY: MEMBER ABC

$\xrightarrow{+} \Sigma F_x = 0$: $C_x - 240\ N = 0$

$C_x = +240\ N$

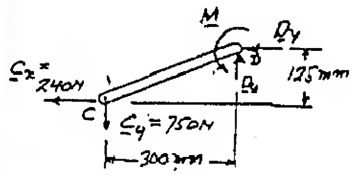
$+↺ \Sigma M_C = 0$:

$(240\ N)(500\ mm) - B(160\ mm) = 0$

$B = +750\ N$

$+\uparrow \Sigma F_y = 0$: $C_y - 750\ N = 0$

$C_y = +750\ N$.



FREE BODY: MEMBER CD

$+↺ \Sigma M_D = 0$:

$M + (750\ N)(300\ mm)$

$-(240\ N)(125\ mm) = 0$

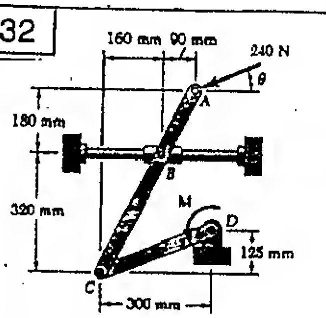
$M = -195 \times 10^3\ N\cdot mm$

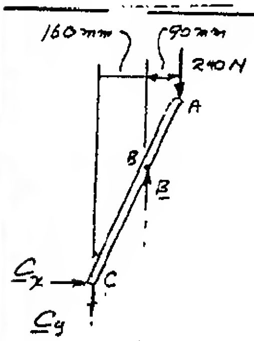
$\underline{M} = 195\ kN\cdot m$ ↻ ◀

6.132

GIVEN: $\theta = 90°$

FIND: COUPLE $\underline{M}$ FOR EQUILIBRIUM





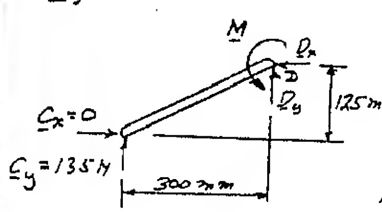
FREE BODY: MEMBER ABC

$\xrightarrow{+} \Sigma F_x = 0$: $C_x = 0$

$+↺ \Sigma M_B = 0$:

$C_y(160\ mm) - (240\ N)(90\ mm) = 0$

$C_y = +135\ N$



FREE BODY: MEMBER CD

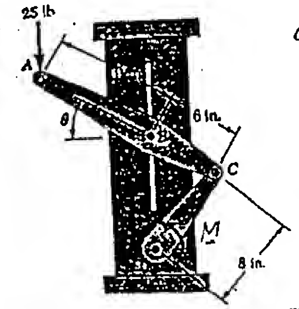
$+↺ \Sigma M_D = 0$

$M - (135\ N)(300\ mm) = 0$

$M = +40.5 \times 10^3\ N\cdot mm$

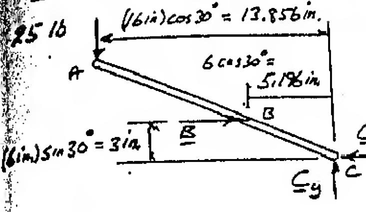
$\underline{M} = 40.5\ kN\cdot m$ ↺ ◀

6.133



GIVEN: $\theta = 30°$

FIND: COUPLE $\underline{M}$ FOR EQUILIBRIUM



FREE BODY: MEMBER ABC

$+\circlearrowleft \Sigma M_C = 0$

$(25\,lb)(13.856\,in.) - B(3\,in.) = 0$

$B = +115.47\,N$

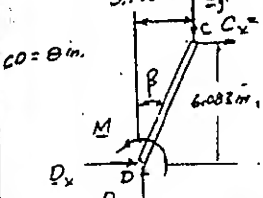
$+\uparrow \Sigma F_y = 0$

$-25\,lb + C_y = 0$

$C_y = +25\,lb$

$\xrightarrow{+} \Sigma F_x = 0: \; 115.47\,N - C_x = 0$

$C_x = +115.47\,lb$



FREE BODY: MEMBER CD

$\beta = \sin^{-1}\frac{5.196}{8}$; $\beta = 40.505°$

$CD\cos\beta = (8\,in.)\cos 40.505° = 6.083\,in.$

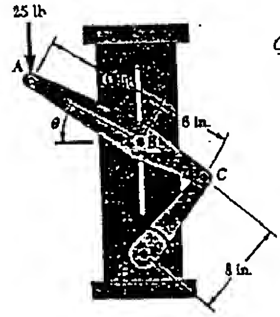
$+\circlearrowleft \Sigma M_D = 0$

$M - (25\,lb)(5.196\,in.) - (115.47\,lb)(6.083\,in.) = 0$

$M = +832.3\,lb \cdot in.$

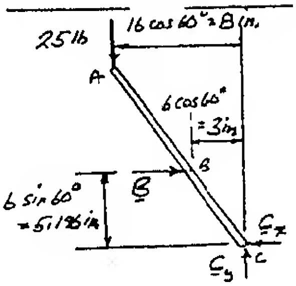
$\underline{M} = 832\,lb \cdot in. \circlearrowleft$ ◀

6.134



GIVEN: $\theta = 60°$

FIND: COUPLE $\underline{M}$ FOR EQUILIBRIUM



FREE BODY: MEMBER ABC

$+\circlearrowleft \Sigma M_C = 0: \; (25\,lb)(8\,in.) - B(5.196\,in.) = 0$

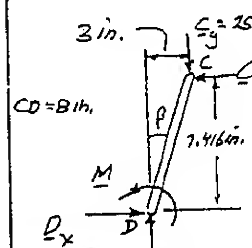
$B = +38.49\,lb$

$\xrightarrow{+} \Sigma F_x = 0: \; 38.49\,lb - C_x = 0$

$C_x = +38.49\,lb$

$+\uparrow \Sigma F_y = 0: \; -25\,lb + C_y = 0$

$C_y = +25\,lb$



FREE BODY: MEMBER CD

$\beta = \sin^{-1}\frac{3}{8}$; $\beta = 22.024°$

$CD\cos\beta = (8\,in.)\cos 22.024° = 7.416\,in.$

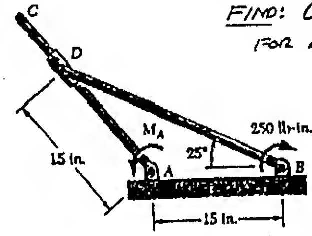
$+\circlearrowleft \Sigma M_D = 0:$

$M - (25\,lb)(3\,in.) - (38.49\,lb)(7.416\,in.) = 0$

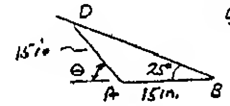
$M = +360.4\,lb \cdot in.$

$\underline{M} = 360\,lb \cdot in. \circlearrowleft$ ◀

6.135



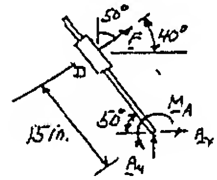
FIND: COUPLE $\underline{M}_A$ FOR EQUILIBRIUM



GEOMETRY: ΔABD IS ISOSCELES

$\therefore \theta = 2\angle ABD = 50°$

$BD = 2[(15\,in.)\cos 25°] = 27.189\,in.$

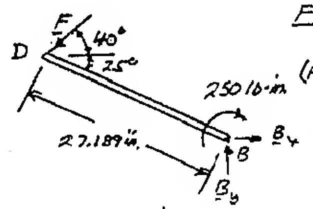


FREE BODY: MEMBER AC

F IS ⊥ TO AD, ∠40°

$+\circlearrowleft \Sigma M_A = 0: \; M_A - F(15\,in.) = 0$

$M_A = (15\,in.)F \circlearrowleft$ (1)



FREE BODY: MEMBER BD

$+\circlearrowleft \Sigma M_B = 0:$

$(F\sin 65°)(27.189\,in.) - 250\,lb \cdot in. = 0$

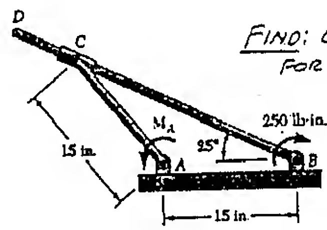
$F = +10.145\,lb$

FROM EQ(1):

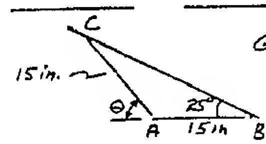
$M_A = (15\,in.)(10.145\,lb)$

$\underline{M}_A = 152.8\,lb \cdot in. \circlearrowleft$ ◀

6.136



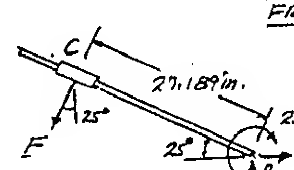
FIND: COUPLE $\underline{M}_A$ FOR EQUILIBRIUM



GEOMETRY: ΔABC IS ISOSCELES

$\therefore \theta = 50°$

$BC = 2[(15\,in.)\cos 25°] = 27.189\,in.$

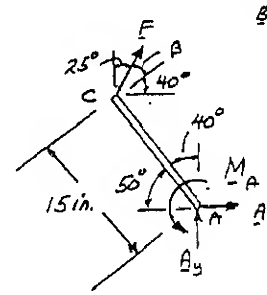


FREE BODY: MEMBER BC

$+\circlearrowleft \Sigma M_B = 0$

$F(27.189\,in.) - 250\,lb \cdot in. = 0$

$F = +9.195\,lb$



FREE BODY: MEMBER AC

$\beta = 90° - 25° - 40° = 25°$

$+\circlearrowleft \Sigma M_A = 0$

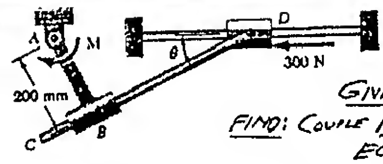
$M_A - (F\cos 25°)(15\,in.) = 0$

$M_A = (9.195\,lb)\cos 25°(15\,in.)$

$M_A = +125.0\,lb \cdot in.$

$\underline{M}_A = 125\,lb \cdot in. \circlearrowleft$ ◀

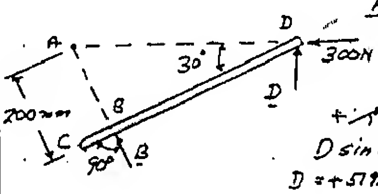
6.137



GIVEN: $\theta = 30°$

FIND: COUPLE $\underline{M}$ FOR EQUILIBRIUM

FREE BODY: ROD CD



IN △ ABD:

$AD = (200\text{ mm})/\sin 30°$

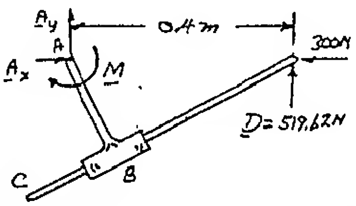
$AD = 400\text{ mm} = 0.4\text{ m}$

$+\nearrow 30°\ \Sigma F = 0$

$D \sin 30° - (300N)\cos 30° = 0$

$D = +519.62N \quad \underline{D} = 519.62N\uparrow$

FREE BODY: ENTIRE FRAME



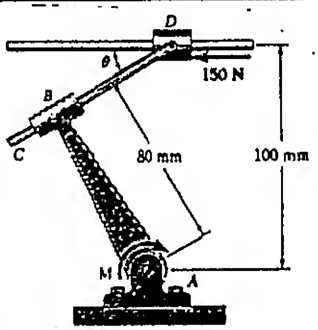
$+\circlearrowleft \Sigma M_A = 0:$

$(519.62N)(0.4m) - M = 0$

$M = +207.8\ N\cdot m$

$\underline{M} = 208\ N\cdot m$ ◄

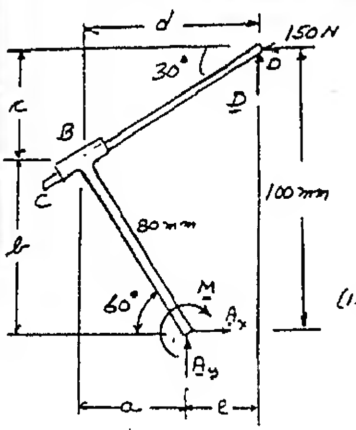
6.138



GIVEN: $\theta = 30°$

FIND: COUPLE $\underline{M}$ FOR EQUILIBRIUM

FREE BODY: ENTIRE FRAME



GEOMETRY

$a = 80\cos 60° = 40\text{ mm}$

$b = 80\sin 60° = 69.282\text{ mm}$

$c = 100 - b = 30.718\text{ mm}$

$d = c/\tan 30° = 53.205\text{ mm}$

$e = d - a = 53.205 - 40$

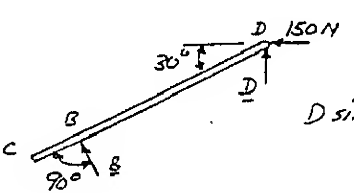
$e = 13.205\text{ mm}$

$+\circlearrowleft \Sigma M_A = 0$

$(150N)(100\text{ mm}) + D(13.205\text{ mm}) - M = 0$

$M = 15N\cdot m + D(13.205\text{ mm}) \quad (1)$

FREE BODY: ROD CD



$+\nearrow 30°\ \Sigma F = 0$

$D \sin 30° - (150N)\cos 30° = 0$

$D = 259.81N$

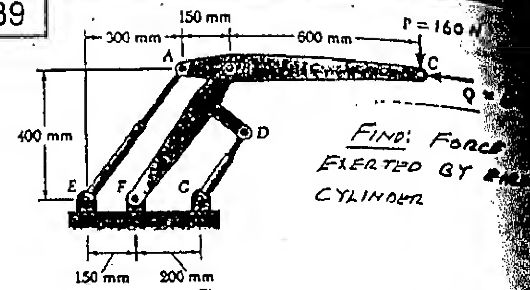
RETURN TO EQ(1):

$M = 15N\cdot m + (259.81N)(0.013205m)$

$M = +18.431\ N\cdot m$

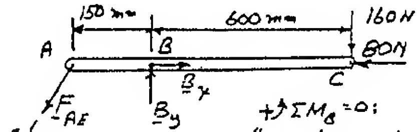
$\underline{M} = 18.43\ N\cdot m$ ◄

6.139



FIND: FORCE EXERTED BY CYLINDER

FREE BODY: MEMBER ABC



$+\circlearrowleft \Sigma M_B = 0:$

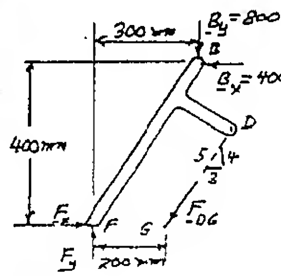
$\frac{4}{5}F_{AE}(150\text{ mm}) - (160N)(600\text{ mm}) = 0$

$F_{AE} = +800N \quad F_{AE} = 800N\ T$ ◄

$\xrightarrow{+} \Sigma F_x = 0: \ -\frac{3}{5}(800N) + B_x - 80N = 0 \quad B_x = +560N$

$+\uparrow \Sigma F_y = 0: \ -\frac{4}{5}(800N) + B_y - 160N = 0 \quad B_y = +800N$

FREE BODY: MEMBER BDF



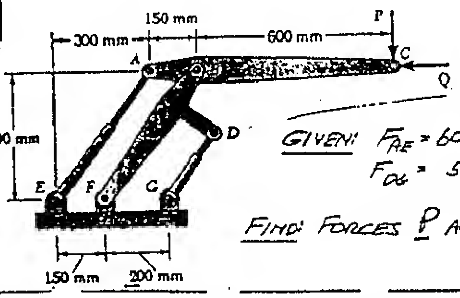
$+\circlearrowleft \Sigma M_F = 0$

$(560N)(400\text{ mm}) - (800N)(300\text{ mm}) - \frac{4}{5}F_{DG}(200\text{ mm}) = 0$

$F_{DG} = -100N$

$F_{DG} = 100N\ C$ ◄

6.140

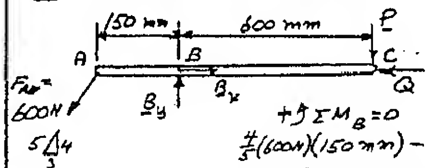


GIVEN: $F_{AE} = 600N\ T$

$F_{DG} = 50N\ T$

FIND: FORCES $\underline{P}$ AND $\underline{Q}$

FREE BODY: MEMBER ABC



$+\circlearrowleft \Sigma M_B = 0$

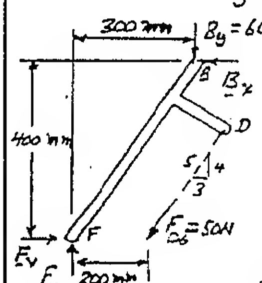
$\frac{4}{5}(600N)(150\text{ mm}) - P(600\text{ mm}) = 0$

$P = +120N \quad \underline{P} = 120N\downarrow$ ◄

$+\circlearrowleft \Sigma M_C = 0: \ \frac{4}{5}(600N)(750\text{ mm}) - B_y(600\text{ mm}) = 0$

$B_y = +600N$

FREE BODY: MEMBER BDF



$+\circlearrowleft \Sigma M_F = 0$

$B_x(400\text{ mm}) - (600N)(300\text{ mm}) - \frac{4}{5}(50N)(200\text{ mm}) = 0$

$B_x = +470N$

RETURN TO FREE BODY: MEMBER ABC

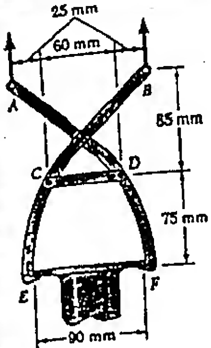
$\xrightarrow{+} \Sigma F_x = 0$

$-\frac{3}{5}(600N) + 470N - Q = 0$

$Q = +110N$

$\underline{Q} = 110N \leftarrow$ ◄

(6.141)

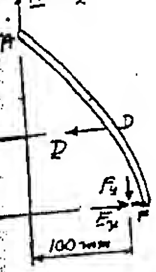


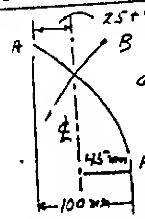
GIVEN: TONGS EXERT UPWARD FORCE OF 45 kN ON PIPECAP.

FIND: FORCES EXERTED AT D AND F ON TONG ADF.

$A = \frac{45\,kN}{2} = 22.5\,kN$

FREE BODY: TONG ADF





$25 + \frac{60}{2} = 55\,mm$

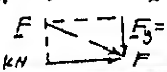
$+\uparrow \Sigma F_y = 0$: $22.5\,kN - F_y = 0$

$F_y = +22.5\,kN$

$\Sigma M_F = 0$: $D(75\,mm) - (22.5\,kN)(100\,mm) = 0$

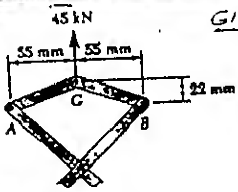
$D = +30\,kN$ $\quad$ $D = 30\,kN \leftarrow$ ◀

$\Sigma F_x = 0$: $-30\,kN + F_x = 0$ $\quad$ $F_x = +30\,kN$



$F = 37.5\,kN \measuredangle 36.9^\circ$ ◀

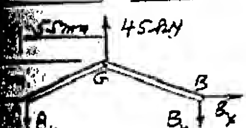
6.142



GIVEN: TOGGLE SHOWN IS ADDED TO TONGS OF PROB. 6.141

FIND: FORCES EXERTED AT D AND F ON TONG ADF

FREE BODY: TOGGLE



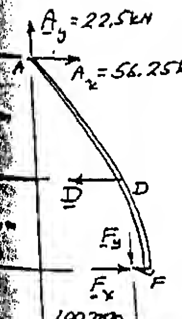
BY SYMMETRY: $A_y = \frac{1}{2}(45\,kN) = 22.5\,kN$

AG IS A TWO-FORCE MEMBER

$$\frac{22.5\,kN}{22\,mm} = \frac{A_x}{55\,mm}$$

$A_x = 56.25\,kN$

FREE BODY: TONG ADF



$+\uparrow \Sigma F_y = 0$: $22.5\,kN - F_y = 0$

$F_y = +22.5\,kN$

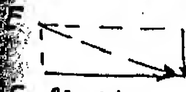
$+\circlearrowleft \Sigma M_F = 0$:

$D(75\,mm) - (22.5\,kN)(100\,mm) - (56.25\,kN)(160\,mm) = 0$

$D = +150\,kN$ $\quad$ $D = 150\,kN \leftarrow$ ◀

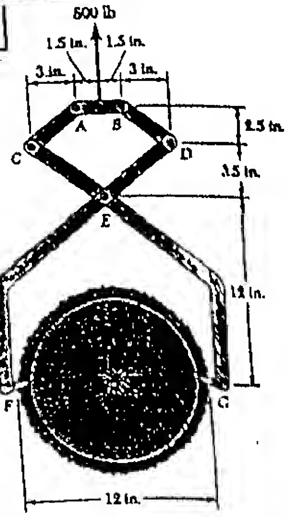
$\pm \Sigma F_x = 0$: $56.25\,kN - 150\,kN + F_x = 0$

$F_x = 93.75\,kN$



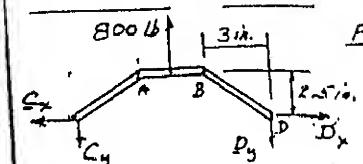
$F = 96.4\,kN \measuredangle 13.5^\circ$ ◀

6.143



GIVEN: WEIGHT OF LOG IS 800 lb

FIND: FORCES EXERTED AT E AND F ON TONG DEF.



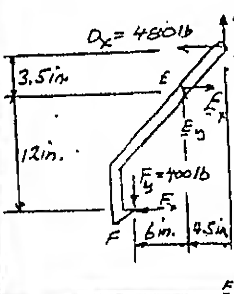
FREE BODY: MEMBERS CA, AB, BD

BY SYMMETRY

$D_y = \frac{1}{2}(800\,lb) = 400\,lb$

SINCE BD IS A TWO-FORCE MEMBER

$$\frac{D_y}{2.5\,in.} = \frac{D_x}{3\,in.};\quad \frac{400\,lb}{2.5\,in.} = \frac{D_x}{3\,in.};\quad D_x = 480\,lb$$



FREE BODY: TONG DEF

$+\uparrow \Sigma F_y = 0$: $400\,lb + E_y - 400\,lb = 0$

$E_y = 0$

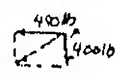
$+\circlearrowleft \Sigma M_F = 0$

$(400\,lb)(10.5\,in.) + (480\,lb)(15.5\,in.) - E_x(12\,in.) = 0$ ◀

$E_x = +970\,lb$ $\quad$ $E = 970\,lb \rightarrow$

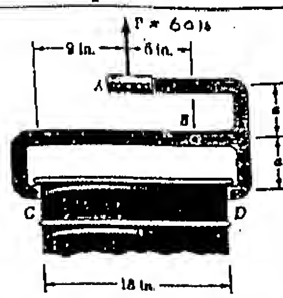
$\pm \Sigma F_x = 0$: $970\,lb - 480\,lb - F_x = 0$

$F_x = 490\,lb$



$F = 633\,lb \measuredangle 39.2^\circ$ ◀

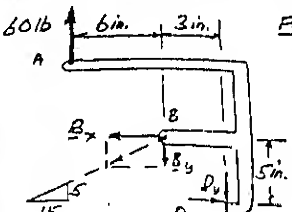
6.144



GIVEN: $a = 5\,in.$ WEIGHT OF BARREL = 60 lb

FIND: FORCES EXERTED AT B AND D ON TONG ABD

WE NOTE THAT BC IS A TWO-FORCE MEMBER.



FREE BODY: TONG ABD

$$\frac{B_x}{15} = \frac{B_y}{5} \qquad B_x = 3B_y$$

$+\circlearrowleft \Sigma M_D = 0$

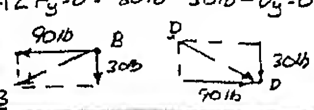
$B_y(3\,in.) + 3B_y(5\,in.) - (60\,lb)(9\,in.) = 0$

$B_y = 30\,lb \uparrow$

$B_x = 3B_y$: $\quad$ $B_x = 90\,lb \leftarrow$

$\pm \Sigma F_x = 0$: $-90\,lb + D_x = 0$. $\quad$ $D_x = 90\,lb \rightarrow$

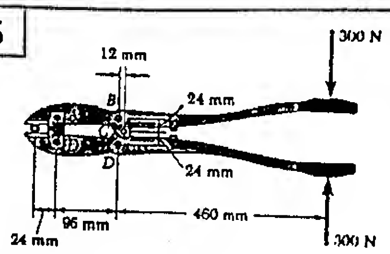
$+\uparrow \Sigma F_y = 0$: $60\,lb - 30\,lb - D_y = 0$ $\quad$ $D_y = 30\,lb \downarrow$



$B = 94.9\,lb \measuredangle 18.4^\circ$ ◀

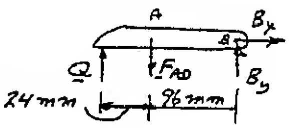
$D = 94.9\,lb \measuredangle 18.4^\circ$ ◀

6.145



FIND: MAGNITUDE OF FORCES EXERTED ON BOLT

WE NOTE THAT AD IS A TWO-FORCE MEMBER



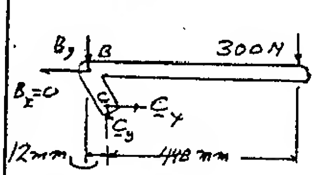
FREE BODY: JAW AB

$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$

$+\circlearrowleft \Sigma M_A = 0$

$B_y(96\,mm) - Q(24\,mm) = 0$

$Q = 4B_y \qquad (1)$



FREE BODY: HANDLE

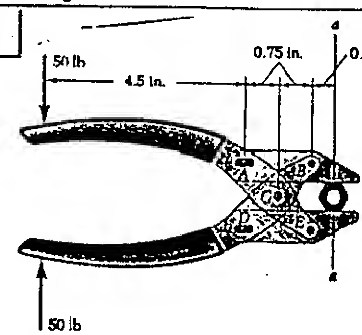
$+\circlearrowleft \Sigma M_C = 0$

$B_y(12\,mm) - (300\,N)(448\,mm) = 0$

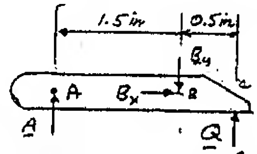
$B_y = 11.2 \times 10^3\,N = 11.2\,kN$

EQ(1): $Q = 4B_y = 4(11.2\,kN) = 44.8\,kN \qquad Q = 44.8\,kN$ ◀

6.146



FIND: GRIPPING FORCES EXERTED ALONG LINE a-a ON THE NUT



FREE BODY: UPPER JAW

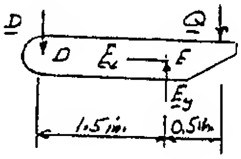
$\xrightarrow{+} \Sigma F_x = 0 \quad B_x = 0$

$+\circlearrowleft \Sigma M_B = 0$

$Q(0.5\,in.) - A(1.5\,in.) = 0$

$A = Q/3$

$+\circlearrowleft \Sigma M_A = 0: \; -B_y(1.5\,in.) + A(2\,in.) = 0 \qquad B_y = \frac{4Q}{3}$



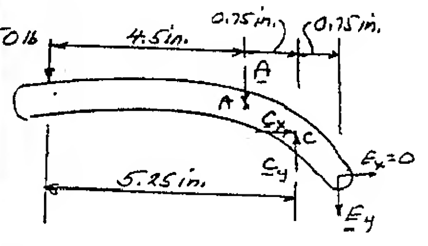
FREE BODY: LOWER JAW

$\xrightarrow{+} \Sigma F_x = 0: \quad E_x = 0$

$+\circlearrowleft \Sigma M_D = 0:$

$E_y(1.5\,in.) - Q(2\,in.) = 0$

$E_y = 4Q/3$



FREE BODY: HANDLE

FROM ABOVE:

$A = Q/3$

$E_y = 4Q/3$

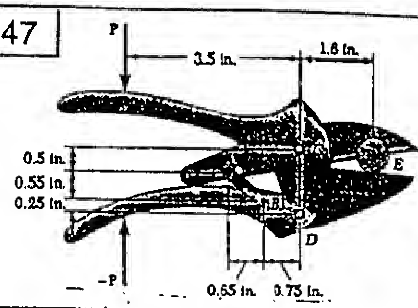
$+\circlearrowleft \Sigma M_C = 0: \; (50\,lb)(5.25\,in.) + A(0.75\,in.) - E_y(0.75\,in.) = 0$

$262.5\,lb\cdot in. + (Q/3)(0.75\,in.) - (4Q/3)(0.75\,in.) = 0$

$262.5\,lb\cdot in. - Q(0.75\,in.) = 0$

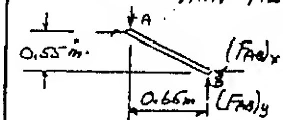
$Q = 350\,lb \qquad Q = 350\,lb$ ◀

6.147



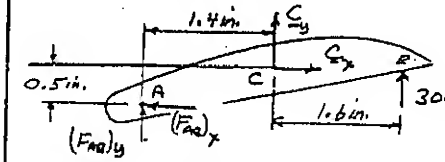
FIND: MAGNITUDE FOR 300-lb VERTICAL CUTTING FORCE ON BRANCH AT E.

WE NOTE THAT AB IS A TWO-FORCE MEMBER



$\frac{(F_{AB})_x}{0.65\,in.} = \frac{(F_{AB})_y}{0.55\,in.}$

$(F_{AB})_y = \frac{11}{13}(F_{AB})_x \qquad (1)$



FREE BODY: BLADE ACE

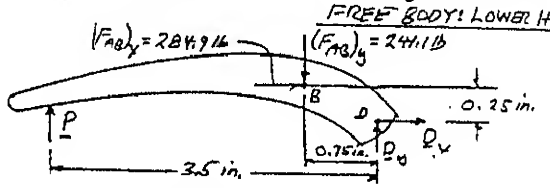
$+\circlearrowleft \Sigma M_C = 0: \; (300\,lb)(1.6\,in.) - (F_{AB})_x(0.5\,in.) - (F_{AB})_y(1.4\,in.) = 0$

USE EQ(1): $(F_{AB})_x(0.5\,in.) + \frac{11}{13}(F_{AB})_x(1.4\,in.) = 480\,lb\cdot in.$

$1.6846(F_{AB})_x = 480 \qquad (F_{AB})_x = 284.9\,lb$

$(F_{AB})_y = \frac{11}{13}(284.9\,lb) \qquad (F_{AB})_y = 241.1\,lb$

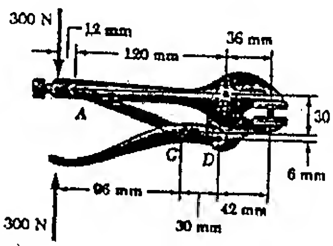
FREE BODY: LOWER HANDLE



$+\circlearrowleft \Sigma M_D = 0: \; (241.1\,lb)(0.75\,in.) - (284.9\,lb)(0.25\,in.) - P(3.5\,in.) = 0$

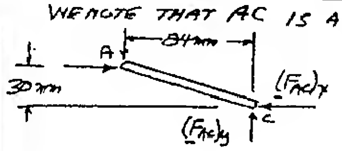
$P = 31.3\,lb$ ◀

6.148



FIND: GRIPPING FORCES PRODUCED BY 300-lb FORCES

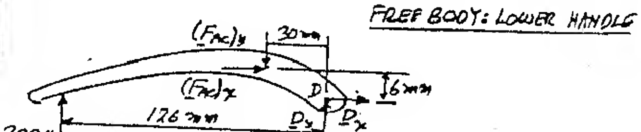
WE NOTE THAT AC IS A TWO-FORCE MEMBER



$\frac{(F_{AC})_x}{84\,mm} = \frac{(F_{AC})_y}{30\,mm}$

$(F_{AC})_x = 2.8(F_{AC})_y$

FREE BODY: LOWER HANDLE



$+\circlearrowleft \Sigma M_D = 0: \; (F_{AC})_y(30\,mm) - (F_{AC})_x(6\,mm) - (300\,N)(126\,mm) = 0$

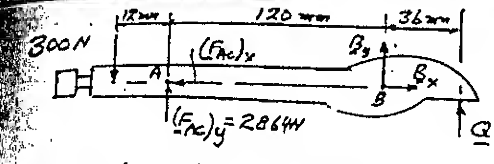
$(F_{AC})_y(30) - 2.8(F_{AC})_y(6) - (300)(126) = 0$

$(F_{AC})_y = 2864\,N$

(CONTINUED)

6.148 CONTINUED

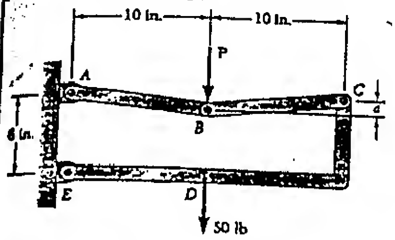
FREE BODY: UPPER HANDLE



$+\circlearrowleft \Sigma M_B = 0$: $(300N)(132mm) - (2864N)(120mm) + Q(36mm) = 0$

$Q = 8450N$ $\quad Q = 8.45 kN$ ◀

6.149 and 6.150

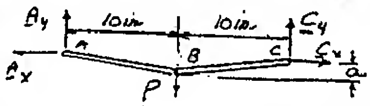


FIND: FORCE P REQUIRED FOR EQUILIBRIUM

PROB. 6.149 WHEN $a = 1$ in.

PROB. 6.150 WHEN $a = 0.5$ in.

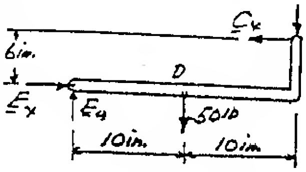
WE NOTE THAT AB AND BC ARE TWO-FORCE MEMBERS



FREE BODY: TOGGLE

BY SYMMETRY: $C_y = P/2$

$$\frac{C_x}{10in.} = \frac{C_y}{a}; \quad C_x = \frac{10}{a} C_y = \frac{10}{a} \cdot \frac{P}{2} = \frac{5P}{a}$$



FREE BODY: MEMBER CDE

$+\circlearrowleft \Sigma M_E = 0$:

$C_x(6in) - C_y(20in) - (50lb)(10in) = 0$

$\frac{5P}{a}(6) - \frac{P}{2}(20) = 500$

$P\left(\frac{30}{a} - 10\right) = 500 \quad (1)$

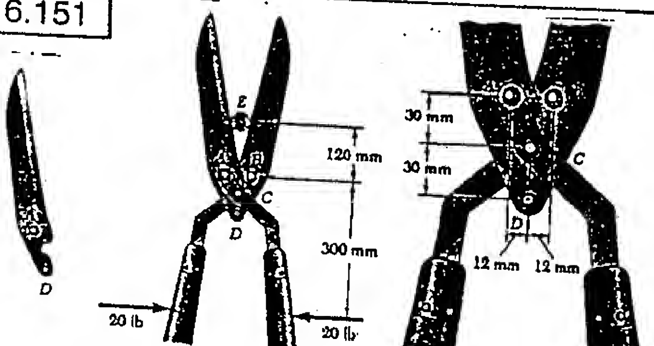
PROB. 6.149: $a = 1$ in.

Eq(1): $P\left(\frac{30}{1} - 10\right) = 500$; $\quad 20P = 500 \quad P = 25 lb \downarrow$ ◀

PROB. 6.150: $a = 0.5$ in.

Eq(1): $P\left(\frac{30}{0.5} - 10\right) = 500$; $\quad 50P = 500 \quad P = 10 lb \downarrow$ ◀

6.151

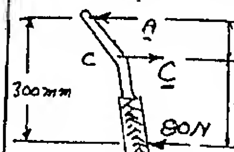


FIND: MAGNITUDE OF FORCES EXERTED AT E.

BY SYMMETRY VERTICAL COMPONENTS C_y, D_y, E_y ARE 0. THEN BY CONSIDERING $\Sigma F_y = 0$ ON THE BLADES OR HANDLES, WE FIND THAT A_y AND B_y ARE 0. THUS FORCES AT A, B, C, D, AND E ARE HORIZONTAL

(CONTINUED)

6.151 CONTINUED



FREE BODY: RIGHT HANDLE

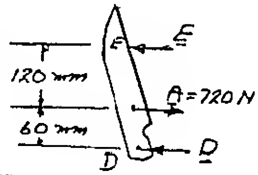
$+\circlearrowleft \Sigma M_C = 0$:

$A(30mm) - (80N)(270mm) = 0$

$A = +720N$

$\xrightarrow{+} \Sigma F_x = 0$: $C - 720N - 80N = 0$

$C = +800N$



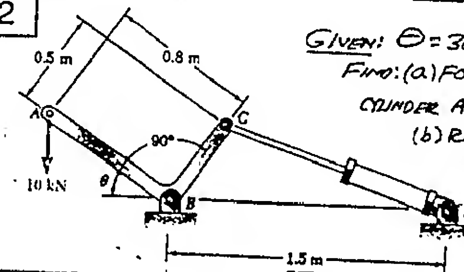
FREE BODY: LEFT BLADE

$+\circlearrowleft \Sigma M_D = 0$

$E(180mm) - (720N)(60mm) = 0$

$E = 240N$ ◀

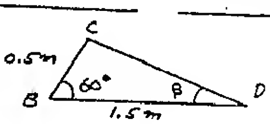
6.152



GIVEN: $\Theta = 30°$

FIND: (a) FORCE OF CYLINDER AT C.

(b) REACTION AT B.



GEOMETRY: IN ΔBCD

LAW OF COSINES

$(CD)^2 = (0.5)^2 + (1.5)^2 - 2(0.5)(1.5)\cos 60°$

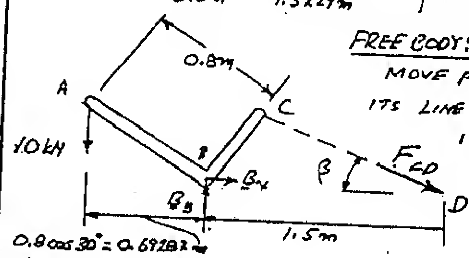
$CD = 1.3229$ m

LAW OF SINES

$$\frac{\sin \beta}{0.5m} = \frac{\sin 60°}{1.3229m}; \quad \sin\beta = 0.3273; \quad \beta = 19.107°$$

FREE BODY: ENTIRE SYSTEM

MOVE FORCE F_{CD} ALONG ITS LINE OF ACTION SO IT ACTS AT D.



(a)

$+\circlearrowleft \Sigma M_B = 0$: $(10 kN)(0.69282m) - F_{CD}\sin\beta(1.5m) = 0$

$6.9282 kN\cdot m - F_{CD}\sin 19.107°(1.5m) = 0$

$F_{CD} = 14.111 kN$

$F_{CD} = 14.11 kN \measuredangle 19.1°$ ◀

(b)

$\xrightarrow{+} \Sigma F_x = 0$: $B_x + F_{CD}\cos\beta = 0$

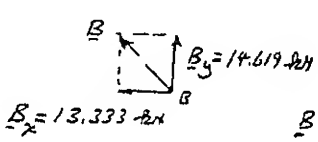
$B_x + (14.11 kN)\cos 19.107° = 0$

$B_x = -13.333 kN \quad B_x = 13.333 kN \leftarrow$

$+\uparrow \Sigma F_y = 0$: $B_y - 10kN - F_{CD}\sin 19.107° = 0$

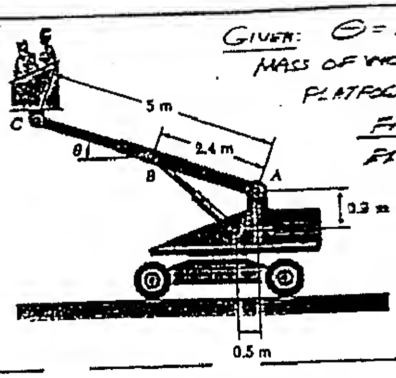
$B_y - 10kN - (14.111kN)\sin 19.107° = 0$

$B_y = +14.619 kN \quad B_y = 14.619 kN \uparrow$

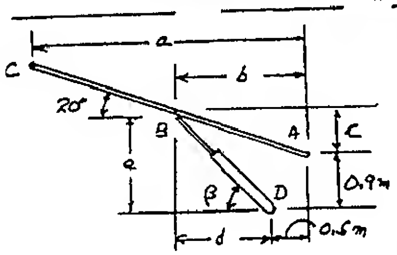


$B = 19.79 kN \measuredangle 47.6°$ ◀

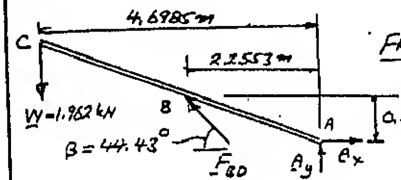
6.153



GIVEN: $\theta = 20°$
MASS OF WORKERS AND PLATFORM = 200 kg
FIND: (a) FORCE EXERTED BY BD AT B.
(b) FORCE EXERTED ON CARRIAGE AT A.



GEOMETRY:
$a = (5\,m)\cos 20° = 4.6985\,m$
$b = (2.4\,m)\cos 20° = 2.2553\,m$
$c = (2.4\,m)\sin 20° = 0.8208\,m$
$d = b - 0.5 = 1.7553\,m$
$e = c + 0.9 = 1.7208\,m$
$\tan\beta = \frac{e}{d} = \frac{1.7208}{1.7553}$; $\beta = 44.43°$

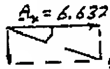


FREE BODY: ARM ABC
WE NOTE THAT BD IS A TWO-FORCE MEMBER

$W = (200\,kg)(9.81\,m/s^2) = 1.962\,kN$

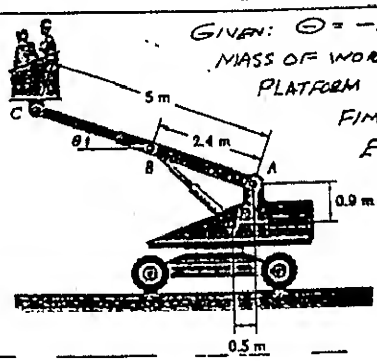
(a)
$+\circlearrowright \Sigma M_A = 0$: $(1.962\,kN)(4.6985\,m) - F_{BD}\sin 44.43°(2.2553\,m) + F_{BD}\cos 44.43°(0.8208\,m) = 0$
$9.2185 - F_{BD}(0.9927) = 0$: $F_{BD} = 9.2867\,kN$
$F_{BD} = 9.29\,kN \measuredangle 44.4°$ ◀

(b)
$\xrightarrow{+} \Sigma F_x = 0$: $A_x - F_{BD}\cos\beta = 0$
$A_x = (9.2867\,kN)\cos 44.43° = 6.632\,kN$ $\quad A_x = 6.632\,kN \longrightarrow$
$+\uparrow \Sigma F_y = 0$: $A_y - 1.962\,kN + F_{BD}\sin\beta = 0$
$A_y = 1.962\,kN - (9.2867\,kN)\sin 44.43° = -4.539\,kN$ $\quad A_y = 4.539\,kN \downarrow$

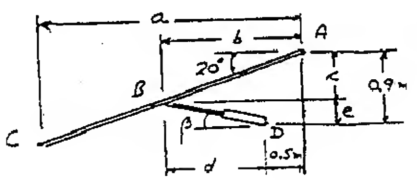


$A = 8.04\,kN \measuredangle 34.4°$ ◀

6.154



GIVEN: $\theta = -20°$
MASS OF WORKERS AND PLATFORM = 200 kg.
FIND: (a) FORCE EXERTED BY BD AT B.
(b) FORCE EXERTED ON CARRIAGE AT A.



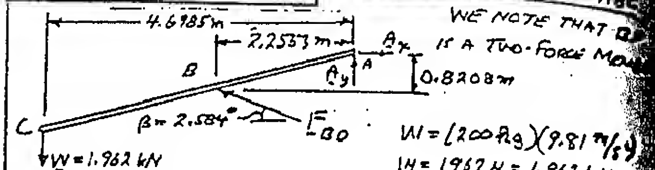
GEOMETRY:
$a = (5\,m)\cos 20° = 4.6985\,m$
$b = (2.4\,m)\cos 20° = 2.2552\,m$
$c = (2.4\,m)\sin 20° = 0.8208\,m$
$d = b - 0.5 = 1.7553\,m$
$e = 0.9 - c = 0.0792\,m$

$\tan\beta = \frac{e}{d} = \frac{0.0792}{1.7552}$; $\beta = 2.584°$

(CONTINUED)

6.154 CONTINUED

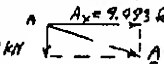
FREE BODY: ARM ABC
WE NOTE THAT BD IS A TWO-FORCE MEMBER



$W = (200\,kg)(9.81\,m/s^2)$
$W = 1962\,N = 1.962\,kN$

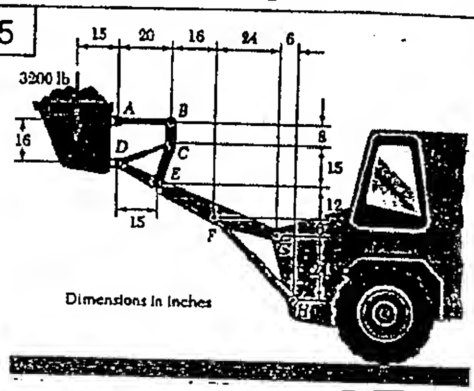
(a) $+\circlearrowright \Sigma M_A = 0$
$(1.962\,kN)(4.6985\,m) - F_{BD}\sin 2.584°(2.2553\,m) - F_{BD}\cos 2.584°(0.8208\,m) = 0$
$9.2185 - F_{BD}(0.9216) = 0$ $\quad F_{BD} = 10.003\,kN$
$F_{BD} = 10.00\,kN \measuredangle 2.58°$ ◀

(b)
$\xrightarrow{+} \Sigma F_x = 0$: $A_x - F_{BD}\cos\beta = 0$
$A_x = (10.003\,kN)\cos 2.583° = 9.993\,kN$ $\quad A_x = 9.993\,kN \longrightarrow$
$+\uparrow \Sigma F_y = 0$: $A_y - 1.962\,kN + F_{BD}\sin\beta = 0$
$A_y = 1.962\,kN - (10.003\,kN)\sin 2.583° = 1.5112\,kN$ $\quad A_y = 1.5112\,kN$

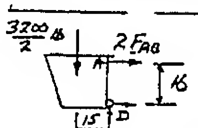


$A = 10.11\,kN \measuredangle 8.6°$ ◀

6.155

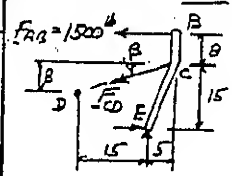


FIND: FORCE EXERTED BY CD AND FH.

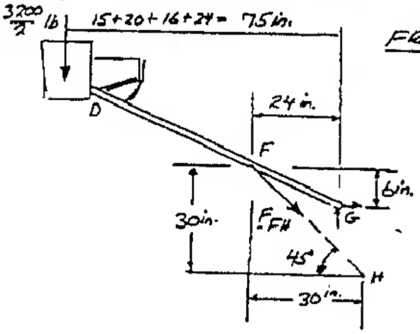


FREE BODY: BUCKET (ONE MECHANISM)
$+\circlearrowright \Sigma M_D = 0$: $(1600\,lb)(15\,in.) - F_{AB}(16\,in.) = 0$
$F_{AB} = 1500\,lb$

NOTE: THERE ARE 2 IDENTICAL SUPPORT MECHANISMS



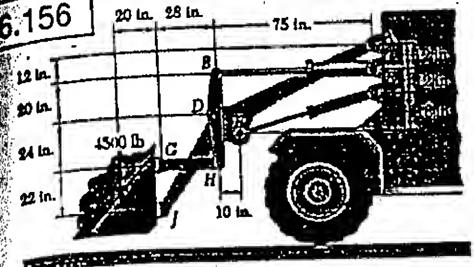
FREE BODY: ONE ARM BCE
$\tan\beta = \frac{8}{20}$; $\beta = 21.8°$
$+\circlearrowright \Sigma M_E = 0$: $(1500\,lb)(23\,in.)$
$+ F_{CD}\cos 21.8°(15\,in.) - F_{CD}\sin 21.8°(5\,in.) = 0$
$F_{CD} = -2858\,lb$
$F_{CD} = 2.86\,kips$ C ◀



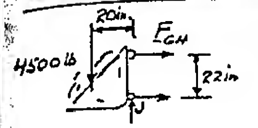
FREE BODY: ARM DFG

$+\circlearrowright \Sigma M_G = 0$:
$(1600\,lb)(75\,in.) + F_{FH}\sin 45°(24\,in.) - F_{FH}\cos 45°(6\,in.) = 0$
$F_{FH} = -9.428\,kips$
$F_{FH} = 9.43\,kips$ C ◀

6.156



FIND: FORCE EXERTED BY
(a) CYLINDER BC
(b) CYLINDER EF

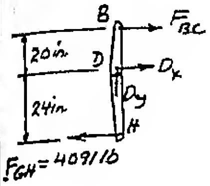


FREE BODY: BUCKET

$+\circlearrowleft \Sigma M_J = 0$:

$(4500\ lb)(20in.) - F_{GH}(22\ in.) = 0$

$F_{GH} = 4091\ lb$



FREE BODY: ARM BDH

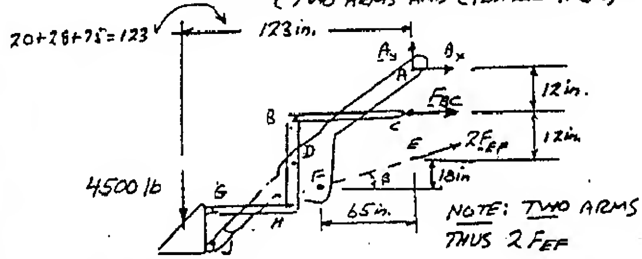
$+\circlearrowleft \Sigma M_D = 0$:

$-(4091\ lb)(24\ in.) - F_{BC}(20\ in.) = 0$

$F_{BC} = -4909\ lb$ $\quad F_{BC} = 4.91$ kips C ◀

FREE BODY: ENTIRE MECHANISM

(TWO ARMS AND CYLINDERS AFJE)



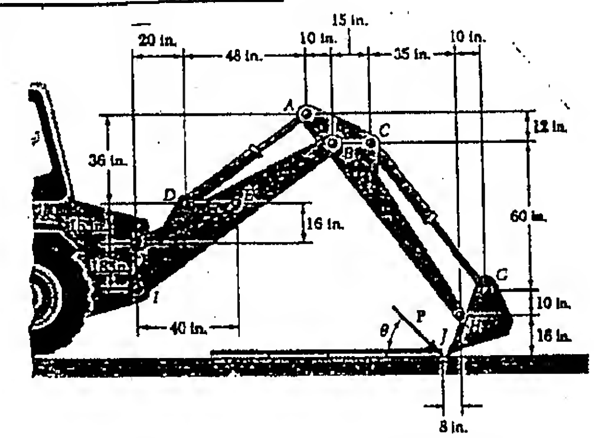
$\tan\beta = \frac{18in.}{65in.}$; $\beta = 15.48°$

$+\circlearrowleft \Sigma M_A = 0$: $(4500\ lb)(123\ in.) + F_{BC}(12in.) + 2F_{EF}\cos\beta(24in.) = 0$

$(4500\ lb)(123in) - (4909\ lb)(12in.) + 2F_{EF}\cos 15.48(24in.) = 0$

$F_{EF} = -10{,}690\ lb$ $\qquad F_{EF} = 10.69$ kips C ◀

6.157 and 6.158



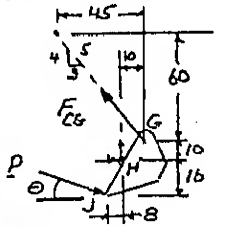
GIVEN: $P = 2$ kips

FIND: FORCE EXERTED BY EACH CYLINDER

PROB. 6.157 WHEN $\theta = 45°$

PROB. 6.158 WHEN $\theta = 0$

6.157 and 6.158 CONTINUED



FREE BODY: BUCKET

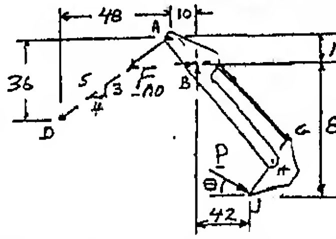
$+\circlearrowleft \Sigma M_H = 0$ (DIMENSIONS IN INCHES)

$\frac{4}{5}F_{CG}(10) + \frac{3}{5}F_{CG}(10)$

$+ P\cos\theta(16) + P\sin\theta(8) = 0$

$F_{CG} = -\frac{P}{14}(16\cos\theta + 8\sin\theta)$ $\qquad (1)$

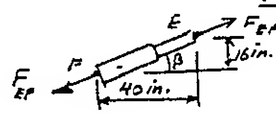
FREE BODY: ARM ABH AND BUCKET (DIMENSIONS IN INCHES)



$+\circlearrowleft \Sigma M_B = 0$: $\frac{4}{5}F_{AD}(12) + \frac{3}{5}F_{AD}(10) + P\cos\theta(86) - P\sin\theta(42) = 0$

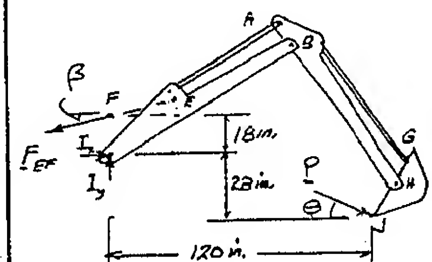
$F_{AD} = -\frac{P}{15.6}(86\cos\theta - 42\sin\theta)$ $\qquad (2)$

FREE BODY: BUCKET AND ARMS IEB + ABH



GEOMETRY OF CYLINDER EF

$\tan\beta = \frac{16in.}{40in.}$ $\qquad \beta = 21.801°$



$+\circlearrowleft \Sigma M_I = 0$

$F_{EF}\cos\beta(18in.) + P\cos\theta(28in.) - P\sin\theta(120in.) = 0$

$F_{EF} = \frac{P(120\sin\theta - 28\cos\theta)}{\cos 21.8°(18)} = \frac{P}{16.7126}(120\sin\theta - 28\cos\theta)$ (3)

PROB. 6.157 $\quad P = 2$ kips, $\theta = 45°$

EQ(1): $F_{CG} = -\frac{2}{14}(16\cos45° + 8\sin45°) = -2.42$ kips

$F_{CG} = 2.42$ kips C ◀

EQ(2): $F_{AD} = -\frac{2}{15.6}(86\cos45° - 42\sin45°) = -3.99$ kips

$F_{AD} = -3.99$ kips C ◀

EQ(3): $F_{EF} = \frac{2}{16.7126}(120\sin45° - 28\cos45°) = +7.79$ kips

$F_{EF} = 7.79$ kips T ◀

PROB. 6.158 $\quad P = 2$ kips, $\theta = 0$

EQ(1): $F_{CG} = -\frac{2}{14}(16\cos0 + 8\sin0) = -2.29$ kips

$F_{CG} = 2.29$ kips C ◀

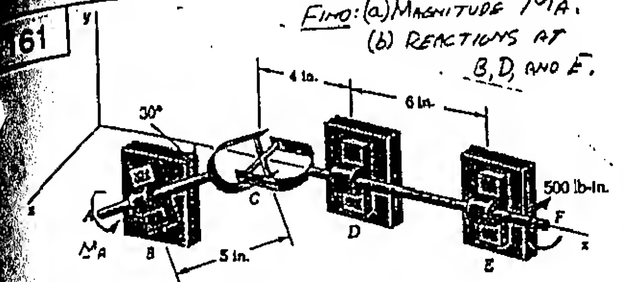
EQ(2): $F_{AD} = -\frac{2}{15.6}(86\cos0 - 42\sin0) = -11.03$ kips

$F_{AD} = 11.03$ kips C ◀

EQ(3): $F_{EF} = \frac{2}{16.7126}(120\sin0 - 28\cos\theta) = -3.35$ kips

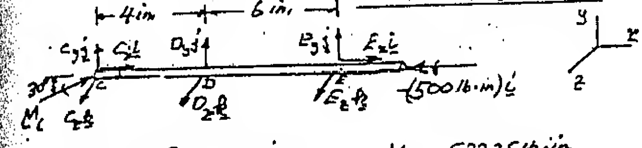
$F_{EF} = 3.35$ kips C ◀

FIND: (a) MAGNITUDE M_A.
(b) REACTIONS AT B, D, AND E.



WE RECALL FROM FIG. 4.10, page 187, THAT A UNIVERSAL JOINT EXERTS ON MEMBERS IT CONNECTS A FORCE OF UNKNOWN DIRECTION AND A COUPLE ABOUT AN AXIS PERPENDICULAR TO THE CROSS PIECE.

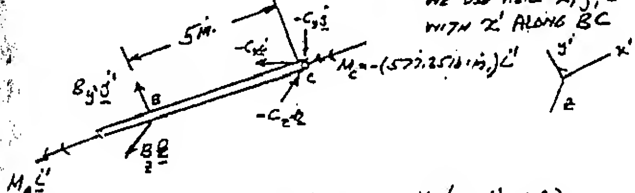
FREE BODY: SHAFT DF



$\Sigma M_x = 0$: $M_C \cos 30° - 500 \text{ lb·in.} = 0$ $M_C = 577.35 \text{ lb·in.}$

FREE BODY: SHAFT BC

WE USE HERE x', y', z WITH x' ALONG BC



$\Sigma M_C = 0$: $-M_A \mathbf{i}' - (577.35 \text{ lb·in.})\mathbf{i}' + (-5 \text{ in.})\mathbf{i}' \times (B_y \mathbf{j}' + B_z \mathbf{k}) = 0$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO:

(i) $M_A - 577.35 \text{ lb·in.} = 0$ $M_A = 577.35 \text{ lb·in.}$ $M_A = 577 \text{ lb·in.}$ ◄

(j) $B_z = 0$
(k) $B_y = 0$ } → $B = 0$ $B = 0$, $C = 0$

$\Sigma \mathbf{F} = 0$: $B + C = 0$, SINCE $B = 0$,

RETURN TO FREE BODY OF SHAFT DF

$\Sigma M_D = 0$ (NOTE THAT $C = 0$ AND $M_C = 577.35 \text{ lb·in.}$)

$(577.35 \text{ lb·in.})(\cos 30° \mathbf{i} + \sin 30° \mathbf{j}) - (500 \text{ lb·in.})\mathbf{i} + (6 \text{ in.})\mathbf{i} \times (E_x \mathbf{i} + E_y \mathbf{j} + E_z \mathbf{k}) = 0$

$(500 \text{ lb·in.})\mathbf{i} + (288.68 \text{ lb·in.})\mathbf{j} - (500 \text{ lb·in.})\mathbf{i} + (6 \text{ in.})E_y \mathbf{k} - (6 \text{ in.})E_z \mathbf{j} = 0$

EQUATE COEFFICIENTS OF UNIT VECTORS TO ZERO:

(j) $288.68 \text{ lb·in.} - (6 \text{ in.})E_z = 0$ $E_z = 48.1 \text{ lb}$

(k) $E_y = 0$

$\Sigma \mathbf{F} = 0$: $\mathbf{C} + \mathbf{D} + \mathbf{E} = 0$

$0 + D_y \mathbf{j} + D_z \mathbf{k} + E_x \mathbf{i} + (48.1 \text{ lb})\mathbf{k} = 0$

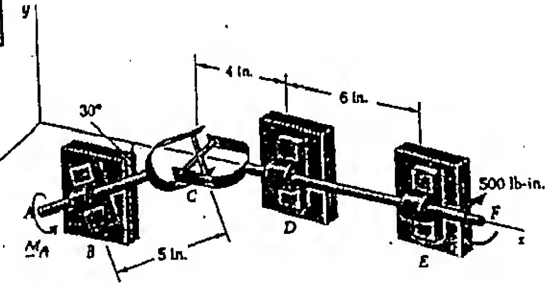
(i) $E_x = 0$

(j) $D_y = 0$

(k) $D_z + 48.1 \text{ lb} = 0$ $D_z = -48.1 \text{ lb}$

REACTIONS ARE: $\mathbf{B} = 0$, $\mathbf{D} = -(48.1 \text{ lb})\mathbf{k}$, $\mathbf{E} = (48.1 \text{ lb})\mathbf{k}$ ◄◄◄

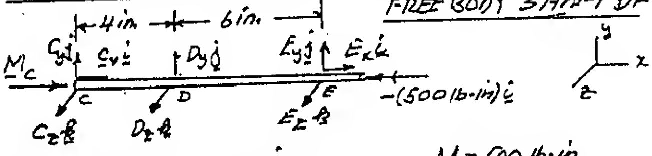
*6.162



GIVEN: ROTATE SHAFT UNTIL CROSSPIECE ATTACHED TO SHAFT CF IS VERTICAL, THEN

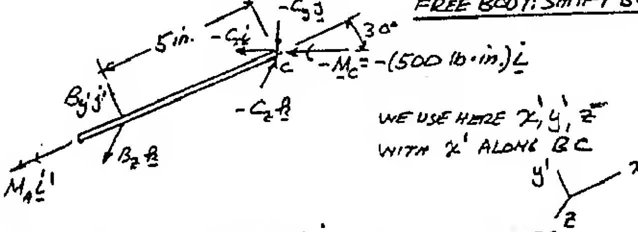
FIND: (a) MAGNITUDE M_A. (b) REACTIONS AT B, D, AND E.

FREE BODY SHAFT DF



$\Sigma M_x = 0$; $M_C - 500 \text{ lb·in.} = 0$ $M_C = 500 \text{ lb·in.}$

FREE BODY: SHAFT BC



WE USE HERE x', y', z WITH x' ALONG BC

WE RESOLVE $-(500 \text{ lb·in.})\mathbf{i}$ INTO COMPONENTS ALONG x' AND y' AXES: $-\mathbf{M}_C = -(500 \text{ lb·in.})(\cos 30° \mathbf{i}' + \sin 30° \mathbf{j}')$

$\Sigma M_C = 0$: $M_A \mathbf{i}' - (500 \text{ lb·in.})(\cos 30° \mathbf{i}' + \sin 30° \mathbf{j}') + (5 \text{ in.})\mathbf{i}' \times (B_y \mathbf{j}' + B_z \mathbf{k}) = 0$

$M_A \mathbf{i}' - (433 \text{ lb·in.})\mathbf{i}' - (250 \text{ lb·in.})\mathbf{j}' + (5 \text{ in.})B_y \mathbf{k} - (5 \text{ in.})B_z \mathbf{j}' = 0$

EQUATE TO ZERO COEFFICIENTS OF UNIT VECTORS:

(i) $M_A - 433 \text{ lb·in.} = 0$ $M_A = 433 \text{ lb·in.}$ ◄

(j) $-250 \text{ lb·in.} - (5 \text{ in.})B_z = 0$ $B_z = -50 \text{ lb}$

(k) $B_{y'} = 0$

REACTION AT B: $\mathbf{B} = -(50 \text{ lb})\mathbf{k}$

$\Sigma \mathbf{F} = 0$: $\mathbf{B} - \mathbf{C} = 0$

$-(50 \text{ lb})\mathbf{k} - \mathbf{C} = 0$ $\mathbf{C} = -(50 \text{ lb})\mathbf{k}$

RETURN TO FREE BODY OF SHAFT DF:

$\Sigma M_D = 0$: $(6 \text{ in.})\mathbf{i} \times (E_x \mathbf{i} + E_y \mathbf{j} + E_z \mathbf{k}) - (4 \text{ in.})\mathbf{i} \times (-50 \text{ lb})\mathbf{k} - (500 \text{ lb·in.})\mathbf{i} + (500 \text{ lb·in.})\mathbf{i} = 0$

$(6 \text{ in.})E_y \mathbf{k} - (6 \text{ in.})E_z \mathbf{j} - (200 \text{ lb·in.})\mathbf{j} = 0$

(k) $E_y = 0$

(j) $-(6 \text{ in.})E_z - 200 \text{ lb·in.} = 0$ $E_z = -33.3 \text{ lb}$

$\Sigma \mathbf{F} = 0$: $\mathbf{C} + \mathbf{D} + \mathbf{E} = 0$

$-(50 \text{ lb})\mathbf{k} + D_y \mathbf{j} + D_z \mathbf{k} + E_x \mathbf{i} - (33.3 \text{ lb})\mathbf{k} = 0$

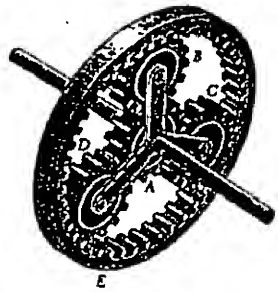
(i) $E_x = 0$

(k) $-50 \text{ lb} - 33.3 \text{ lb} + D_z = 0$

$D_z = 83.3 \text{ lb}$

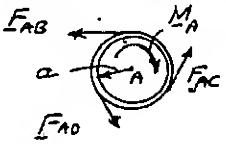
REACTIONS ARE: $\mathbf{B} = -(50 \text{ lb})\mathbf{k}$, $\mathbf{D} = (83.3 \text{ lb})\mathbf{k}$, $\mathbf{E} = -(33.3 \text{ lb})\mathbf{k}$ ◄◄◄

6.159



GIVEN: RADIUS OF GEAR A IS $a = 18$ mm, GEAR B IS b, $M_A = 10$ N·m APPLIED TO GEAR A. $M_S = 50$ N·m APPLIED TO BCD.

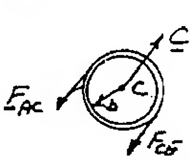
FIND: (a) VALUE OF b, (b) COUPLE M_E APPLIED TO GEAR E.



GEAR A: BY SYMMETRY $F_{AB} = F_{AC} = F_{AD}$

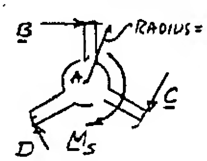
$+\circlearrowleft \Sigma M_A = 0: \; -M_A + 3F_{AC}\,a = 0$

$F_{AC} = \dfrac{M_A}{3a}$



GEAR C: $+\circlearrowleft \Sigma M_C = 0: \; F_{AC}b - F_{CE}b = 0$

$F_{CE} = F_{AC} = \dfrac{M_A}{3a}$



SPIDER BCD:

BY SYMMETRY: $B = C = D$

$+\circlearrowleft \Sigma M_A = 0$

$-M_S + 3C(a+b) = 0$

$-M_S + 3\left(\dfrac{M_A}{3a}\right)(a+b) = 0$

$\dfrac{M_S}{M_A} = 2\,\dfrac{a+b}{a} = 2\left(1 + \dfrac{b}{a}\right)$

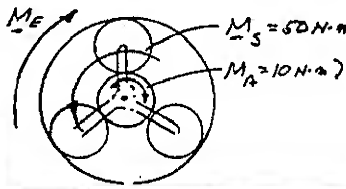
(a)

GIVEN: $M_S = 50$ N·m AND $M_A = 10$ N·m

$\dfrac{50 \text{ N·m}}{10 \text{ N·m}} = 2\left(1 + \dfrac{b}{a}\right)$

$\dfrac{b}{a} = 1.5$ FOR $a = 18$ mm, $b = 1.5(18 \text{ mm})$; $b = 27$ mm ◀

(b)



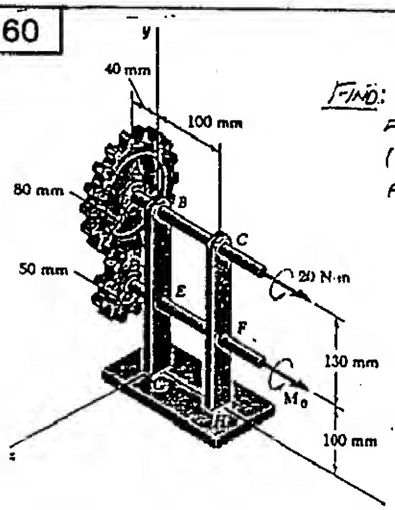
FREE BODY: ENTIRE SYSTEM

$\circlearrowleft \Sigma M = 0$

$10 \text{ N·m} - 50 \text{ N·m} + M_E = 0$

$M_E = 40$ N·m ◀

6.160

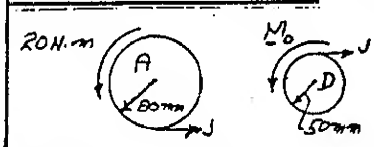


FIND: (a) COUPLE M_0 FOR EQUILIBRIUM (b) REACTIONS AT G AND H.

(CONTINUED)

6.160 CONTINUED

PROJECTIONS ON yz PLANE



GEAR A: $+\circlearrowleft \Sigma M_A = 0: \; 20 \text{ N·m} - J(0.08 \text{ m}) = 0$

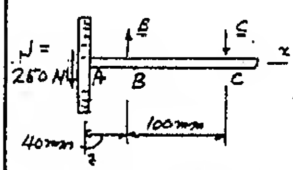
$J = 250$ N

GEAR D: $+\circlearrowleft \Sigma M_D = 0: \; M_0 - J(0.05 \text{ m}) = 0$

$M_0 = (250 \text{ N})(0.05 \text{ m}) = 0$

(a) $M_0 = 12.5$ N·m $\quad \mathbf{M}_0 = (12.5 \text{ N·m})\mathbf{i}$ ◀

(b) PROJECTIONS ON xz PLANE $\quad$ GEAR A AND AXLE AC:



$+\circlearrowleft \Sigma M_B = 0:$

$(250 \text{ N})(40 \text{ mm}) - C(100 \text{ mm}) = 0$

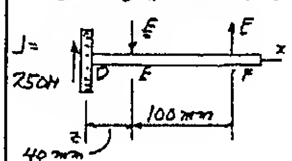
$C = 100$ N

$+\circlearrowleft \Sigma M_C = 0:$

$(250 \text{ N})(140 \text{ mm}) - B(100 \text{ mm}) = 0$

$B = 350$ N

GEAR D AND AXLE DF:



$+\circlearrowleft \Sigma M_E = 0:$

$-(250 \text{ N})(40 \text{ mm}) + F(100 \text{ mm}) = 0$

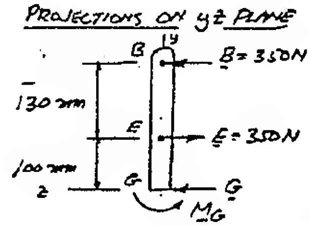
$F = 100$ N

$+\circlearrowleft \Sigma M_F = 0:$

$-(250 \text{ N})(140 \text{ mm}) + E(100 \text{ mm}) = 0$

$E = 350$ N

PROJECTIONS ON yz PLANE $\quad$ BRACKET BG:



$\Sigma F_z = 0:$

$350 \text{ N} - 350 \text{ N} + G = 0$

$G = 0$ ◀

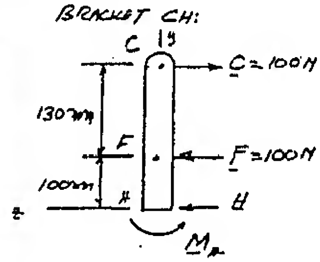
$\Sigma M_x = 0:$

$(350 \text{ N})(130 \text{ mm}) + M_G = 0$

$M_G = -45\,500$ N·mm

$\mathbf{M}_G = -(45.5 \text{ N·m})\mathbf{i}$ ◀

BRACKET CH:



$\Sigma F_z = 0:$

$100 \text{ N} - 100 \text{ N} + H = 0$

$H = 0$ ◀

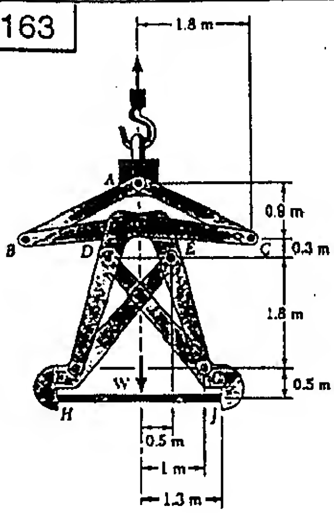
$\Sigma M_x = 0:$

$-(100 \text{ N})(130 \text{ mm}) + M_H = 0$

$M_H = 13\,000$ N·mm

$\mathbf{M}_H = (13.3 \text{ N·m})\mathbf{i}$ ◀

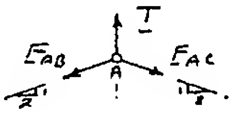
*6.163



GIVEN: MASS OF SLAB HJ IS 7500 kg.

FIND: COMPONENTS OF FORCES ACTING ON MEMBER EFH.

FREE BODY: PIN A

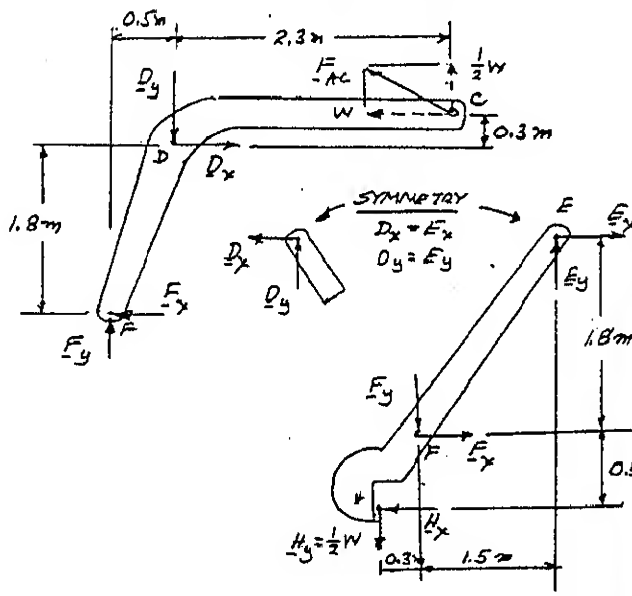


$T = W = mg = (7500\ \text{kg})(9.81\ \text{m/s}^2) = 73.575\ \text{kN}$

$\Sigma F_x = 0: \quad (F_{AB})_x = (F_{AC})_x$

$\Sigma F_y = 0: \quad (F_{AB})_y = (F_{AC})_y = \frac{1}{2}W$

ALSO: $(F_{AC})_x = 2(F_{AC})_y = W$



FREE BODY: MEMBER CDE

$+\circlearrowleft \Sigma M_D = 0: \quad W(0.3) + \frac{1}{2}W(2.3) - F_x(1.8) - F_y(0.5\ \text{m}) = 0$

OR: $1.8F_x + 0.5F_y = 1.45W$ (1)

$\xrightarrow{+} \Sigma F_x = 0: \quad D_x - F_x - W = 0;$ OR $E_x - F_x = W$ (2)

$+\uparrow \Sigma F_y = 0: \quad F_y - D_y + \frac{1}{2}W = 0;$ OR $E_y - F_y = \frac{1}{2}W$ (3)

FREE BODY: MEMBER EFH

$+\circlearrowleft \Sigma M_E = 0: \quad F_x(1.8) + F_y(1.5) - H_x(2.3) + \frac{1}{2}W(1.8\ \text{m}) = 0$

OR $1.8F_x + 1.5F_y = 2.3H_x - 0.9W$ (4)

$\xrightarrow{+} \Sigma F_x = 0: \quad E_x + F_x - H_x = 0$ OR $E_x + F_x = H_x$ (5)

(CONTINUED)

* 6.163 CONTINUED

SUBTRACT (2) FROM (5): $2F_x = H_x - W$ (6)

SUBTRACT (4) FROM 3x(1): $3.6F_x = 5.25W - 2.3H_x$ (7)

ADD (7) TO 2.3x(6): $\therefore 8.2F_x = 2.95W$

$F_x = 0.35976\,W$ (8)

SUBSTITUTE FROM (8) INTO (1):

$(1.8)(0.35976W) + 0.5F_y = 1.45W$

$0.5F_y = 1.45W - 0.64756W = 0.80244W$

$F_y = 1.6049\,W$ (9)

SUBSTITUTE FROM (8) INTO (2):

$E_x - 0.35976W = W; \quad E_x = 1.35976W$

SUBSTITUTE FROM (9) INTO (3):

$E_y - 1.6049W = \frac{1}{2}W \qquad E_y = 2.1049\,W$

FROM (5): $H_x = E_x + F_x = 1.35976W + 0.35976W = 1.71952\,W$

RECALL THAT: $H_y = \frac{1}{2}W$

SINCE ALL EXPRESSIONS OBTAINED ARE POSITIVE, ALL FORCES ARE DIRECTED AS SHOWN ON THE FREE-BODY DIAGRAMS.

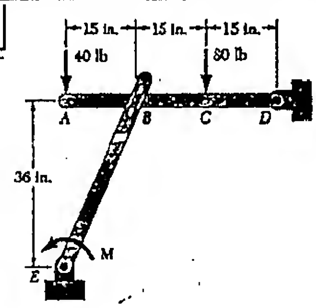
SUBSTITUTE $W = 73.575\ \text{kN}$:

$E_x = 100.0\ \text{kN} \rightarrow \qquad E_y = 154.9\ \text{kN} \uparrow$ ◀

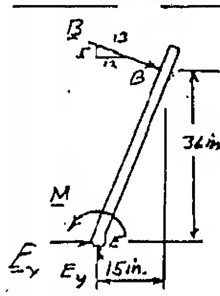
$F_x = 26.5\ \text{kN} \rightarrow \qquad F_y = 118.1\ \text{kN} \downarrow$ ◀

$H_x = 126.5\ \text{kN} \leftarrow \qquad H_y = 36.8\ \text{kN} \downarrow$ ◀

6.164

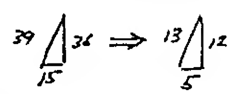


FIND: COUPLE M FOR EQUILIBRIUM



FREE BODY: MEMBER BE

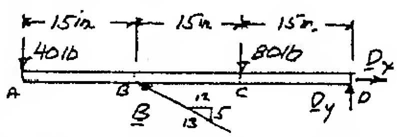
$BE = (15^2 + 36^2)^{1/2} = 39\ \text{in.}$



$+\circlearrowleft \Sigma M_E = 0: \quad M - B(39\ \text{in.}) = 0$

$M = B(39\ \text{in.})$ (1)

FREE BODY: MEMBER AD

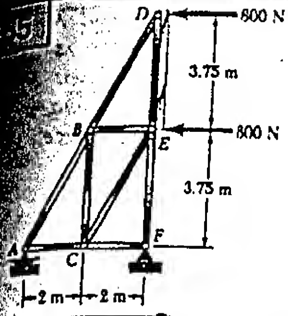


$+\circlearrowleft \Sigma M_D = 0: \quad (40\ \text{lb})(45\ \text{in.}) + (80\ \text{lb})(15\ \text{in.}) - \frac{5}{13}B(30\ \text{in.}) = 0$

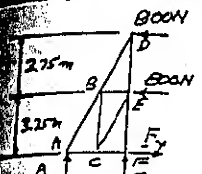
$B = 260\ \text{lb}$

EQ(1) $M = B(39\ \text{in.}) = (260\ \text{lb})(39\ \text{in.}) = 10{,}140\ \text{lb}\cdot\text{in.}$

$M = 10.14\ \text{kip}\cdot\text{in.}\ \circlearrowleft$ ◀



Find: Force in each member



Free body entire truss

$+\circlearrowleft \Sigma M_F = 0$

$(800N)(7.5m) + (800N)(3.75m) - A(4m) = 0$

$A = +2250N \quad A = 2250N \uparrow$

$+\uparrow \Sigma F_y = 0: \; 2250N + F_y = 0$

$F_y = -2250N \quad F_y = 2250N \downarrow$

$\xrightarrow{+} \Sigma F_x = 0: \; -800N - 800N + F_x = 0$

$F_x = +1600N \quad F_x = 1600N \rightarrow$

Joint D:

$\frac{800N}{8} = \frac{F_{DE}}{15} = \frac{F_{BD}}{17}$

$F_{BD} = 1700N \; C$

$F_{DE} = 1500N \; T$

Joint A:

$\frac{2250N}{15} = \frac{F_{AB}}{17} = \frac{F_{AC}}{8}$

$F_{AB} = 2550N \; C$

$F_{AC} = 1200N \; T$

Joint F:

$\xrightarrow{+} \Sigma F_x = 0: \; 1600N - F_{CF} = 0$

$F_{CF} = +1600N \quad F_{CF} = 1600N \; T$

$+\uparrow \Sigma F_y = 0: \; F_{EF} - 2250N = 0$

$F_{EF} = +2250N \quad F_{EF} = 2250N \; T$

Joint C:

$\xrightarrow{+} \Sigma F_x = 0: \; \frac{8}{17} F_{CE} - 1200N + 1600N = 0$

$F_{CE} = -850N \quad F_{CE} = 850N \; C$

$+\uparrow \Sigma F_y = 0: \; F_{BC} + \frac{15}{17} F_{CE} = 0$

$F_{BC} = -\frac{15}{17} F_{CE} = -\frac{15}{17}(-850N)$

$F_{BC} = +750N \quad F_{BC} = 750N \; T$

Joint E:

$\xrightarrow{+} \Sigma F_x = 0:$

$-F_{BE} - 800N + \frac{8}{17}(850N) = 0$

$F_{BE} = -400N \quad F_{BE} = 400N \; C$

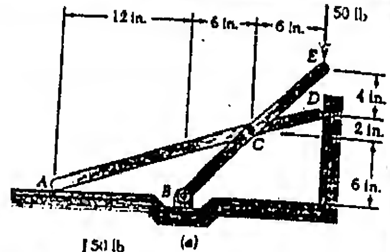
$+\uparrow \Sigma F_y = 0:$

$1500N - 2250N + \frac{15}{17}(850N) = 0$

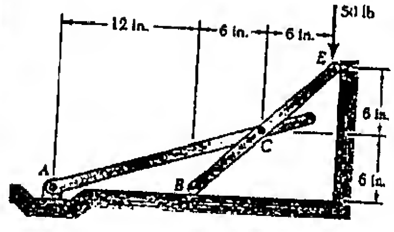
$0 = 0$

(checks)

6.166



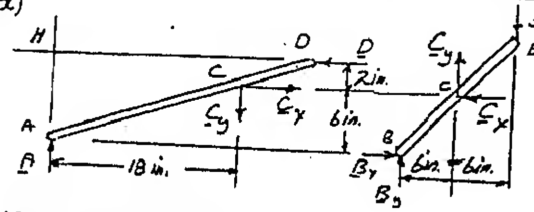
(a)



(b)

Find: For each frame, the forces exerted at B and C on member BCE

(a)



Free body of member ACD

$+\circlearrowleft \Sigma M_H = 0: \; C_x(2 \text{ in.}) - C_y(18 \text{ in.}) = 0 \qquad C_x = 9C_y \qquad (1)$

Free body of member BCE

$+\circlearrowleft \Sigma M_B = 0: \; C_x(6 \text{ in.}) + C_y(6 \text{ in.}) - (50 \text{ lb})(12 \text{ in.}) = 0$

Substitute from (1): $9C_y(6) + C_y(6) - 600 = 0$

$C_y = +10 \text{ lb}; \quad C_x = 9C_y = 9(10) = +90 \text{ lb}$

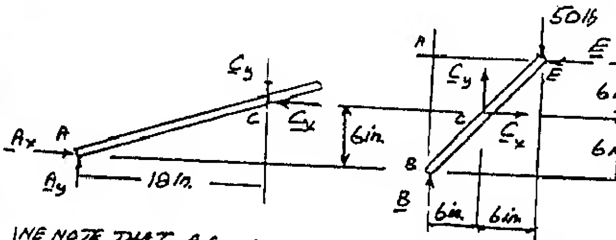
$C = 90.6 \text{ lb} \; \measuredangle \; 6.3°$

$\xrightarrow{+} \Sigma F_x = 0: \; B_x - 90 \text{ lb} = 0 \qquad B_x = 90 \text{ lb}$

$+\uparrow \Sigma F_y = 0: \; B_y + 10 \text{ lb} - 50 \text{ lb} = 0 \qquad B_y = 40 \text{ lb}$

$B = 98.5 \text{ lb} \; \measuredangle \; 24.0°$

(b)



We note that AC is a two-force member

$\frac{C_x}{18 \text{ in.}} = \frac{C_y}{6 \text{ in.}} \qquad C_x = 3C_y \qquad (1)$

On free body of member BCE

$+\circlearrowleft \Sigma M_B = 0: \; C_x(6 \text{ in.}) + C_y(6 \text{ in.}) - (50 \text{ lb})(12 \text{ in.}) = 0$

Substitute from (1):

$3C_y(6) + C_y(6) - 600 = 0$

$C_y = +25 \text{ lb}; \quad C_x = 3C_y = 3(25 \text{ lb}) = 75 \text{ lb}$

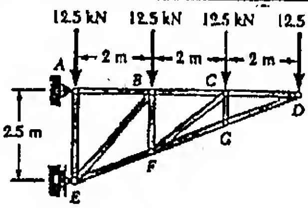
$C = 79.1 \text{ lb} \; \measuredangle \; 18.4°$

$+\uparrow \Sigma F_y = 0: \; B + C_y - 50 \text{ lb} = 0$

$B + 25 \text{ lb} - 50 \text{ lb} = 0$

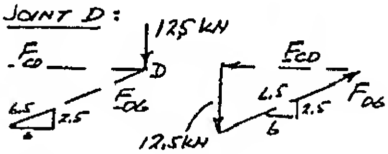
$B = +25 \text{ lb} \qquad B = 25 \text{ lb} \uparrow$

6.167



FIND: THE FORCE IN EACH MEMBER

JOINT D:



$$\frac{12.5\,kN}{2.5} = \frac{F_{CD}}{6} = \frac{F_{DG}}{6.5}$$

$F_{CD} = 30\,kN\ T$ ◀

$F_{DG} = 32.5\,kN\ C$ ◀

JOINT G:

$\nwarrow \Sigma F = 0:\ F_{CG} = 0$ ◀

$\nearrow \Sigma F = 0:\ F_{FG} = 32.5\,kN\ C$ ◀

JOINT C: $BF = \frac{2}{3}(2.5\,m) = 1.6667\,m;\ \beta = \angle BCF = \tan^{-1}\frac{BF}{2} = 39.81°$

$+\uparrow \Sigma F_y = 0:\ -12.5\,kN - F_{CF}\sin\beta = 0$

$-12.5\,kN - F_{CF}\sin 39.81° = 0$

$F_{CF} = -19.526\,kN \quad F_{CF} = 19.53\,kN\ C$ ◀

$\xrightarrow{+} \Sigma F_x = 0:\ 30\,kN - F_{BC} - F_{CF}\cos\beta = 0$

$30\,kN - F_{BC} - (-19.526\,kN)\cos 39.81° = 0$

$F_{BC} = +45.0\,kN \quad F_{BC} = 45.0\,kN\ T$ ◀

JOINT F:

$F_{CF} = 19.526\,kN$, $\beta = 39.81°$, $F_{FG} = 32.5\,kN$

$\xrightarrow{+} \Sigma F_x = 0:\ -\frac{6}{6.5}F_{EF} - \frac{6}{6.5}(32.5\,kN) - F_{CF}\cos\beta = 0$

$F_{EF} = -32.5\,kN - \left(\frac{6.5}{6}\right)(19.526\,kN)\cos 39.81°$

$F_{EF} = -48.75\,kN \quad F_{EF} = 48.8\,kN\ C$ ◀

$+\uparrow \Sigma F_y = 0:\ F_{BF} - \frac{2.5}{6.5}F_{EF} - \frac{2.5}{6.5}(32.5\,kN) - (19.526\,kN)\sin 39.81° = 0$

$F_{BF} - \frac{2.5}{6.5}(-48.75\,kN) - 12.5\,kN - 12.5\,kN = 0$

$F_{BF} = +6.25\,kN \quad F_{BF} = 6.25\,kN\ T$ ◀

JOINT B:

$F_{BC} = 45.0\,kN$, $F_{BF} = 6.25\,kN$, 12.5 kN

$\tan\alpha = \frac{2.5\,m}{2\,m};\ \gamma = 51.34°$

$+\uparrow \Sigma F_y = 0:$

$-12.5\,kN - 6.25\,kN - F_{BE}\sin 51.34° = 0$

$F_{BE} = -24.0\,kN \quad F_{BE} = 24.0\,kN\ C$ ◀

$\xrightarrow{+} \Sigma F_x = 0:\ 45.0\,kN - F_{AB} - (24.0\,kN)\cos 51.34° = 0$

$F_{AB} = +30\,kN \quad F_{AB} = 30.0\,kN\ T$ ◀

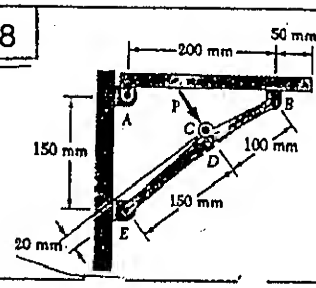
JOINT E:

$F_{BE} = 24\,kN$, $F_{EF} = 48.75\,kN$, $\gamma = 51.34°$

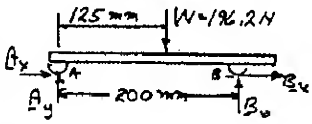
$\xrightarrow{+} \Sigma F_y = 0:\ F_{AE} - (24\,kN)\sin 51.34° - (48.75\,kN)\frac{2.5}{6.5} = 0$

$F_{AE} = +37.5\,kN \quad F_{AE} = 37.5\,kN\ T$ ◀

6.168



GIVEN: MASS OF SHELF IS 20 kg.
FIND: FORCE P REQUIRED TO RELEASE BRACE



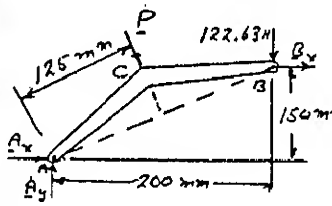
FREE BODY: SHELF

$W = (20\,kg)(9.81\,m/s^2) = 196.2\,N$

$+\circlearrowleft \Sigma M_A = 0$

$B_y(200\,mm) - (196.2\,N)(125\,mm) = 0$

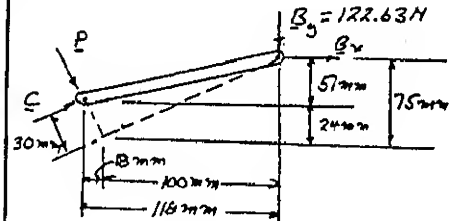
$B_y = 122.63\,N$



FREE BODY: PORTION ACB

$+\circlearrowleft \Sigma M_A = 0:\ -B_x(150\,mm) - P(125\,mm) - (122.63\,N)(200\,mm) = 0$

$B_x = -163.5 - 0.8333P \quad (1)$

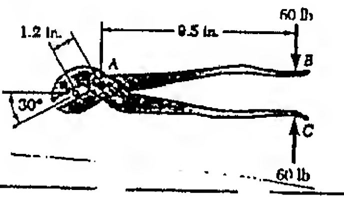


$+\circlearrowleft \Sigma M_C = 0:\ +(122.63\,N)(118\,mm) + B_x(51\,mm) = 0$

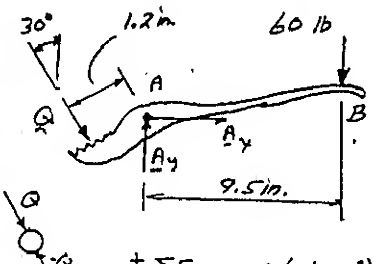
$+(122.63\,N)(118\,mm) + (-163.5 - 0.8333P)(51\,mm) = 0$

$P = 144.28\,N \quad P = 144.3\,N$ ◀

6.169



FIND: (a) MAGNITUDE OF FORCES EXERTED ON ROD.
(b) FORCE EXERTED AT A ON PORTION AB OF PLIERS



FREE BODY: PORTION AB

$+\circlearrowleft \Sigma M_A = 0:$

$Q(1.2\,in) - (60\,lb)(9.5\,in) = 0$

$Q = 475\,lb$ ◀

$\xrightarrow{+} \Sigma F_x = 0:\ Q(\sin 30°) + A_x = 0$

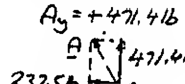
$(475\,lb)(\sin 30°) + A_x = 0$

$A_x = -237.5\,lb \quad A_x = 237.5\,lb \leftarrow$

$+\uparrow \Sigma F_y = 0:\ -Q(\cos 30°) + A_y - 60\,lb = 0$

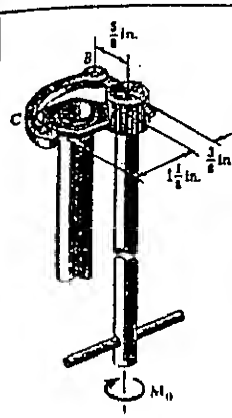
$-(475\,lb)(\cos 30°) + A_y - 60\,lb = 0$

$A_y = +471.4\,lb \quad A_y = 471.4\,lb \uparrow$



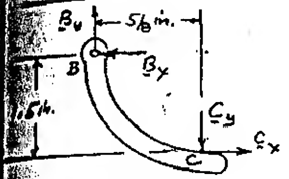
$A = 528\,lb \measuredangle 63.3°$ ◀

6.170



GIVEN: FORCES EXERTED ON THE NUT ARE EQUIVALENT TO A COUPLE OF MAGNITUDE 135 lb·in. ↻.

FIND: (a) MAGNITUDE OF FORCE AT B ON JAW BC, (b) THE COUPLE M_0



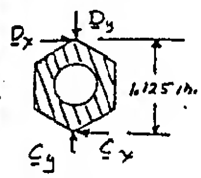
FREE BODY: JAW BC

THIS IS A TWO-FORCE MEMBER

$\frac{C_y}{1.5\text{ in.}} = \frac{C_x}{5/8\text{ in.}}$ $\quad C_y = 2.4C_x$

$\Sigma F_x = 0: \quad B_x = C_x \qquad (1)$

$\Sigma F_y = 0: \quad B_y = C_y = 2.4C_x \qquad (2)$



FREE BODY: NUT

$\Sigma F_x = 0 \quad C_x = D_x$

$\circlearrowright \Sigma M = 135\text{ lb·in.}$

$C_x(1.125\text{ in.}) = 135\text{ lb·in.}$

$C_x = 120\text{ lb}$

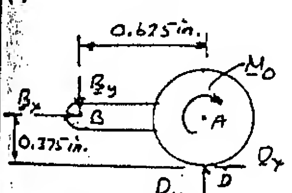
(a)

EQ. (1): $B_x = C_x = 120\text{ lb}$

EQ. (2): $B_y = C_y = 2.4(120\text{ lb}) = 288\text{ lb}$

$B = (B_x^2 + B_y^2)^{1/2} = (120^2 + 288^2)^{1/2}; \qquad B = 312\text{ lb}$ ◀

(b)



FREE BODY: ROD

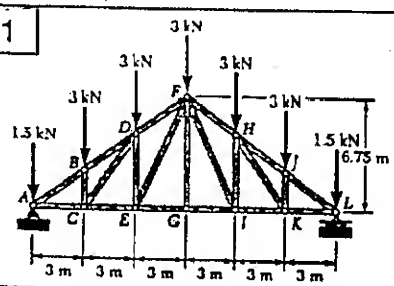
$+\circlearrowleft \Sigma M_A = 0:$

$-M_0 + B_y(0.625\text{ in}) - B_x(0.375\text{ in}) = 0$

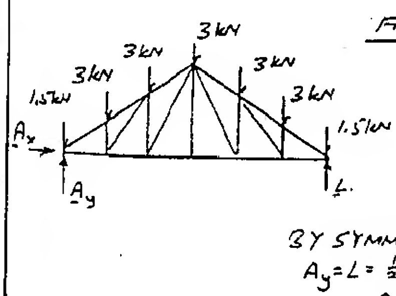
$-M_0 + (288)(0.625) - (120)(0.375) = 0$

$M_0 = 135\text{ lb·in.}$ ↻ ◀

6.171



FIND: FORCE IN MEMBERS CE, DE, AND DF



FREE BODY: ENTIRE TRUSS

$\Sigma F_x = 0: A_x = 0$

TOTAL LOAD $= 5(3\text{ kN}) + 2(1.5\text{ kN}) = 18\text{ kN}$

BY SYMMETRY:

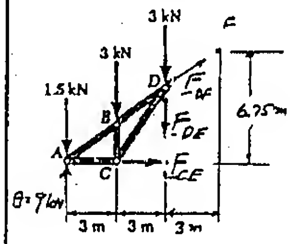
$A_y = L = \frac{1}{2}(18\text{ kN})$

$A = L = 9\text{ kN}\uparrow$

(CONTINUED)

6.171 CONTINUED

FREE BODY: PORTION ACD



NOTE: SLOPE OF ABDF IS $\frac{6.75}{9.00} = \frac{3}{4}$ (3-4-5)

FORCE IN CE:

$+\circlearrowleft \Sigma M_D = 0: \quad F_{CE}(\frac{2}{3} \times 6.75\text{ m}) - (9\text{ kN})(6\text{ m}) + (1.5\text{ kN})(6\text{ m}) + (3\text{ kN})(3\text{ m}) = 0$

$F_{CE}(4.5\text{ m}) - 36\text{ kN·m} = 0$

$F_{CE} = +8\text{ kN} \qquad F_{CE} = 8\text{ kN T}$ ◀

FORCE IN DE:

$+\circlearrowleft \Sigma M_A = 0: \quad F_{DE}(6\text{ m}) + (3\text{ kN})(6\text{ m}) + (3\text{ kN})(3\text{ m}) = 0$

$F_{DE} = -4.5\text{ kN} \qquad F_{DE} = 4.5\text{ kN C}$ ◀

FORCE IN DF:

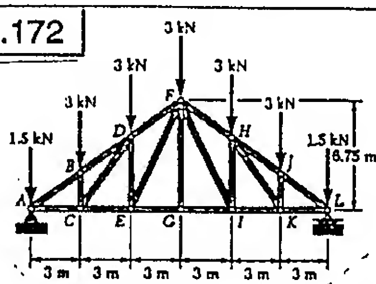
SUM MOMENTS ABOUT E WHERE F_{CE} AND F_{DE} INTERSECT

$+\circlearrowleft \Sigma M_E = 0: \quad (1.5\text{ kN})(6\text{ m}) - (9\text{ kN})(6\text{ m}) + (3\text{ kN})(3\text{ m}) + \frac{4}{5}F_{DF}(\frac{2}{3} \times 6.75\text{ m}) = 0$

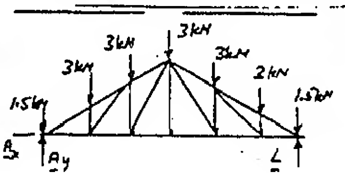
$\frac{4}{5}F_{DF}(4.5\text{ m}) - 36\text{ kN·m}$

$F_{DF} = -10.00\text{ kN} \qquad F_{DF} = 10.00\text{ kN C}$ ◀

6.172



FIND: FORCE IN MEMBERS FH, FI, AND GI.



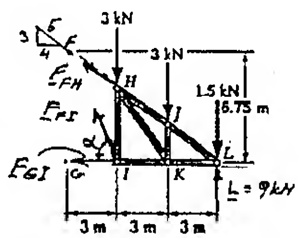
FREE BODY: ENTIRE TRUSS

$\Sigma F_x = 0: \quad A_x = 0$

TOTAL LOAD $= 5(3\text{ kN}) + 2(1.5\text{ kN}) = 18\text{ kN}$

BY SYMMETRY

$A_y = L = \frac{1}{2}(18) = 9\text{ kN}\uparrow$



FREE BODY: PORTION HIL

SLOPE OF FHJL

$\frac{6.75}{9.00} = \frac{3}{4}$ (3-4-5)

$\tan\alpha = \frac{FG}{GI} = \frac{6.75\text{ m}}{3\text{ m}} \qquad \alpha = 66.04°$

FORCE IN FH:

$+\circlearrowleft \Sigma M_I = 0: \quad \frac{4}{5}F_{FH}(\frac{2}{3} \times 6.75\text{ m}) + (9\text{ kN})(6\text{ m}) - (1.5\text{ kN})(6\text{ m}) - (3\text{ kN})(3\text{ m}) = 0$

$\frac{4}{5}F_{FH}(4.5\text{ m}) + 36\text{ kN·m}$

$F_{FH} = -10.00\text{ kN} \qquad F_{FH} = 10.00\text{ kN C}$ ◀

FORCE IN FI:

$+\circlearrowleft \Sigma M_L = 0: \quad F_{FI}\sin\alpha(6\text{ m}) - (3\text{ kN})(6\text{ m}) - (3\text{ kN})(3\text{ m}) = 0$

$F_{FI}\sin 66.04°(6\text{ m}) = 27\text{ kN·m}$

$F_{FI} = +4.92\text{ kN} \qquad F_{FI} = 4.92\text{ kN T}$ ◀

FORCE IN GI:

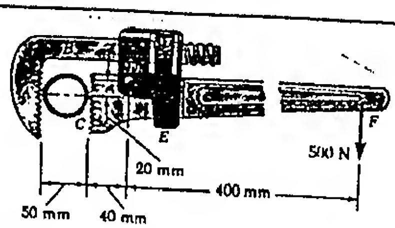
$+\circlearrowleft \Sigma M_H = 0:$

$F_{GI}(6.75\text{ m}) + (3\text{ kN})(3\text{ m}) + (3\text{ kN})(6\text{ m}) + (1.5\text{ kN})(9\text{ m}) - (9\text{ kN})(9\text{ m}) = 0$

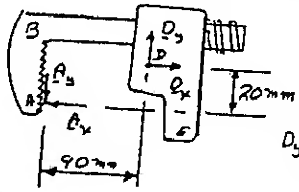
$F_{GI}(6.75\text{ m}) = +40.5\text{ kN·m}$

$F_{GI} = +6.00\text{ kN} \qquad F_{GI} = 6.00\text{ kN T}$ ◀

6.173



FIND: COMPONENTS OF FORCES ON PIPE AT A AND AT C.

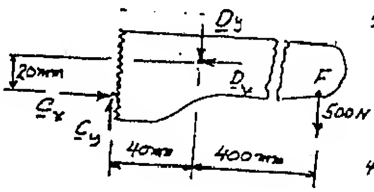


FREE BODY: PORTION ABDE

THIS IS A TWO-FORCE MEMBER

$\frac{A_y}{20\text{ mm}} = \frac{A_x}{90\text{ mm}}$; $A_x = 4.5A_y$

$D_y = A_y$: $D_x = A_x = 4.5D_y$ (1)



FREE BODY: PORTION CF

$+\circlearrowleft \Sigma M_C = 0$

$D_x(20\text{ mm}) - D_y(40\text{ mm}) - (500\text{ N})(440\text{ mm}) = 0$

SUBSTITUTE FROM (1)

$4.5D_y(20) - D_y(40) - 220\times10^3 = 0$

$D_y = 4400\text{ N} = +4.4\text{ kN}$

$D_x = 4.5(D_y) = +19.8\text{ kN}$

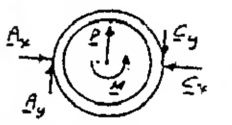
$+\uparrow\Sigma F_y = 0$: $C_y - 4.4\text{ kN} - 0.5\text{ kN} = 0$; $C_y = +4.9\text{ kN}$

$\xrightarrow{+}\Sigma F_x = 0$: $C_x - 19.8\text{ kN} = 0$; $C_x = +19.8\text{ kN}$

FROM PORTION ABDE:

$A_x = D_x = +19.8\text{ kN}$ $A_y = D_y = +4.4\text{ kN}$

WE NOTE THAT ALL COMPONENTS FOUND ABOVE ACT IN DIRECTIONS DRAWN. COMPONENTS ON THE PIPE ARE EQUAL AND OPPOSITE TO THOSE ON WRENCH.

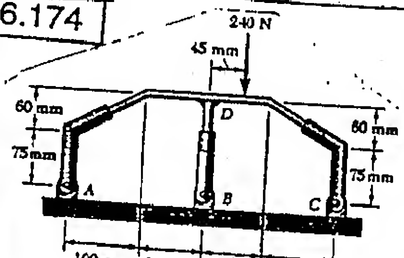


$\underline{A}_x = 19.8\text{ kN} \rightarrow$ $\underline{A}_y = 4.4\text{ kN}\uparrow$ ◀

$\underline{C}_x = 19.8\text{ kN} \leftarrow$ $\underline{C}_y = 4.9\text{ kN}\downarrow$ ◀

NOTE: FREE BODY OF PIPE ALSO INCLUDES REACTIONS, $\underline{P}$ AND $\underline{M}$, EXERTED BY GROUND.

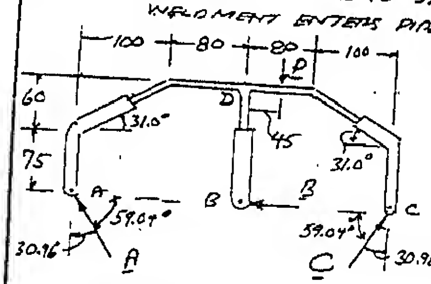
6.174



FIND: REACTIONS AT A, B, AND C.

NOTE:

EACH REACTION IS $\perp$ TO SLOPE OF PIPE WHERE WELDMENT ENTERS PIPE.



FREE BODY: ENTIRE FRAME

DIMENSIONS IN mm

$P = 240\text{ N}$

$\tan^{-1}\frac{60}{100} = 30.96°$

(CONTINUED)

6.174 CONTINUED

$+\circlearrowleft\Sigma M_A = 0$: $C\cos 30.96°(360\text{ mm}) - (240\text{ N})(225\text{ mm}) = 0$

$C = +174.92\text{ N}$ $\underline{C} = 174.9\text{ N} \measuredangle 59.0°$

$+\circlearrowleft\Sigma M_C = 0$: $-A\cos 30.96°(360\text{ mm}) + (240\text{ N})(135\text{ mm}) = 0$

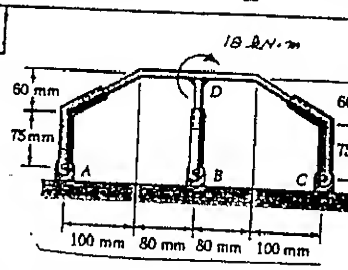
$A = 104.95\text{ N}$ $\underline{A} = 105.0\text{ N} \measuredangle 59.0°$ ◀

$\xrightarrow{+}\Sigma F_x = 0$: $-A\sin 30.96° + C\sin 30.96° - B = 0$

$B = (C - A)\sin 30.96° = (174.92\text{ N} - 104.95\text{ N})\sin 30.96°$

$B = +36.0\text{ N}$ $\underline{B} = 36.0\text{ N} \leftarrow$ ◀

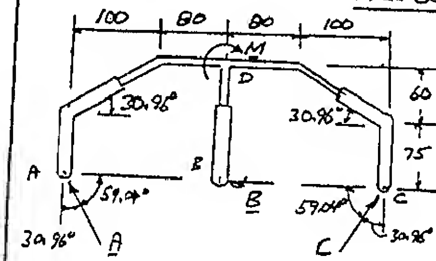
6.175



FIND: THE REACTIONS AT A, B, AND C.

NOTE: EACH REACTION IS $\perp$ TO SLOPE OF PIPE WHERE WELDMENT ENTERS PIPE.

FREE BODY: ENTIRE FRAME



DIMENSIONS IN mm

$M = 18\text{ kN}\cdot\text{m}$

$\tan^{-1}\frac{60}{100} = 30.96°$

$+\circlearrowleft\Sigma M_A = 0$: $M - C\cos 30.96°(360\text{ mm}) = 0$

$18\text{ kN}\cdot\text{m} - C\cos 30.96°(0.36\text{ m}) = 0$

$C = +58.31\text{ kN}$ $\underline{C} = 58.3\text{ kN} \measuredangle 59.0°$ ◀

$+\circlearrowleft\Sigma M_C = 0$: $M + A\cos 30.96°(360\text{ mm}) = 0$

$18\text{ kN}\cdot\text{m} + A\cos 30.96°(0.36\text{ m}) = 0$

$A = -58.31\text{ kN}$ $\underline{A} = 58.3\text{ kN} \measuredangle 59.0°$ ◀

$\xrightarrow{+}\Sigma F_x = 0$: $-A\sin 30.96° - B + C\sin 30.96° = 0$

$B = (C - A)\sin 30.96°$

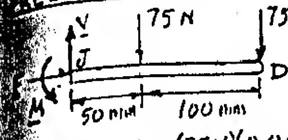
$B = (58.31\text{ kN} - (-58.31\text{ kN}))\sin 30.96°$

$B = +60.0\text{ kN}$ $\underline{B} = 60\text{ kN} \leftarrow$ ◀

7.1

GIVEN: FRAME AND LOADING OF PROB. 6.75.
FIND: INTERNAL FORCES AT POINT J.

WE CUT MEMBER BCD AT POINT J AND CONSIDER THE FREE BODY JD:



$\xrightarrow{+} \Sigma F_x = 0$: $F = 0$ ◄

$+\uparrow \Sigma F_y = 0$: $V - 75N - 75N = 0$

$V = +150.0N$ $\underline{V} = 150.0N\uparrow$ ◄

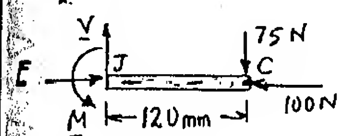
$+\circlearrowleft \Sigma M_J = 0$: $M - (75N)(0.05m) - (75N)(0.15m) = 0$

$M = +15.00$ N·m $\underline{M} = 15.00 N{\cdot}m$ ◄

7.2

GIVEN: FRAME AND LOADING OF PROB. 6.76.
FIND: INTERNAL FORCES AT POINT J.

WE CUT MEMBER ABC AT POINT J AND CONSIDER THE FREE BODY JC. WE RECALL FROM THE SOLUTION OF PROB. 6.76 THAT THE REACTION AT C IS $\underline{C} = 125 N \measuredangle 36.9°$ OR $\underline{C}_x = 100 N \leftarrow$, $\underline{C}_y = 75 N\downarrow$.



$\xrightarrow{+} \Sigma F_x = 0$: $\underline{F} = 100.0 N \rightarrow$ ◄

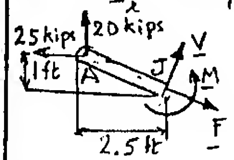
$+\uparrow \Sigma F_y = 0$: $\underline{V} = 75.0 N\uparrow$ ◄

$+\circlearrowleft \Sigma M_J = 0$: $M - (75N)(0.120m) = 0$ $\underline{M} = 9.00$ N·m ◄

7.3

GIVEN: FRAME AND LOADING OF PROB. 6.81.
FIND: INTERNAL FORCES AT POINT J LOCATED HALFWAY BETWEEN A AND B.

WE CUT MEMBER ABC AT POINT J AND CONSIDER THE FREE BODY AJ. WE RECALL FROM THE SOLUTION OF PROB. 6.81 THAT THE COMPONENTS OF THE REACTION AT A ARE $\underline{A}_x = 25$ kips $\leftarrow$ AND $\underline{A}_y = 20$ kips $\uparrow$.



$\searrow^{+} \Sigma F_x = 0$: $F - \frac{2.5}{\sqrt{1^2+2.5^2}}(25) - \frac{1}{\sqrt{1^2+2.5^2}}(20) = 0$

$F = +30.6$ kips, $\underline{F} = 30.6$ kips ◄

$+\nearrow \Sigma F_y = 0$: $V - \frac{1}{\sqrt{1^2+2.5^2}}(25) + \frac{2.5}{\sqrt{1^2+2.5^2}}(20) = 0$

$V = -9.28$ kips $\underline{V} = 9.28$ kips ◄

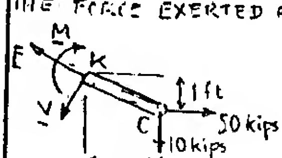
$+\circlearrowleft \Sigma M_J = 0$: $M + (25 kips)(1 ft) - (20 kips)(2.5 ft) = 0$

$M = +25.0$ kip·ft $\underline{M} = 25.0$ kip·ft ◄

7.4

GIVEN: FRAME AND LOADING OF PROB. 6.81.
FIND: INTERNAL FORCES AT POINT K LOCATED HALFWAY BETWEEN B AND C.

WE DISCONNECT MEMBER ABC AND CUT IT AT POINT K. WE CONSIDER THE FREE BODY KC. WE RECALL FROM THE SOLUTION OF PROB. 6.81 THAT THE COMPONENTS OF THE FORCE EXERTED AT C ON KC ARE $\underline{C}_x = 50$ kips $\rightarrow$, $\underline{C}_y = 10$ kips $\downarrow$



$\xrightarrow{+} \Sigma F_x = 0$: $-F + \frac{2.5}{\sqrt{1^2+2.5^2}}(50) + \frac{1}{\sqrt{1^2+2.5^2}}(10) = 0$

$F = +50.14$ kips, $\underline{F} = 50.1$ kips ◄

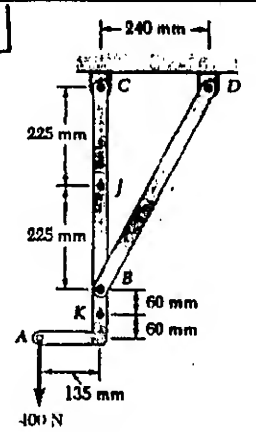
$+\nearrow F_y = 0$: $-V + \frac{1}{\sqrt{1^2+2.5^2}}(50) - \frac{2.5}{\sqrt{1^2+2.5^2}}(10) = 0$

$V = +9.285$ kips. $\underline{V} = 9.28$ kips ◄

$+\circlearrowleft \Sigma M_K = 0$: $-M + (50 kips)(1 ft) - (10 kips)(2.5 ft) = 0$

$M = +25.0$ kip·ft $\underline{M} = 25.0$ kip·ft ◄

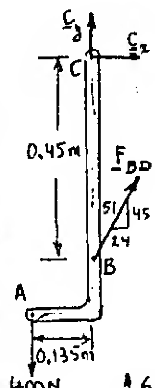
7.5



GIVEN: STRUCTURE AND LOADING SHOWN.

FIND: INTERNAL FORCES AT POINT J.

FREE BODY: MEMBER ABC



$+\circlearrowleft \Sigma M_C = 0$:

$(400 N)(0.135 m) + (\frac{24}{51} F_{BD})(0.45 m) = 0$

$F_{BD} = -255$ N $\underline{F}_{BD} = 255$ N C

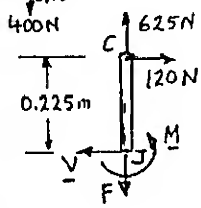
$\xrightarrow{+} \Sigma F_x = 0$: $C_x + \frac{24}{51}(-255N) = 0$

$C_x = +120$ N $\underline{C}_x = 120$ N $\rightarrow$ ◁

$+\uparrow \Sigma F_y = 0$: $C_y - 400 N + \frac{45}{51}(-255N) = 0$

$C_y = +625$ N $\underline{C}_y = 625$ N $\uparrow$ ◁

FREE BODY: CJ



$+\uparrow \Sigma F_y = 0$: $-F + 625 N = 0$

$F = +625$ N $\underline{F} = 625$ N $\downarrow$ ◄

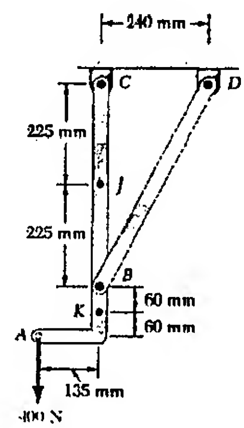
$\xrightarrow{+} \Sigma F_x = 0$: $-V + 120 N = 0$

$V = +120$ N $\underline{V} = 120$ N $\leftarrow$ ◄

$+\circlearrowleft \Sigma M_J = 0$: $M - (120 N)(0.225 m) = 0$

$M = +27.0$ N·m $M = 27.0$ N·m ◄

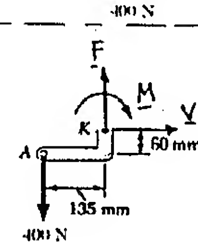
7.6



GIVEN: STRUCTURE AND LOADING SHOWN.

FIND: INTERNAL FORCES AT POINT K.

FREE BODY: AK



$+\uparrow \Sigma F_y = 0$: $F - 400 N = 0$

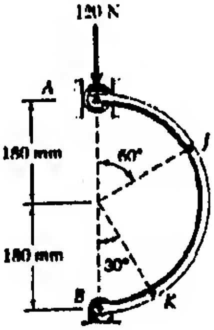
$F = +400$ N $\underline{F} = 400$ N $\uparrow$ ◄

$\xrightarrow{+} \Sigma F_x = 0$: $V = 0$ $\underline{V} = 0$ ◄

$+\circlearrowleft \Sigma M_K = 0$: $(400 N)(0.135 m) - M = 0$

$M = +54.0$ N·m $\underline{M} = 54.0$ N·m ◄

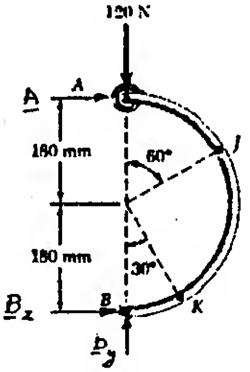
7.7



GIVEN:
SEMICIRCULAR ROD LOADED AS SHOWN

FIND:
INTERNAL FORCES AT POINT J

FREE BODY: ROD AB



$+\circlearrowleft \Sigma M_B = 0: -A(360\text{mm}) = 0$

$A = 0$ ◁

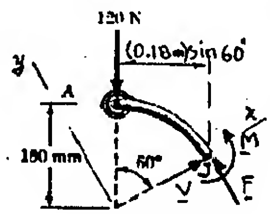
$\xrightarrow{+} \Sigma F_x = 0: \; B_x + A = 0$

$B_x = -A = 0$

$+\uparrow \Sigma F_y = 0: \; B_y - 120\text{ N} = 0$

$B_y = 120\text{ N}, \quad \underline{B} = 120\text{N}\uparrow$ ◁

FREE BODY: AJ



$+\nwarrow \Sigma F_y = 0: F - (120\text{N})\sin 60° = 0$

$F = +103.9\text{N} \quad \underline{F} = 103.9\text{N}\nwarrow$ ◀

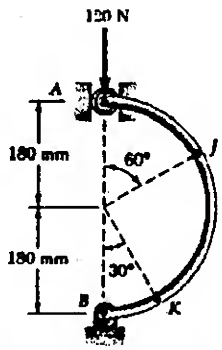
$+\swarrow \Sigma F_x = 0: V - (120\text{N})\cos 60° = 0$

$V = +60.0\text{N} \quad \underline{V} = 60.0\text{N}\nearrow$ ◀

$+\circlearrowleft \Sigma M_J = 0: M - (120\text{N})(0.18\text{m})\sin 60° = 0$

$M = +18.71\text{N·m} \quad \underline{M} = 18.71\text{N·m}\circlearrowleft$ ◀

7.8

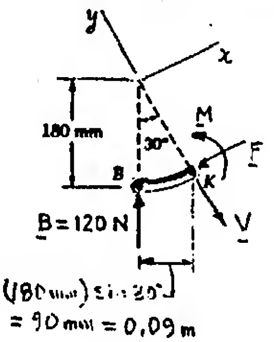


GIVEN:
SEMICIRCULAR ROD LOADED AS SHOWN

FIND:
INTERNAL FORCES AT POINT K.

REACTION AT B: (SEE SOLUTION OF PROB. 7.7) $\underline{B} = 120\text{N}\uparrow$ ◁

FREE BODY: BK



$(180\text{mm})\sin 30° = 90\text{mm} = 0.09\text{m}$

$\xrightarrow{+} \Sigma F_x = 0: -F + (120\text{N})\sin 30° = 0$

$F = +60.0\text{N} \quad \underline{F} = 60.0\text{N}\swarrow$ ◀

$+\nwarrow \Sigma F_y = 0: -V + (120\text{N})\cos 30° = 0$

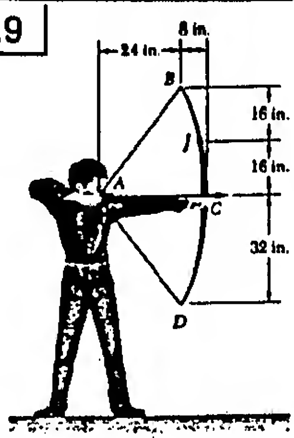
$V = +103.9\text{N} \quad \underline{V} = 103.9\text{N}\searrow$ ◀

$+\circlearrowleft \Sigma M_K = 0: M - (120\text{N})(0.09\text{m}) = 0$

$M = +10.80\text{ N·m}$

$\underline{M} = 10.80\text{N·m}\circlearrowleft$ ◀

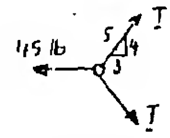
7.9



GIVEN:
ARCHER PULLING WITH A 45-lb FORCE ON THE BOWSTRING.

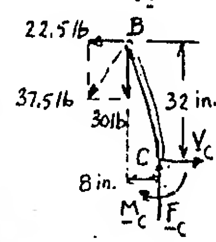
FIND:
INTERNAL FORCES AT POINT J.
(ASSUME THAT THE SHAPE OF THE BOW IS A PARABOLA)

FREE BODY: POINT A



$\xrightarrow{+} \Sigma F_x = 0: \; 2(\tfrac{3}{5}T) - 45\text{ lb} = 0 \qquad T = 37.5\text{ lb}$ ◁

FREE BODY: PORTION OF BOW BC



$+\uparrow \Sigma F_y = 0: \; F_C - 30\text{ lb} = 0$

$\underline{F}_C = 30\text{lb}\uparrow$ ◁

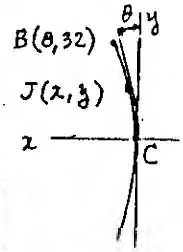
$\xrightarrow{+} \Sigma F_x = 0: \; V_C - 22.5\text{ lb} = 0$

$\underline{V}_C = 22.5\text{lb}\rightarrow$ ◁

$+\circlearrowleft \Sigma M_C = 0: \; (22.5\text{lb})(32\text{in.}) + (30\text{lb})(8\text{in}) - M_C = 0$

$\underline{M}_C = 960\text{ lb·in.}\circlearrowleft$ ◁

EQUATION OF PARABOLA



$x = ky^2$

AT B: $\quad 8 = k(32)^2 \qquad k = \frac{1}{128}$

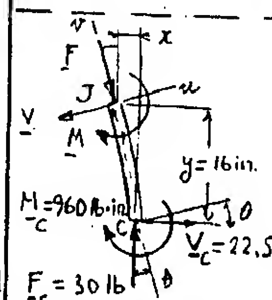
THEREFORE, EQUATION IS

$$x = \frac{y^2}{128} \qquad (1)$$

THE SLOPE AT J IS OBTAINED BY DIFFERENTIATING (1):

$$dx = \frac{2y\,dy}{128}, \quad \tan\theta = \frac{dx}{dy} = \frac{y}{64} \qquad (2)$$

FREE BODY: PORTION OF BOW CJ



MAKING $y = 16\text{in.}$ IN EQS (1) AND (2):

$x = \frac{(16)^2}{128} = 2.00\text{in.}$

$\tan\theta = \frac{16}{64} = 0.25 \qquad \theta = 14.04°$

$+\nwarrow \Sigma F_t = 0: -F + (30\text{lb})\cos 14.04° - (22.5\text{lb})\sin 14.04° = 0$

$F = +23.6\text{ lb} \qquad \underline{F} = 23.6\text{lb}\nwarrow$ ◀

$+\nearrow \Sigma F_u = 0: -V + (30\text{ lb})\sin 14.04° + (22.5\text{ lb})\cos 14.04° = 0$

$V = +29.1\text{ lb} \qquad \underline{V} = 29.1\text{lb}\swarrow$ ◀

$+\circlearrowleft \Sigma M_J = 0: -M - 960\text{ lb·in.} + (30\text{lb})(2\text{in.}) + (22.5\text{lb})(16\text{in.}) = 0$

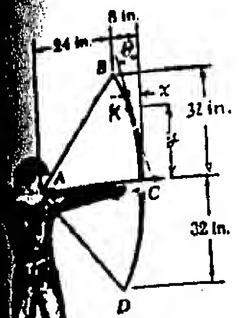
$M = -540\text{ lb·in.} \qquad \underline{M} = 540\text{ lb·in.}\circlearrowright$ ◀

7.10

GIVEN: ARCHER AND BOW OF PROB. 7.9, WITH ARCHER PULLING WITH A 45-lb FORCE ON BOWSTRING.

FIND: MAGNITUDE AND LOCATION IN THE BOW OF THE MAXIMUM (a) AXIAL FORCE, (b) SHEARING FORCE, (c) BENDING MOMENT.

THE FOLLOWING RESULTS WERE OBTAINED IN THE FIRST PART OF THE SOLUTION OF PROB. 7.9:



INTERNAL FORCES AT C (ON BC)

$\underline{F}_C = 30\,\text{lb}\uparrow,\ \underline{V}_C = 22.5\,\text{lb}\rightarrow,\ \underline{M}_C = 960\,\text{lb}\cdot\text{in.}$

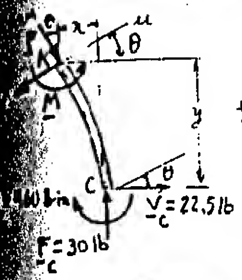
EQUATION OF PARABOLA (BOW)

$$x = \frac{y^2}{128} \qquad (1)$$

SLOPE (ANGLE θ)

$$\tan\theta = \frac{dx}{dy} = \frac{y}{64} \qquad (2)$$

FREE BODY: PORTION OF BOW CK



(a) MAXIMUM AXIAL FORCE

$\nwarrow \sum F_v = 0:\ -F + (30\text{ lb})\cos\theta - (22.5\text{ lb})\sin\theta = 0$

$F = 30\cos\theta - 22.5\sin\theta$

F IS LARGEST AT C ($\theta = 0$)

$F_m = 30.0$ lb AT C

(b) MAXIMUM SHEARING FORCE

$+\nearrow \sum F_u = 0:\ -V + (30\text{ lb})\sin\theta + (22.5\text{ lb})\cos\theta = 0$

$V = 30\sin\theta + 22.5\cos\theta$

V IS LARGEST AT B (AND D)

WHERE $\theta = \theta_{max} = \tan^{-1}(1/2) = 26.56°$

$V_m = 30\sin 26.56° + 22.5\cos 26.56°$

$V_m = 33.5$ lb AT B AND D

(c) MAXIMUM BENDING MOMENT

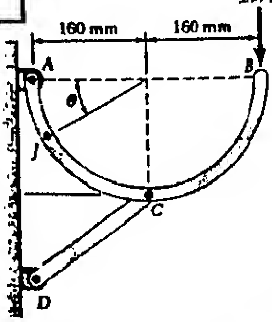
$+\circlearrowleft \sum M_K = 0:\ M - 960\text{ lb}\cdot\text{in.} + (30\text{ lb})x + (22.5\text{ lb})y = 0$

$M = 960 - 30x - 22.5y$

M IS LARGEST AT C, WHERE $x = y = 0$.

$M_m = 960$ lb·in. at C

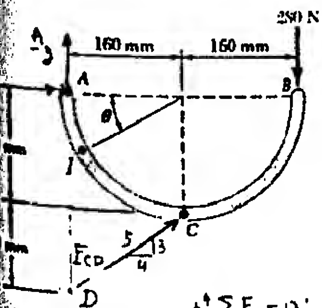
7.11



GIVEN: SEMICIRCULAR ROD LOADED AS SHOWN.

FIND: INTERNAL FORCES AT POINT J WHERE $\theta = 30°$

FREE BODY: ROD ACB



$+\circlearrowleft \sum M_A = 0:$

$(\tfrac{4}{5}F_{CD})(0.16\text{ m}) + (\tfrac{3}{5}F_{CD})(0.16\text{ m}) - (280\text{ N})(0.32\text{ m}) = 0$

$\underline{F}_{CD} = 400\text{ N}\nearrow$

$\xrightarrow{+} \sum F_x = 0:\ A_x + \tfrac{4}{5}(400\text{ N}) = 0$

$A_x = -320\text{ N} \qquad \underline{A}_x = 320\text{ N}\leftarrow$

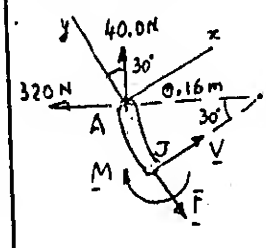
$+\uparrow \sum F_y = 0:\ A_y + \tfrac{3}{5}(400\text{ N}) - 280\text{ N} = 0$

$A_y = +40.0\text{ N} \qquad \underline{A}_y = 40.0\text{ N}\uparrow$

(CONTINUED)

7.11 CONTINUED

FREE BODY: AJ



$+\nwarrow \sum F_y = 0:\ (320\text{ N})\sin 30° + (40.0\text{ N})\cos 30° - F = 0$

$F = +194.6\text{ N} \qquad \underline{F} = 194.6\text{ N}\ \measuredangle 60°$

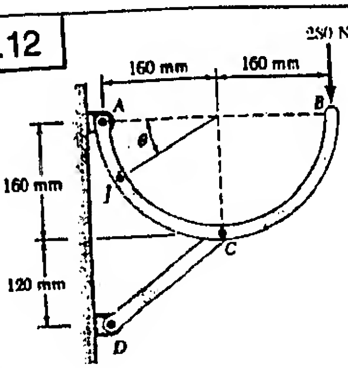
$+\nearrow \sum F_x = 0:\ (40.0\text{ N})\sin 30° - (320\text{ N})\cos 30° + V = 0$

$V = +257\text{ N} \qquad \underline{V} = 257\text{ N}\ \measuredangle 30°$

$+\circlearrowleft \sum M_J = 0:\ (320\text{ N})(0.16\text{ m})\sin 30° - (40.0\text{ N})(0.16\text{ m})(1-\cos 30°) - M = 0$

$M = +24.7\text{ N}\cdot\text{m} \qquad \underline{M} = 24.7\text{ N}\cdot\text{m}$

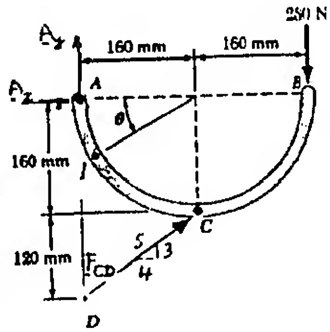
7.12



GIVEN: SEMICIRCULAR ROD LOADED AS SHOWN.

FIND: MAGNITUDE AND LOCATION OF MAXIMUM BENDING MOMENT IN THE ROD.

FREE BODY: ROD ACB



$+\circlearrowleft \sum M_A = 0:$

$(\tfrac{4}{5}F_{CD})(0.16\text{ m}) + (\tfrac{3}{5}F_{CD})(0.16\text{ m}) - (280\text{ N})(0.32\text{ m}) = 0$

$\underline{F}_{CD} = 400\text{ N}\nearrow$

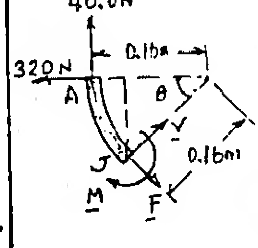
$\xrightarrow{+} \sum F_x = 0:\ A_x + \tfrac{4}{5}(400\text{ N}) = 0$

$A_x = -320\text{ N} \qquad \underline{A}_x = 320\text{ N}\leftarrow$

$+\uparrow \sum F_y = 0:\ A_y + \tfrac{3}{5}(400\text{ N}) - 280\text{ N} = 0$

$A_y = +40.0\text{ N} \qquad \underline{A}_y = 40.0\text{ N}\uparrow$

FREE BODY: AJ (FOR θ < 90°)



$+\circlearrowleft \sum M_J = 0:\ (320\text{ N})(0.16\text{ m})\sin\theta - (40.0\text{ N})(0.16\text{ m})(1-\cos\theta) - M = 0$

$$M = 51.2\sin\theta + 6.4\cos\theta - 6.4 \qquad (1)$$

FOR MAXIMUM VALUE BETWEEN A AND C

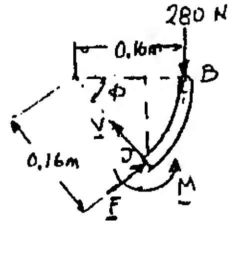
$\dfrac{dM}{d\theta} = 0:\quad 51.2\cos\theta - 6.4\sin\theta = 0$

$\tan\theta = \dfrac{51.2}{6.4} = 8 \qquad \theta = 82.87°$

CARRYING INTO (1):

$M = 51.2\sin 82.87° + 6.4\cos 82.87° - 6.4 = +45.20\text{ N}\cdot\text{m}$

FREE BODY: BJ (FOR θ > 90°)



$+\circlearrowleft \sum M_J = 0:$

$M - (280\text{ N})(0.16\text{ m})(1-\cos\phi) = 0$

$M = (44.8\text{ N}\cdot\text{m})(1-\cos\phi)$

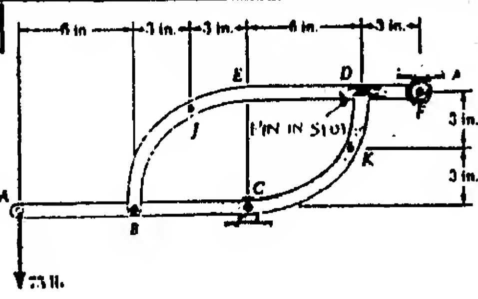
LARGEST VALUE OCCURS FOR $\phi = 90°$, THAT IS, AT C, AND IS

$M_C = 44.8\text{ N}\cdot\text{m}$

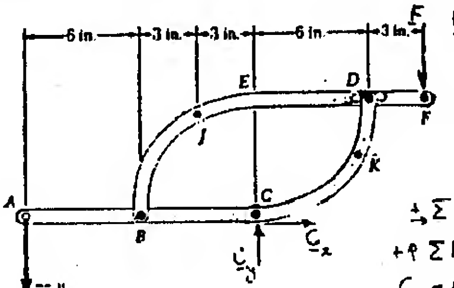
WE CONCLUDE THAT

$M_{max} = 45.2\text{ N}\cdot\text{m}$ FOR $\theta = 82.9°$

7.13



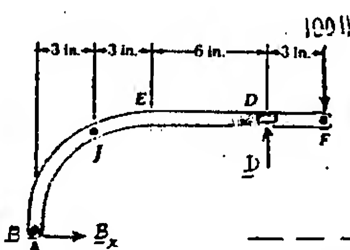
GIVEN: TWO MEMBERS, CONSISTING EACH OF A STRAIGHT AND A QUARTER-CIRCULAR ROD, SUPPORT A 75-lb LOAD.
FIND: INTERNAL FORCES AT POINT J.



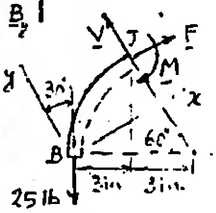
FREE BODY: ENTIRE FRAME

$+\circlearrowleft \Sigma M_C = 0:$
$(75\,lb)(12\,in.) - F(9\,in.) = 0$
$F = 100\,lb\downarrow$ ◁

$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$
$+\uparrow \Sigma F_y = 0: C_y - 75\,lb - 100\,lb = 0$
$C_y = +175\,lb, \; C = 175\,lb\uparrow$ ◁



FREE BODY: MEMBER BEDF
$+\circlearrowleft \Sigma M_B = 0:$
$D(12\,in.) - (100\,lb)(15\,in.) = 0$
$D = 125\,lb\uparrow$ ◁
$\xrightarrow{+} \Sigma F_x = 0: \; B_x = 0$
$+\uparrow \Sigma F_y = 0: B_y + 125\,lb - 100\,lb = 0$
$B_y = -25\,lb \quad B = 25\,lb\downarrow$ ◁

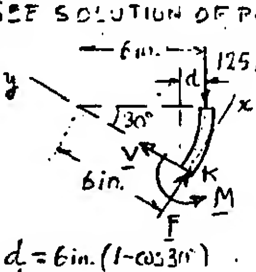


FREE BODY: BJ
$+\nearrow \Sigma F_x = 0: \; F - (25\,lb)\sin 30° = 0$
$F = 12.50\,lb \measuredangle 30°$ ◀
$+\nwarrow \Sigma F_y = 0: \; V - (25\,lb)\cos 30° = 0$
$V = 21.7\,lb \measuredangle 60°$ ◀
$+\circlearrowleft \Sigma M_J = 0: \; -M + (25\,lb)(3\,in.) = 0$
$M = 75.0\,lb\cdot in.$ ◀

7.14

(SEE FIGURE OF PROB. 7.13)
GIVEN: TWO MEMBERS, CONSISTING EACH OF A STRAIGHT AND A QUARTER-CIRCULAR ROD, SUPPORT A 75-lb LOAD.
FIND: INTERNAL FORCES AT POINT K.

SEE SOLUTION OF PROB. 7.13 UP TO DASHED LINE.



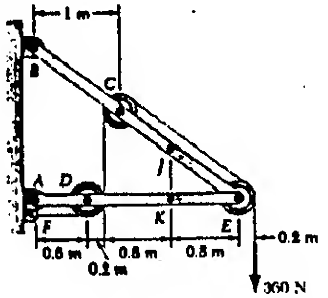
FREE BODY: DK
WE FOUND IN PROB. 7.13 THAT $D = 125\,lb\uparrow$ ON BEDF, THUS
$D = 125\,lb\downarrow$ ON DK. ◁

$+\nearrow \Sigma F_x = 0: \; F - (125\,lb)\cos 30° = 0$
$F = 108.3\,lb \measuredangle 60°$ ◀
$+\nwarrow \Sigma F_y = 0: \; V - (125\,lb)\sin 30° = 0$
$V = 62.5\,lb \measuredangle 30°$ ◀

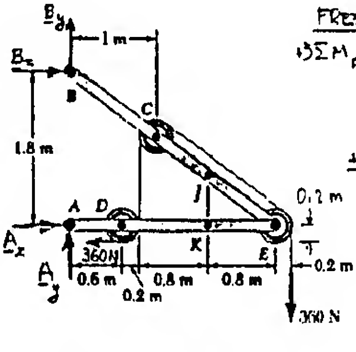
$d = 6\,in.(1 - \cos 30°)$
$d = 0.8038\,in.$

$+\circlearrowleft \Sigma M_K = 0: \; M - (125\,lb)d = 0$
$M = (125\,lb)d = (125\,lb)(0.8038\,in.) = 100.5\,lb\cdot in.$
$M = 100.5\,lb\cdot in.$ ◀

7.15

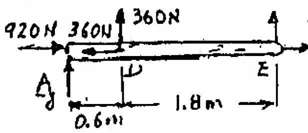


GIVEN: FRAME AND LOADING SHOWN. FOR EACH PULLEY R = 200 mm
FIND: INTERNAL FORCES AT POINT J.

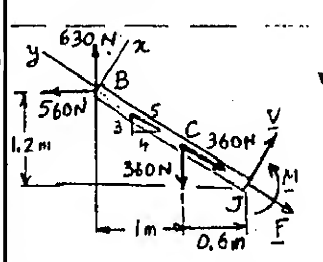


FREE BODY: FRAME AND PULLEYS
$+\circlearrowleft \Sigma M_A = 0: \; -B_x(1.8\,m) - (360\,N)(0.2\,m) - (360\,N)(2.6\,m) = 0$
$B_x = -560\,N \quad B_x = 560\,N \leftarrow$ ◁
$\xrightarrow{+} \Sigma F_x = 0: \; A_x - 560\,N - 360\,N = 0$
$A_x = +920\,N \quad A_x = 920\,N \rightarrow$ ◁
$+\uparrow \Sigma F_y = 0: \; A_y + B_y - 360\,N = 0$
$A_y + B_y = 360\,N \quad (1)$

FREE BODY: MEMBER AE
WE RECALL FROM PROB. 6.90 THAT THE FORCES APPLIED TO A PULLEY MAY BE APPLIED DIRECTLY TO THE AXLE OF THE PULLEY.

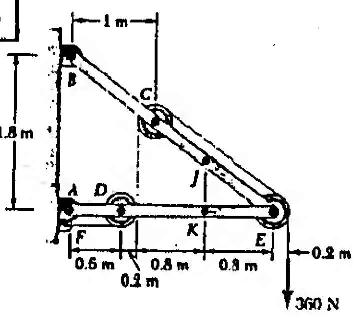


$+\circlearrowleft \Sigma M_E = 0: -A_y(2.4\,m) - (360\,N)(1.8\,m) = 0$
$A_y = -270\,N \quad A_y = 270\,N\downarrow$ ◁
FROM (1): $B_y = 360\,N + 270\,N$
$B_y = 630\,N \quad B_y = 630\,N\uparrow$ ◁



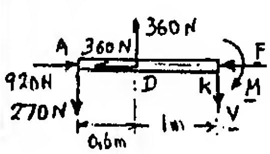
FREE BODY: BJ
$+\nwarrow \Sigma F_y = 0: \; \frac{3}{5}(630\,N) + \frac{4}{5}(560\,N) - 360\,N - \frac{3}{5}(360\,N) - F = 0$
$F = +250\,N \quad F = 250\,N \searrow$ ◀
$+\nearrow \Sigma F_x = 0: \; \frac{4}{5}(630\,N) - \frac{3}{5}(560\,N) - \frac{4}{5}(360\,N) + V = 0$
$V = 120.0\,N \quad V = 120.0\,N \nearrow$ ◀
$+\circlearrowleft \Sigma M_J = 0: \; (560\,N)(1.2\,m) - (630\,N)(1.6\,m) + (360\,N)(0.6\,m) + M = 0$
$M = +120.0\,N\cdot m \quad M = 120.0\,N\cdot m$ ◀

7.16



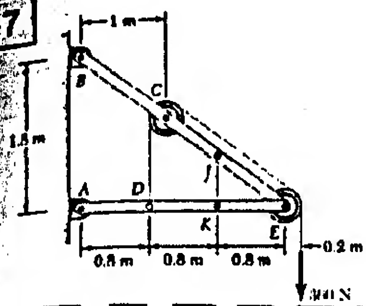
GIVEN: FRAME AND LOADING SHOWN. FOR EACH PULLEY R = 200 mm.
FIND: INTERNAL FORCES AT POINT K.

SEE SOLUTION OF PROB. 7.15 UP TO DASHED LINE, WE FOUND
$A_x = 920\,N \rightarrow, \quad A_y = 270\,N\downarrow$ ◁



FREE BODY: AK
$\xrightarrow{+} \Sigma F_x = 0: \; 920\,N - 360\,N - F = 0$
$F = +560\,N \quad F = 560\,N \leftarrow$ ◀
$+\uparrow \Sigma F_y = 0: \; 360\,N - 270\,N - V = 0$
$V = +90.0\,N \quad V = 90.0\,N\downarrow$ ◀
$+\circlearrowleft \Sigma M_K = 0: \; (270\,N)(1.6\,m) - (360\,N)(1\,m) - M = 0$
$M = +72.0\,N\cdot m \quad M = 72.0\,N\cdot m$ ◀

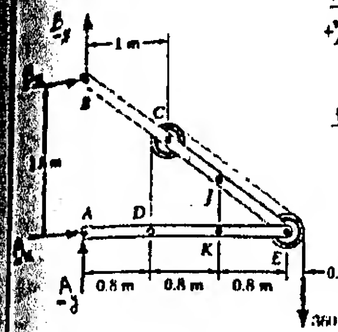
7.17



GIVEN: FRAME AND LOADING SHOWN. FOR EACH PULLEY $R = 200$ mm

FIND: INTERNAL FORCES AT POINT J.

FREE BODY: FRAME AND PULLEYS



$+\circlearrowleft \Sigma M_A = 0: -B_x(1.8\,m) - (360\,N)(2.6\,m) = 0$

$B_x = -520\,N \qquad B_x = 520\,N \leftarrow$ ◁

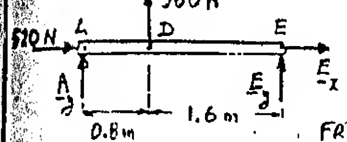
$\xrightarrow{+} \Sigma F_x = 0: \; A_x - 520\,N = 0$

$A_x = +520\,N \qquad A_x = 520\,N \rightarrow$ ◁

$+\uparrow \Sigma F_y = 0: \; A_y + B_y - 360\,N = 0$

$A_y + B_y = 360\,N \qquad (1)$

FREE BODY: MEMBER AE



$+\circlearrowleft \Sigma M_E = 0:$

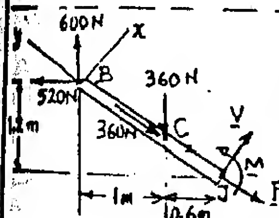
$-A_y(2.4\,m) - (360\,N)(1.6\,m) = 0$

$A_y = -240\,N \qquad A_y = 240\,N \downarrow$ ◁

FROM (1): $B_y = 360\,N + 240\,N$

$B_y = +600\,N \qquad B_y = 600\,N \uparrow$ ◁

FREE BODY: BJ



WE RECALL FROM PROB. 6.90 THAT THE FORCES APPLIED TO A PULLEY MAY BE APPLIED DIRECTLY TO ITS AXLE.

$\nearrow^{+} \Sigma F_y = 0: \; \frac{3}{5}(600\,N) + \frac{4}{5}(520\,N) - 360\,N - \frac{3}{5}(360\,N) - F = 0$

$F = +200\,N \qquad F = 200\,N \searrow$ ◀

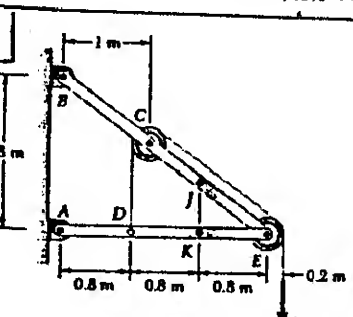
$+\nearrow \Sigma F_x = 0: \; \frac{4}{5}(600\,N) - \frac{3}{5}(520\,N) - \frac{4}{5}(360\,N) + V = 0$

$V = +120.0\,N \qquad V = 120.0\,N \nearrow$ ◀

$+\circlearrowleft \Sigma M_J = 0: (520\,N)(1.2\,m) - (600\,N)(1.6\,m) + (360\,N)(0.6\,m) + M = 0$

$M = +120.0\,N{\cdot}m \qquad M = 120.0\,N{\cdot}m \circlearrowleft$ ◀

7.18



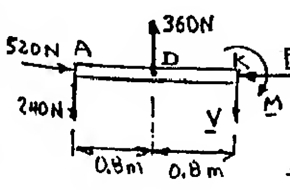
GIVEN: FRAME AND LOADING SHOWN. FOR EACH PULLEY $R = 200$ mm.

FIND: INTERNAL FORCES AT POINT K.

SEE SOLUTION OF PROB. 7.17 UP TO DASHED LINE.

WE FOUND $A_x = 520\,N \rightarrow, \; A_y = 240\,N \downarrow$ ◁

FREE BODY: AK



$\xrightarrow{+} \Sigma F_x = 0: \; 520\,N - F = 0$

$F = +520\,N \qquad F = 520\,N \leftarrow$ ◀

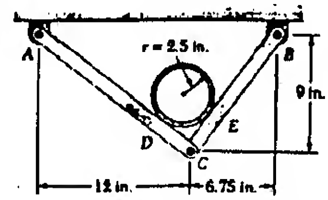
$+\uparrow \Sigma F_y = 0: \; 360\,N - 240\,N - V = 0$

$V = +120.0\,N \qquad V = 120.0\,N \downarrow$ ◀

$+\circlearrowleft \Sigma M_K = 0: (240\,N)(1.6\,m) - (360\,N)(0.8\,m) - M = 0$

$M = +96.0\,N{\cdot}m \qquad M = 96.0\,N{\cdot}m \circlearrowleft$ ◀

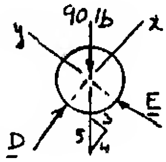
7.19



GIVEN: PIPE SUPPORTED EVERY 9 ft BY FRAME SHOWN. PIPE AND CONTENTS WEIGH 10 lb/ft.

FIND: MAGNITUDE AND LOCATION OF M_{max} IN AC

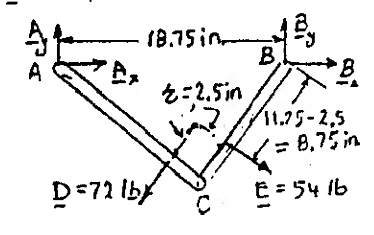
FREE BODY: 10-ft SECTION OF PIPE



$+\nearrow \Sigma F_x = 0: \; D - \frac{4}{5}(90\,lb) = 0 \qquad D = 72\,lb \nearrow$ ◁

$+\nwarrow \Sigma F_y = 0: \; E - \frac{3}{5}(90\,lb) = 0 \qquad E = 54\,lb \nwarrow$ ◁

FREE BODY: FRAME



$+\circlearrowleft \Sigma M_B = 0: \; -A_y(18.75\,in.) + (72\,lb)(2.5\,in.) + (54\,lb)(8.75\,in.) = 0$

$A_y = +34.8\,lb, \quad A_y = 34.8\,lb \uparrow$ ◁

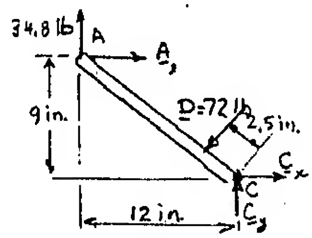
$+\uparrow \Sigma F_y = 0:$

$B_y + 34.8\,lb - \frac{4}{5}(72\,lb) - \frac{3}{5}(54\,lb) = 0$

$B_y = +55.2\,lb, \quad B_y = 55.2\,lb \uparrow$ ◁

$\xrightarrow{+} \Sigma F_x = 0: \; A_x + B_x - \frac{3}{5}(72\,lb) + \frac{4}{5}(54\,lb) = 0. \qquad A_x + B_x = 0 \qquad (1)$

FREE BODY: MEMBER AC



$+\circlearrowleft \Sigma M_C = 0:$

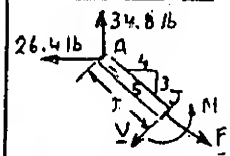
$(72\,lb)(2.5\,in.) - (34.8\,lb)(12\,in.) - A_x(9\,in.) = 0$

$A_x = -26.4\,lb, \quad A_x = 26.4\,lb \leftarrow$ ◁

FROM (1): $B_x = -A_x = +26.4\,lb$

$B_x = 26.4\,lb \rightarrow$ ◁

FREE BODY: PORTION AJ

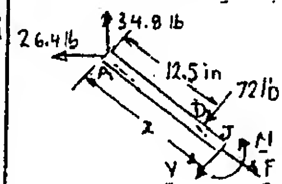


FOR $x \le 12.5$ in. ($AJ \le AD$):

$+\circlearrowleft \Sigma M_J = 0: (26.4\,lb)\frac{3}{5}x - (34.8\,lb)\frac{4}{5}x + M = 0$

$M = 12x, \quad M_{max} = 150\,lb{\cdot}in.$ for $x = 12.5$ in.

$M_{max} = 150.0\,lb{\cdot}in.$ at D ◁



FOR $x > 12.5$ in. ($AJ > AD$)

$+\circlearrowleft \Sigma M_J = 0: (26.4\,lb)\frac{3}{5}x - (34.8\,lb)\frac{4}{5}x + (72\,lb)(x - 12.5) + M = 0$

$M = 900 - 60x, \; M_{max} = 150\,lb{\cdot}in.$ for $x = 12.5$ in

THUS: $M_{max} = 150.0\,lb{\cdot}in.$ at D. ◀

7.20

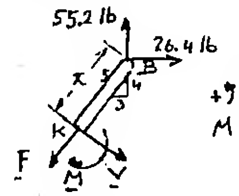
GIVEN: FRAME OF PROB. 7.19.

FIND: MAGNITUDE AND LOCATION OF M_{max} IN BC.

SEE SOLUTION OF PROB. 7.19 UP TO DASHED LINE.

WE FOUND $B_x = 26.4\,lb \rightarrow, \; B_y = 55.2\,lb \uparrow$ ◁

FREE BODY: PORTION BK

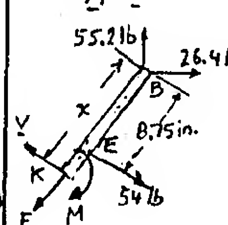


FOR $x \le 8.75$ in. ($BK \le BE$)

$+\circlearrowleft \Sigma M_K = 0: (55.2\,lb)\frac{3}{5}x - (26.4\,lb)\frac{4}{5}x - M = 0$

$M = 12x, \quad M_{max} = 105.0\,lb{\cdot}in.$ for $x = 8.75$ in.

$M_{max} = 105.0\,lb{\cdot}in.$ at E ◁



FOR $x > 8.75$ in. ($BK > BE$)

$+\circlearrowleft \Sigma M_K = 0: (55.2\,lb)\frac{3}{5}x - (26.4\,lb)\frac{4}{5}x - (54\,lb)(x - 8.75\,in.) - M = 0$

$M = 472.5 - 42x, \; M_{max} = 105.0\,lb{\cdot}in.$ for $x = 8.75$ in

THUS:

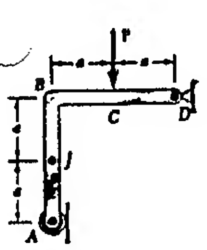
$M_{max} = 105.0\,lb{\cdot}in.$ at E ◀

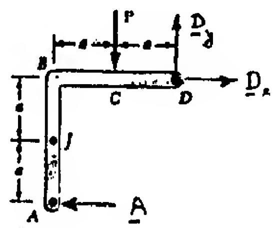
7.21

GIVEN: BENT ROD SUPPORTED AND LOADED AS SHOWN.
FIND: FOR EACH CASE, THE INTERNAL FORCES AT J.

CASE (a)

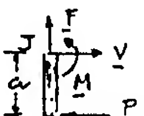
FREE-BODY DIAGRAM





$+\circlearrowleft \Sigma M_D = 0: \; Pa - A(2a) = 0 \qquad \underline{A} = \frac{P}{2} \leftarrow$

FREE BODY: AJ



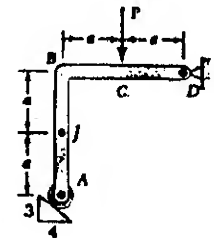
$+\uparrow \Sigma F_y = 0: \qquad \underline{F} = 0$

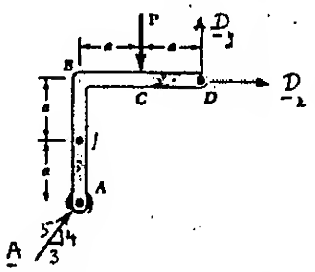
$\xrightarrow{+} \Sigma F_x = 0: \; V - \frac{P}{2} = 0 \qquad \underline{V} = \frac{P}{2} \rightarrow$

$+\circlearrowleft \Sigma M_J = 0: \; M - \frac{P}{2}a = 0 \qquad \underline{M} = \frac{1}{2}Pa$

CASE (b)

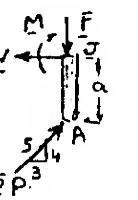
FREE-BODY DIAGRAM





$+\circlearrowleft \Sigma M_D = 0: \; Pa - (\frac{4}{5}A)(2a) + (\frac{3}{5}A)(2a) = 0 \qquad \underline{A} = \frac{5}{2}P$

FREE BODY: AJ



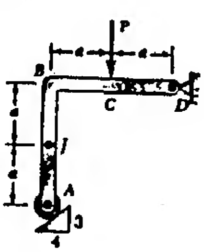
$+\uparrow \Sigma F_y = 0: \; -F + \frac{4}{5}(\frac{5}{2}P) = 0 \qquad \underline{F} = 2P\downarrow$

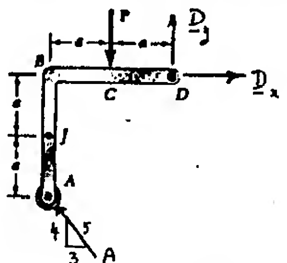
$\xrightarrow{+} \Sigma F_x = 0: \; -V + \frac{3}{5}(\frac{5}{2}P) = 0 \qquad \underline{V} = \frac{3}{2}P \leftarrow$

$+\circlearrowleft \Sigma M_J = 0: \; -M + \frac{3}{5}(\frac{5}{2}P)a = 0 \qquad \underline{M} = \frac{3}{2}Pa$

CASE (c)

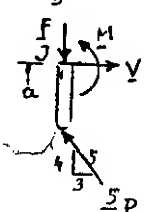
FREE-BODY DIAGRAM





$+\circlearrowleft \Sigma M_D = 0: \; Pa - (\frac{4}{5}A)(2a) - (\frac{3}{5}A)(2a) = 0 \qquad \underline{A} = \frac{5}{14}P$

FREE BODY: AJ



$+\uparrow \Sigma F_y = 0: \; -F + \frac{4}{5}(\frac{5}{14}P) = 0 \qquad \underline{F} = \frac{2}{7}P\downarrow$

$\xrightarrow{+} \Sigma F_x = 0: \; V - \frac{3}{5}(\frac{5}{14}P) = 0 \qquad \underline{V} = \frac{3}{14}P \rightarrow$

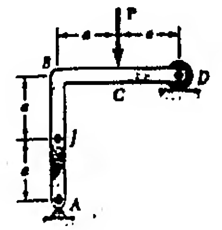
$+\circlearrowleft \Sigma M_J = 0: \; M - \frac{3}{5}(\frac{5}{14}P)a = 0 \qquad \underline{M} = \frac{3}{14}Pa$

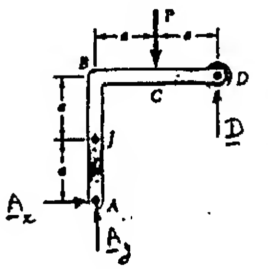
7.22

GIVEN: BENT ROD SUPPORTED AND LOADED AS SHOWN.
FIND: FOR EACH CASE, THE INTERNAL FORCES AT J.

CASE (a)

FREE-BODY DIAGRAM



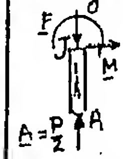


$+\circlearrowleft \Sigma M_A = 0: \; D(2a) - Pa = 0 \qquad \underline{D} = \frac{P}{2}\uparrow$

$\xrightarrow{+} \Sigma F_x = 0: \; A_x = 0$

$+\uparrow \Sigma F_y = 0: \; A_y - P + \frac{P}{2} = 0 \qquad A_y = +\frac{P}{2} \qquad \underline{A} = \frac{P}{2}\uparrow$

FREE BODY: AJ



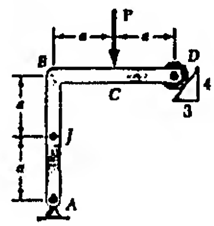
$+\uparrow \Sigma F_y = 0: \; \frac{P}{2} - F = 0 \qquad \underline{F} = \frac{P}{2}\downarrow$

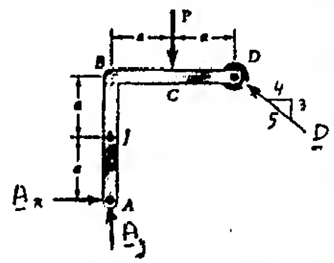
$\xrightarrow{+} \Sigma F_x = 0: \; V = 0 \qquad \underline{V} = 0$

$+\circlearrowleft \Sigma M_J = 0: \; M = 0 \qquad \underline{M} = 0$

CASE (b)

FREE-BODY DIAGRAM



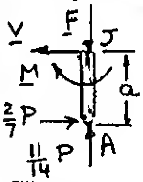


$+\circlearrowleft \Sigma M_A = 0: (\frac{4}{5}D)(2a) + (\frac{3}{5}D)(2a) - Pa = 0 \qquad \underline{D} = \frac{5}{14}P$

$\xrightarrow{+} \Sigma F_x = 0: \; A_x - \frac{4}{5}(\frac{5}{14}P) = 0 \quad A_x = +\frac{2}{7}P \qquad \underline{A_x} = \frac{2}{7}P \rightarrow$

$+\uparrow \Sigma F_y = 0: \; A_y + \frac{3}{5}(\frac{5}{14}P) - P = 0, \; A_y = +\frac{11}{14}P \qquad \underline{A_y} = \frac{11}{14}P\uparrow$

FREE BODY: AJ



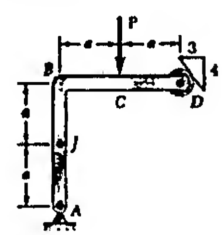
$+\uparrow \Sigma F_y = 0: \; -F + \frac{11}{14}P = 0 \qquad \underline{F} = \frac{11}{14}P\downarrow$

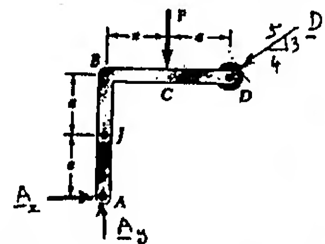
$\xrightarrow{+} \Sigma F_x = 0: \; -V + \frac{2}{7}P = 0 \qquad \underline{V} = \frac{2}{7}P \leftarrow$

$+\circlearrowleft \Sigma M_J = 0: \; -M + (\frac{2}{7}P)a = 0 \qquad \underline{M} = \frac{2}{7}Pa$

CASE (c)

FREE-BODY DIAGRAM



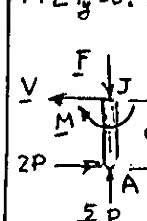


$+\circlearrowleft \Sigma M_A = 0: (\frac{4}{5}D)(2a) - (\frac{3}{5}D)(2a) - Pa = 0 \qquad \underline{D} = \frac{5}{2}P$

$\xrightarrow{+} \Sigma F_x = 0: \; A_x - \frac{4}{5}(\frac{5}{2}P) = 0 \quad A_x = +2P \qquad \underline{A_x} = 2P \rightarrow$

$+\uparrow \Sigma F_y = 0: \; A_y - \frac{3}{5}(\frac{5}{2}P) - P = 0, \; A_y = +\frac{5}{2}P \qquad \underline{A_y} = \frac{5}{2}P\uparrow$

FREE BODY: AJ



$+\uparrow \Sigma F_y = 0: \; -F + \frac{5}{2}P = 0 \qquad \underline{F} = \frac{5}{2}P\downarrow$

$\xrightarrow{+} \Sigma F_x = 0: \; -V + 2P = 0 \qquad \underline{V} = 2P \leftarrow$

$+\circlearrowleft \Sigma M_J = 0: \; -M + (2P)a = 0 \qquad \underline{M} = 2Pa$

7.23 AN

THUS: M=

7.23

7.24

ALTERNA

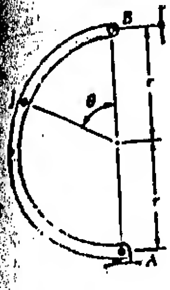
THUS:
M= Ws

MAKIN

M=

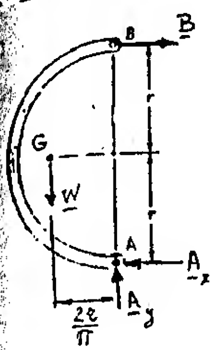
7.23 AND 7.24

GIVEN: SEMICIRCULAR ROD OF WEIGHT W AND UNIFORM CROSS SECTION SUPPORTED AS SHOWN.



FIND: BENDING MOMENT AT J WHEN
$\theta = 60°$ (PROB. 7.23)
$\theta = 150°$ (PROB. 7.24)

FREE BODY: ROD

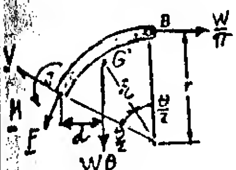


$+\circlearrowleft \Sigma M_A = 0: \; W\left(\frac{2r}{\pi}\right) - B(2r) = 0 \qquad \underline{B} = \frac{W}{\pi} \rightarrow$

$\xrightarrow{+} \Sigma F_x = 0: \; \frac{W}{\pi} - A_x = 0 \qquad \underline{A}_x = \frac{W}{\pi} \leftarrow$

$+\uparrow \Sigma F_y = 0: \; A_y - W = 0 \qquad \underline{A}_y = W \uparrow$

FREE BODY: PORTION BJ



$+\circlearrowleft \Sigma M_J = 0:$

$M - \frac{W}{\pi} r(1-\cos\theta) - \frac{W\theta}{\pi} d = 0$

$M = \frac{W}{\pi} r(1-\cos\theta) + \frac{W\theta}{\pi} d$

BUT $d = r\sin\theta - \bar{r}\sin\frac{\theta}{2}$

$= r\sin\theta - \frac{r\sin\frac{\theta}{2}}{\frac{\theta}{2}}\sin\frac{\theta}{2}$

$= r\sin\theta - \frac{r}{\theta}\, 2\sin^2\frac{\theta}{2}$

$= r\sin\theta - \frac{r}{\theta}(1-\cos\theta)$

THUS: $M = \frac{W}{\pi} r(1-\cos\theta) + \frac{W}{\pi} r\,\theta\sin\theta - \frac{W}{\pi} r(1-\cos\theta)$

$M = \frac{Wr}{\pi}\theta\sin\theta \qquad (1)$

7.23

MAKING $\theta = 60° = \frac{\pi}{3}$ IN EQ. (1):

$M = \frac{Wr}{\pi}\frac{\pi}{3}\sin 60° = Wr\frac{\sin 60°}{3} \qquad \underline{M = 0.289 Wr} \circlearrowleft$

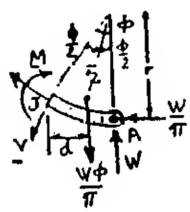
7.24

MAKING $\theta = 150° = \frac{5\pi}{6}$ IN EQ. (1):

$M = \frac{Wr}{\pi}\frac{5\pi}{6}\sin 150° = \frac{5}{12}Wr \qquad \underline{M = 0.417 Wr} \circlearrowleft$ (ON BJ)

ALTERNATIVE SOLUTION TO PROB. 7.24:

FREE BODY: AJ



$+\circlearrowleft \Sigma M_J = 0:$

$-M + Wr\sin\phi - \frac{W}{\pi} r(1-\cos\phi) - \frac{W\phi}{\pi} d = 0$

$M = Wr\sin\phi - \frac{W}{\pi} r(1-\cos\phi) - \frac{W\phi}{\pi} d$

BUT $d = r\sin\phi - \bar{r}\sin\frac{\phi}{2}$

$= r\sin\phi - \frac{r\sin\frac{\phi}{2}}{\phi/2}\sin\frac{\phi}{2}$

$= r\sin\phi - \frac{r}{\phi}\, 2\sin^2\frac{\phi}{2}$

$= r\sin\phi - \frac{r}{\phi}(1-\cos\phi)$

THUS:
$M = Wr\sin\phi - \frac{W}{\pi} r(1-\cos\phi) - \frac{W}{\pi} r\,\phi\sin\phi + \frac{W}{\pi} r(1-\cos\phi)$

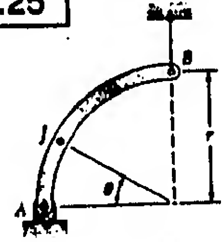
$M = Wr\left(1-\frac{\phi}{\pi}\right)\sin\phi \qquad (2)$

MAKING $\phi = 180° - 150° = 30° = \pi/6$ IN EQ. (2):

$M = Wr\left(1-\frac{1}{6}\right)\sin 30° = \frac{5}{12}Wr \qquad \underline{M = 0.417 Wr} \circlearrowleft$ (ON AJ)

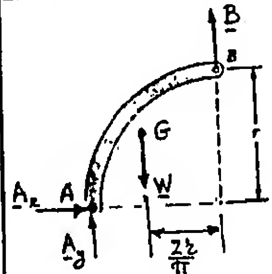
7.25

GIVEN: QUARTER CIRCULAR ROD OF WEIGHT W AND UNIFORM CROSS SECTION SUPPORTED AS SHOWN



FIND: BENDING MOMENT AT J WHEN $\theta = 30°$.

FREE BODY: ROD



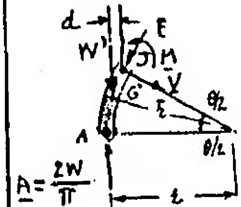
$\xrightarrow{+} \Sigma F_x = 0: \qquad A_x = 0$

$+\circlearrowleft \Sigma M_B = 0:$

$W\left(\frac{2r}{\pi}\right) - A_y r = 0$

$A_y = \frac{2W}{\pi} \qquad \underline{A} = \frac{2W}{\pi}\uparrow$

FREE BODY: PORTION AJ



$+\circlearrowleft \Sigma M_J = 0:$

$M + W'd - \frac{2W}{\pi} r(1-\cos\theta) = 0$

$M = \frac{2W}{\pi} r(1-\cos\theta) - W'd \qquad (1)$

BUT $W' = W\frac{\theta}{\pi/2} = \frac{2W\theta}{\pi} \qquad (2)$

AND

$d = \bar{r}\cos\frac{\theta}{2} - r\cos\theta$

$= r\frac{\sin\theta/2}{\theta/2}\cos\frac{\theta}{2} - r\cos\theta$

$= r\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\theta} - r\cos\theta$

$d = r\left(\frac{\sin\theta}{\theta} - \cos\theta\right) \qquad (3)$

SUBSTITUTING FROM (2) AND (3) INTO (1):

$M = \frac{2W}{\pi} r(1-\cos\theta) - \frac{2W\theta}{\pi} r\left(\frac{\sin\theta}{\theta} - \cos\theta\right)$

$M = \frac{2Wr}{\pi}(1-\cos\theta - \sin\theta + \theta\cos\theta) \qquad (4)$

MAKING $\theta = 30° = \frac{\pi}{6}$ IN EQ (4):

$M = 2Wr\left[\frac{1-\cos 30° - \sin 30°}{\pi} + \frac{1}{6}\cos 30°\right]$

$\underline{M = 0.0557 Wr} \circlearrowleft$

THE SOLUTIONS OF PROBS. 7.26 AND 7.27 ARE GIVEN ON THE NEXT PAGE

7.28

GIVEN: ROD OF PROB. 7.25.

FIND: MAGNITUDE AND LOCATION OF MAXIMUM BENDING MOMENT.

WE RECALL EQ. (4) OF PROB. 7.25:

$M = \frac{2Wr}{\pi}(1-\cos\theta - \sin\theta + \theta\cos\theta) \qquad (4)$

$\frac{dM}{d\theta} = \frac{2Wr}{\pi}(\sin\theta - \cancel{\cos\theta} + \cancel{\cos\theta} - \theta\sin\theta)$

SETTING $\frac{dM}{d\theta} = 0$:

$\sin\theta\,(1-\theta) = 0$

THE ROOTS OF THIS EQUATION FOR $0 \le \theta \le \frac{\pi}{2}$ ARE

$\theta = 0$ AND $\theta = 1$ RAD $= 57.3°$

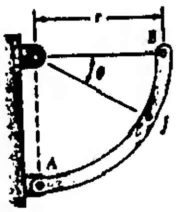
FOR $\theta = 0$, $M = 0$. FOR $\theta = 1$ RAD $= 57.3°$, EQ. (4) YIELDS

$M = \frac{2Wr}{\pi}(1 - \cos 57.3° - \sin 57.3° + 1 \times \cos 57.3°)$

$= \frac{2Wr}{\pi}(1 - \sin 57.3°) = 0.1009\, Wr$

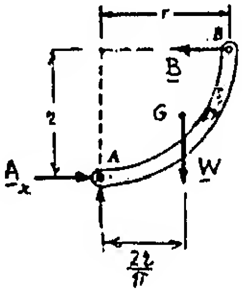
THUS: $M_{max} = 0.1009\, Wr$ for $\theta = 57.3°$

7.26



GIVEN:
QUARTER CIRCULAR ROD OF WEIGHT W AND UNIFORM CROSS SECTION SUPPORTED AS SHOWN.
FIND:
BENDING MOMENT AT J WHEN $\theta = 30°$

FREE BODY: ROD

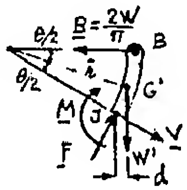


$+\circlearrowleft \Sigma M_A = 0:$

$B r - W\left(\frac{2r}{\pi}\right) = 0$

$B = \frac{2W}{\pi} \leftarrow$ ◁

FREE BODY: PORTION BJ



$+\circlearrowleft \Sigma M_J = 0:$

$\frac{2W}{\pi} r \sin\theta - W'd - M = 0$

$M = \frac{2W}{\pi} r \sin\theta - W'd \qquad (1)$

BUT $W' = W\frac{\theta}{\pi/2} = \frac{2W\theta}{\pi} \qquad (2)$

AND

$d = \bar{r}\cos\frac{\theta}{2} - r\cos\theta$

$= r\,\frac{\sin\theta/2}{\theta/2}\cos\frac{\theta}{2} - r\cos\theta$

$= r\,\frac{2\sin\theta/2\cos\theta/2}{\theta} - r\cos\theta$

$d = r\left(\frac{\sin\theta}{\theta} - \cos\theta\right) \qquad (3)$

SUBSTITUTING FROM (2) AND (3) INTO (1):

$M = \frac{2W}{\pi} r\sin\theta - \frac{2W\theta}{\pi} r\left(\frac{\sin\theta}{\theta} - \cos\theta\right)$

$M = \frac{2Wr}{\pi}\theta\cos\theta \qquad (4)$ ◁

MAKING $\theta = 30° = \frac{\pi}{6}$ IN EQ. (4):

$M = \frac{2Wr}{\pi}\left(\frac{\pi}{6}\right)\cos 30° = \frac{Wr}{3}\cos 30° \qquad M = 0.289\,Wr$ ◀

7.27

GIVEN: ROD OF PROB. 7.26.
FIND: MAGNITUDE AND LOCATION OF MAXIMUM BENDING MOMENT.

WE RECALL EQ. (4) OF PROB. 7.26:

$M = \frac{2Wr}{\pi}\theta\cos\theta \qquad (4)$

$\frac{dM}{d\theta} = 0: \qquad \cos\theta - \theta\sin\theta = 0$

$\tan\theta = \frac{1}{\theta}$

SOLVING BY SUCCESSIVE APPROXIMATIONS:

$\theta = 49.293° = 0.86033$ RAD

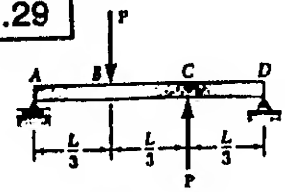
SUBSTITUTING INTO EQ. (4):

$M = \frac{2Wr}{\pi}(0.86033\text{ RAD})\cos 49.293° = 0.3572\,Wr$

THUS: $M_{max} = 0.357\,Wr$ for $\theta = 49.3°$ ◀

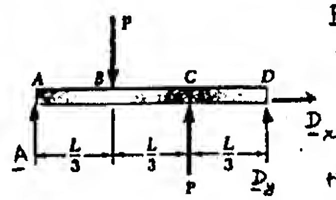
THE SOLUTION OF PROB. 7.28 IS GIVEN ON THE PRECEDING PAGE

7.29



GIVEN:
BEAM AND LOADING
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$, $|M|_{max}$.

FREE BODY: ENTIRE BEAM



$+\circlearrowleft \Sigma M_D = 0:$

$P\left(\frac{2L}{3}\right) - P\left(\frac{L}{3}\right) - AL = 0$

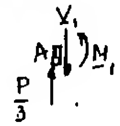
$A = P/3$

$\Sigma F_x = 0: D_x = 0$

$+\uparrow \Sigma F_y = 0: \frac{P}{3} - P + P + D_y = 0$

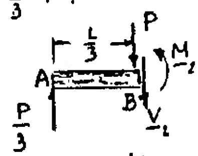
$D_y = -P/3 \qquad D = P/3 \downarrow$

(a) SHEAR AND BENDING MOMENT. SINCE THE LOADING CONSISTS OF CONCENTRATED LOADS, THE SHEAR DIAGRAM IS MADE OF HORIZONTAL STRAIGHT-LINE SEGMENTS AND THE B.M. DIAGRAM IS MADE OF OBLIQUE STRAIGHT-LINE SEGMENTS. WE SHALL DETERMINE V AND M JUST TO THE RIGHT OF A, B, AND C.



$+\uparrow \Sigma F_y = 0: \; -V_1 + \frac{P}{3} = 0 \qquad V_1 = +P/3$

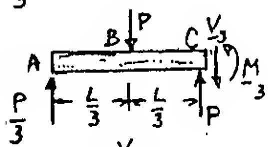
$+\circlearrowleft \Sigma M_1 = 0: \; M_1 - \frac{P}{3}(0) = 0 \qquad M_1 = 0$



$+\uparrow \Sigma F_y = 0: -V_2 + \frac{P}{3} - P = 0, \; V_2 = -2P/3$

$+\circlearrowleft \Sigma M_2 = 0: M_2 - \frac{P}{3}\left(\frac{L}{3}\right) + P(0) = 0$

$M_2 = +PL/9$

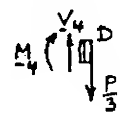


$+\uparrow \Sigma F_y = 0: \; \frac{P}{3} - P + P - V_3 = 0$

$V_3 = +P/3$

$+\circlearrowleft \Sigma M_3 = 0: M_3 - \frac{P}{3}\left(\frac{2L}{3}\right) + P\frac{L}{3} - P(0) = 0$

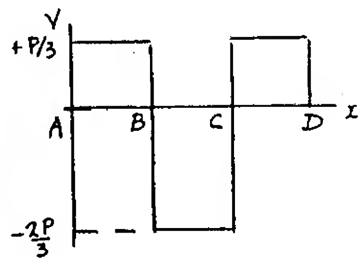
$M_3 = -PL/9$

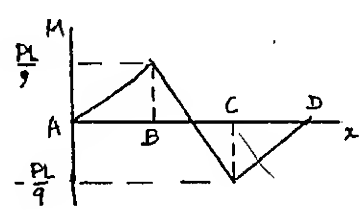


JUST TO THE LEFT OF D:

$+\uparrow \Sigma F_y = 0: \; V_4 - \frac{P}{3} = 0 \qquad V_4 = +\frac{P}{3}$

$+\circlearrowleft \Sigma M_4 = 0: \; -M_4 - \frac{P}{3}(0) = 0 \qquad M_4 = 0$





(b) $|V|_{max} = 2P/3 \; ; \; |M|_{max} = PL/9$ ◀

GIVEN:
BEAM AND LOADING SHOWN
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

SUPPORTS. BECAUSE OF SYMMETRY OF LOADING:
$(A+D) = \frac{1}{2}(w\frac{L}{2})$ $\quad A = D = \frac{1}{4}wL$ ◁

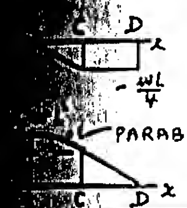
(a) SHEAR AND BENDING MOMENT

JUST TO THE RIGHT OF A:
$V_1 = +\frac{1}{4}wL$ ◁
$M_1 = 0$ ◁

AT B: $+\uparrow\Sigma F_y = 0: \frac{1}{4}wL - V_2 = 0, \quad V_2 = +\frac{1}{4}wL$ ◁
$+\circlearrowleft\Sigma M_B = 0: M_2 - (\frac{1}{4}wL)(\frac{L}{4}) = 0$
$M_2 = +wL^2/16$ ◁

AT CENTER LINE:
$+\uparrow\Sigma F_y = 0: \frac{1}{4}wL - \frac{1}{4}wL - V_3 = 0, \quad V_3 = 0$ ◁
$+\circlearrowleft\Sigma M_3 = 0:$
$M_3 - (\frac{1}{4}wL)(\frac{L}{2}) + (\frac{wL}{4})(\frac{L}{8}) = 0, \quad M_3 = \frac{3}{32}wL^2$ ◁

THE REMAINDERS OF THE DIAGRAMS ARE OBTAINED FROM SYMMETRY.



(b) $|V|_{max} = wL/4$ ◀

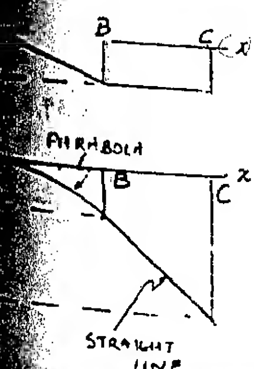
$|M|_{max} = 3wL^2/32$ ◀

GIVEN:
BEAM AND LOADING SHOWN
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

SHEAR AND BENDING MOMENT.

AT A: $V_A = M_A = 0$ ◁
$+\uparrow\Sigma F_y = 0: -V_B - \frac{wL}{2} = 0, \quad V_B = -\frac{wL}{2}$ ◁
$+\circlearrowleft\Sigma M_B = 0: M_B + (\frac{wL}{2})(\frac{L}{4}) = 0$
$M_B = -wL^2/8$ ◁

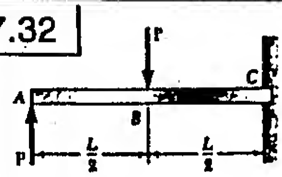
$+\uparrow\Sigma F_y = 0: \quad V_C = -\frac{wL}{2}$ ◁
$+\circlearrowleft\Sigma M_C = 0:$
$M_C + (\frac{wL}{2})(\frac{3L}{4}) = 0, \quad M_C = -\frac{3wL^2}{8}$ ◁



(b)
$|V|_{max} = wL/2$ ◀

$|M|_{max} = 3wL^2/8$ ◀

7.32



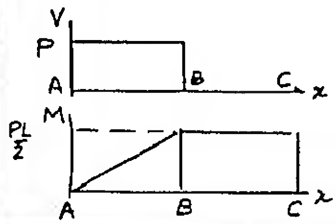
GIVEN:
BEAM AND LOADING SHOWN.
(a) DRAW V AND M DIAGRAMS.
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

(a) SHEAR AND BENDING MOMENT

JUST TO THE RIGHT OF A: $V_1 = +P, \ M_1 = 0$ ◁

JUST TO THE RIGHT OF B:
$+\uparrow\Sigma F_y = 0: P - P - V_2 = 0 \quad V_2 = 0$ ◁
$+\circlearrowleft\Sigma M_2 = 0: M_2 - P(\frac{L}{2}) = 0 \quad M_2 = +PL/2$ ◁

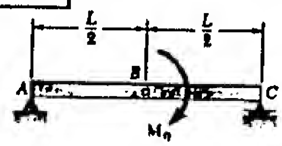
JUST TO THE LEFT OF C:
$+\uparrow\Sigma F_y = 0: P - P - V_3 = 0, \ V_3 = 0$ ◁
$+\circlearrowleft\Sigma M_3 = 0:$
$M_3 + P(\frac{L}{2}) - PL = 0, \ M_3 = +PL/2$ ◁



(b) $|V|_{max} = P$ ◀

$|M|_{max} = PL/2$ ◀

7.33

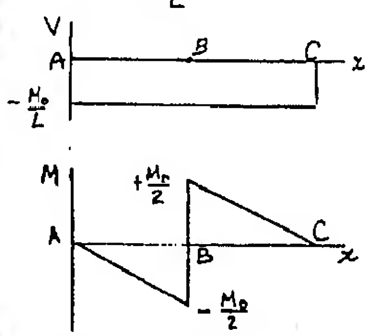


GIVEN:
BEAM AND LOADING SHOWN.
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

FREE BODY: ENTIRE BEAM
$+\circlearrowleft\Sigma M_A = 0: \ CL - M_0 = 0$
$C = M_0/L \uparrow$ ◁
$\xrightarrow{+}\Sigma F_x = 0: \ A_x = 0$
$+\uparrow\Sigma F_y = 0: A_y + \frac{M_0}{L} = 0, \ A_y = -\frac{M_0}{L}$
$A = M_0/L \downarrow$ ◁

(a) SHEAR AND BENDING MOMENT:

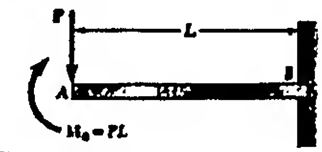
JUST TO THE RIGHT OF A:
$V_1 = -M_0/L, \quad M_1 = 0$ ◁
JUST TO THE LEFT OF B:
$V_2 = -M_0/L, \quad M_2 = -\frac{M_0}{L}\frac{L}{2} = -\frac{M_0}{2}$ ◁
JUST TO THE RIGHT OF B:
$V_3 = -M_0/L, \quad M_3 = +\frac{M_0}{L}\frac{L}{2} = +\frac{M_0}{2}$ ◁
JUST TO THE LEFT OF C:
$V_4 = -M_0/L, \quad M_4 = 0$ ◁



(b) $|V|_{max} = M_0/L$ ◀

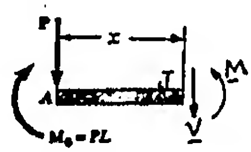
$|M|_{max} = M_0/2$ ◀

7.34



GIVEN:
BEAM AND LOADING SHOWN.
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

FREE BODY: PORTION AJ

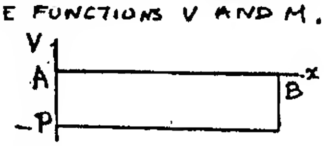


$+\uparrow \Sigma F_y = 0: \; -P - V = 0, \quad V = -P$ ◁

$+\circlearrowleft \Sigma M_J = 0: \; M + Px - PL = 0$

$M = P(L-x)$ ◁

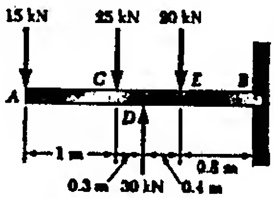
(a) THE V AND M DIAGRAMS ARE OBTAINED BY PLOTTING THE FUNCTIONS V AND M.



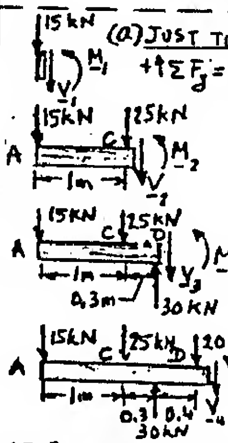
(b) $|V|_{max} = P$ ◀

$|M|_{max} = PL$ ◀

7.35



GIVEN:
BEAM AND LOADING SHOWN
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.



(a) JUST TO THE RIGHT OF A:

$+\uparrow \Sigma F_y = 0: \; V_1 = -15\,kN \qquad M_1 = 0$ ◁

JUST TO THE RIGHT OF C:

$V_2 = -40\,kN \qquad M_2 = -15\,kN{\cdot}m$ ◁

JUST TO THE RIGHT OF D:

$V_3 = -10\,kN, \; M_3 = -27\,kN{\cdot}m$ ◁

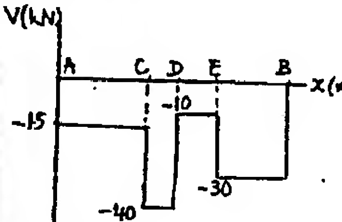
JUST TO THE RIGHT OF E:

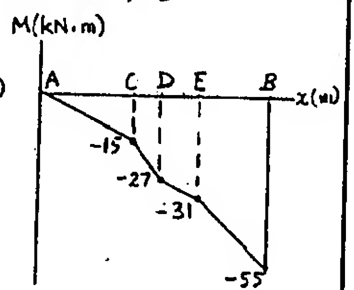
$V_4 = 30\,kN - 60\,kN, \; V_4 = -30\,kN$ ◁

$M_4 = 30 \times 0.4 - 15 \times 1.7 - 25 \times 0.7$

$M_4 = -31\,kN{\cdot}m$ ◁

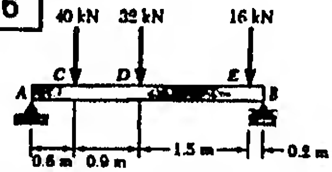
AT B: $M_B = 30 \times 1.2 - 15 \times 2.5 - 25 \times 1.5 - 20 \times 0.8, \; M_B = -55\,kN$ ◁





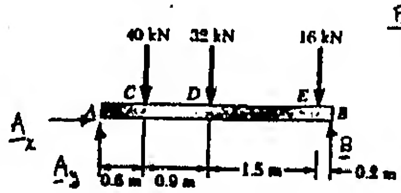
(b) $|V|_{max} = 40.0\,kN; \; |M|_{max} = 55.0\,kN{\cdot}m$ ◀

7.36



GIVEN:
BEAM AND LOADING SHOWN
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

FREE BODY: ENTIRE BEAM



$+\circlearrowleft \Sigma M_A = 0:$

$B(3.2\,m) - (40\,kN)(0.6\,m) - (32\,kN)(1.5\,m) - (16\,kN)(3\,m) = 0 \quad B = +37.5\,kN$

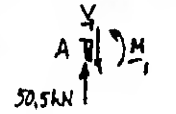
$B = 37.5\,kN\uparrow$ ◁

$\Sigma F_x = 0: \; A_x = 0$

$+\uparrow \Sigma F_y = 0: \; A_y + 37.5\,kN - 40\,kN - 32\,kN - 16\,kN = 0 \quad A_y = +50.5\,kN$

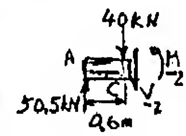
$A = 50.5\,kN\uparrow$ ◁

(a) SHEAR AND BENDING MOMENT.



JUST TO THE RIGHT OF A:

$V_1 = 50.5\,kN \quad M_1 = 0$ ◁



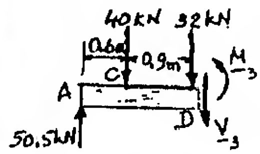
JUST TO THE RIGHT OF C:

$+\uparrow \Sigma F_y = 0: \; 50.5\,kN - 40\,kN - V_2 = 0$

$V_2 = +10.5\,kN$ ◁

$+\circlearrowleft \Sigma M_2 = 0: \; M_2 - (50.5\,kN)(0.6\,m) = 0$

$M_2 = +30.3\,kN{\cdot}m$ ◁



JUST TO THE RIGHT OF D:

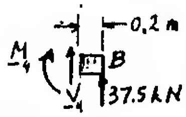
$+\uparrow \Sigma F_y = 0: \; 50.5 - 40 - 32 - V_3 = 0$

$V_3 = -21.5\,kN$ ◁

$+\circlearrowleft \Sigma M_3 = 0:$

$M_3 - (50.5)(1.5) + (40)(0.9) = 0$

$M_3 = +39.8\,kN{\cdot}m$ ◁



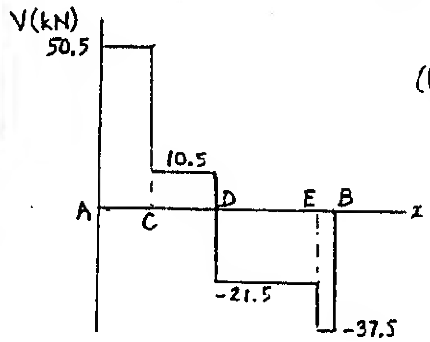
JUST TO THE RIGHT OF E:

$+\uparrow \Sigma F_y = 0: \; V_4 + 37.5 = 0 \quad V_4 = -37.5\,kN$ ◁

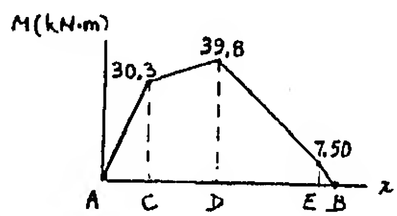
$+\circlearrowleft \Sigma M_4 = 0: \; -M_4 + (37.5)(0.2) = 0$

$M_4 = +7.50\,kN{\cdot}m$ ◁

AT B: $V_B = M_B = 0$ ◁



(b) $|V|_{max} = 50.5\,kN$ ◀



$|M|_{max} = 39.8\,kN{\cdot}m$ ◀

ADING SHOW
ND M DIAGR
E. $|V|_{max}$

TIRE BEAM

$(0\,kN)(0.6\,m)$
$)-(16\,kN)(3$
$7.5\,kN$
$5\,kN\uparrow$
$=0$
$=+50.5\,kN$
$5\,kN\uparrow$

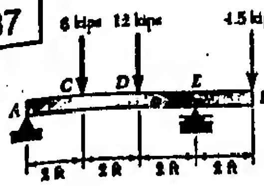
$10.5\,kN$
$=0$
$N\cdot m$
$E\ D$:
$-V_3=0$
$21.5\,kN$

$3\ kN\cdot m$
E:
$37.5\,kN$
$)\ kN\cdot m$

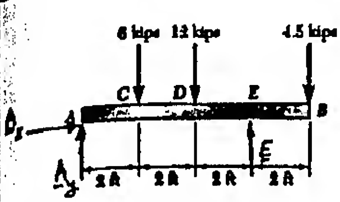
$0.5\,kN$

$kN\cdot m$

7.37



GIVEN:
BEAM AND LOADING SHOWN
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.



FREE BODY: ENTIRE BEAM

$+\circlearrowleft \Sigma M_A = 0:\ E(6\,ft) - (6\,kips)(2\,ft) - (12\,kips)(4\,ft) - (4.5\,kips)(8\,ft) = 0$

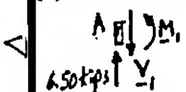
$E = +16\,kips \qquad \underline{E} = 16\,kips\uparrow$ ◁

$\xrightarrow{+} \Sigma F_x = 0:\ A_x = 0$

$+\uparrow \Sigma F_y = 0:\ A_y + 16\,kips - 6\,kips - 12\,kips - 4.5\,kips = 0$

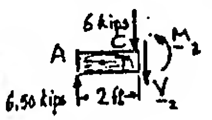
$A_y = +6.50\,kips \qquad \underline{A} = 6.50\,kips\uparrow$ ◁

(a) SHEAR AND BENDING MOMENT



JUST TO THE RIGHT OF A:

$V_1 = +6.50\,kips,\ M_1 = 0$ ◁



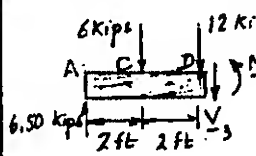
JUST TO THE RIGHT OF C:

$+\uparrow \Sigma F_y = 0:\ 6.50\,kips - 6\,kips - V_2 = 0$

$V_2 = +0.50\,kips$ ◁

$+\circlearrowleft \Sigma M_2 = 0:$

$M_2 - (6.50\,kips)(2\,ft) = 0,\ M_2 = +13\,kip\cdot ft$ ◁



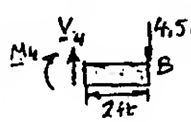
JUST TO THE RIGHT OF D:

$+\uparrow \Sigma F_y = 0:\ 6.50 - 6 - 12 - V_3 = 0$

$V_3 = +11.5\,kips$ ◁

$+\circlearrowleft \Sigma M_3 = 0:$

$M_3 - (6.50)(4) - (6)(2) = 0,\ M_3 = +14\,kip\cdot ft$ ◁

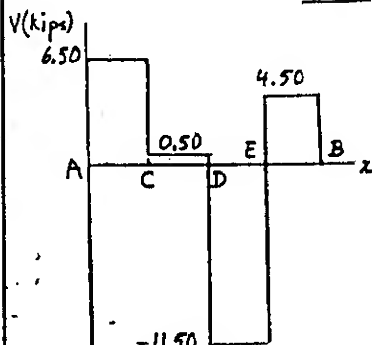


JUST TO THE RIGHT OF E:

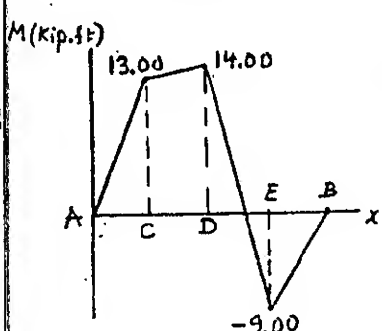
$+\uparrow \Sigma F_y = 0:\ V_4 - 4.5 = 0 \quad V_4 = +4.5\,kips$ ◁

$+\circlearrowleft \Sigma M_4 = 0:\ -M_4 - (4.5)2 = 0 \quad M_4 = -9\,kip\cdot ft$ ◁

AT B: $V_B = M_B = 0$ ◁

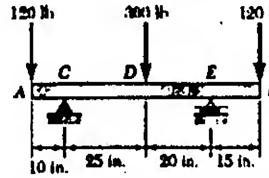


(b)
$|V|_{max} = 11.50\,kips$ ◀



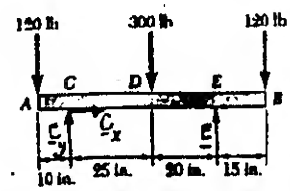
$|M|_{max} = 14.00\,kip\cdot ft$ ◀

7.38



GIVEN:
BEAM AND LOADING SHOWN
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

FREE BODY: ENTIRE BEAM



$+\circlearrowleft \Sigma M_C = 0:\ (120\,lb)(10\,in.) - (300\,lb)(25\,in.) + E(45\,in.) - (120\,lb)(60\,in.) = 0$

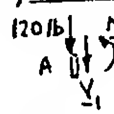
$E = +300\,lb \qquad \underline{E} = 300\,lb\uparrow$ ◁

$\Sigma F_x = 0:\ C_x = 0$

$+\uparrow \Sigma F_y = 0:\ C_y + 300\,lb - 120\,lb - 300\,lb - 120\,lb = 0 \qquad C_y = +240\,lb$

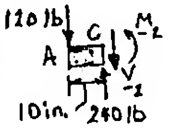
$\underline{C} = 240\,lb\uparrow$ ◁

(a) SHEAR AND BENDING MOMENT



JUST TO THE RIGHT OF A:

$+\uparrow \Sigma F_y = 0:\ -120\,lb - V_1 = 0,\ V_1 = -120\,lb,\ M_1 = 0$ ◁

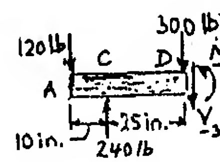


JUST TO THE RIGHT OF C:

$+\uparrow \Sigma F_y = 0:\ 240\,lb - 120\,lb - V_2 = 0,\ V_2 = +120\,lb$ ◁

$+\circlearrowleft \Sigma M_C = 0:\ M_2 + (120\,lb)(10\,in.) = 0$

$M_2 = -1200\,lb\cdot in.$ ◁



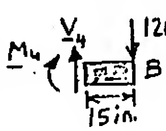
JUST TO THE RIGHT OF D:

$+\uparrow \Sigma F_y = 0:\ 240 - 120 - 300 - V_3 = 0$

$V_3 = -180\,lb$ ◁

$+\circlearrowleft \Sigma M_3 = 0:$

$M_3 + (120)(35) - (240)(25) = 0,\ M_3 = +1800\,lb\cdot in.$ ◁



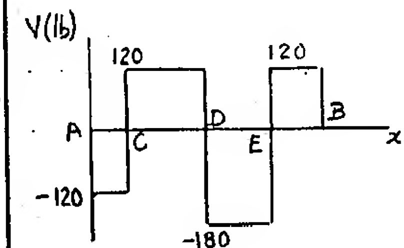
JUST TO THE RIGHT OF E:

$+\Sigma F_y = 0:\ V_4 - 120\,lb = 0 \qquad V_4 = +120\,lb$ ◁

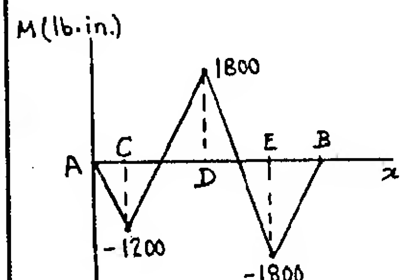
$+\circlearrowleft \Sigma M_4 = 0:\ -M_4 - (120\,lb)(15\,in.) = 0$

$M_4 = -1800\,lb\cdot in.$ ◁

AT B: $V_B = M_B = 0$ ◁

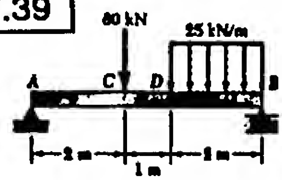


(b)
$|V|_{max} = 180.0\,lb$ ◀

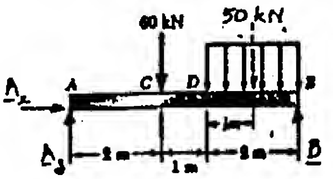


$|M|_{max} = 1800\,lb\cdot in.$ ◀

7.39



GIVEN: BEAM AND LOADING SHOWN.
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.



FREE BODY: ENTIRE BEAM

$+\circlearrowleft \Sigma M_A = 0$:

$B(5\,m) - (60\,kN)(2\,m) - (50\,kN)(4\,m) = 0$

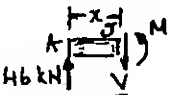
$B = +64.0\,kN,\ B = 64.0\,kN\uparrow$ ◁

$\Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y + 64.0\,kN - 60\,kN - 50\,kN = 0$, $A_y = +46.0\,kN$

$A = 46.0\,kN\uparrow$ ◁

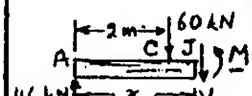
(a) SHEAR AND BENDING-MOMENT DIAGRAMS.



FROM A TO C:

$+\uparrow \Sigma F_y = 0$: $46 - V = 0$ $\quad V = +46\,kN$ ◁

$+\circlearrowleft \Sigma M_J = 0$: $M - 46x = 0$, $M = (46x)\,kN\cdot m$ ◁



FROM C TO D:

$+\uparrow \Sigma F_y = 0$: $46 - 60 - V = 0$ $\quad V = -14\,kN$ ◁

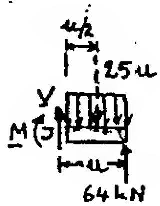
$+\circlearrowleft \Sigma M_J = 0$:

$M - 46x + 60(x-2) = 0$

$M = (120 - 14x)\,kN\cdot m$

FOR $x = 2\,m$: $M_C = +92.0\,kN\cdot m$ ◁

FOR $x = 3\,m$: $M_D = +78.0\,kN\cdot m$ ◁



FROM D TO B:

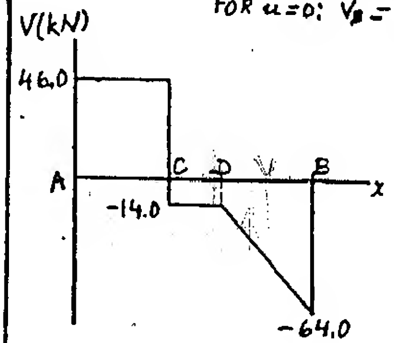
$+\uparrow \Sigma F_y = 0$: $V + 64 - 25u = 0$

$V = (25u - 64)\,kN$

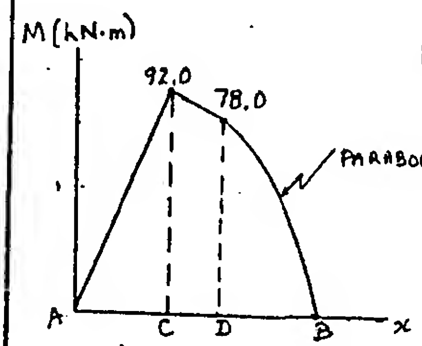
$+\circlearrowleft \Sigma M_J = 0$: $64u - (25u)(\frac{u}{2}) - M = 0$

$M = (64u - 12.5u^2)\,kN\cdot m$

FOR $u = 0$: $V_B = -64\,kN$, $M_B = 0$ ◁

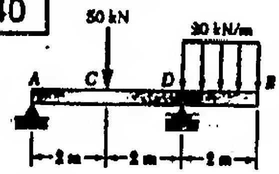


(b)
$|V|_{max} = 64.0\,kN$ ◀

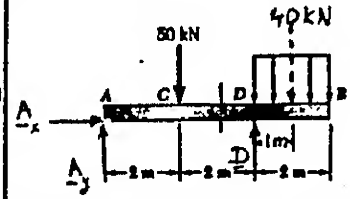


$|M|_{max} = 92.0\,kN\cdot m$ ◀

7.40



GIVEN: BEAM AND LOADING SHOWN.
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.



FREE BODY: ENTIRE BEAM

$+\circlearrowleft \Sigma M_A = 0$:

$D(4\,m) - (50\,kN)(2\,m) - (40\,kN)(5\,m) = 0$

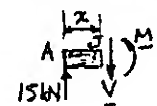
$D = +75.0\,kN,\ D = 75.0\,kN\uparrow$ ◁

$\Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y + 75.0\,kN - 50\,kN - 40\,kN = 0$, $A_y = +15.00\,kN$

$A = 15.00\,kN\uparrow$ ◁

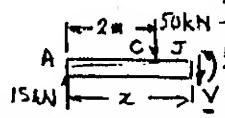
(a) SHEAR AND BENDING-MOMENT DIAGRAMS



FROM A TO C:

$+\uparrow \Sigma F_y = 0$: $15 - V = 0$ $\quad V = +15.00\,kN$, ◁

$+\circlearrowleft \Sigma M_J = 0$: $M - 15x = 0$, $M = (15x)\,kN\cdot m$ ◁



FROM C TO D:

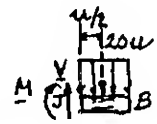
$+\uparrow \Sigma F_y = 0$: $15 - 50 - V = 0$

$V = -35.0\,kN$ ◁

$+\circlearrowleft \Sigma M_J = 0$: $M - 15x + 50(x-2) = 0$ $\quad M = (100 - 35x)\,kN\cdot m$

FOR $x = 2\,m$: $M_C = +30.0\,kN\cdot m$ ◁

FOR $x = 4\,m$: $M_D = -40.0\,kN\cdot m$ ◁



FROM D TO B:

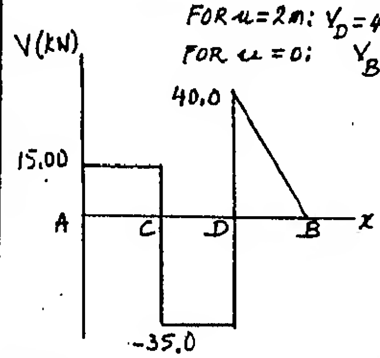
$+\uparrow \Sigma F_y = 0$: $V - 20u = 0$, $\quad V = (20u)\,kN$

$+\circlearrowleft \Sigma M_J = 0$: $-M - (20u)(\frac{u}{2}) = 0$

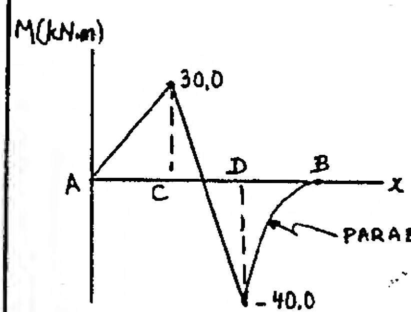
$M = (-10u^2)\,kN\cdot m$

FOR $u = 2\,m$: $V_D = 40\,kN$, $M_D = -40\,kN\cdot m$ ◁

FOR $u = 0$: $V_B = M_B = 0$ ◁



(b)
$|V|_{max} = 40.0\,kN$ ◀

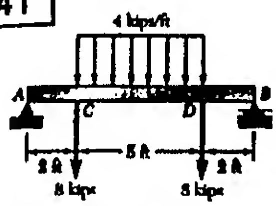


$|M|_{max} = 40.0\,kN\cdot m$ ◀

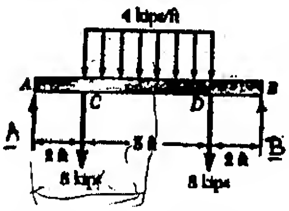
D LOADING SHOWN
V AND M DIAGRAMS
RMINE $|V|_{max}$ AND

Y: ENTIRE BEAM
O:
0kN)(2m) − (40kN)(5m) = 0
N, $D = 75.0$ kN ↑
$A_x = 0$
$A_y = +15.00$ kN
= 15.00 kN ↑
RAMS
= +15.00 kN
M = (15x) kN·m
−V = 0
V = −35.0 kN
(100−35x) kN·m
0 kN·m
0 kN·m
V = (20u) kN
kN·m
−40 kN·m
= 40.0 kN
0 kN·m

7.41



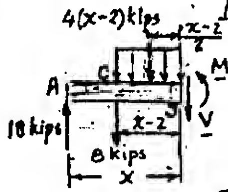
GIVEN:
BEAM AND LOADING SHOWN.
(a) DRAW V AND M DIAGRAMS.
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.



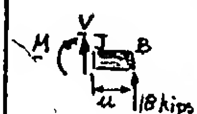
FREE BODY: ENTIRE BEAM
BECAUSE OF SYMMETRY OF LOADING:
$A = B = \frac{1}{2}$ (TOTAL LOAD)
$= \frac{1}{2}(8 + 8 + 4 \times 5)$ kips
$A = B = 18$ kips ↑ ◁

(a) SHEAR AND BENDING-MOMENT DIAGRAMS

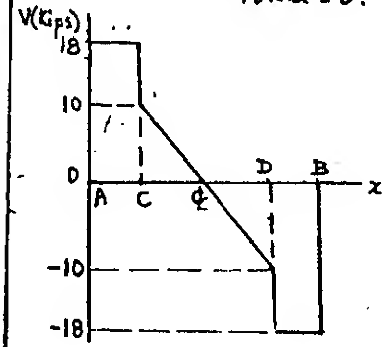
FROM A TO C:
$+\uparrow \Sigma F_y = 0:\ 18 \text{ kips} - V = 0 \qquad V = +18 \text{ kips}$ ◁
$+\circlearrowleft \Sigma M_J = 0:\ M - 18x = 0 \qquad M = +(18x)$ kip·ft ◁



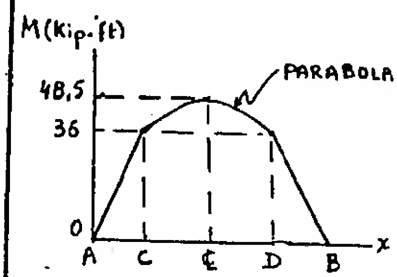
FROM C TO D:
$+\uparrow \Sigma F_y = 0:\ 18 - 8 - 4(x-2) - V = 0$
$V = 18 - 4x$
$+\circlearrowleft \Sigma M_J = 0:$
$M + 8(x-2) + 4(x-2)\frac{x-2}{2} - 18x = 0$
$M = 18x - 8(x-2) - 2(x-2)^2$
FOR $x = 2$: $V_C = +10$ kips, $M_C = +36$ kip·ft ◁
FOR $x = 4.5$: $V_{℄} = 0$, $M_{℄} = +48.5$ kip·ft ◁
FOR $x = 7$: $V_D = -10$ kips, $M_D = +36$ kip·ft ◁



FROM D TO B:
$+\uparrow \Sigma F_y = 0:\ V + 18 = 0 \qquad V = -18$ kips ◁
$+\circlearrowleft \Sigma M_J = 0:\ 18u - M = 0 \qquad M = (18u)$ kip·ft
FOR $u = 2$ ft: $M_D = +36$ kip·ft ◁
FOR $u = 0$: $M_B = 0$ ◁

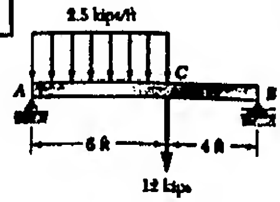


(b)
$|V|_{max} = 18.00$ kips ◀

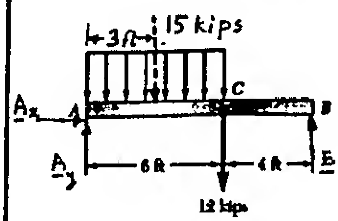


$|M|_{max} = 48.5$ kip·ft ◀

7.42

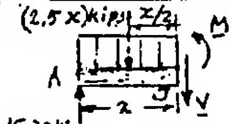


GIVEN:
BEAM AND LOADING SHOWN.
(a) DRAW V AND M DIAGRAMS.
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

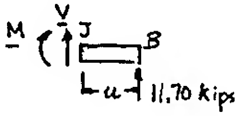


FREE BODY: ENTIRE BEAM
$+\circlearrowleft \Sigma M_A = 0:$
$B(10 \text{ ft}) - (15 \text{ kips})(3 \text{ ft}) - (12 \text{ kips})(6 \text{ ft}) = 0$
$B = +11.70$ kips, $B = 11.70$ kips
$\Sigma F_x = 0:\ A_x = 0$
$+\uparrow \Sigma F_y = 0:\ A_y - 15 - 12 + 11.70 = 0$
$A_y = +15.30$ kips, $A = 15.30$ kips

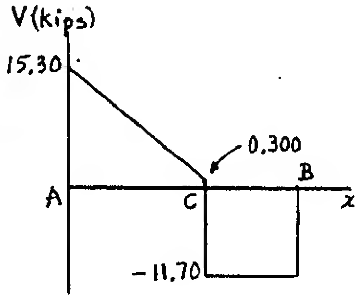
(a) SHEAR AND BENDING-MOMENT DIAGRAMS



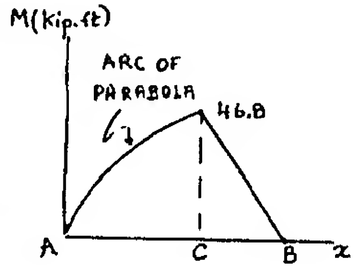
FROM A TO C:
$+\uparrow \Sigma F_y = 0:\ 15.30 - 2.5x - V = 0$
$V = (15.30 - 2.5x)$ kips
$+\circlearrowleft \Sigma M_J = 0:\ M + (2.5x)(\frac{x}{2}) - 15.30x = 0$
$M = 15.30x - 1.25x^2$
FOR $x = 0$: $V_A = +15.30$ kips, $M_A = 0$
FOR $x = 6$ ft: $V_C = +0.300$ kip, $M_C = +46.8$ kip·ft



FROM C TO B:
$+\uparrow \Sigma F_y = 0:\ V + 11.70 = 0$
$V = -11.70$ kips
$+\circlearrowleft \Sigma M_J = 0:\ 11.70u - M = 0$
$M = (11.70u)$ kip·ft
FOR $u = 4$ ft: $M_C = +46.8$ kip·ft
FOR $u = 0$: $M_B = 0$

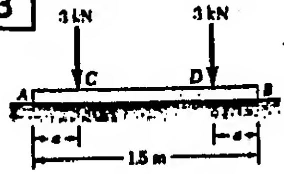


(b)
$|V|_{max} = 15.30$ kips

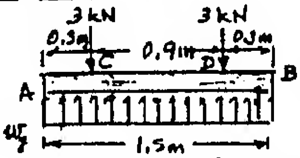


$|M|_{max} = 46.8$ kip·ft

7.43



GIVEN:
BEAM RESTING ON GROUND AND LOADED AS SHOWN ($a = 0.3$ m).
(a) DRAW V AND M DIAGRAMS.
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.



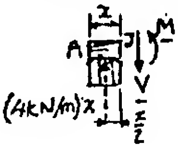
FREE BODY: ENTIRE BEAM

$+\uparrow \Sigma F_y = 0:$

$w_g(1.5\text{m}) - 3\text{kN} - 3\text{kN} = 0$

$w_g = 4\text{ kN/m}$ ◁

(a) SHEAR AND BENDING-MOMENT



FROM A TO C:

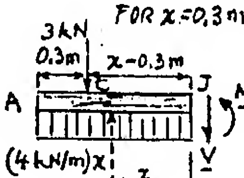
$+\uparrow \Sigma F_y = 0:\ 4x - V = 0 \qquad V = (4x)\text{ kN}$

$+\circlearrowleft \Sigma M_J = 0:\ M - (4x)\frac{x}{2} = 0.$

$M = (2x^2)\text{ kN·m}$

FOR $x = 0$: $V_A = M_A = 0$ ◁

FOR $x = 0.3$ m: $V_C = 1.2$ kN, $M_C = 0.18$ kN·m ◁



FROM C TO D:

$+\uparrow \Sigma F_y = 0:\ 4x - 3\text{kN} - V = 0$

$V = (4x - 3)\text{ kN}$

$+\circlearrowleft \Sigma M_J = 0:$

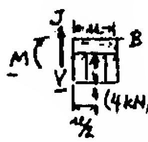
$M + (3\text{kN})(x - 0.3) - (4x)\frac{x}{2} = 0$

$M = (2x^2 - 3x + 0.9)\text{ kN·m}$

FOR $x = 0.3$ m: $V_C = -1.8$ kN, $M_C = +0.18$ kN·m ◁

FOR $x = 0.75$ m: $V_{℄} = 0$, $M_{℄} = -0.225$ kN·m ◁

FOR $x = 1.2$ m: $V_D = +1.8$ kN, $M_D = +0.18$ kN·m ◁



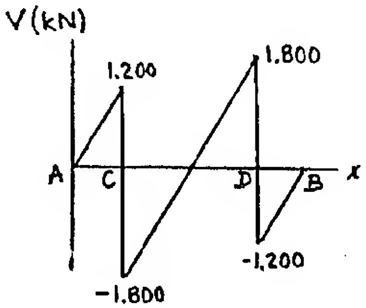
FROM D TO B:

$+\uparrow \Sigma F_y = 0:\ V + 4u = 0 \qquad V = -(4u)\text{ kN}$

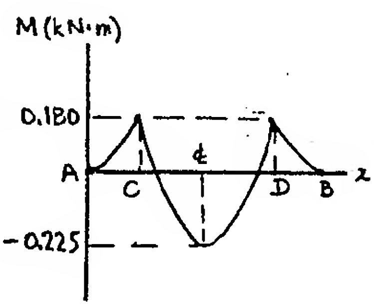
$+\circlearrowleft \Sigma M_J = 0:\ (4u)\frac{u}{2} - M = 0, \quad M = 2u^2$

FOR $u = 0$: $V_B = M_B = 0$ ◁

FOR $u = 0.3$ m: $V_D = -1.2$ kN, $M_D = +0.18$ kN·m ◁

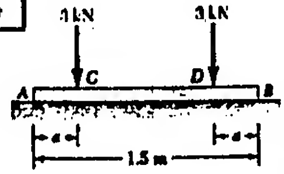


(b) $|V|_{max} = 1.800$ kN ◀

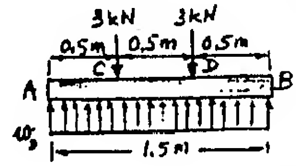


$|M|_{max} = 0.225$ kN·m ◀

7.44



GIVEN:
BEAM RESTING ON GROUND AND LOADED AS SHOWN ($a = 0.5$ m).
(a) DRAW V AND M DIAGRAMS.
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.



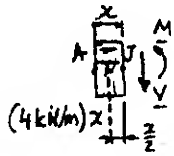
FREE BODY: ENTIRE BEAM

$+\uparrow \Sigma F_y = 0:$

$w_g(1.5\text{m}) - 3\text{kN} - 3\text{kN} = 0$

$w_g = 4\text{ kN/m}$ ◁

(a) SHEAR AND BENDING MOMENT



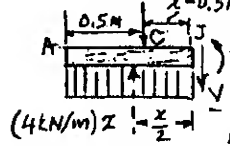
FROM A TO C:

$+\uparrow \Sigma F_y = 0:\ 4x - V = 0 \qquad V = (4x)\text{ kN}$

$+\circlearrowleft \Sigma M_J = 0:\ M - (4x)\frac{x}{2} = 0, \quad M = (2x^2)\text{ kN·m}$

FOR $x = 0$: $V_A = M_A = 0$ ◁

FOR $x = 0.5$ m: $V_C = 2$ kN, $M_C = 0.500$ kN·m ◁



FROM C TO D:

$+\uparrow \Sigma F_y = 0:\ 4x - 3\text{kN} - V = 0$

$V = (4x - 3)\text{ kN}$

$+\circlearrowleft \Sigma M_J = 0:$

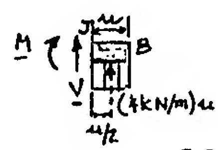
$M + (3\text{kN})(x - 0.5) - (4x)\frac{x}{2} = 0$

$M = (2x^2 - 3x + 1.5)\text{ kN·m}$

FOR $x = 0.5$ m: $V_C = -1.00$ kN, $M_C = 0.500$ kN·m ◁

FOR $x = 0.75$ m: $V_{℄} = 0$, $M_{℄} = 0.375$ kN·m ◁

FOR $x = 1.0$ m: $V_C = 1.00$ kN, $M_C = 0.500$ kN·m ◁



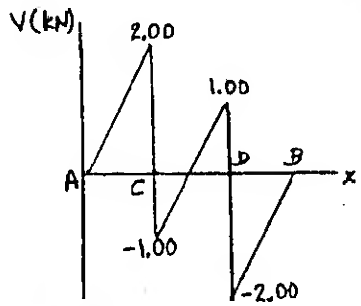
FROM D TO B:

$+\uparrow \Sigma F_y = 0:\ V + 4u = 0 \qquad V = -(4u)\text{ kN}$

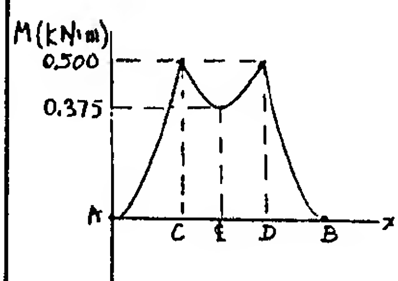
$+\circlearrowleft \Sigma M_J = 0:\ (4u)\frac{u}{2} - M = 0, \quad M = 2u^2$

FOR $u = 0$: $V_B = M_B = 0$ ◁

FOR $u = 0.5$ m: $V_D = -2$ kN, $M_D = 0.500$ kN·m ◁

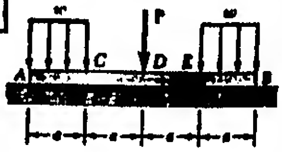


(b) $|V|_{max} = 2.00$ kN ◀



$|M|_{max} = 0.500$ kN·m ◀

7.47

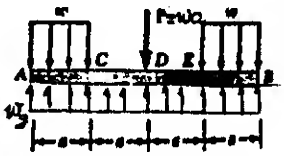


GIVEN: BEAM AND LOADING SHOWN WITH $P = wa$.

(a) DRAW V AND M DIAGRAMS.

(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$

FREE BODY: ENTIRE BEAM

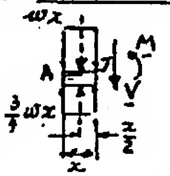


$+\uparrow \Sigma F_y = 0:$

$w_g(4a) - 2wa - wa = 0$

$w_g = \frac{3}{4}w$ ◁

(a) SHEAR AND BENDING-MOMENT DIAGRAMS



FROM A TO C:

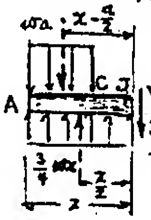
$+\uparrow \Sigma F_y = 0: \ \frac{3}{4}wx - wx - V = 0, \quad V = -\frac{1}{4}wx$

$+\circlearrowleft \Sigma M_J = 0: \ M + (wx)\frac{x}{2} - (\frac{3}{4}wx)\frac{x}{2} = 0$

$M = -\frac{1}{8}wx^2$

FOR $x = 0$: $V_A = M_A = 0$ ◁

FOR $x = a$: $V_C = -\frac{1}{4}wa, \quad M_C = -\frac{1}{8}wa^2$ ◁



FROM C TO D:

$+\uparrow \Sigma F_y = 0: \ \frac{3}{4}wx - wa - V = 0$

$V = (\frac{3}{4}x - a)w$

$+\circlearrowleft \Sigma M_J = 0:$

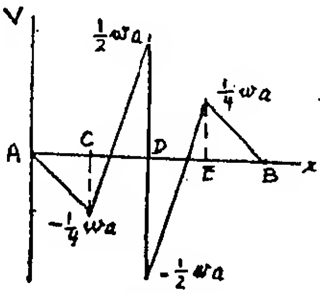
$M + wa(x - \frac{a}{2}) - \frac{3}{4}wx(\frac{x}{2}) = 0$

$M = \frac{3}{8}wx^2 - wa(x - \frac{a}{2}) \quad (1)$

FOR $x = a$: $V_C = -\frac{1}{4}wa, \ M_C = -\frac{1}{8}wa^2$ ◁

FOR $x = 2a$: $V_D = +\frac{1}{2}wa, \ M_D = 0$ ◁

BECAUSE OF THE SYMMETRY OF THE LOADING, WE CAN DEDUCE THE VALUES OF V AND M FOR THE RIGHT-HAND HALF OF THE BEAM FROM THE VALUES OBTAINED FOR ITS LEFT-HAND HALF.

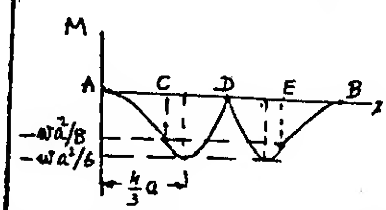


(b) $|V|_{max} = \frac{1}{2}wa$ ◀

TO FIND $|M|_{max}$, WE DIFFERENTIATE EQ.(1) AND SET $\frac{dM}{dx} = 0$:

$\frac{dM}{dx} = \frac{3}{4}wx - wa = 0, \quad x = \frac{4}{3}a, \quad M = \frac{3}{8}w(\frac{4}{3}a)^2 - wa^2(\frac{4}{3} - \frac{1}{2}) = -\frac{wa^2}{6}$

$|M|_{max} = \frac{1}{6}wa^2$ ◀



B.M. DIAGRAM CONSISTS OF FOUR DISTINCT ARCS OF PARABOLA.

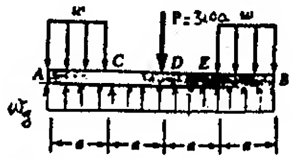
7.48

GIVEN: BEAM AND LOADING SHOWN WITH $P = 3wa$.

(a) DRAW V AND M DIAGRAMS

(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$

FREE BODY: ENTIRE BEAM

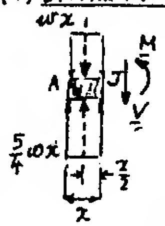


$+\uparrow \Sigma F_y = 0:$

$w_g(4a) - 2wa - 3wa = 0$

$w_g = \frac{5}{4}w$ ◁

(a) SHEAR AND BENDING-MOMENT DIAGRAMS



FROM A TO C:

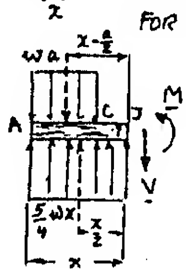
$+\uparrow \Sigma F_y = 0: \ \frac{5}{4}wx - wx - V = 0, \quad V = +\frac{1}{4}wx$

$+\circlearrowleft \Sigma M_J = 0: \ M + (wx)\frac{x}{2} - (\frac{5}{4}wx)\frac{x}{2} = 0$

$M = +\frac{1}{8}wx^2$

FOR $x = 0$: $V_A = M_A = 0$ ◁

FOR $x = a$: $V_C = +\frac{1}{4}wa, \ M_C = +\frac{1}{8}wa^2$ ◁



FROM C TO D:

$+\uparrow \Sigma F_y = 0: \ \frac{5}{4}wx - wa - V = 0$

$V = (\frac{5}{4}x - a)w$

$+\circlearrowleft \Sigma M_J = 0:$

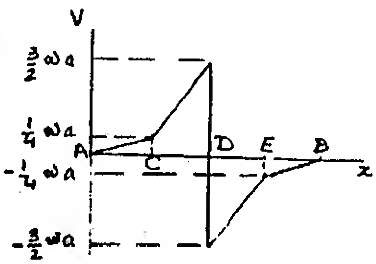
$M + wa(x - \frac{a}{2}) - \frac{5}{4}wx(\frac{x}{2}) = 0$

$M = \frac{5}{8}wx^2 - wa(x - \frac{a}{2}) \quad (1)$

FOR $x = a$: $V_C = +\frac{1}{4}wa, \ M_C = +\frac{1}{8}wa^2$ ◁

FOR $x = 2a$: $V_D = +\frac{3}{2}wa, \ M_D = +wa^2$ ◁

BECAUSE OF THE SYMMETRY OF THE LOADING, WE CAN DEDUCE THE VALUES OF V AND M FOR THE RIGHT-HAND HALF OF THE BEAM FROM THE VALUES OBTAINED FOR ITS LEFT-HAND HALF.



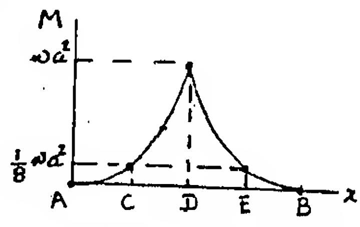
(b) $|V|_{max} = \frac{3}{2}wa$ ◀

TO FIND $|M|_{max}$, WE DIFFERENTIATE EQ.(1) AND SET $\frac{dM}{dx} = 0$:

$\frac{dM}{dx} = \frac{5}{4}wx - wa = 0, \quad x = \frac{4}{5}a < a$ (OUTSIDE PORTION CD)

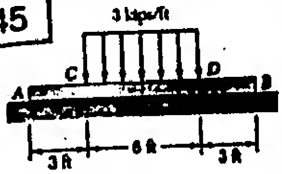
THE MAX. VALUE OF $|M|$ OCCURS AT D:

$|M|_{max} = wa^2$ ◀



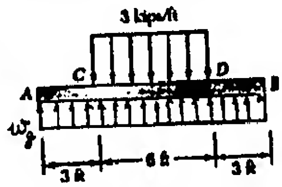
B.M. DIAGRAM CONSISTS OF FOUR DISTINCT ARCS OF PARABOLA.

7.45



GIVEN: BEAM RESTING ON GROUND AND LOADED AS SHOWN.
(a) DRAW V AND M DIAGRAMS.
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

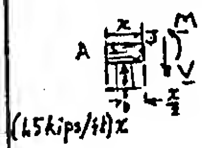
FREE BODY: ENTIRE BEAM



$+\uparrow \Sigma F_y = 0$:
$w_g(12\text{ ft}) - (3\text{ kips/ft})(6\text{ ft}) = 0$
$w_g = 1.5\text{ kips/ft}$ ◁

(a) SHEAR AND BM DIAGRAMS

FROM A TO C:



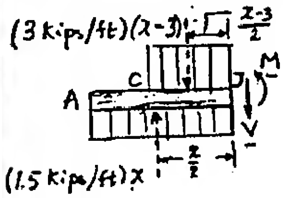
$+\uparrow \Sigma F_y = 0$: $1.5x - V = 0$ $\quad V = (1.5x)\text{ kips}$

$+\circlearrowleft \Sigma M_J = 0$: $M - (1.5x)\frac{x}{2}$ $\quad M = (0.75x^2)\text{ kip}\cdot\text{ft}$ ◁

FOR $x = 0$: $V_A = M_A = 0$

FOR $x = 3$ ft: $V_C = 4.5$ kips, $M_C = 6.75$ kip·ft ◁

FROM C TO D:

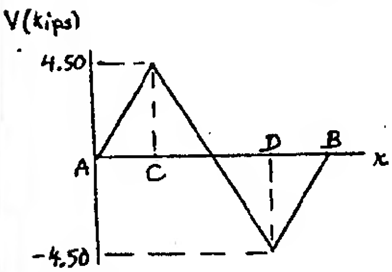


$+\uparrow \Sigma F_y = 0$: $1.5x - 3(x-3) - V = 0$
$V = (9 - 1.5x)\text{ kips}$

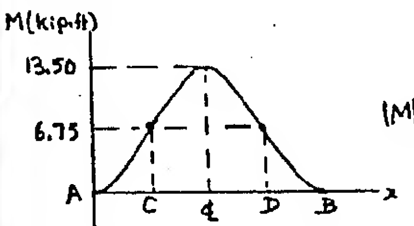
$+\circlearrowleft \Sigma M_J = 0$:
$M + 3(x-3)\frac{x-3}{2} - (1.5x)\frac{x}{2} = 0$
$M = [0.75x^2 - 1.5(x-3)^2]\text{ kip}\cdot\text{ft}$

FOR $x = 3$ ft: $V_C = 4.5$ kips, $M_C = 6.75$ kip·ft ◁
FOR $x = 6$ ft: $V_{℄} = 0$, $M_{℄} = 13.50$ kip·ft ◁
FOR $x = 9$ ft: $V_D = -4.5$ kips, $M_D = 6.75$ kip·ft ◁

AT B: $V_B = M_B = 0$ ◁



(b) $|V|_{max} = 4.50$ kips ◀

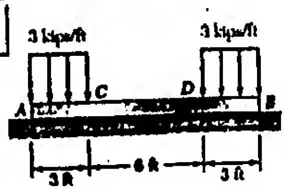


$|M|_{max} = 13.50$ kip·ft ◀

B.M. DIAGRAM CONSISTS OF THREE DISTINCT ARCS OF PARABOLA, ALL LOCATED ABOVE THE x AXIS.

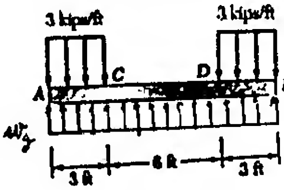
THUS: $M \geq 0$ EVERYWHERE ◀

7.46



GIVEN: BEAM RESTING ON GROUND AND LOADED AS SHOWN.
(a) DRAW V AND M DIAGRAMS.
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

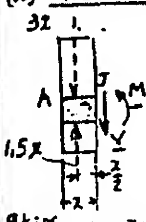
FREE BODY: ENTIRE BEAM



$+\uparrow \Sigma F_y = 0$:
$w_g(12\text{ ft}) - (3\text{ kips/ft})(6\text{ ft}) = 0$
$w_g = 1.5\text{ kips/ft}$ ◁

(a) SHEAR AND BENDING-MOMENT DIAGRAMS.

FROM A TO C:



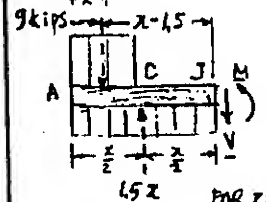
$+\uparrow \Sigma F_y = 0$: $1.5x - 3x - V = 0$ $\quad V = (-1.5x)\text{ kips}$

$+\circlearrowleft \Sigma M_J = 0$: $M + (3x)\frac{x}{2} - (1.5x)\frac{x}{2} = 0$
$M = (-0.75x^2)\text{ kip}\cdot\text{ft}$ ◁

FOR $x = 0$: $V_A = M_A = 0$ ◁

FOR $x = 3$ ft: $V_C = -4.5$ kips, $M_C = -6.75$ kip·ft ◁

FROM C TO D:

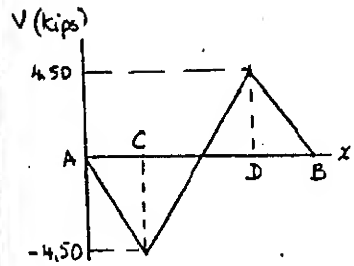


$+\uparrow \Sigma F_y = 0$: $1.5x - 9 - V = 0$, $\quad V = (1.5x - 9)\text{ kips}$

$+\circlearrowleft \Sigma M_J = 0$:
$M + 9(x - 1.5) - (1.5x)\frac{x}{2} = 0$
$M = 0.75x^2 - 9x + 13.5$

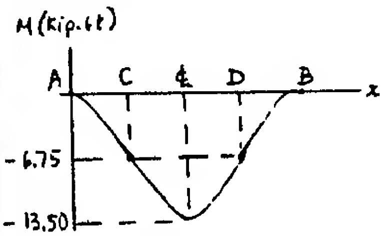
FOR $x = 3$ ft: $V_C = -4.5$ kips, $M_C = -6.75$ kip·ft ◁
FOR $x = 6$ ft: $V_{℄} = 0$, $M_{℄} = -13.50$ kip·ft ◁
FOR $x = 9$ ft: $V_D = 4.5$ kips, $M_D = -6.75$ kip·ft ◁

AT B: $V_B = M_B = 0$ ◁



(b) $|V_{max}| = 4.50$ kips ◀

B.M. DIAGRAM CONSISTS OF THREE DISTINCT ARCS OF PARABOLA.



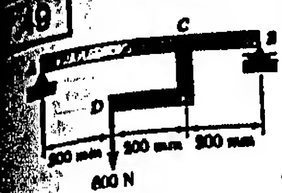
$|M|_{max} = 13.50$ kip·ft ◀

SINCE ENTIRE DIAGRAM IS BELOW THE x AXIS:

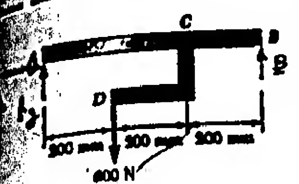
$M \leq 0$ EVERYWHERE ◀

7.49

GIVEN:
BEAM AND LOADING SHOWN.
DRAW V AND M DIAGRAMS AND DETERMINE V AND M
(a) JUST TO THE LEFT OF C.
(b) JUST TO THE RIGHT OF C



FREE BODY: ENTIRE BEAM



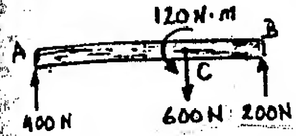
$+\circlearrowleft \Sigma M_A = 0$: $B(0.6\,m) - (600\,N)(0.2\,m) = 0$

$B = +200\,N$ $\quad B = 200\,N\uparrow$ ◁

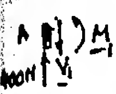
$\Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y - 600\,N + 200\,N = 0$

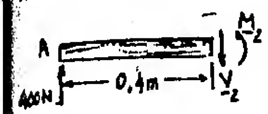
$A_y = +400\,N$ $\quad A = 400\,N\uparrow$ ◁



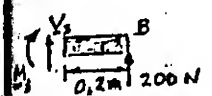
WE REPLACE THE 600-N LOAD BY AN EQUIVALENT FORCE-COUPLE SYSTEM AT C



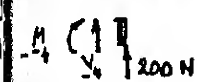
JUST TO THE RIGHT OF A:
$V_1 = +400\,N, \; M_1 = 0$ ◁



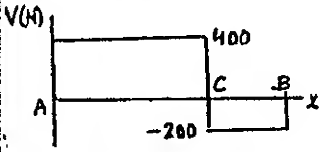
(a) JUST TO THE LEFT OF C:
$V_2 = +400\,N$ ◀
$M_2 = (400\,N)(0.4\,m)$ $\quad M_2 = +160.0\,N\cdot m$ ◀

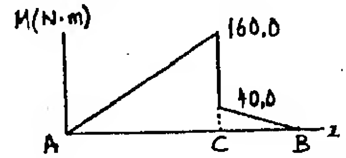


(b) JUST TO THE RIGHT OF C:
$V_3 = -200\,N$ ◀
$M_3 = (200\,N)(0.2\,m)$ $\quad M_3 = +40.0\,N\cdot m$ ◀

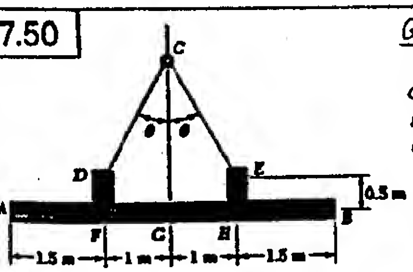


JUST TO THE LEFT OF B:
$V_4 = -200\,N, \; M_4 = 0$ ◁



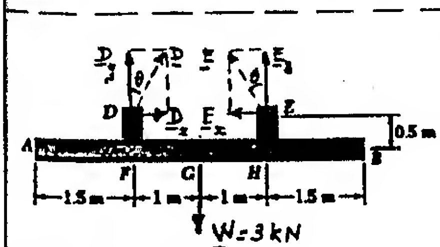


7.50



GIVEN:
STRUCTURAL MEMBER CONSISTING OF 3-kN BEAM AB AND TWO CHANNELS OF NEGLIGIBLE WEIGHT IS LIFTED WITH $\theta = 30°$.
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

FREE BODY: BEAM AND CHANNELS



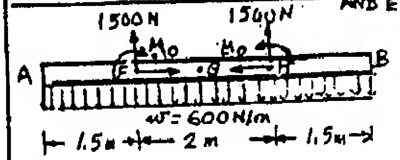
FROM SYMMETRY:
$E_y = D_y$
THUS:
$E_x = D_x = D_y \tan\theta$ (1)

$+\uparrow \Sigma F_y = 0$: $D_y + E_y - 3\,kN = 0$ $\qquad D_y = E_y = 1.5\,kN\uparrow$ ◁

FROM (1): $D_x = (1.5\,kN)\tan\theta \rightarrow$, $\quad E_x = (1.5\,kN)\tan\theta \leftarrow$ ◁

(CONTINUED)

7.50 CONTINUED

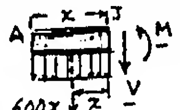


WE REPLACE THE FORCES AT D AND E BY EQUIVALENT FORCE-COUPLE SYSTEMS AT F AND H, WHERE

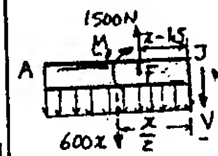
$M_0 = (1.5\,kN\tan\theta)(0.5\,m)$
$= (750\,N\cdot m)\tan\theta$ (2)

WE NOTE THAT THE WEIGHT OF THE BEAM PER UNIT LENGTH IS
$$w = \frac{W}{L} = \frac{3\,kN}{5\,m} = 0.6\,kN/m = 600\,N/m$$

(a) SHEAR AND BENDING-MOMENT DIAGRAMS



FROM A TO F:
$+\uparrow \Sigma F_y = 0$: $-V - 600x = 0$ $\quad V = (-600x)\,N$
$+\circlearrowleft \Sigma M_J = 0$: $M + (600x)\frac{x}{2} = 0$, $\quad M = (-300x^2)\,N\cdot m$
FOR $x = 0$: $V_A = M_A = 0$ ◁
FOR $x = 1.5\,m$: $V_F = -900\,N$, $M_F = -675\,N\cdot m$ ◁

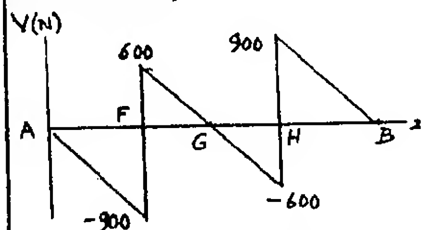


FROM F TO H:
$+\uparrow \Sigma F_y = 0$: $1500 - 600x - V = 0$
$V = (1500 - 600x)\,N$
$+\circlearrowleft \Sigma M_J = 0$:
$M + (600x)\frac{x}{2} - 1500(x - 1.5) - M_0 = 0$
$M = M_0 - 300x^2 + 1500(x - 1.5)$ N·m

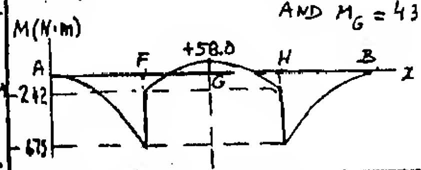
FOR $x = 1.5\,m$: $V_F = +600\,N$, $\quad M_F = (M_0 - 675)\,N\cdot m$ ◁
FOR $x = 2.5\,m$: $V_G = 0$, $\quad M_G = (M_0 - 375)\,N\cdot m$ ◁

FROM G TO B, THE V AND M DIAGRAMS WILL BE OBTAINED BY SYMMETRY.

(b) $|V|_{max} = 900\,N$ ◀



RECALLING THAT $\theta = 30°$, EQ.(2) YIELDS $M_0 = 433\,N\cdot m$
THUS, JUST TO THE RIGHT OF F: $M = 433 - 675 = -242\,N\cdot m$ ◁
AND $M_G = 433 - 375 = +58.0\,N\cdot m$ ◁

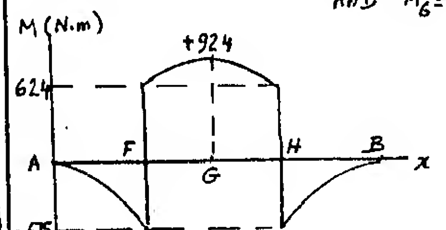


$|M|_{max} = 675\,N\cdot m$ ◀

7.51

SOLVE PROB. 7.50 WHEN $\theta = 60°$.

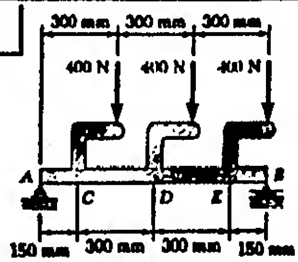
SEE SOLUTION OF PROB. 7.50 UP TO DASHED LINE (INCLUDING SHEAR DIAGRAM).
MAKING $\theta = 60°$ IN EQ.(2): $M_0 = 750\tan 60° = 1299\,N\cdot m$
THUS, JUST TO THE RIGHT OF F: $M = 1299 - 675 = 624\,N\cdot m$ ◁
AND $M_G = 1299 - 375 = 924\,N\cdot m$ ◁



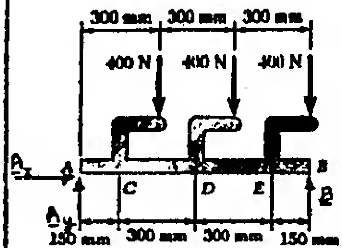
(b) $|V|_{max} = 900\,N$ ◀

$|M|_{max} = 924\,N\cdot m$ ◀

7.52



GIVEN: BEAM AND LOADING SHOWN. DRAW V AND M DIAGRAMS. DETERMINE $|V|_{max}$ AND $|M|_{max}$.



FREE BODY: ENTIRE BEAM

$+\circlearrowleft \Sigma M_A = 0:$

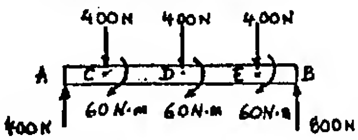
$B(0.9\,m) - (400\,N)(0.3\,m) - (400\,N)(0.6\,m) - (400\,N)(0.9\,m) = 0$

$B = +800\,N \qquad \underline{B} = 800\,N\uparrow$ ◁

$\Sigma F_x = 0: \quad A_x = 0$

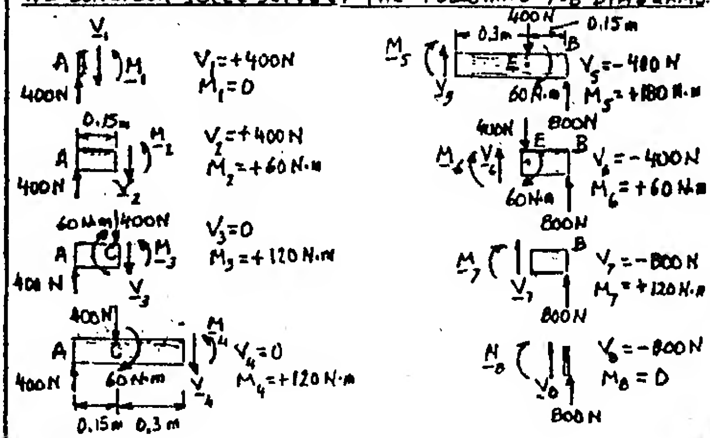
$+\uparrow \Sigma F_y = 0: \; A_y + 800\,N - 3(400\,N) = 0$

$A_y = +400\,N \qquad \underline{A} = 400\,N\uparrow$ ◁



WE REPLACE THE LOADS BY EQUIVALENT FORCE-COUPLE SYSTEMS AT C, D, AND E.

WE CONSIDER SUCCESSIVELY THE FOLLOWING F-B DIAGRAMS.



$V_1 = +400\,N$, $M_1 = 0$

$V_2 = +400\,N$, $M_2 = +60\,N\cdot m$

$V_3 = 0$, $M_3 = +120\,N\cdot m$

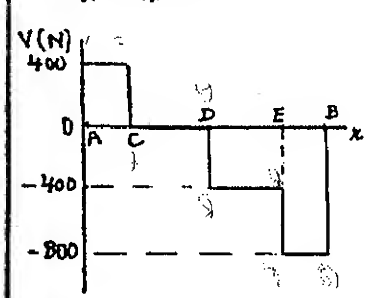
$V_4 = 0$, $M_4 = +120\,N\cdot m$

$V_5 = -400\,N$, $M_5 = +180\,N\cdot m$

$V_6 = -400\,N$, $M_6 = +60\,N\cdot m$

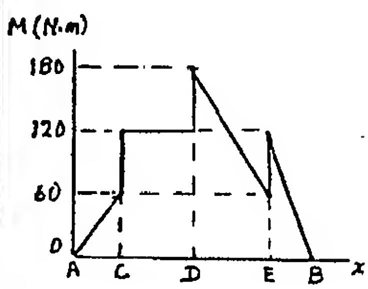
$V_7 = -800\,N$, $M_7 = +120\,N\cdot m$

$V_8 = -800\,N$, $M_8 = 0$



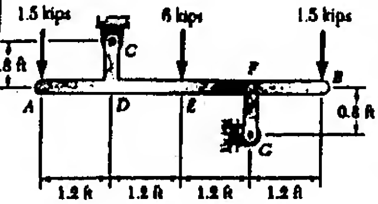
(b)

$|V|_{max} = 800\,N$ ◀

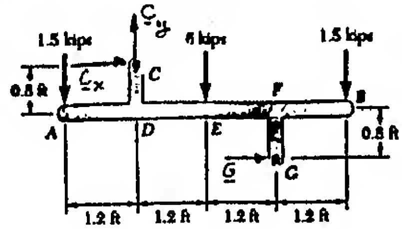


$|M|_{max} = 180.0\,N\cdot m$ ◀

7.53



GIVEN: BEAM AND LOADING SHOWN. DRAW V AND M DIAGRAMS. DETERMINE $|V|_{max}$ AND $|M|_{max}$.



FREE BODY: ENTIRE BEAM

$+\circlearrowleft \Sigma M_C = 0: (1.5\,kips)(1.2\,ft) - (6\,kips)(1.2\,ft) + G(1.6\,ft) - (1.5\,kips)(3.6\,ft) = 0$

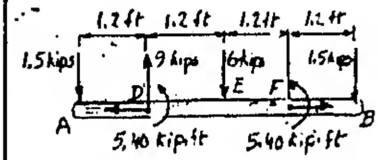
$\underline{G} = 6.75\,kips \rightarrow$ ◁

$\overset{+}{\rightarrow} \Sigma F_x = 0: C_x + 6.75\,kips = 0$

$\underline{C}_x = 6.75\,kips \leftarrow$ ◁

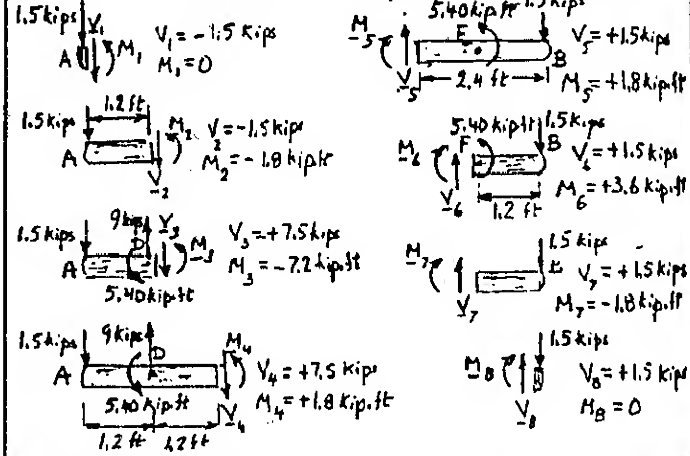
$+\uparrow \Sigma F_y = 0: C_y - 1.5 - 6 - 1.5 = 0$

$\underline{C}_y = 9.00\,kips\uparrow$ ◁



WE REPLACE THE REACTIONS BY EQUIVALENT FORCE-COUPLE SYSTEMS AT D AND F.

WE CONSIDER SUCCESSIVELY THE FOLLOWING F-B DIAGRAMS:



$V_1 = -1.5\,kips$, $M_1 = 0$

$V_2 = -1.5\,kips$, $M_2 = -1.8\,kip\cdot ft$

$V_3 = +7.5\,kips$, $M_3 = -7.2\,kip\cdot ft$

$V_4 = +7.5\,kips$, $M_4 = +1.8\,kip\cdot ft$

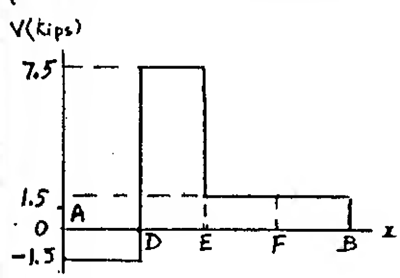
$V_5 = +1.5\,kips$, $M_5 = +1.8\,kip\cdot ft$

$V_6 = +1.5\,kips$, $M_6 = +3.6\,kip\cdot ft$

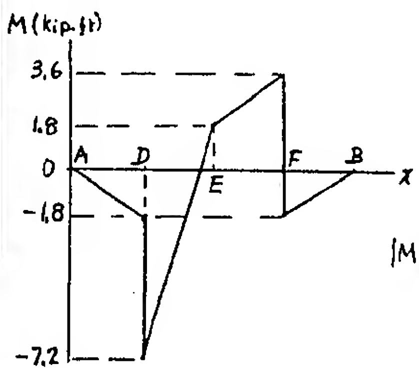
$V_7 = +1.5\,kips$, $M_7 = -1.8\,kip\cdot ft$

$V_8 = +1.5\,kips$, $M_8 = 0$

(AXIAL FORCES HAVE BEEN OMITTED FROM F-B DIAGRAMS)



$|V|_{max} = 7.50\,kips$ ◀



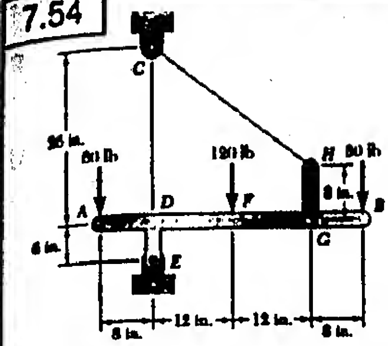
$|M|_{max} = 7.20\,kip\cdot ft$ ◀

7.54

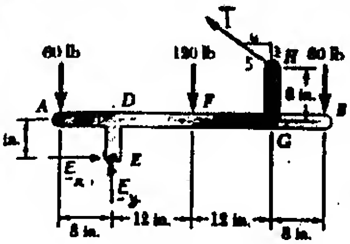
GIVEN: BEAM AND LOADING SHOWN

DRAW V AND M DIAGRAMS

DETERMINE $|V|_{max}$ AND $|M|_{max}$.



FREE BODY: ENTIRE BEAM



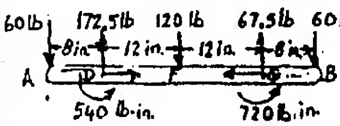
$+\circlearrowleft \Sigma M_E = 0$: $(60\text{ lb})(8\text{ in.}) - (120\text{ lb})(12\text{ in.}) - (60\text{ lb})(32\text{ in.}) + (\tfrac{4}{5}T)(14\text{ in.}) + (\tfrac{3}{5}T)(24\text{ in.}) = 0$

$T = 112.5\text{ lb}$ ◁

$\xrightarrow{+} \Sigma F_x = 0$: $E_x - \tfrac{4}{5}(112.5\text{ lb}) = 0$

$E_x = 90.0\text{ lb} \rightarrow$ ◁

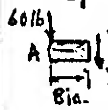
$+\uparrow \Sigma F_y = 0$: $E_y - 240\text{ lb} + \tfrac{3}{5}(112.5\text{ lb}) = 0$, $E_y = 172.5\text{ lb}\uparrow$ ◁



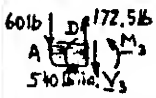
WE REPLACE THE REACTIONS AT E AND H BY EQUIVALENT FORCE-COUPLE SYSTEMS AT D AND G, RESPECTIVELY.

WE CONSIDER SUCCESSIVELY THE FOLLOWING F-B DIAGRAMS

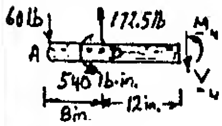
$V_1 = -60\text{ lb}$
$M_1 = 0$



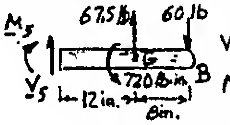
$V_2 = -60\text{ lb}$
$M_2 = -480\text{ lb}\cdot\text{in}$



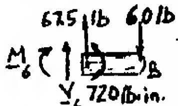
$V_3 = +112.5\text{ lb}$
$M_3 = -1020\text{ lb}\cdot\text{in}$



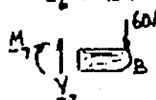
$V_4 = +112.5\text{ lb}$
$M_4 = +330\text{ lb}\cdot\text{in}$



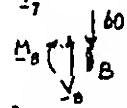
$V_5 = -7.5\text{ lb}$
$M_5 = +330\text{ lb}\cdot\text{in}$



$V_6 = -7.5\text{ lb}$
$M_6 = +240\text{ lb}\cdot\text{in}$

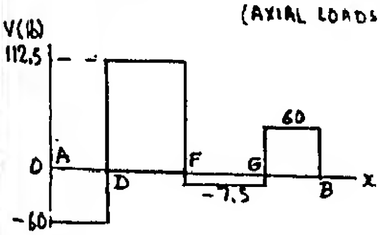


$V_7 = +60\text{ lb}$
$M_7 = -480\text{ lb}\cdot\text{in}$

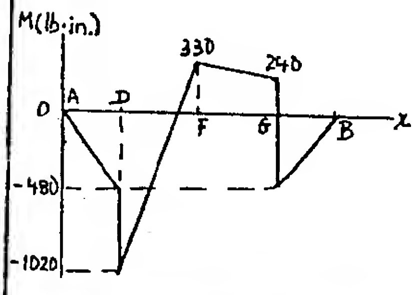


$V_8 = +60\text{ lb}$
$M_8 = 0$

(AXIAL LOADS HAVE BEEN OMITTED)



$|V|_{max} = 112.5\text{ lb}$ ◀



$|M|_{max} = 1020\text{ lb}\cdot\text{in}$ ◀

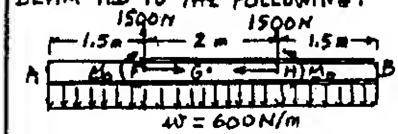
7.55

GIVEN: STRUCTURAL MEMBER OF PROB. 7.50.

FIND: (a) ANGLE θ FOR WHICH $|M|_{max}$ IS AS SMALL AS POSSIBLE

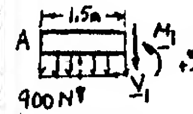
(b) CORRESPONDING VALUE OF $|M|_{max}$.

SEE SOLUTION OF PROB. 7.50 FOR REDUCTION OF LOADING ON BEAM AB TO THE FOLLOWING:



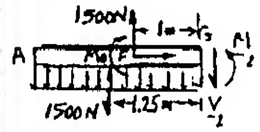
WHERE $M_0 = (750\text{ N}\cdot\text{m})\tan\theta$ ◁
[EQUATION (2)]

THE LARGEST NEGATIVE B.M. OCCURS JUST TO THE LEFT OF F:



$+\circlearrowleft \Sigma M_1 = 0$: $M_1 + (900\text{ N})(\tfrac{1.5\text{ m}}{2}) = 0$, $M_1 = -675\text{ N}\cdot\text{m}$ ◁

THE LARGEST POSITIVE B.M. OCCURS AT G:



$+\circlearrowleft \Sigma M_2 = 0$: $M_2 - M_0 + (1500\text{ N})(1.25\text{ m} - 1\text{ m}) = 0$

$M_2 = M_0 - 375\text{ N}\cdot\text{m}$ ◁

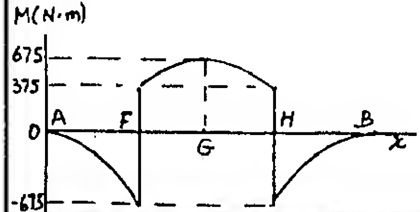
EQUATING M_2 AND $-M_1$:

$M_0 - 375 = +675$

$M_0 = 1050\text{ N}\cdot\text{m}$

FROM EQ. (2):

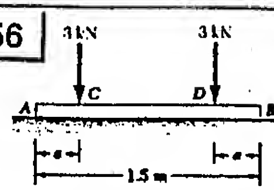
$\tan\theta = \dfrac{1050}{750} = 1.400$



(a) $\theta = 54.5°$ ◀

(b) $|M|_{max} = 675\text{ N}\cdot\text{m}$ ◀

7.56



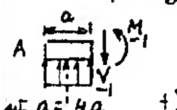
GIVEN: BEAM RESTING ON GROUND AND LOADED AS SHOWN.

FIND: (a) DISTANCE a FOR WHICH $|M|_{max}$ IS AS SMALL AS POSSIBLE

(b) CORRESPONDING VALUE OF $|M|_{max}$

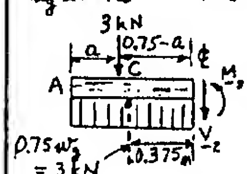
FORCE PER UNIT LENGTH EXERTED BY GROUND:

$w_g = \dfrac{6\text{ kN}}{1.5\text{ m}} = 4\text{ kN/m}$



THE LARGEST POSITIVE B.M. OCCURS JUST TO THE LEFT OF C:

$+\circlearrowleft \Sigma M_1 = 0$: $M_1 = (4a)\tfrac{a}{2}$ $\quad M_1 = 2a^2$ ◁



THE LARGEST NEGATIVE B.M. OCCURS AT THE CENTER LINE:

$+\circlearrowleft \Sigma M_2 = 0$: $M_2 + 3(0.75 - a) - 3(0.375) = 0$

$M_2 = -(1.125 - 3a)$ ◁

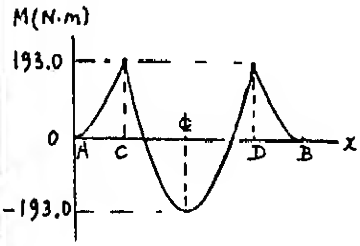
EQUATING M_1 AND $-M_2$:

$2a^2 = 1.125 - 3a$

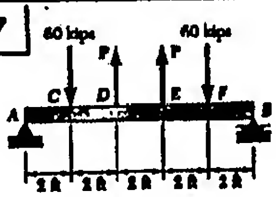
$a^2 + 1.5a - 0.5625 = 0$

(a) SOLVING THE QUADRATIC EQ.: $a = 0.31066$, $a = 0.311\text{ m}$ ◀

(b) SUBSTITUTING: $|M|_{max} = M_1 = 2(0.31066)^2$ $\quad |M|_{max} = 193.0\text{ N}\cdot\text{m}$ ◀

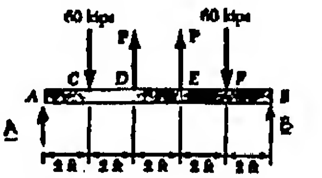


7.57



GIVEN:
BEAM AND LOADING SHOWN

FIND:
(a) VALUE OF P FOR WHICH $|M|_{max}$ IS AS SMALL AS POSSIBLE
(b) CORRESPONDING VALUE OF $|M|_{max}$.

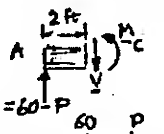


FREE BODY: ENTIRE BEAM

BECAUSE OF SYMMETRY OF BEAM AND LOADING, WE HAVE

$A = B = (60\text{ kips} - P)\uparrow$ ◁

LARGEST POSITIVE B.M. OCCURS AT C:



$+\circlearrowleft \Sigma M_C = 0:\ M_C - (60-P)(2\text{ ft}) = 0$

$M_C = 120 - 2P$ (1) ◁

LARGEST NEGATIVE B.M. OCCURS AT CENTER LINE:

$+\circlearrowleft \Sigma M_{ℓ} = 0:\ M_{ℓ} + (60)(3) - (60-P)(5) - P(1) = 0$

$M_{ℓ} = 120 - 4P$ ◁

(a) EQUATING M_C AND $-M_{ℓ}$:

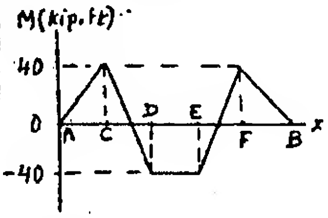
$120 - 2P = -(120 - 4P)$

$6P = 240,\quad P = 40.0\text{ kips}$ ◀

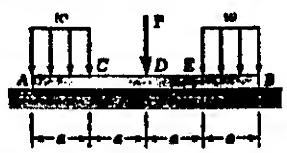
(b) SUBSTITUTING INTO (1):

$|M|_{max} = M_C = 120 - 2(40)$

$|M|_{max} = 40\text{ kip·ft}$ ◀



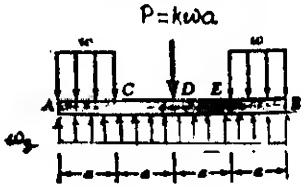
7.58



GIVEN:
BEAM AND LOADING SHOWN (SAME AS FOR PROBS 7.47 & 7.48)

FIND:
(a) RATIO $k = P/wa$ FOR WHICH $|M|_{max}$ IS AS SMALL AS POSSIBLE
(b) CORRESPONDING VALUE OF $|M|_{max}$.

$P = kwa$



FREE BODY: ENTIRE BEAM

$+\uparrow \Sigma F_y = 0:$

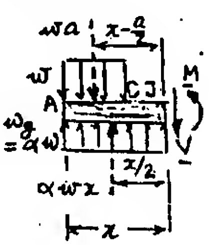
$w_g(4a) - 2wa - kwa = 0$

$w_g = \frac{w}{4}(2+k)$

SETTING $w_g/w = \alpha$ (1)

WE HAVE $k = 4\alpha - 2$ (2)

MINIMUM VALUE OF B.M. FOR M TO HAVE NEGATIVE VALUES, WE MUST HAVE $w_g < w$. WE VERIFY THAT M WILL THEN BE NEGATIVE AND KEEP DECREASING IN THE PORTION AC OF THE BEAM. THEREFORE, M_{min} WILL OCCUR BETWEEN C AND D.



FROM C TO D:

$+\circlearrowleft \Sigma M_J = 0:\ M + wa\left(x - \frac{a}{2}\right) - \alpha w x\left(\frac{x}{2}\right) = 0$

$M = \frac{1}{2}w(\alpha x^2 - 2ax + a^2)$ (3)

WE DIFFERENTIATE AND SET $\frac{dM}{dx} = 0$:

$\alpha x - a = 0 \qquad x_{min} = \frac{a}{\alpha}$ (4)

SUBSTITUTING IN (3):

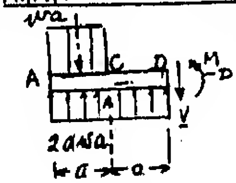
$M_{min} = \frac{1}{2}wa^2\left(\frac{1}{\alpha} - \frac{2}{\alpha} + 1\right)$

$M_{min} = -wa^2\,\frac{1-\alpha}{2\alpha}$ (5)

(CONTINUED)

7.58 CONTINUED

MAXIMUM VALUE OF B.M. OCCURS AT D



$+\circlearrowleft \Sigma M_D = 0:\ M_D + wa\left(\frac{3a}{2}\right) - (2\alpha wa)a = 0$

$M_{max} = M_D = wa^2\left(2\alpha - \frac{3}{2}\right)$ (6)

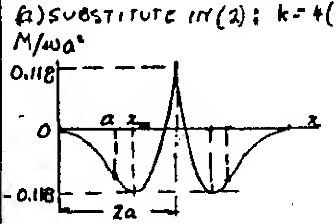
EQUATING $-M_{min}$ AND M_{max}:

$wa^2\,\frac{1-\alpha}{2\alpha} = wa^2\left(2\alpha - \frac{3}{2}\right)$

$4\alpha^2 - 2\alpha - 1 = 0$

$\alpha = \frac{2+\sqrt{20}}{8} \qquad \alpha = \frac{1+\sqrt{5}}{4} = 0.809$

(a) SUBSTITUTE IN (2): $k = 4(0.809) - 2 \qquad k = 1.236$ ◀



(b) SUBSTITUTE FOR α IN (5):

$|M|_{max} = -M_{min} = -wa^2\,\frac{1-0.809}{2(0.809)}$

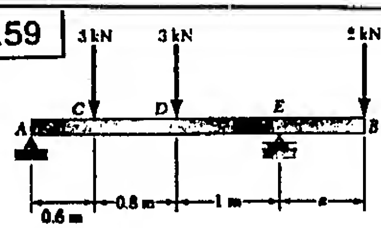
$|M|_{max} = 0.1180\,wa^2$ ◀

SUBSTITUTE FOR x IN (4):

$x_{min} = \frac{a}{0.809} = 1.236\,a$ ◁

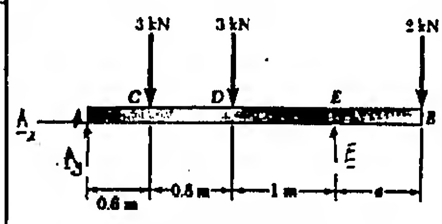
B.M. DIAGRAM CONSISTS OF 4 ARCS OF PARABOLA.
COMPARE THIS DIAGRAM WITH THOSE OF PROB. 7.47 AND 7.48

7.59



GIVEN: BEAM AND LOADING SHOWN.

FIND:
(a) DISTANCE a FOR WHICH $|M|_{max}$ IS AS SMALL AS POSSIBLE
(b) CORRESPONDING VALUE OF $|M|_{max}$.



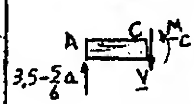
FREE BODY: ENTIRE BEAM

$\Sigma F_x = 0:\ A_x = 0$

$+\circlearrowleft \Sigma M_E = 0:\ -A_y(1.8) + (3)(1.8) + 3(1) - (2)a = 0$

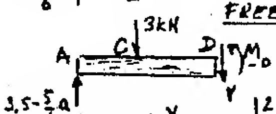
$A_y = 3.5\text{ kN} - \frac{5}{6}a$

$A = 3.5\text{ kN} - \frac{5}{6}a\ \uparrow$ ◁



FREE BODY: AC

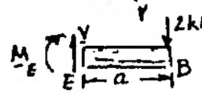
$+\circlearrowleft \Sigma M_C = 0:\ M_C - (3.5 - \frac{5}{6}a)(0.6\text{ m}) = 0, \quad M_C = +2.1 - \frac{a}{2}$ ◁



FREE BODY: AD: $+\circlearrowleft \Sigma M_D = 0:$

$M_D - (3.5 - \frac{5}{6}a)(1.4\text{ m}) + (3\text{ kN})(0.8\text{ m}) = 0$

$M_D = +2.5 - \frac{7}{6}a$ ◁



FREE BODY: EB

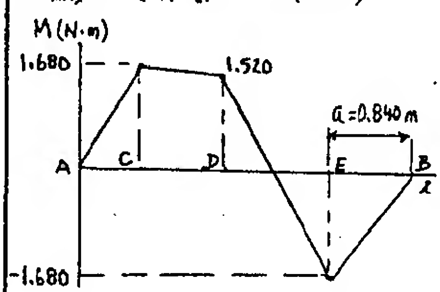
$+\circlearrowleft \Sigma M_E = 0:\ -M_E - (2\text{ kN})a = 0$

$M_E = -2a$ ◁

WE SHALL ASSUME THAT $M_C > M_D$ AND, THUS, THAT $M_{max} = M_C$.
WE SET $M_{max} = |M_{min}|$ OR $M_C = |M_E|$:

$2.1 - \frac{a}{2} = 2a \qquad a = 0.840\text{ m}$ ◁

$|M|_{max} = M_C = |M_E| = 2a = 2(0.840) \qquad |M|_{max} = 1.680\text{ N·m}$ ◁



WE MUST CHECK OUR ASSUMPTION.

$M_D = 2.5 - \frac{7}{6}(0.840)$

$= 1.520\text{ N·m}$

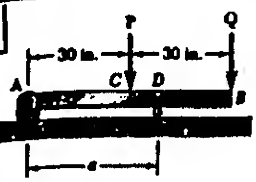
THUS, $M_C > M_D$, O.K.

THE ANSWERS ARE

(a) $a = 0.840\text{ m}$ ◀

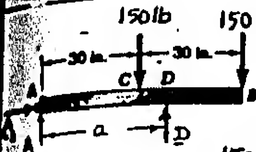
(b) $|M|_{max} = 1.680\text{ N·m}$ ◀

7.60



GIVEN: BEAM SHOWN WITH $P = Q = 150$ lb.

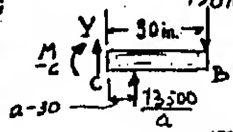
FIND: (a) DISTANCE a FOR WHICH $|M|_{max}$ IS AS SMALL AS POSSIBLE
(b) CORRESPONDING VALUE OF $|M|_{max}$.



FREE BODY: ENTIRE BEAM

$+\circlearrowleft \Sigma M_A = 0$: $Da - (150)(30) - (150)(60) = 0$

$D = \dfrac{13500}{a}$ ◁

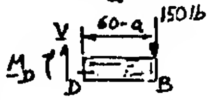


FREE BODY: CB

$+\circlearrowleft \Sigma M_C = 0$:

$-M_C - (150)(30) + \dfrac{13500}{a}(a-30) = 0$

$M_C = 9000\left(1 - \dfrac{45}{a}\right)$ ◁



FREE BODY: DB

$+\circlearrowleft \Sigma M_D = 0$: $-M_D - (150)(60-a) = 0$

$M_D = -150(60-a)$ ◁

(a) WE SET $M_{max} = |M_{min}|$ OR $M_C = -M_D$: $9000\left(1 - \dfrac{45}{a}\right) = 150(60-a)$

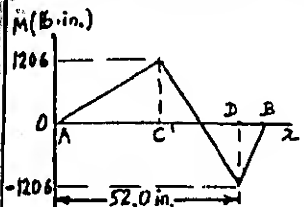
$60 - \dfrac{2700}{a} = 60 - a$

$a^2 = 2700 \qquad a = 51.96$ in.

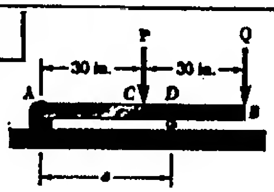
$a = 52.0$ in. ◀

(b) $|M|_{max} = -M_D = 150(60 - 51.96)$

$|M|_{max} = 1206$ lb·in. ◀

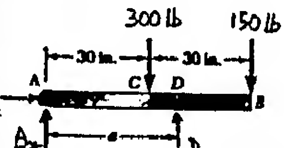


7.61



GIVEN: BEAM SHOWN WITH $P = 300$ lb AND $Q = 150$ lb.

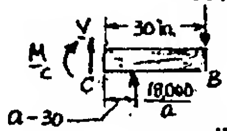
FIND: DISTANCE a FOR WHICH $|M|_{max}$ IS AS SMALL AS POSSIBLE
(b) CORRESPONDING VALUE OF $|M|_{max}$.



FREE BODY: ENTIRE BEAM

$+\circlearrowleft \Sigma M_A = 0$: $Da - (300)(30) - (150)(60) = 0$

$D = \dfrac{18{,}000}{a}$ ◁

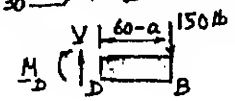


FREE BODY: CB

$+\circlearrowleft \Sigma M_C = 0$:

$-M_C - (150)(30) + \dfrac{18{,}000}{a}(a-30) = 0$

$M_C = 13{,}500\left(1 - \dfrac{40}{a}\right)$ ◁



FREE BODY: DB

$+\circlearrowleft \Sigma M_D = 0$: $-M_D - (150)(60-a) = 0$

$M_D = -150(60-a)$ ◁

(a) WE SET $M_{max} = |M_{min}|$ OR $M_C = -M_D$: $13{,}500\left(1 - \dfrac{40}{a}\right) = 150(60-a)$

$90 - \dfrac{3600}{a} = 60 - a$

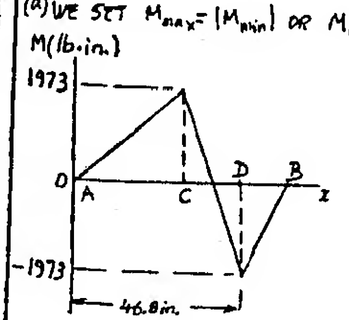
$a^2 + 30a - 3600 = 0$

$a = \dfrac{-30 + \sqrt{15{,}300}}{2} = 46.847$

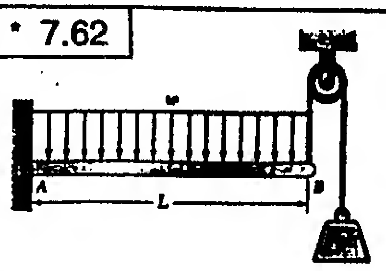
$a = 46.8$ in ◀

(b) $|M|_{max} = -M_D = 150(60 - 46.847)$

$|M|_{max} = 1973$ lb·in. ◀



* 7.62

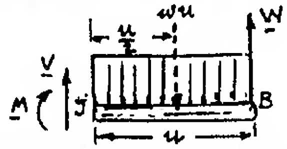


GIVEN: BEAM WITH COUNTERWEIGHT

FIND: W FOR WHICH $|M|_{max}$ IS AS SMALL AS POSSIBLE AND CORRESPONDING $|M|_{max}$.

CONSIDER FOLLOWING CASES:
(a) LOAD w PERMANENTLY APPLIED TO BEAM
(b) LOAD w MAY BE APPLIED OR REMOVED

(a) LOAD w PERMANENTLY APPLIED.



FREE BODY: PORTION JB

$+\circlearrowleft \Sigma M_J = 0$:

$Wu - (wu)\dfrac{u}{2} - M = 0$

$M = Wu - \frac{1}{2}wu^2 \qquad (1)$

MAXIMUM (POSITIVE) VALUE OF M: DIFFERENTIATE (1) AND SET $dM/du = 0$:

$\dfrac{dM}{du} = W - wu = 0 \qquad u_m = \dfrac{W}{w} \qquad (2)$

SUBSTITUTE INTO EQ. (1):

$M_{max} = W\left(\dfrac{W}{w}\right) - \frac{1}{2}w\left(\dfrac{W}{w}\right)^2 \qquad M_{max} = \dfrac{1}{2}\dfrac{W^2}{w} \qquad (3)$

LARGEST NEGATIVE VALUE OF M OCCURS AT A

SETTING $u = L$ IN EQ. (1): $M_A = WL - \frac{1}{2}wL^2 \qquad (4)$

SETTING $M_{max} = |M_{min}|$, OR $M_{max} = -M_A$:

$\dfrac{1}{2}\dfrac{W^2}{w} = -WL + \frac{1}{2}wL^2$

$W^2 + 2wLW - w^2L^2 = 0 \qquad W = \dfrac{-2wL + \sqrt{8w^2L^2}}{2} = (\sqrt{2} - 1)wL$

$W = 0.4142\,wL, \quad W = 0.414\,wL$ ◀

CARRYING INTO (2) AND (4): $u_m = 0.414L$ (FROM B)

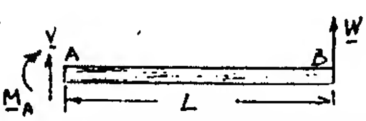
$|M|_{max} = -M_A = -(0.4142\,wL^2 - 0.5\,wL^2)$

$|M|_{max} = 0.0858\,wL^2$ ◀

(b) LOAD w MAY BE APPLIED OR REMOVED

WITH NO LOAD w: M_{max} OCCURS AT A:

$M_{max} = M_A = WL \qquad (5)$



WE MUST CONSIDER THE FOLLOWING POSSIBILITIES:
(SUB "w" MEANS THAT LOAD w IS APPLIED; SUB "NL", THAT IT IS NOT)

① $(M_{max})_{NL} = (M_{max})_w$ OR $WL = \dfrac{1}{2}\dfrac{W^2}{w} \qquad W = 2wL$ ◁

WITH THIS VALUE OF W, WE HAVE

$|M|_{max} = WL = (2wL)L = 2wL^2$

② $(M_{max})_{NL} = |M_{min}|_w$ OR $(M_A)_{NL} = -(M_A)_w$

$WL = -(WL - \frac{1}{2}wL^2)$

$2WL = \frac{1}{2}wL^2 \qquad W = 0.250\,wL$ ◁

WITH THIS VALUE OF W, WE HAVE

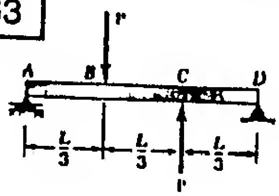
$|M|_{max} = WL = (0.250\,wL)L = 0.250\,wL^2$

THE COUNTERWEIGHT, THEREFORE, SHOULD BE

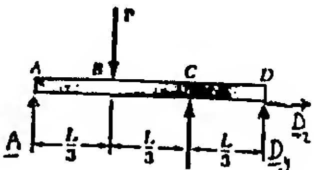
$W = 0.250\,wL$ ◀

WITH $|M|_{max} = 0.250\,wL^2$ ◀

7.63



GIVEN:
BEAM AND LOADING SHOWN
(1) DRAW V AND M DIAGRAMS
(2) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

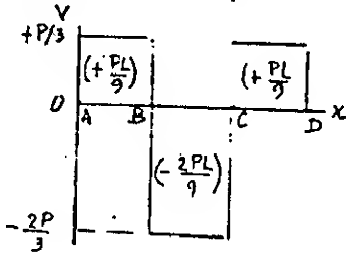


FREE BODY: ENTIRE BEAM

$+\circlearrowleft \Sigma M_D = 0$:

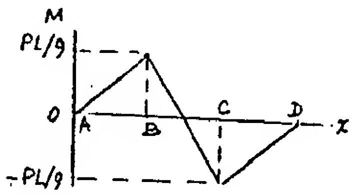
$P\left(\frac{2L}{3}\right) - P\left(\frac{L}{3}\right) - AL = 0$

$\underline{A} = P/3 \uparrow$



SHEAR DIAGRAM
WE NOTE THAT
$V_A = A = +P/3$

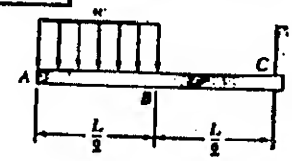
$|V|_{max} = 2P/3$



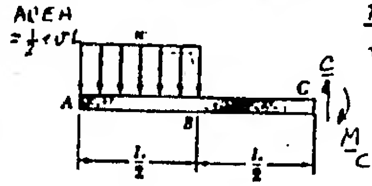
B.M. DIAGRAM
WE NOTE THAT $M_A = 0$

$|M|_{max} = PL/9$

7.65



GIVEN:
BEAM AND LOADING SHOWN
(1) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$



FREE BODY: ENTIRE BEAM

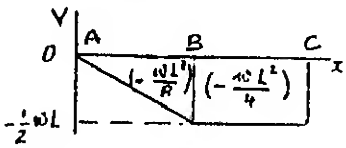
$+\uparrow \Sigma F_y = 0: \; C - w\frac{L}{2} = 0$

$\underline{C} = \frac{1}{2}wL \uparrow$

$+\circlearrowleft \Sigma M_C = 0$:

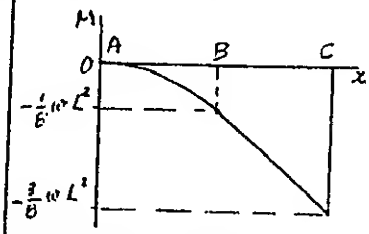
$\left(\frac{1}{2}wL\right)\left(\frac{3L}{4}\right) - M_C = 0$

$\underline{M}_C = \frac{3}{8}wL^2 \circlearrowleft$



SHEAR DIAGRAM
AT A: $V_A = 0$

$|V|_{max} = \frac{1}{2}wL$



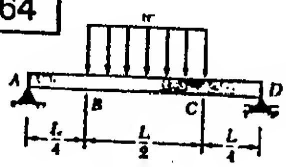
B.M. DIAGRAM
AT A: $M = 0, \frac{dM}{dx} = V = 0$

$|M|_{max} = \frac{3}{8}wL^2$

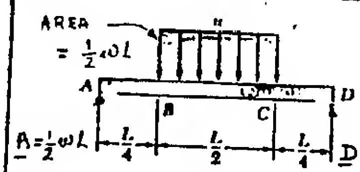
FROM A TO B:
ARC OF PARABOLA

SINCE V HAS NO DISCONTINUITY AT B, THE SLOPE OF THE PARABOLA AT B IS EQUAL TO THE SLOPE OF THE STRAIGHT-LINE SEGMENT.

7.64



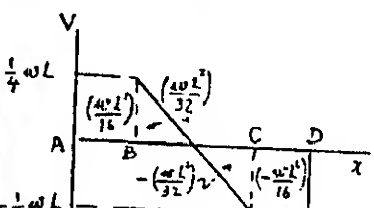
GIVEN:
BEAM AND LOADING SHOWN.
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$



REACTIONS AT A AND D
BECAUSE OF THE SYMMETRY OF THE SUPPORTS AND LOADING,

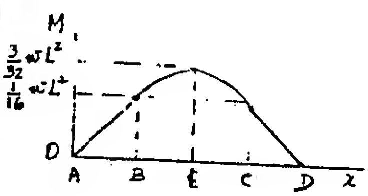
$A = D = \frac{1}{2}\left(w\frac{L}{2}\right) = \frac{1}{4}wL$

$\underline{A} = \underline{D} = \frac{1}{4}wL \uparrow$



SHEAR DIAGRAM
AT A: $V_A = +\frac{1}{4}wL$
FROM B TO C:
OBLIQUE STRAIGHT LINE

$|V|_{max} = \frac{1}{4}wL$

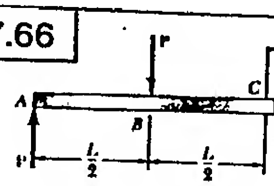


B.M. DIAGRAM
AT A: $M_A = 0$
FROM B TO C:
ARC OF PARABOLA

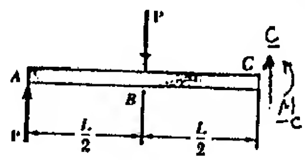
$|M|_{max} = \frac{3}{32}wL^2$

SINCE V HAS NO DISCONTINUITY AT B NOR C, THE SLOPE OF THE PARABOLA AT THESE POINTS IS THE SAME AS THE SLOPE OF THE ADJOINING STRAIGHT-LINE SEGMENT.

7.66



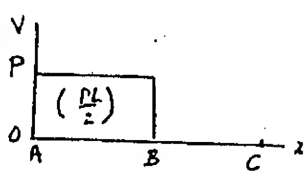
GIVEN:
BEAM AND LOADING SHOWN
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.



FREE BODY: ENTIRE BEAM

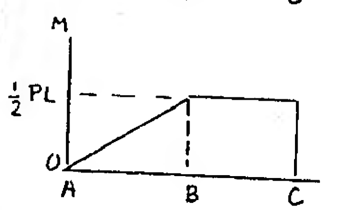
$\Sigma F_y = 0: \; \underline{C} = 0$

$\Sigma M_C = 0: \; \underline{M}_C = \frac{1}{2}PL \circlearrowleft$



SHEAR DIAGRAM:
AT A: $V_A = +P$

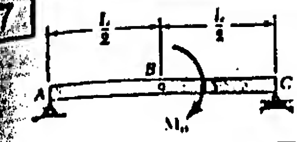
$|V|_{max} = P$



B.M. DIAGRAM
AT A: $M_A = 0$

$|M|_{max} = \frac{1}{2}PL$

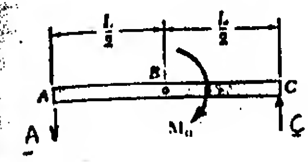
7.67



GIVEN: BEAM AND LOADING SHOWN

(a) DRAW V AND M DIAGRAMS.

(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$

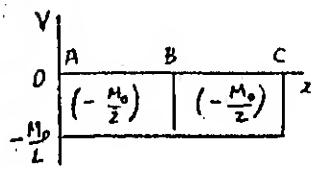


FREE BODY: ENTIRE BEAM

$\Sigma F_y = 0: \quad A = C$

$+\circlearrowleft \Sigma M_C = 0: \; AL - M_0 = 0$

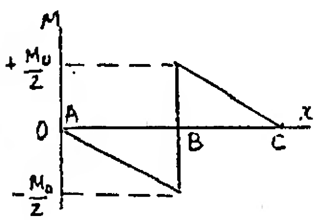
$A = C = \dfrac{M_0}{L}$



SHEAR DIAGRAM

AT A: $V_A = -\dfrac{M_0}{L}$

$|V|_{max} = \dfrac{M_0}{L}$



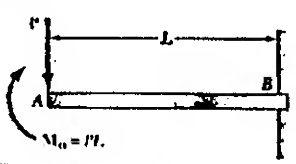
B.M. DIAGRAM

AT A: $M_A = 0$

AT B, M INCREASES BY M_0 ON ACCOUNT OF APPLIED COUPLE.

$|M|_{max} = M_0/2$

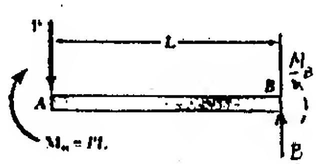
7.68



GIVEN: BEAM AND LOADING SHOWN.

(a) DRAW V AND M DIAGRAMS.

(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.



FREE BODY: ENTIRE BEAM

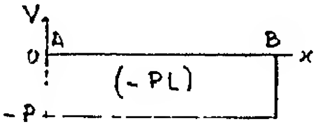
$+\uparrow \Sigma F_y = 0: \; B - P = 0$

$B = P\uparrow$

$+\circlearrowleft \Sigma M_B = 0:$

$M_B - \cancel{M_0} + \cancel{PL} = 0$

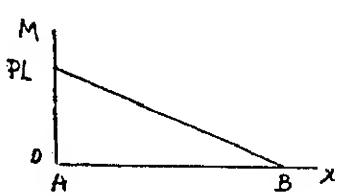
$M_B = 0$



SHEAR DIAGRAM

AT A: $V_A = -P$

$|V|_{max} = P$

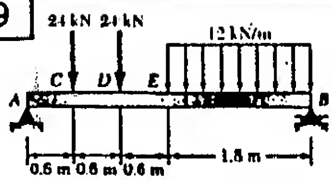


B.M. DIAGRAM

AT A: $M_A = M_0 = PL$

$|M|_{max} = PL$

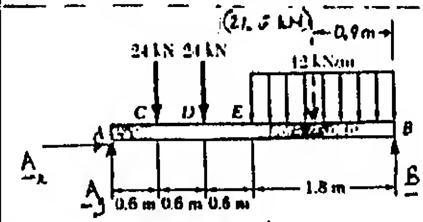
7.69



GIVEN: BEAM AND LOADING SHOWN.

(a) DRAW V AND M DIAGRAMS

(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.



FREE BODY: ENTIRE BEAM

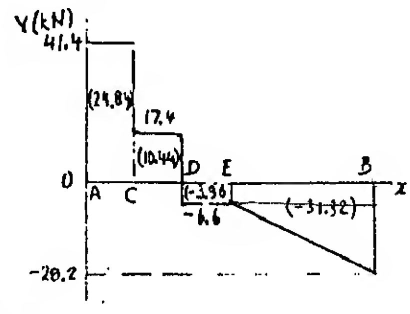
$+\circlearrowleft \Sigma M_B = 0: (24\,kN)(3\,m)$

$+ (24\,kN)(2.4\,m) + (21.6\,kN)(0.9\,m)$

$- A_y(3.6\,m) = 0$

$A_y = +41.4\,kN$

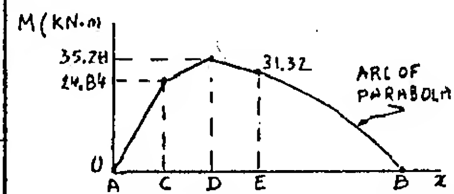
$\Sigma F_x = 0: \quad A_x = 0$



SHEAR DIAGRAM

AT A: $V_A = A_y = +41.4\,kN$

$|V|_{max} = 41.4\,kN$



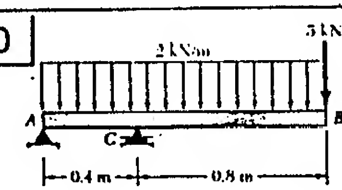
B.M. DIAGRAM

AT A: $M_A = 0$

$|M|_{max} = 35.3\,kN \cdot m$

THE SLOPE OF THE PARABOLA AT E IS THE SAME AS THAT OF THE SEGMENT DE

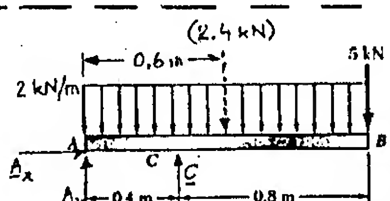
7.70



GIVEN: BEAM AND LOADING SHOWN

(a) DRAW V AND M DIAGRAMS

(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.



FREE BODY: BEAM

$+\circlearrowleft \Sigma M_A = 0:$

$C(0.4\,m) - (2.4\,kN)(0.6\,m)$

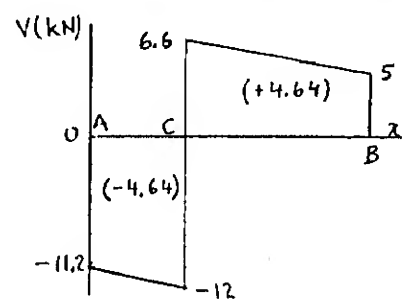
$- (5\,kN)(1.2\,m) = 0$

$C = 18.6\,kN\uparrow$

$\Sigma F_x = 0: \; A_x = 0$

$+\uparrow \Sigma F_y = 0: \; A_y + 18.6 - 2.4 - 5 = 0$

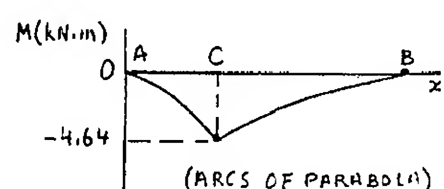
$A_y = -11.2\,kN$



SHEAR DIAGRAM

AT A: $V_A = A_y = -11.2\,kN$

$|V|_{max} = 12.00\,kN$

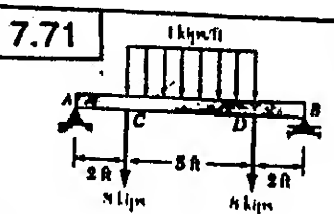


B.M. DIAGRAM

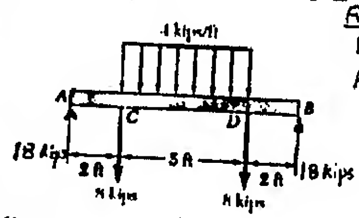
AT A: $M_A = 0$

$|M|_{max} = 4.64\,kN \cdot m$

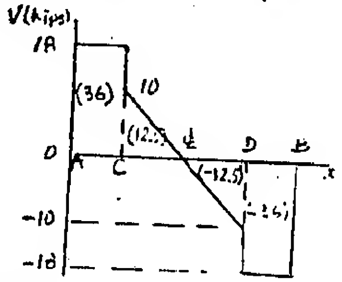
7.71



GIVEN:
BEAM AND LOADING SHOWN
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

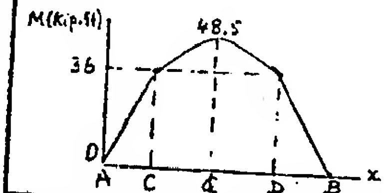


REACTIONS AT SUPPORTS
BECAUSE OF THE SYMMETRY:
$A = B = \frac{1}{2}(8+8+4\times5)$ kips
$A = B = 18$ kips ↑ ◁



SHEAR DIAGRAM.
AT A: $V_A = +18$ kips

$|V|_{max} = 18$ kips ◀

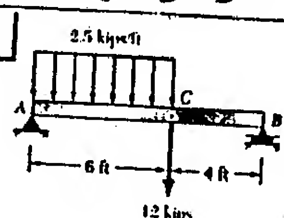


B.M. DIAGRAM
AT A: $M_A = 0$

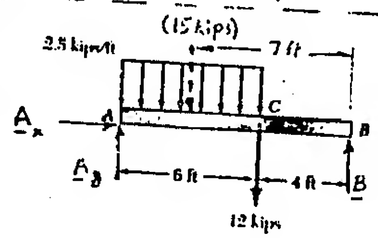
$|M|_{max} = 48.5$ kip·ft ◀

DISCONTINUITIES IN SLOPE AT C AND D, DUE TO THE DISCONTINUITIES OF V.

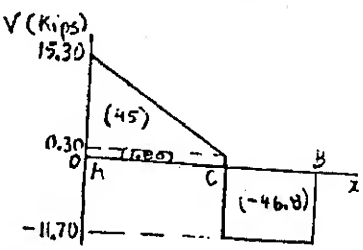
7.72



GIVEN:
BEAM AND LOADING SHOWN.
(a) DRAW V AND M DIAGRAMS.
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

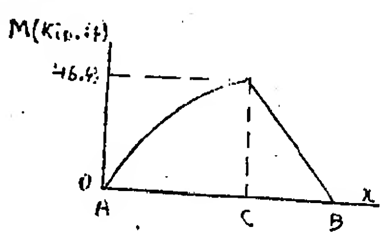


FREE BODY: BEAM
$\Sigma F_x = 0$: $A_x = 0$
$+\circlearrowleft \Sigma M_B = 0$: $(12 \text{ kips})(4 \text{ ft})$
$+(15 \text{ kips})(7 \text{ ft}) - A_y(10 \text{ ft}) = 0$
$A_y = +15.3$ kips ◁
$+\uparrow \Sigma F_y = 0$: $B + 15.3 - 15 - 12 = 0$
$B = +11.7$ kips ◁



SHEAR DIAGRAM
AT A: $V_A = A_y = 15.3$ kips

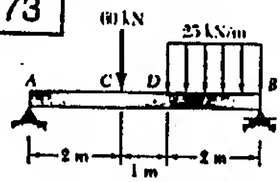
$|V|_{max} = 15.30$ kips ◀



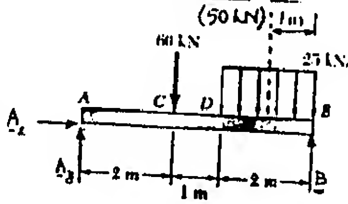
B.M. DIAGRAM
AT A: $M_A = 0$

$|M|_{max} = 46.9$ kip·ft ◀

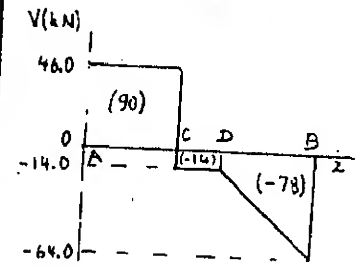
7.73



GIVEN:
BEAM AND LOADING SHOWN
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

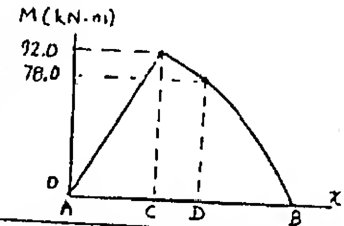


FREE BODY: BEAM
$\Sigma F_x = 0$: $A_x = 0$
$+\circlearrowleft \Sigma M_B = 0$:
$(60 \text{ kN})(3 \text{ m}) + (50 \text{ kN})(1 \text{ m}) - A_y(5 \text{ m}) = 0$
$A_y = +46.0$ kN
$+\uparrow \Sigma F_y = 0$: $B + 46.0 \text{ kN} - 60 \text{ kN} - 50 \text{ kN} = 0$
$B = +64.0$ kN



SHEAR DIAGRAM
AT A: $V_A = A_y = +46.0$ kN

$|V|_{max} = 64.0$ kN

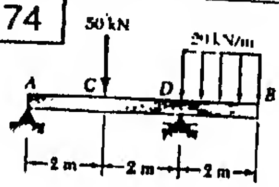


B.M. DIAGRAM
AT A: $M_A = 0$

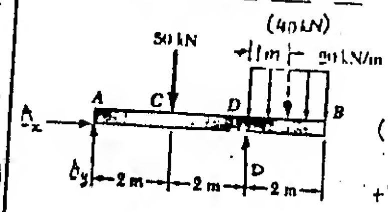
$|M|_{max} = 92.0$ kN·m

PARABOLA FROM D TO B. ITS SLOPE AT D IS SAME AS THAT OF STRAIGHT-LINE SEGMENT CD SINCE V HAS NO DISCONTINUITY AT D.

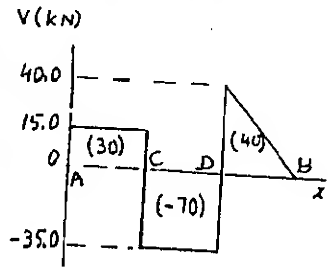
7.74



GIVEN:
BEAM AND LOADING SHOWN.
(a) DRAW V AND M DIAGRAMS.
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.

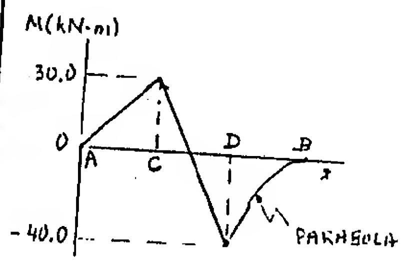


FREE BODY: BEAM
$\Sigma F_x = 0$: $A_x = 0$
$+\circlearrowleft \Sigma M_D = 0$:
$(50 \text{ kN})(2 \text{ m}) - (40 \text{ kN})(1 \text{ m}) - A_y(4 \text{ m}) = 0$
$A_y = +15.00$ kN ◁
$+\uparrow \Sigma F_y = 0$: $D + 15.00 - 50 - 40 = 0$
$D = +75.0$ kN ◁



SHEAR DIAGRAM
AT A: $V_A = A_y = +15.00$ kN

$|V|_{max} = 70.0$ kN ◀

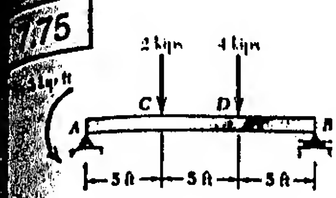


B.M. DIAGRAM
AT A: $M_A = 0$

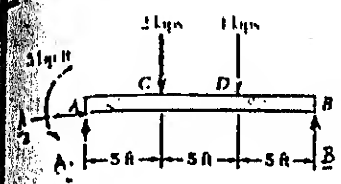
$|M|_{max} = 40.0$ kN·m ◀

AT B:
SLOPE $= \frac{dM}{dx} = V_B = 0$

7.75



GIVEN:
BEAM AND LOADING SHOWN
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.



FREE BODY: BEAM

$+\circlearrowleft \Sigma M_B = 0$:

$5\text{ kip·ft} + (2\text{ kips})(10\text{ ft}) + (4\text{ kips})(5\text{ ft}) - A_y(15\text{ ft}) = 0$

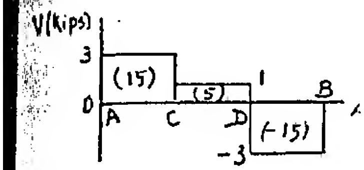
$A_y = +3.00\text{ kips}$

$\Sigma F_x = 0$: $A_x = 0$

SHEAR DIAGRAM

AT A: $V_A = A_y = +3.00\text{ kips}$

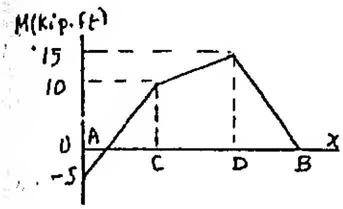
$|V|_{max} = 3.00\text{ kips}$



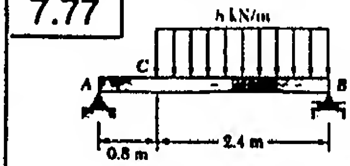
B.M. DIAGRAM

AT A: $M_A = -5\text{ kip·ft}$

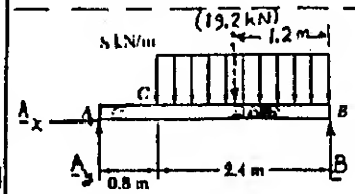
$|M|_{max} = 15.00\text{ kip·ft}$



7.77



GIVEN:
BEAM AND LOADING SHOWN
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE MAGNITUDE AND LOCATION OF $|M|_{max}$.



FREE BODY: BEAM

$\Sigma F_x = 0$: $A_x = 0$

$+\circlearrowleft \Sigma M_B = 0$:

$(19.2\text{ kN})(1.2\text{ m}) - A_y(3.2\text{ m}) = 0$

$A_y = +7.20\text{ kN}$

SHEAR DIAGRAM

$V_A = V_C = A_y = +7.20\text{ kN}$

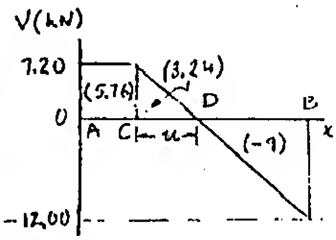
TO DETERMINE POINT D WHERE $V=0$, WE WRITE

$V_D - V_C = -wu$

$0 - 7.20\text{ kN} = -(8\text{ kN/m})u$

$u = 0.9\text{ m}$

WE NEXT COMPUTE ALL AREAS

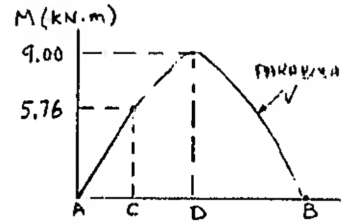


B.M. DIAGRAM

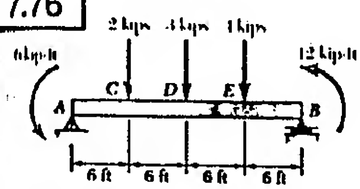
AT A: $M_A = 0$

LARGEST VALUE OCCURS AT D, WITH $AD = 0.8 + 0.9 = 1.700\text{ m}$

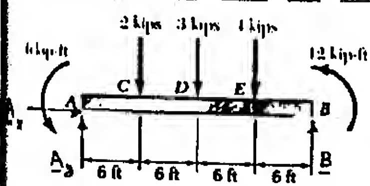
$|M|_{max} = 9.00\text{ kN·m}$,
1.700 m FROM A



7.76



GIVEN:
BEAM AND LOADING SHOWN
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE $|V|_{max}$ AND $|M|_{max}$.



FREE BODY: BEAM

$+\circlearrowleft \Sigma M_B = 0$:

$6\text{ kip·ft} + 12\text{ kip·ft} + (2\text{ kips})(18\text{ ft}) + (3\text{ kips})(12\text{ ft}) + (4\text{ kips})(6\text{ ft}) - A_y(24\text{ ft}) = 0$

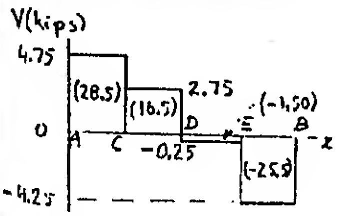
$A_y = +4.75\text{ kips}$

$\Sigma F_x = 0$: $A_x = 0$

SHEAR DIAGRAM

AT A: $V_A = A_y = +4.75\text{ kips}$

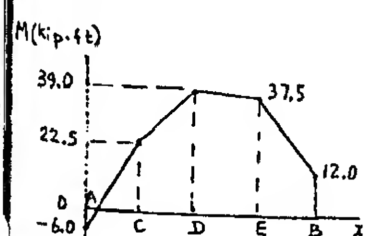
$|V|_{max} = 4.75\text{ kips}$



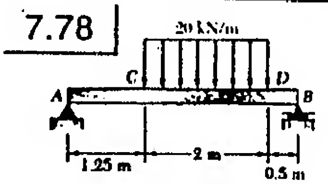
B.M. DIAGRAM

AT A: $M_A = -6\text{ kip·ft}$

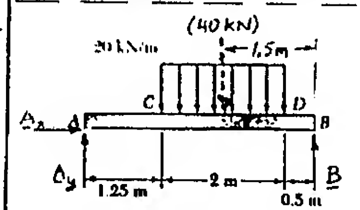
$|M|_{max} = 39.0\text{ kip·ft}$



7.78



GIVEN:
BEAM AND LOADING SHOWN
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE MAGNITUDE AND LOCATION OF $|M|_{max}$.



FREE BODY: BEAM

$\Sigma F_x = 0$: $A_x = 0$

$+\circlearrowleft \Sigma M_B = 0$:

$(40\text{ kN})(1.5\text{ m}) - A_y(3.75\text{ m}) = 0$

$A_y = +16.00\text{ kN}$

SHEAR DIAGRAM

$V_A = V_C = A_y = +16.00\text{ kN}$

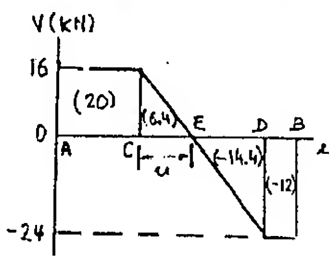
TO DETERMINE POINT E WHERE $V=0$, WE WRITE

$V_E - V_C = -wu$

$0 - 16\text{ kN} = -(20\text{ kN/m})u$

$u = 0.800\text{ m}$

WE NEXT COMPUTE ALL AREAS



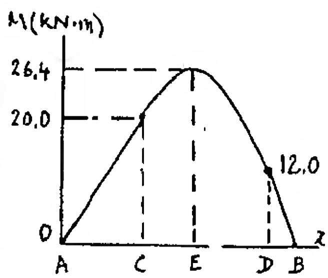
B.M. DIAGRAM

AT A: $M_A = 0$

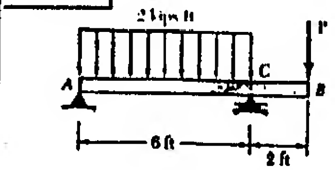
LARGEST VALUE OCCURS AT E WITH $AE = 1.25 + 0.8 = 2.05\text{ m}$

$|M|_{max} = 26.4\text{ kN·m}$,
2.05 m FROM A

FROM A TO C AND D TO B: STRAIGHT-LINE SEGMENTS
FROM C TO D: PARABOLA



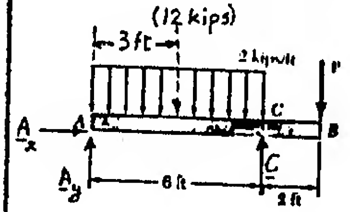
7.79



GIVEN

BEAM AND LOADING SHOWN

DRAW V AND M DIAGRAMS AND DETERMINE MAGNITUDE AND LOCATION OF $|M|_{max}$ FOR (a) $P = 6$ kips, (b) $P = 3$ kips.



FREE BODY: BEAM

$\Sigma F_x = 0$: $A_x = 0$

$+\circlearrowleft \Sigma M_A = 0$:

$C(6\text{ ft}) - (12\text{ kips})(3\text{ ft}) - P(8\text{ ft}) = 0$

$C = 6\text{ kips} + \frac{4}{3}P$ (1)

$\Sigma F_y = 0$:

$A_y + (6 + \frac{4}{3}P) - 12 - P = 0$

$A_y = 6\text{ kips} - \frac{1}{3}P$ (2)

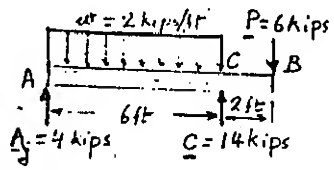
(a) $P = 6$ kips.

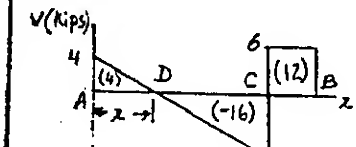
LOAD DIAGRAM

SUBSTITUTING FOR P IN EQS. (2) AND (1):

$A_y = 6 - \frac{1}{3}(6) = 4$ kips

$C = 6 + \frac{4}{3}(6) = 14$ kips





SHEAR DIAGRAM

$V_A = A_y = +4$ kips

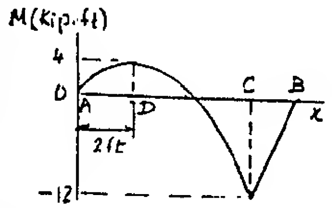
TO DETERMINE POINT D WHERE $V = 0$:

$V_D - V_A = -wx$

$0 - 4\text{ kips} = (2\text{ kips/ft})x$

$x = 2$ ft

WE COMPUTE ALL AREAS



B.M. DIAGRAM

AT A: $M_A = 0$

$|M|_{max} = 12.00$ kip·ft, AT C

PARABOLA FROM A TO C

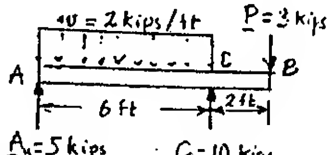
(b) $P = 3$ kips

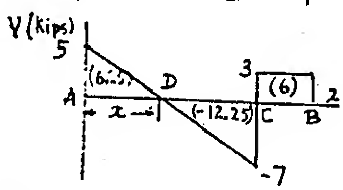
LOAD DIAGRAM

SUBSTITUTING FOR P IN EQS. (2) AND (1):

$A = 6 - \frac{1}{3}(3) = 5$ kips

$C = 6 + \frac{4}{3}(3) = 10$ kips





SHEAR DIAGRAM

$V_A = A_y = +5$ kips

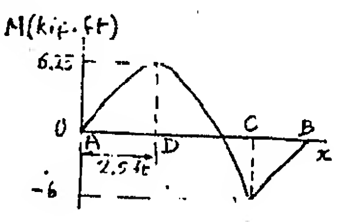
TO DETERMINE D WHERE $V = 0$:

$V_D - V_A = -wx$

$0 - (5\text{ kips}) = -(2\text{ kips/ft})x$

$x = 2.5$ ft

WE COMPUTE ALL AREAS



B.M. DIAGRAM

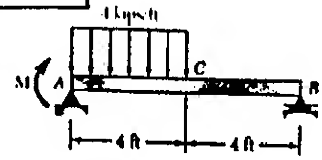
AT A: $M_A = 0$

$|M|_{max} = 6.25$ kip·ft,

2.50 ft FROM A

PARABOLA FROM A TO C.

7.80



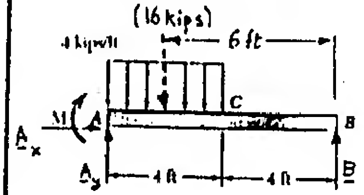
GIVEN:

BEAM AND LOADING

DRAW V AND M DIAGRAMS

DETERMINE MAGNITUDE AND LOCATION OF $|M|_{max}$

(a) $M = 0$, (b) $M = 24$ kip·ft



FREE BODY: BEAM

$\Sigma F_x = 0$: $A_x = 0$

$+\circlearrowleft \Sigma M_B = 0$:

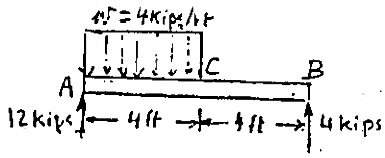
$(16\text{ kips})(6\text{ ft}) - A_y(8\text{ ft}) ...$

$A_y = 12\text{ kips} - \frac{1}{8}M$

$+\uparrow \Sigma F_y = 0$: $B + 12 - \frac{1}{8}M ...$

$B = 4\text{ kips} + \frac{1}{8}M$

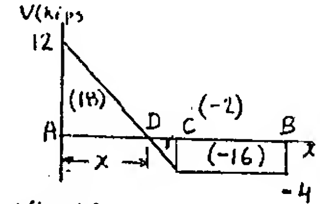
(a) $M = 0$.



LOAD DIAGRAM

MAKING $M = 0$ IN (1)

$A_y = +12$ kips, $B = 4$ ...



SHEAR DIAGRAM

$V_A = A_y = +12$ kips

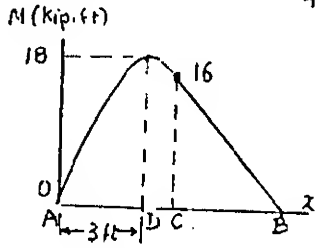
TO DETERMINE POINT D WHERE $V = 0$:

$V_D - V_A = -wx$

$0 - 12\text{ kips} = -(4\text{ kips/ft})x$

$x = 3$ ft

WE COMPUTE ALL AREAS



B.M. DIAGRAM

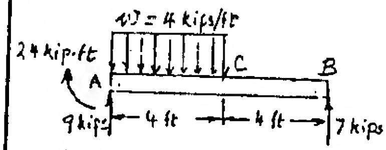
AT A: $M_A = 0$

$|M|_{max} = 18.00$ kip·ft,

3 ft FROM A

PARABOLA FROM A TO C.

(b) $M = 24$ kip·ft

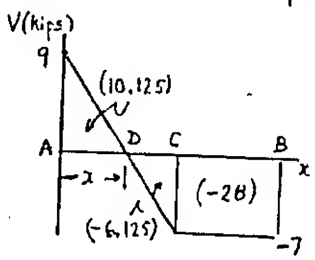


LOAD DIAGRAM

MAKING $M = 24$ kip·ft IN (1)

$A_y = 12 - \frac{1}{8}(24) = +9$ kips

$B = 4 + \frac{1}{8}(24) = +7$ kips



SHEAR DIAGRAM

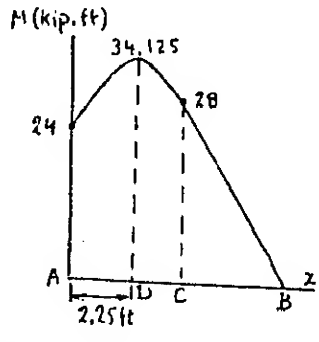
$V_A = A_y = +9$ kips

TO DETERMINE POINT D WHERE $V = 0$:

$V_D - V_A = -wx$

$0 - 9\text{ kips} = -(4\text{ kips/ft})x$

$x = 2.25$ ft



B.M. DIAGRAM

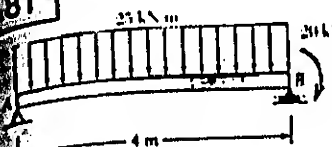
AT A: $M_A = +24$ kip·ft

$|M|_{max} = 34.1$ kip·ft,

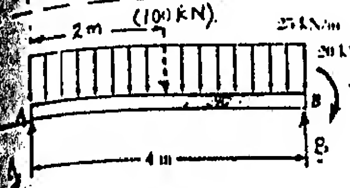
2.25 ft FROM A

PARABOLA FROM A TO C

7.81



GIVEN:
BEAM AND LOADING SHOWN.
(a) DRAW V AND M DIAGRAMS
(b) DETERMINE MAGNITUDE AND LOCATION OF $|M|_{max}$.



FREE BODY: BEAM

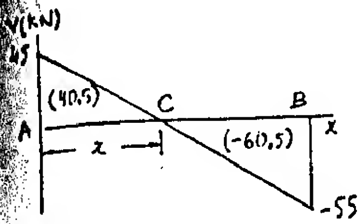
$+\circlearrowleft \Sigma M_A = 0$: $B(4m) - (100kN)(2m) - 20kN\cdot m = 0$

$B = +55\ kN$ ◁

$\Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y + 55 - 100 = 0$

$A_y = +45\ kN$ ◁



SHEAR DIAGRAM

AT A: $V_A = A_y = +45\ kN$

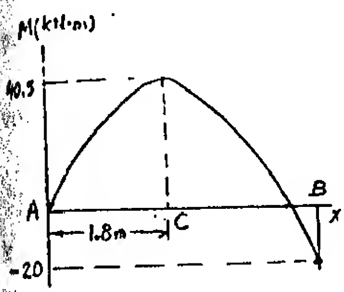
TO DETERMINE POINT C WHERE V=0:

$V_C - V_A = -wx$

$0 - 45\ kN = -(25\ kN/m)x$

$x = 1.8\ m$ ◁

WE COMPUTE ALL AREAS



B.M. DIAGRAM

AT A: $M_A = 0$

AT B: $M_B = -20\ kN\cdot m$

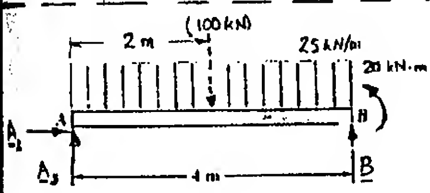
$|M|_{max} = 40.5\ kN\cdot m$, ◀

1.800 m FROM A. ◀

SINGLE ARC OF PARABOLA

7.82

SOLVE PROB. 7.81, ASSUMING THAT 20-kN·m COUPLE AT B IS COUNTERCLOCKWISE.



FREE BODY: BEAM

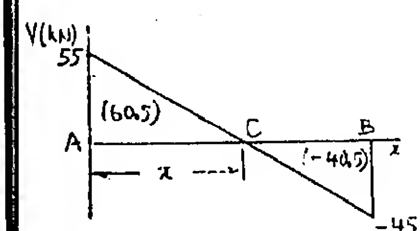
$+\circlearrowleft \Sigma M_A = 0$: $B(4m) - (100kN)(2m) + 20kN\cdot m = 0$

$B = +45\ kN$ ◁

$\Sigma F_x = 0$: $A_x = 0$

$+\Sigma F_y = 0$: $A_y + 45 - 100 = 0$

$A_y = +55\ kN$ ◁



SHEAR DIAGRAM

AT A: $V_A = A_y = +55\ kN$

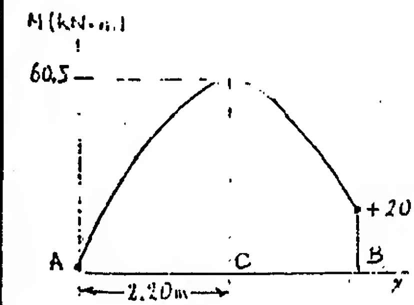
TO DETERMINE POINT C WHERE V=0:

$V_C - V_A = -wx$

$0 - 55\ kN = -(25\ kN/m)x$

$x = 2.20\ m$ ◁

WE COMPUTE ALL AREAS



B.M. DIAGRAM

AT A: $M_A = 0$

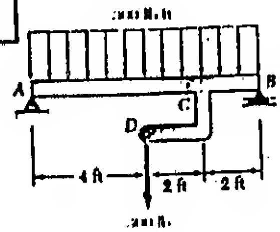
AT B: $M_B = +20\ kN\cdot m$

$|M|_{max} = 60.5\ kN\cdot m$, ◀

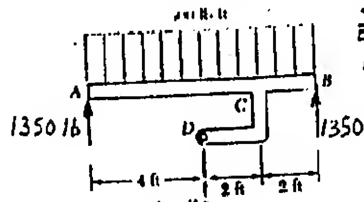
2.20 m FROM A ◀

SINGLE ARC OF PARABOLA

7.83



GIVEN:
STRUCTURE AND LOADING SHOWN
(a) DRAW V AND M DIAGRAMS FOR BEAM AB.
(b) DETERMINE MAGNITUDE AND LOCATION OF $|M|_{max}$.

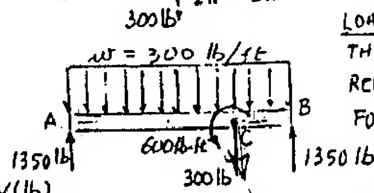


REACTIONS AT SUPPORTS

BECAUSE OF SYMMETRY OF LOAD:

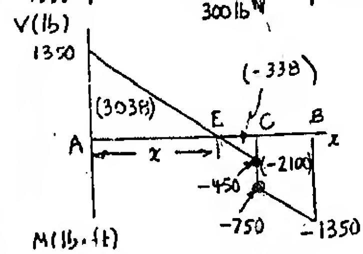
$A = B = \frac{1}{2}(300 \times 8 + 300)$

$A = B = 1350\ lb\uparrow$ ◁



LOAD DIAGRAM FOR AB

THE 300-lb FORCE AT D IS REPLACED BY AN EQUIVALENT FORCE-COUPLE SYSTEM AT C.



SHEAR DIAGRAM

AT A: $V_A = A = 1350\ lb$

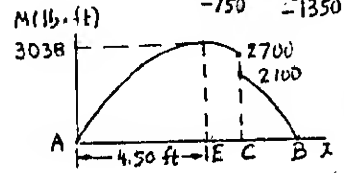
TO DETERMINE POINT E WHERE V=0:

$V_E - V_A = -wx$

$0 - 1350\ lb = -(300\ lb/ft)x$

$x = 4.50\ ft$ ◁

WE COMPUTE ALL AREAS



B.M. DIAGRAM

AT A: $M_A = 0$

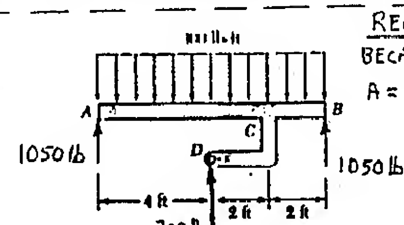
NOTE 600-lb·ft DROP AT C DUE TO COUPLE

$|M|_{max} = 3040\ lb\cdot ft$, ◀

4.50 ft FROM A. ◀

7.84

SOLVE PROB. 7.83, ASSUMING THAT 300-lb FORCE APPLIED AT D IS DIRECTED UPWARD.

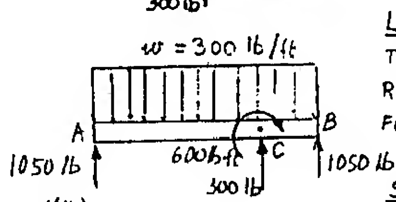


REACTIONS AT SUPPORTS

BECAUSE OF SYMMETRY OF LOAD:

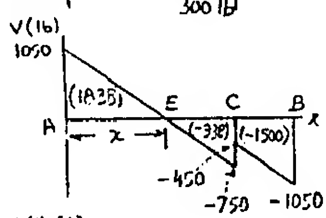
$A = B = \frac{1}{2}(300 \times 8 - 300)$

$A = B = 1050\ lb\uparrow$ ◁



LOAD DIAGRAM

THE 300-lb FORCE AT D IS REPLACED BY AN EQUIVALENT FORCE-COUPLE SYSTEM AT C



SHEAR DIAGRAM

AT A: $V_A = A = 1050\ lb$

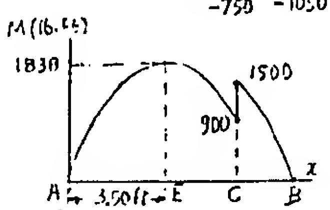
TO DETERMINE POINT E WHERE V=0:

$V_E - V_A = -wx$

$0 - 1050\ lb = -(300\ lb/ft)x$

$x = 3.50\ ft$ ◁

WE COMPUTE ALL AREAS



B.M. DIAGRAM

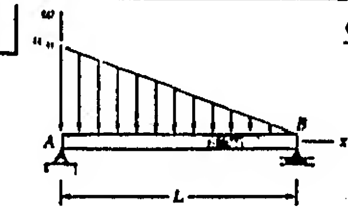
AT A: $M_A = 0$

NOTE 600-lb INCREASE AT C DUE TO COUPLE.

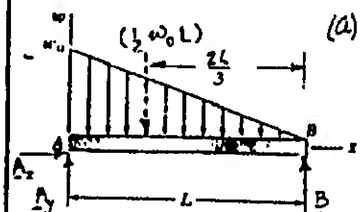
$|M|_{max} = 1838\ lb\cdot ft$ ◀

3.50 ft FROM A ◀

7.85



GIVEN: BEAM AND LOADING SHOWN.
(a) WRITE EQUATIONS FOR $V(x)$ AND $M(x)$
(b) DETERMINE MAGNITUDE AND LOCATION OF M_{max}



(a) FREE BODY: BEAM

$\Sigma F_x = 0$: $A_x = 0$

$+\circlearrowleft \Sigma M_B = 0$:

$(\frac{1}{2} w_0 L)(\frac{2L}{3}) - A_y L = 0$

$A_y = \frac{1}{3} w_0 L$

THUS: $V_A = A_y = +\frac{1}{3} w_0 L, \quad M_A = 0 \quad (1)$

LOAD: $w(x) = w_0 (1 - \frac{x}{L})$

SHEAR: FROM EQ.(7.2): $V(x) - V_A = -\int_0^x w(x)dx = -w_0 \int_0^x (1 - \frac{x}{L})dx$

INTEGRATING AND RECALLING (1):

$$V(x) - \frac{1}{3} w_0 L = -w_0 (x - \frac{x^2}{2L})$$

$$V(x) = \frac{w_0}{6L}(3x^2 - 6Lx + 2L^2) \quad (2)$$

BENDING MOMENT: FROM EQ.(7.4) AND RECALLING THAT $M_A = 0$,

$M(x) - M_A = \int_0^x V(x)\,dx \qquad M(x) = \frac{w_0}{6L}(x^3 - 3Lx^2 + 2L^2 x) \quad (3)$

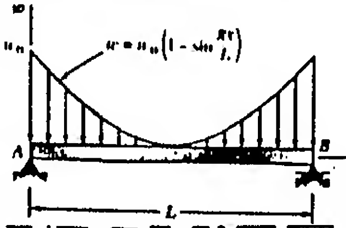
(b) MAXIMUM BENDING MOMENT

$\frac{dM}{dx} = V = 0$. EQ.(2): $3x^2 - 6Lx + 2L^2 = 0$

$$x = \frac{6 - \sqrt{36 - 24}}{6} L = 0.42265\,L$$

CARRYING INTO (3): $M_{max} = 0.0642\, w_0 L^2$, AT $x = 0.423\,L$

7.87



GIVEN: BEAM AND LOADING SHOWN.
(a) WRITE EQUATIONS FOR $V(x)$ AND $M(x)$
(b) DETERMINE MAGNITUDE AND LOCATION OF M_{max}

(a) REACTIONS AT SUPPORTS: $A = B = \frac{1}{2}W$, WHERE W = TOTAL LOAD

$W = \int_0^L w\,dx = w_0 \int_0^L (1 - \sin\frac{\pi x}{L})dx = w_0 \left[x + \frac{L}{\pi}\cos\frac{\pi x}{L}\right]_0^L = w_0 L(1 - \frac{2}{\pi})$

THUS $V_A = A = \frac{1}{2}W = \frac{1}{2} w_0 L (1 - \frac{2}{\pi}), \qquad M_A = 0$

LOAD: $w(x) = w_0 (1 - \sin\frac{\pi x}{L})$

SHEAR: FROM EQ.(7.2): $V(x) - V_A = -\int_0^x w(x)dx = -w_0 \int_0^x (1 - \sin\frac{\pi x}{L})dx$

INTEGRATING AND RECALLING FIRST OF EQS.(1),

$$V(x) - \frac{1}{2} w_0 L (1 - \frac{2}{\pi}) = -w_0 \left[x + \frac{L}{\pi}\cos\frac{\pi x}{L}\right]_0^x$$

$$V(x) = \frac{1}{2} w_0 L (1 - \frac{2}{\pi}) - w_0 (x + \frac{L}{\pi}\cos\frac{\pi x}{L}) + w_0 \frac{L}{\pi}$$

$$V(x) = w_0 (\frac{L}{2} - x - \frac{L}{\pi}\cos\frac{\pi x}{L}) \quad (2)$$

BENDING MOMENT: FROM EQ.(7.4) AND RECALLING THAT $M_A = 0$

$$M(x) - M_A = \int_0^x V(x)dx = w_0 \left[\frac{L}{2}x - \frac{1}{2}x^2 - (\frac{L}{\pi})^2 \sin\frac{\pi x}{L}\right]_0^x$$

$$M(x) = \frac{1}{2} w_0 (Lx - x^2 - \frac{2L^2}{\pi^2}\sin\frac{\pi x}{L}) \quad (3)$$

(b) MAXIMUM BENDING MOMENT

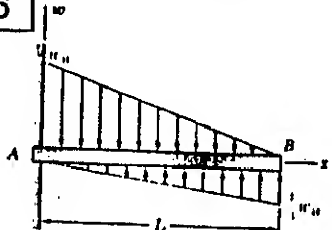
$\frac{dM}{dx} = V = 0$. THIS OCCURS AT $x = \frac{L}{2}$ AS WE MAY CHECK FROM (2):

$$V(\tfrac{L}{2}) = w_0 (\tfrac{L}{2} - \tfrac{L}{2} - \tfrac{L}{\pi}\cos\tfrac{\pi}{2}) = 0$$

FROM (3): $M(\frac{L}{2}) = \frac{1}{2} w_0 (\frac{L^2}{2} - \frac{L^2}{4} - \frac{2L^2}{\pi^2}\sin\frac{\pi}{2}) = \frac{1}{8} w_0 L^2 (1 - \frac{8}{\pi^2}) = 0.0237 w_0 L^2$

$M_{max} = 0.0237 w_0 L^2$, AT $x = L/2$

7.86



GIVEN: BEAM AND LOADING SHOWN.
(a) WRITE EQUATIONS FOR $V(x)$ AND $M(x)$.
(b) DETERMINE MAGNITUDE AND LOCATION OF M_{max}.

(a) WE NOTE THAT AT B $(x = L)$: $V_B = 0, \; M_B = 0 \quad (1)$

LOAD: $w(x) = w_0 (1 - \frac{x}{L}) - \frac{1}{3} w_0 (\frac{x}{L}) = w_0 (1 - \frac{4x}{3L})$

SHEAR: WE USE EQ.(7.2) BETWEEN $C (x = x)$ AND $B (x = L)$:

$V_B - V_C = -\int_x^L w(x)dx \qquad 0 - V(x) = -\int_x^L w(x)\,dx$

$$V(x) = w_0 \int_x^L (1 - \frac{4x}{3L})dx = w_0 \left[x - \frac{2x^2}{3L}\right]_x^L = w_0 (L - \frac{2L}{3} - x + \frac{2x^2}{3L})$$

$$V(x) = \frac{w_0}{3L}(2x^2 - 3Lx + L^2) \quad (2)$$

BENDING MOMENT: WE USE EQ.(7.4) BETWEEN $C (x = x)$ AND $B (x = L)$:

$M_B - M_C = \int_x^L V(x)dx \qquad 0 - M(x) = \frac{w_0}{3L}\int_x^L (2x^2 - 3Lx + L^2)dx$

$$M(x) = -\frac{w_0}{3L}\left[\frac{2}{3}x^3 - \frac{3}{2}Lx^2 + L^2 x\right]_x^L = -\frac{w_0}{18L}\left[4x^3 - 9Lx^2 + 6L^2 x\right]_x^L$$

$$= -\frac{w_0}{18L}\left[(4L^3 - 9L^3 + 6L^3) - (4x^3 - 9Lx^2 + 6L^2 x)\right]$$

$$M(x) = \frac{w_0}{18L}(4x^3 - 9Lx^2 + 6L^2 x - L^3) \quad (3)$$

(b) MAXIMUM BENDING MOMENT

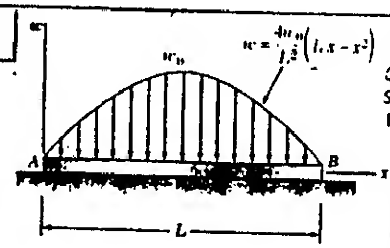
$\frac{dM}{dx} = V = 0$. EQ.(2): $2x^2 - 3Lx + L^2 = 0$

$$x = \frac{3 - \sqrt{9 - 8}}{4} L = \frac{L}{2}$$

CARRYING INTO (3):

$M_{max} = w_0 L^2/72$, AT $x = L/2$

7.88



GIVEN: BEAM RESTING ON GROUND AND SUPPORTS PARABOLIC LOAD SHOWN.
(a) WRITE EQUATIONS FOR $V(x)$ AND $M(x)$
(b) DETERMINE MAGNITUDE AND LOCATION OF M_{max}.

(a) FROM FIG. 5.8A: TOTAL LOAD $= W = \frac{2}{3} w_0 L$

GROUND PRESSURE $= w_g = \frac{W}{L} = \frac{2}{3} w_0$.

WE ALSO NOTE THAT $V_A = M_A = 0 \quad (1)$

NET LOAD: $w_N(x) = w - w_g = \frac{4w_0}{L^2}(Lx - x^2 - \frac{L^2}{6})$

SHEAR: FROM EQ.(7.2): $V(x) - V(A) = -\int_0^x w_N(x)\,dx$

RECALLING (1): $V(x) = -\frac{4w_0}{L^2}\int_0^x (-\frac{L^2}{6} + Lx - x^2)dx$

$V(x) = -\frac{4w_0}{L^2}(-\frac{L^2}{6} + \frac{L}{2}x^2 - \frac{1}{3}x^3), \quad V(x) = \frac{2w_0}{3L^2}(L^2 x - 3Lx^2 + 2x^3) \quad (2)$

BENDING MOMENT: FROM EQ.(7.4), WITH $M_A = 0$:

$$M(x) - 0 = \int_0^x V(x)\,dx = \frac{2w_0}{3L^2}(\frac{1}{2}L^2 x^2 - Lx^3 + \frac{1}{2}x^4)$$

$$M(x) = \frac{w_0}{3L^2}(L^2 x^2 - 2Lx^3 + x^4) \quad (3)$$

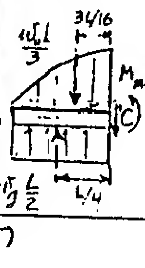
(b) MAXIMUM BENDING MOMENT

$\frac{dM}{dx} = V = 0$. THIS OCCURS AT $x = \frac{L}{2}$ AS WE MAY CHECK FROM (2):

$$V(\tfrac{L}{2}) = \frac{2w_0}{3L^2}(\frac{L^3}{2} - 3\frac{L^3}{4} + 2\frac{L^3}{8}) = 0$$

FROM (3): $M(\frac{L}{2}) = \frac{w_0}{3L^2}(\frac{L^4}{4} - 2\frac{L^4}{8} + \frac{L^4}{16}) = w_0 L^2/48$

$M_{max} = w_0 L^2/48$, AT $x = L/2$



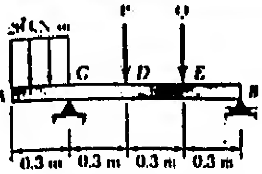
CHECK

$+\circlearrowleft \Sigma M_C = 0$: $M_{max} + \frac{w_0 L}{3}(\frac{3L}{16}) - \frac{w_g L}{2}(\frac{L}{4}) = 0$

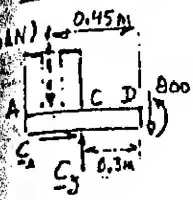
RECALLING THAT $w_g = \frac{2}{3} w_0$,

$M_{max} = (\frac{1}{3} w_0 L)(\frac{L}{4}) - (\frac{w_0 L}{3})(\frac{3L}{16}) = w_0 L^2/48 \quad$ (O.K.)

7.89



GIVEN:
BEAM AND LOADING SHOWN.
WE KNOW THAT $M_D = +800$ N·m AND $M_E = +1300$ N·m.
(a) FIND P AND Q.
(b) DRAW V AND M DIAGRAMS



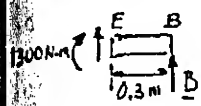
(a) FREE BODY: PORTION AD

$\Sigma F_x = 0$: $\quad C_x = 0$

$+\circlearrowleft \Sigma M_D = 0$:

$-C_y(0.3\text{ m}) + 0.800\text{ kN·m} + (6\text{ kN})(0.45\text{ m}) = 0$

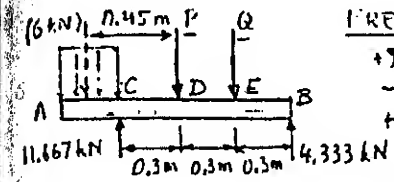
$C_y = +11.667$ kN $\quad \underline{C = 11.667\text{ kN}\uparrow}$ ◁



FREE BODY: PORTION EB

$+\circlearrowleft \Sigma M_E = 0$: $B(0.3\text{ m}) - 1.300\text{ kN·m} = 0$

$\underline{B} = 4.333\text{ kN}\uparrow$ ◁



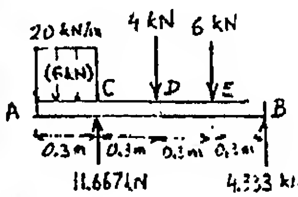
FREE BODY: ENTIRE BEAM

$+\circlearrowleft \Sigma M_D = 0$: $(6\text{ kN})(0.45\text{ m}) - (11.667\text{ kN})(0.3\text{ m}) - Q(0.3\text{ m}) + (4.333\text{ kN})(0.6\text{ m}) = 0$

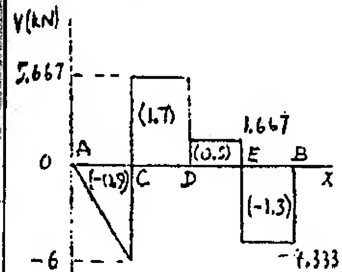
$\underline{Q} = 6.00\text{ kN}\downarrow$ ◀

$+\uparrow \Sigma F_y = 0$: $11.667\text{ kN} + 4.333\text{ kN} - 6\text{ kN} - P - 6\text{ kN} = 0$

$\underline{P} = 4.00\text{ kN}\downarrow$ ◀



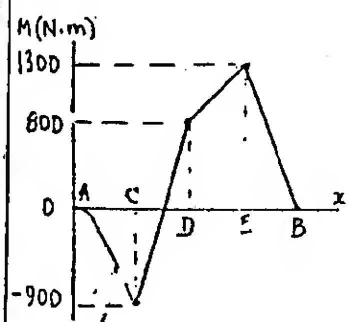
LOAD DIAGRAM



SHEAR DIAGRAM

AT A: $V_A = 0$

$|V|_{max} = 6$ kN ◁



B.M. DIAGRAM

AT A: $M_A = 0$

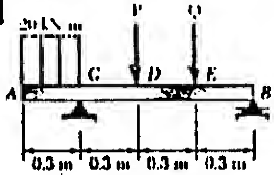
$|M|_{max} = 1300$ N·m ◁

WE CHECK THAT $M_D = +800$ N·m AND $M_E = +1300$ N·m AS GIVEN.

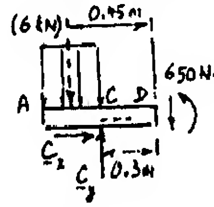
AT C: $M_C = -900$ N·m ◀

PARABOLA WITH HORIZONTAL TANGENT AT A.

7.90



GIVEN:
BEAM AND LOADING SHOWN.
WE KNOW THAT $M_D = +650$ N·m AND $M_E = +1450$ N·m.
(a) FIND P AND Q.
(b) DRAW V AND M DIAGRAMS



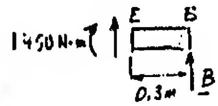
(a) FREE BODY: PORTION AD

$\Sigma F_x = 0$: $\quad C_x = 0$

$+\circlearrowleft \Sigma M_D = 0$:

$-C(0.3\text{ m}) + 0.650\text{ kN·m} + (6\text{ kN})(0.45\text{ m}) = 0$

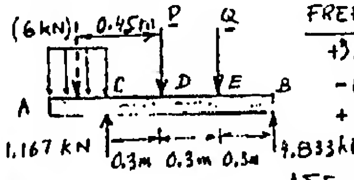
$C_y = +11.167$ kN $\quad \underline{C = 11.167\text{ kN}\uparrow}$ ◁



FREE BODY: PORTION EB

$+\circlearrowleft \Sigma M_E = 0$: $B(0.3\text{ m}) - 1.450\text{ kN·m} = 0$

$\underline{B} = 4.833\text{ kN}\uparrow$ ◁



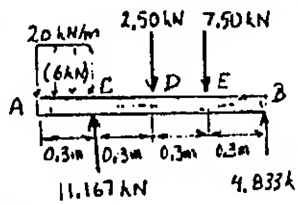
FREE BODY: ENTIRE BEAM

$+\circlearrowleft \Sigma M_D = 0$: $(6\text{ kN})(0.45\text{ m}) - (11.167\text{ kN})(0.3\text{ m}) - Q(0.3\text{ m}) + (4.833\text{ kN})(0.6\text{ m}) = 0$

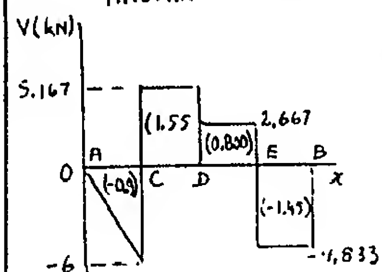
$\underline{Q} = 7.50\text{ kN}\downarrow$ ◀

$+\uparrow \Sigma F_y = 0$: $11.167\text{ kN} + 4.833\text{ kN} - 6\text{ kN} - P - 7.50\text{ kN} = 0$

$\underline{P} = 2.50\text{ kN}\downarrow$ ◀



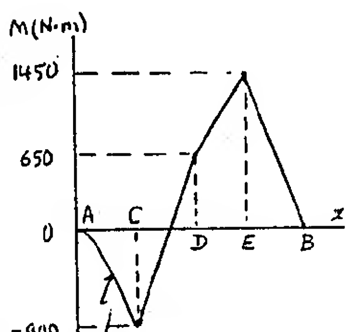
LOAD DIAGRAM



SHEAR DIAGRAM

AT A: $V_A = 0$

$|V|_{max} = 6$ kN ◁



B.M. DIAGRAM

AT A: $M_A = 0$

$|M|_{max} = 1450$ N·m ◁

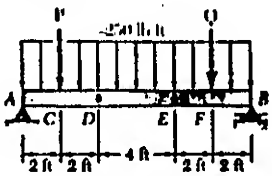
WE CHECK THAT $M_D = +650$ N·m AND $M_E = +1450$ N·m AS GIVEN.

AT C: $M_C = -900$ N·m ◀

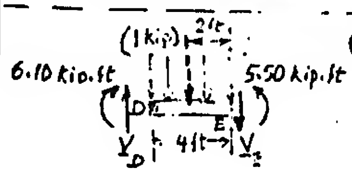
PARABOLA WITH HORIZONTAL TANGENT AT A.

* 7.91

GIVEN:
BEAM AND LOADING SHOWN.
WE KNOW THAT $M_D = +6.10$ kip·ft
AND $M_E = +5.50$ kip·ft
(a) FIND P AND Q.
(b) DRAW V AND M DIAGRAMS



(a) FREE BODY: PORTION DE



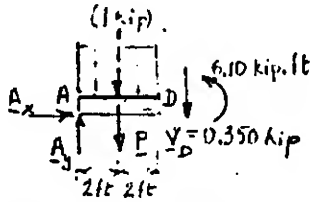
$+\circlearrowleft \Sigma M_E = 0$: 5.50 kip·ft $- 6.10$ kip·ft $+ (1\text{ kip})(2\text{ ft}) - V_D(4\text{ ft}) = 0$

$V_D = +0.350$ kip

$+\uparrow \Sigma F_y = 0$: 0.350 kip $- 1$ kip $- V_E = 0$

$V_E = -0.650$ kip

FREE BODY: PORTION AD



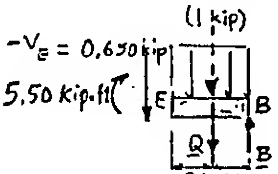
$+\circlearrowleft \Sigma M_A = 0$: 6.10 kip·ft $- P(2\text{ ft}) - (1\text{ kip})(2\text{ ft}) - (0.350\text{ kip})(4\text{ ft}) = 0$

$P = 1.350$ kips $\downarrow$ ◀

$\Sigma F_x = 0$: $A_x = 0$ $\quad +\uparrow \Sigma F_y = 0$: $A_y - 1$ kip $- 1.350$ kip $- 0.350$ kip $= 0$

$A_y = +2.70$ kips $\quad A = 2.70$ kips $\uparrow$ ◁

FREE BODY: PORTION EB



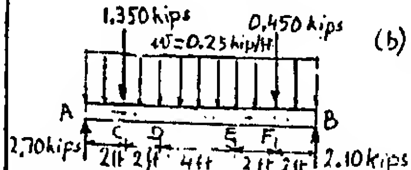
$+\circlearrowleft \Sigma M_B = 0$: $(0.650\text{ kip})(4\text{ ft}) + (1\text{ kip})(2\text{ ft}) + Q(2\text{ ft}) - 5.50$ kip·ft $= 0$

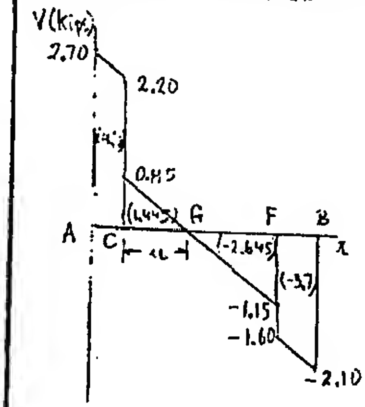
$Q = 0.450$ kip $\downarrow$ ◀

$+\uparrow \Sigma F_y = 0$: $B - 0.450 - 1 - 0.650 = 0$

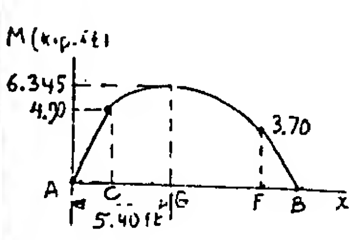
$B = 2.10$ kips $\uparrow$ ◁

(b) LOAD DIAGRAM





SHEAR DIAGRAM
AT A: $V_A = A = +2.70$ kips
TO DETERMINE POINT G WHERE $V = 0$, WE WRITE
$V_G - V_C = -wu$
$0 - (0.85\text{ kips}) = -(2.5\text{ kip/ft})u$
$u = 0.340$ ft ◁
WE NEXT COMPUTE ALL AREAS
$|V|_{max} = 2.70$ kips at A ◀

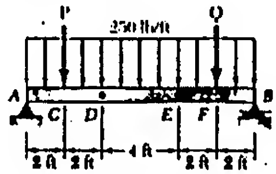


B.M. DIAGRAM
AT A: $M_A = 0$
LARGEST VALUE OCCURS AT G WITH $AG = 2 + 3.40 = 5.40$ ft
$|M|_{max} = 6.345$ kip·ft, ◀
5.40 ft FROM A ◀

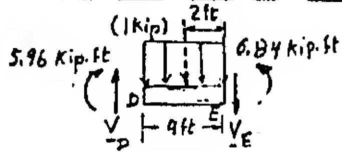
B.M. DIAGRAM CONSISTS OF 3 DISTINCT ARCS OF PARABOLAS.

* 7.92

GIVEN:
BEAM AND LOADING SHOWN.
WE KNOW THAT $M_D = +5.96$ kip·ft
AND $M_E = +6.84$ kip·ft
(a) FIND P AND Q
(b) DRAW V AND M DIAGRAMS



(a) FREE BODY: PORTION DE



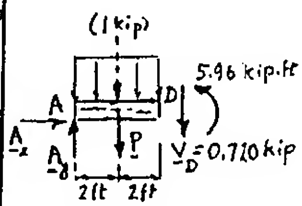
$+\circlearrowleft \Sigma M_E = 0$: 6.84 kip·ft $- 5.96$ kip·ft $+ (1\text{ kip})(2\text{ ft}) - V_D(4\text{ ft}) = 0$

$V_D = +0.720$ kip

$+\uparrow \Sigma F_y = 0$: 0.720 kip $- 1$ kip $- V_E = 0$

$V_E = -0.280$ kip

FREE BODY: PORTION AD



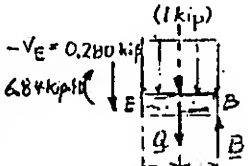
$+\circlearrowleft \Sigma M_A = 0$: 5.96 kip·ft $- P(2\text{ ft}) - (1\text{ kip})(2\text{ ft}) - (0.720\text{ kip})(4\text{ ft}) = 0$

$P = 0.540$ kip $\downarrow$

$\Sigma F_x = 0$: $A_x = 0$, $+\uparrow \Sigma F_y = 0$: $A_y - 1$ kip $- 0.540$ kip $- 0.720$ kip $= 0$

$A_y = +2.26$ kips $\quad A = 2.26$ kips $\uparrow$

FREE BODY: PORTION EB



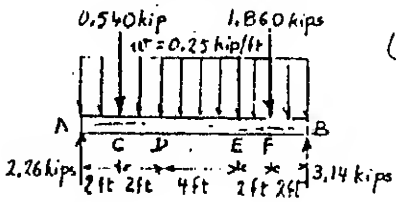
$+\circlearrowleft \Sigma M_B = 0$: $(0.280\text{ kip})(4\text{ ft}) + (1\text{ kip})(2\text{ ft}) + Q(2\text{ ft}) - 6.84$ kip·ft $= 0$

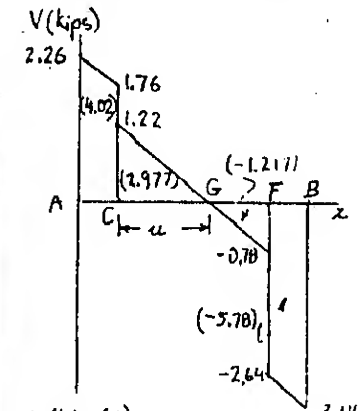
$Q = 1.860$ kips $\downarrow$

$+\uparrow \Sigma F_y = 0$: $B - 1.860 - 1 - 0.280 = 0$

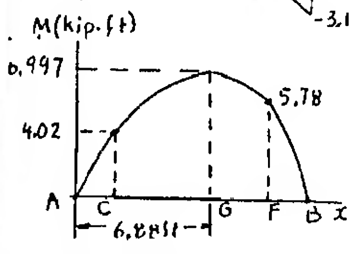
$B = 3.14$ kips $\uparrow$

(b) LOAD DIAGRAM





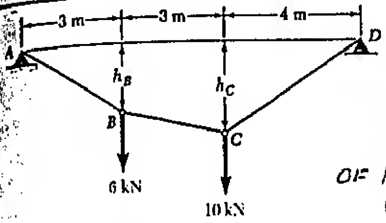
SHEAR DIAGRAM
AT A: $V_A = A = +2.26$ kips
TO DETERMINE POINT G WHERE $V = 0$, WE WRITE
$V_G - V_C = -wu$
$0 - (1.22\text{ kips}) = -(0.25\text{ kip/ft})u$
$u = 4.88$ ft
WE NEXT COMPUTE ALL AREAS.
$|V|_{max} = 3.14$ kips at B ◀



B.M. DIAGRAM
AT A: $M_A = 0$
LARGEST VALUE OCCURS AT G WITH $AG = 2 + 4.88 = 6.88$ ft
$|M|_{max} = 6.997$ kip·ft, ◀
6.88 ft FROM A ◀

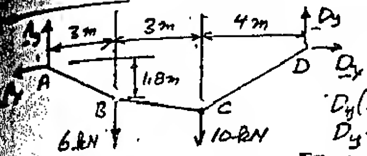
B.M. DIAGRAM CONSISTS OF 3 DISTINCT ARCS OF PARABOLAS.

7.93



GIVEN: $h_B = 1.8\,m$

FIND:
(a) DISTANCE h_C
(b) COMPONENTS OF REACTION AT D
(c) T_{max}



FREE BODY: ENTIRE CABLE

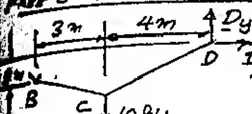
$+\circlearrowleft \Sigma M_A = 0$

$D_y(10\,m) - (6\,kN)(3\,m) - (10\,kN)(6\,m) = 0$

$D_y = 7.8\,kN \qquad D_y = 7.8\,kN \uparrow$ ◀

$\Sigma F_y = 0$: $A_y + 7.8 - 6 - 10 = 0$; $A_y = 8.2\,kN \uparrow$

FREE BODY: PORTION BCD



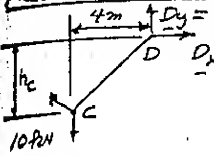
$+\circlearrowleft \Sigma M_B = 0$

$(7.8\,kN)(7\,m) - (10\,kN)(3\,m) - D_x(1.8\,m) = 0$

$D_x = 13.667\,kN \qquad D_x = 13.67\,kN \rightarrow$ ◀

$A_x = D_x = 13.667\,kN$

FREE BODY: PORTION CD

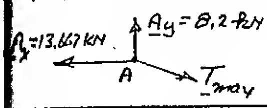


$+\circlearrowleft \Sigma M_C = 0$

$(7.8\,kN)(4\,m) - (13.667\,kN)h_c = 0$

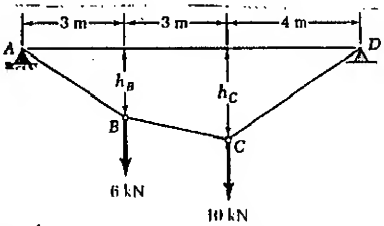
$h = 2.28\,m$ ◀

MAX. TENSION OCCURS AT A (LARGEST SLOPE OF CABLE)



$T_{max} = (8.2^2 + 13.667^2)^{1/2} \qquad T_{max} = 15.94\,kN$ ◀

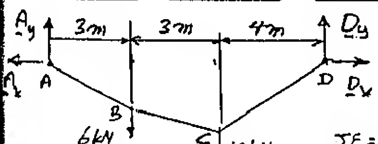
7.94



GIVEN: $T_{max} = 15\,kN$

FIND:
(a) h_B
(b) h_C

FREE BODY: ENTIRE CABLE



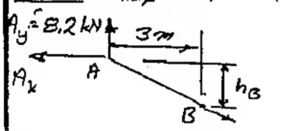
$+\circlearrowleft \Sigma M_A = 0$: $-(6\,kN)(3\,m) - (10\,kN)(6\,m) + D_y(10\,m) = 0$

$D_y = 7.8\,kN$

$\Sigma F_y = 0$: $A_y + 7.8\,kN - 6\,kN - 10\,kN = 0$

$A_y = 8.2\,kN$

ASSUME $T_{max} = 15\,kN$ OCCURS IN PORTION AB



FREE BODY: PORTION AB

$T_m^2 = A_x^2 + A_y^2$

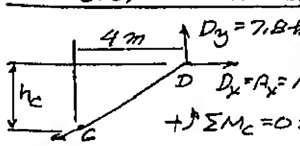
$(15\,kN)^2 = A_x^2 + (8.2\,kN)^2$

$A_x = 12.56\,kN$

$+\circlearrowleft \Sigma M_B = 0$: $(12.56\,kN)h_B - (8.2\,kN)(3\,m) = 0$

$h_B = 1.959\,m$ ◀

FREE BODY: PORTION CD



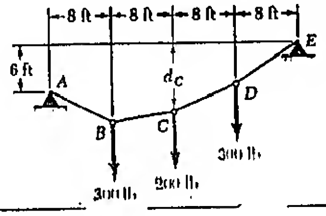
$+\circlearrowleft \Sigma M_C = 0$: $(7.8\,kN)(4\,m) - (12.56\,kN)h_c = 0$

$h_c = 2.48\,kN$ ◀

$T_{CD} = (7.8^2 + 12.56^2)^{1/2} = 14.78\,kN < 15\,kN$

OK T_{max} OCCURS AT A

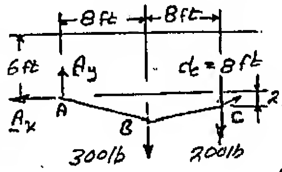
7.95



GIVEN: $d_c = 8\,ft$.

FIND: REACTION
(a) AT A
(b) AT E

FREE BODY: PORTION ABC

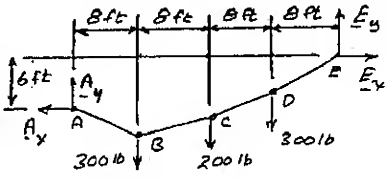


$+\circlearrowleft \Sigma M_C = 0$:

$2A_x - 16A_y + 300 \times 8 = 0$

$A_x = 8A_y - 1200 \qquad (1)$

FREE BODY: ENTIRE CABLE



$+\circlearrowleft \Sigma M_E = 0$: $+6A_x + 32A_y - (300\,lb + 200\,lb + 300\,lb)16\,ft = 0$

$3A_x + 16A_y - 6400 = 0$

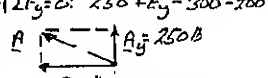
SUBSTITUTE FROM EQ(1):

$3(8A_y - 1200) + 16A_y - 6400 = 0 \qquad A_y = 250\,lb \uparrow$

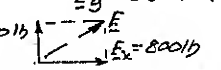
EQ(1) $A_x = 8(250) - 1200 \qquad A_x = 800\,lb \leftarrow$

$\xrightarrow{+} \Sigma F_x = 0$: $-A_x + E_x = 0$; $-800\,lb + E_x = 0$; $\qquad E_x = 800\,lb \rightarrow$

$+\uparrow \Sigma F_y = 0$: $250 + E_y - 300 - 200 - 300 = 0 \qquad E_y = 550\,lb \uparrow$

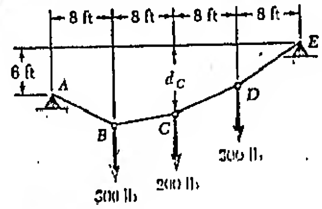


$A = 838\,lb \measuredangle 17.4°$ ◀



$E = 971\,lb \measuredangle 34.5°$ ◀

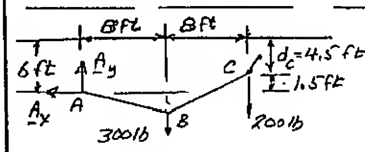
7.96



GIVEN: $d_c = 4.5\,ft$.

FIND: REACTION
(a) AT A,
(b) AT E.

FREE BODY: PORTION ABC

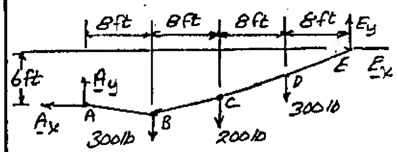


$+\circlearrowleft \Sigma M_C = 0$

$-1.5A_x - 16A_y + 300 \times 8 = 0$

$A_x = (2400 - 16A_y)/1.5 \qquad (1)$

FREE BODY: ENTIRE CABLE



$+\circlearrowleft \Sigma M_E = 0$: $+6A_x + 32A_y - (300\,lb + 200\,lb + 300\,lb)16\,ft = 0$

$3A_x + 16A_y - 6400 = 0$

SUBSTITUTE FROM EQ.(1)

$3(2400 - 16A_y)/1.5 + 16A_y - 6400 = 0$

$A_y = -100\,lb$; THUS A_y ACTS DOWNWARD, $A_y = 100\,lb \downarrow$

EQ.(1) $A_x = (2400 - 16(-100))/1.5 = 2667\,lb \qquad A_x = 2667\,lb \leftarrow$

$\xrightarrow{+} \Sigma F_x = 0$: $-A_x + E_x = 0$; $-2667 + E_x = 0 \qquad E_x = 2667\,lb \rightarrow$

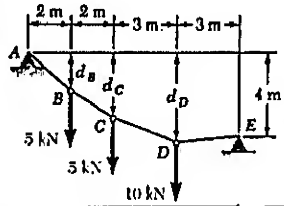
$+\uparrow \Sigma F_y = 0$: $A_y + E_y - 300 - 200 - 300 = 0$

$-100\,lb + E_y - 800\,lb = 0 \qquad E_y = 900\,lb \uparrow$

$A = 2670\,lb \measuredangle 2.1°$ ◀

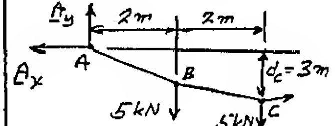
$E = 2810\,lb \measuredangle 18.6°$ ◀

7.97



GIVEN: $d_C = 3\,m$

FIND: (a) d_B AND d_D
(b) REACTION AT E

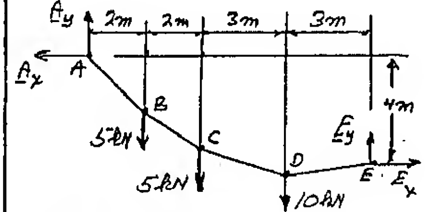


FREE BODY: PORTION ABC

$+\circlearrowleft \Sigma M_C = 0$

$3A_x - 4A_y + (5\,kN)(2\,m) = 0$

$A_x = \frac{4}{3}A_y - \frac{10}{3}$ (1)



FREE BODY: ENTIRE CABLE

$+\circlearrowleft \Sigma M_E = 0$: $4A_x - 10A_y + (5\,kN)(8\,m) + (5\,kN)(6\,m) + (10\,kN)(3\,m) = 0$

$4A_x - 10A_y + 100 = 0$

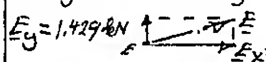
SUBSTITUTE FROM EQ.(1): $4(\frac{4}{3}A_y - \frac{10}{3}) - 10A_y + 100 = 0$

$A_y = +18.571\,kN$; $\quad A_y = 18.571\,kN \uparrow$

EQ.(1) $A_x = \frac{4}{3}(18.571) - \frac{10}{3} = +21.429\,kN$; $\quad A_x = 21.429\,kN \leftarrow$

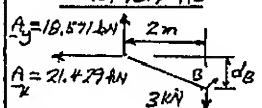
$\xrightarrow{+} \Sigma F_x = 0$: $-A_x + E_x = 0$; $-21.429 + E_x = 0$; $E_x = 21.429\,kN \rightarrow$

$+\uparrow \Sigma F_y = 0$: $18.571\,kN + E_y + 5\,kN + 5\,kN + 10\,kN = 0$; $E_y = 1.429\,kN \uparrow$



$E = 21.5\,kN \angle 3.8°$ ◀

PORTION AB

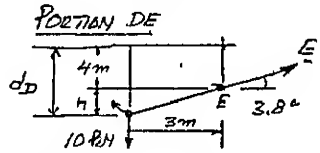


$+\circlearrowleft \Sigma M_B = 0$:

$(18.571\,kN)(2\,m) - (21.429\,kN)\,d_B = 0$

$d_B = 1.733\,m$ ◀

PORTION DE



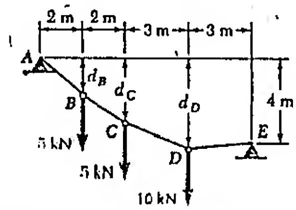
GEOMETRY

$h = (3\,m)\tan 3.8° = 0.199\,m$

$d_D = 4\,m + 0.199\,m$

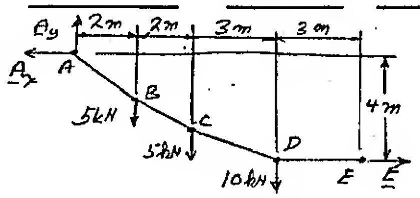
$d_D = 4.20\,m$ ◀

7.98



GIVEN: PORTION DE IS HORIZONTAL

FIND: (a) d_C
(b) REACTION AT A AND E



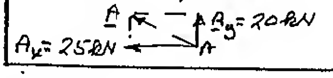
FREE BODY: ENTIRE CABLE

$+\uparrow \Sigma F_y = 0$: $A_y - 5\,kN - 5\,kN - 10\,kN = 0$ $\quad A_y = 20\,kN \uparrow$

$+\circlearrowleft \Sigma M_A = 0$: $E(4\,m) - (5\,kN)(2\,m) - (5\,kN)(4\,m) - (10\,kN)(7\,m) = 0$

$E = +25\,kN$ $\quad E = 25\,kN \rightarrow$ ◀

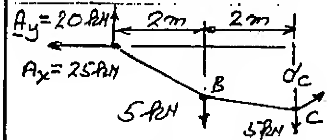
$\Sigma F_x = 0$: $-A_x + 25\,kN = 0$ $\quad A_x = 25\,kN \leftarrow$



$A = 32.0\,kN \angle 38.7°$ ◀

(CONTINUED)

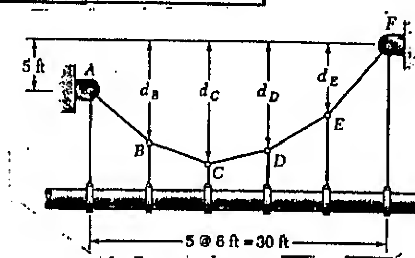
7.98 CONTINUED



FREE BODY: PORTION A...

$+\circlearrowleft \Sigma M_C = 0$: $(25\,kN)d_C$
$-(20\,kN)(4\,m) + (5\,kN)(2...$
$25\,d_C - 70 = 0$
$d_C = 2.80\,m$

7.99 and 7.100

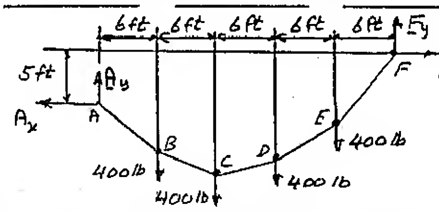


GIVEN: TENSION IN EACH HANGER = 400 lb

FIND: (a) MAXIMUM TENSION, (b) DISTANCE d_D

PROB. 7.99 FOR $d_C = 12$ ft

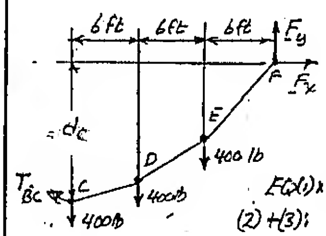
PROB. 7.100 FOR $d_C = 9$ ft



FREE BODY: ENTIRE CABLE

$+\circlearrowleft \Sigma M_A = 0$: $5F_x - 30F_y + 400 \times 6 + 400 \times 12 + 400 \times 18 + 400 \times 24 = 0$

$5F_x - 30F_y + 24000 = 0$ (1)



FREE BODY: PORTION CDEF

$+\circlearrowleft \Sigma M_C = 0$

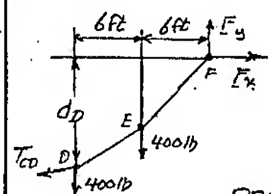
$d_C F_x - 18F_y + 400 \times 6 + 400 \times 12 = 0$

$d_C F_x - 18F_y + 7200 = 0$ (2)

EQ(1)×(−0.6): $-3F_x + 18F_y - 14400 = 0$ (3)

(2)+(3): $d_C F_x - 3F_x - 7200 = 0$

$F_x = \frac{7200}{d_C - 3}$ (4)



FREE BODY: PORTION DEF

$+\circlearrowleft \Sigma M_D = 0$: $d_D F_x - 12F_y + 400 \times 6 = 0$

$d_D = (12F_y - 2400)/F_x$ (5)

PROB 7.99 FOR $d_C = 12$ ft

EQ(4): $F_x = \frac{7200}{12-3} = 800\,lb$

EQ(1): $5(800) - 30F_y + 24000 = 0$; $F_y = 933.3$

EQ(5): $d_D = [12(933.3) - 2400]/800$ $\quad d_D = 11.00$ ft ◀

$F = 1229.3$ lb; $T_{max} = T_{EF} = F$

$\therefore T_{max} = 1229\,lb$ ◀

PROB 7.100 FOR $d_C = 9$ ft

EQ(4): $F_x = 7200/(9-3) = 1200\,lb$

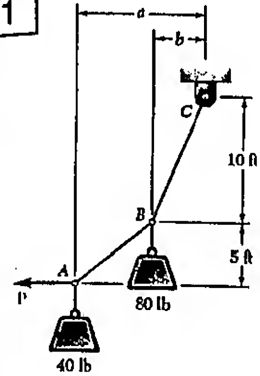
EQ(1): $5(1200) - 30F_y + 24000 = 0$; $F_y = 1000\,lb$

EQ(5): $d_D = [12(1000) - 2400]/1200$; $\quad d_D = 8.00$ ft ◀

$F = 1562$ lb; $T_{max} = T_{EF} = F$

$T_{max} = 1562$ lb ◀

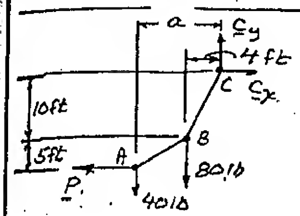
7.101



GIVEN: $b = 4\text{ ft}$

FIND: (a) FORCE P, (b) DISTANCE a.

FREE BODY: ENTIRE CABLE

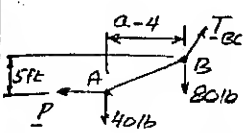


$+\circlearrowleft \Sigma M_C = 0$

$(80\text{ lb})(4\text{ ft}) + (40\text{ lb})a - P(15\text{ ft}) = 0$

$15P - 320 - 40a = 0 \quad (1)$

FREE BODY: PORTION AB



$+\circlearrowleft \Sigma M_B = 0: \; P(5\text{ ft}) - (40\text{ lb})(a - 4\text{ ft}) = 0$

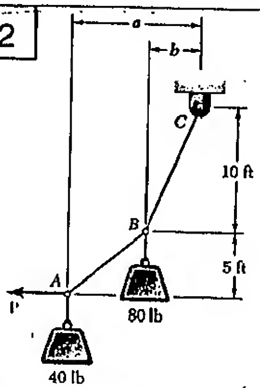
$a = 4 + \frac{P}{8} \quad (2)$

EQ(1): $15P - 320 - 40\left(4 + \frac{P}{8}\right) = 0$

$10P - 480 = 0 \qquad P = 48\text{ lb}$ ◀

EQ(2): $a = 4 + \frac{48}{8} = 4 + 6 = 10\text{ ft} \qquad a = 10\text{ ft}$ ◀

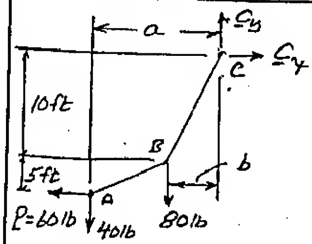
7.102



GIVEN: $P = 60\text{ lb}$.

FIND: DISTANCES a AND b.

FREE BODY: ENTIRE CABLE

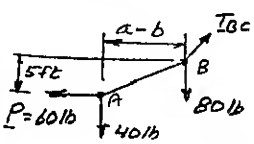


$+\circlearrowleft \Sigma M_C = 0:$

$(60\text{ lb})(15\text{ ft}) - (40\text{ lb})a - (80\text{ lb})b = 0$

$a = 22.5 - 2b \quad (1)$

FREE BODY: PORTION AB



$+\circlearrowleft \Sigma M_B = 0: (60\text{ lb})(5\text{ ft}) - (40\text{ lb})(a - b) = 0$

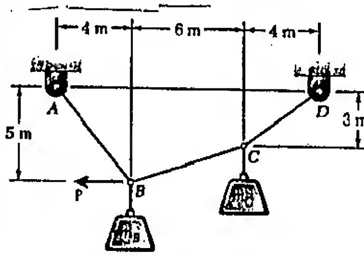
$b = a - 7.5\text{ ft} \quad (2)$

EQ(1): $a = 22.5 - 2(a - 7.5)$

$3a = 37.5 \qquad a = 12.5\text{ ft}$ ◀

EQ(2): $b = 12.5\text{ ft} - 7.5\text{ ft} \qquad b = 5\text{ ft}$ ◀

7.103 and 7.104

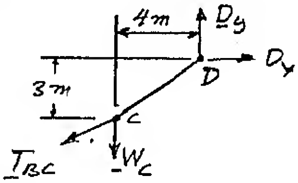


FIND: FORCE P TO MAINTAIN EQUILIBRIUM

PROB. 7.103: $m_B = 70\text{ kg}, m_C = 25\text{ kg}$

PROB. 7.104: $m_B = 18\text{ kg}, m_C = 10\text{ kg}$

FREE BODY: PORTION CD

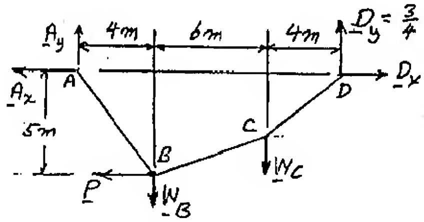


$+\circlearrowleft \Sigma M_C = 0$

$D_y(4\text{ m}) - D_x(3\text{ m}) = 0$

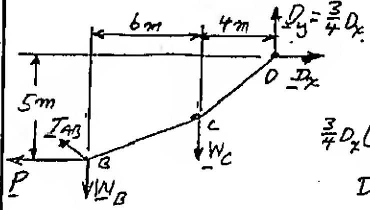
$D_y = \frac{3}{4} D_x$

FREE BODY: ENTIRE CABLE.



$+\circlearrowleft \Sigma M_A = 0: \; \frac{3}{4}D_x(14\text{ m}) - W_B(4\text{ m}) - W_C(10\text{ m}) - P(5\text{ m}) = 0 \quad (1)$

FREE BODY: PORTION BCD



$+\circlearrowleft \Sigma M_B = 0$

$\frac{3}{4}D_x(10\text{ m}) - D_x(5\text{ m}) - W_C(6\text{ m}) = 0$

$D_x = 2.4 W_C \quad (2)$

PROB. 7.103: $m_B = 70\text{ kg} \quad m_C = 25\text{ kg}$

$g = 9.81\text{ m/s}^2: \quad W_B = 70g \quad W_C = 25g$

EQ(2): $D_x = 2.4W_C = 2.4(25g) = 60g$

EQ(1): $\frac{3}{4}60g(14) - 70g(4) - 25g(10) - 5P = 0$

$100g - 5P = 0; \quad P = 20g$

$P = 20(9.81) = 196.2\text{ N}$

$P = 196.2\text{ N}$ ◀

PROB. 7.104: $m_B = 18\text{ kg} \quad m_C = 10\text{ kg}$

$g = 9.81\text{ m/s}^2: \quad W_B = 18g \quad W_C = 10g$

EQ(2): $D_x = 2.4W_C = 2.4(10g) = 24g$

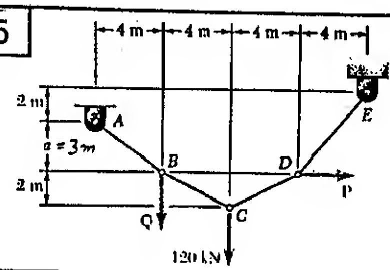
EQ(1): $\frac{3}{4}24g(14) - (18g)(4) - (10g)(10) - 5P = 0$

$80g = 5P: \quad P = 16g$

$P = 16(9.81) = 156.96\text{ N}$

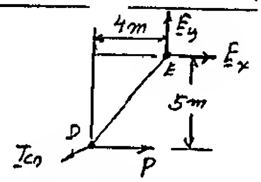
$P = 157.0\text{ N}$ ◀

7.105



GIVEN: $a = 3\,m$

FIND: MAGNITUDES OF P AND Q

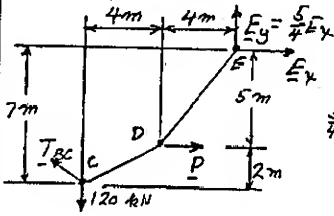


FREE BODY: PORTION DE

$+\circlearrowleft \Sigma M_D = 0$

$E_y(4m) - E_x(5m) = 0$

$E_y = \frac{5}{4}E_x$

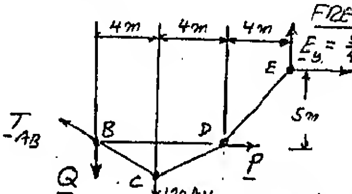


FREE BODY: PORTION CDE

$+\circlearrowleft \Sigma M_C = 0:$

$\frac{5}{4}E_x(8m) - E_x(7m) - P(2m) = 0$

$E_x = \frac{2}{3}P \qquad (1)$



FREE BODY: PORTION BCDE

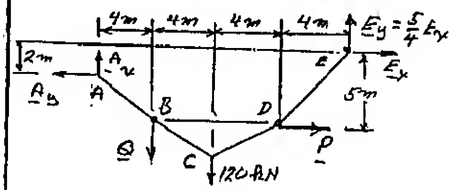
$+\circlearrowleft \Sigma M_B = 0:$

$\frac{5}{4}E_x(12m) - E_x(5m) - (120\,kN)(4m) = 0$

$10E_x - 480 = 0; \; E_x = 48\,kN$

EQ(1): $48\,kN = \frac{2}{3}P$

$P = 72\,kN$ ◀



FREE BODY: ENTIRE CABLE

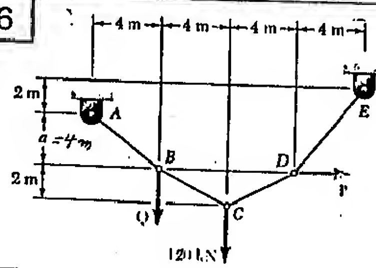
$+\circlearrowleft \Sigma M_A = 0: \; \frac{5}{4}E_x(16m) - E_x(2m) + P(3m) - Q(4m) - (120\,kN)(8m) = 0$

$(48\,kN)(20m - 2m) + (72\,kN)(3m) - Q(4m) - 960\,kN\cdot m = 0$

$4Q = 120$

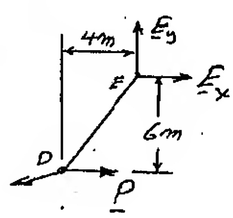
$Q = 30\,kN$ ◀

7.106



GIVEN: $a = 4\,m$

FIND: MAGNITUDES OF P AND Q



FREE BODY: PORTION DE

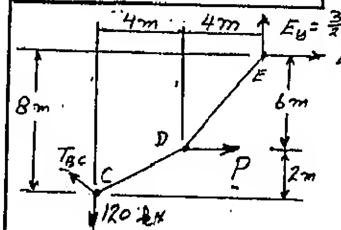
$+\circlearrowleft \Sigma M_D = 0$

$E_y(4m) - E_x(6m) = 0$

$E_y = \frac{3}{2}E_x$

(CONTINUED)

7.106 CONTINUED

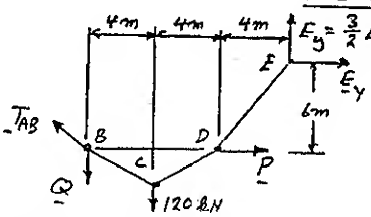


FREE BODY: PORTION CDE

$+\circlearrowleft \Sigma M_C = 0$

$\frac{3}{2}E_x(8m) - E_x(8m) - P(2m) = 0$

$E_x = \frac{1}{2}P \qquad (1)$



FREE BODY: PORTION BCDE

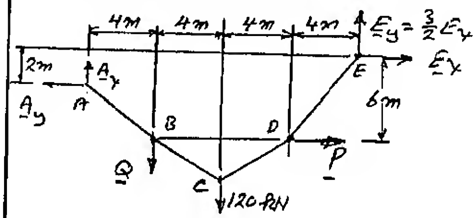
$+\circlearrowleft \Sigma M_B = 0: \; \frac{3}{2}E_x(12m) - E_x(6m) + (120\,kN)(4m) = 0$

$12E_x = 480$

$E_x = 40\,kN$

EQ(1): $E_x = \frac{1}{2}P; \; 40\,kN = \frac{1}{2}P$

$P = 80\,kN$ ◀



FREE BODY: ENTIRE CABLE

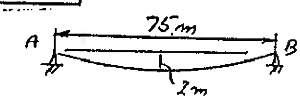
$+\circlearrowleft \Sigma M_A = 0: \; \frac{3}{2}E_x(16m) - E_x(2m) + P(4m) - Q(4m) - (120\,kN)(8m) = 0$

$(40\,kN)(24m - 2m) + (80\,kN)(4m) - Q(4m) - 960\,kN\cdot m = 0$

$4Q = 240$

$Q = 60\,kN$ ◀

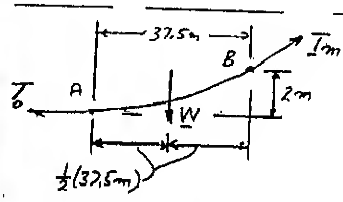
7.107



GIVEN: MASS/METER $= 0.8\,kg/m$

FIND: (a) T_{max}

(b) LENGTH OF CABLE



$w = (0.8\,kg/m)(9.81\,m/s^2)$

$= 7.848\,N/m$

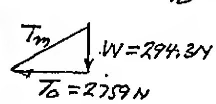
$W = (7.848\,N/m)(37.5\,m)$

$W = 294.3\,N$

(a) $+\circlearrowleft \Sigma M_B = 0 \quad T_0(2m) - W(\frac{1}{2}\,37.5m) = 0$

$T_0(2m) - (294.3\,N)\frac{1}{2}(37.5\,m) = 0$

$T_0 = 2759\,N$



$T_m^2 = (294.3\,N)^2 + (2759\,N)^2$

$T_m = 2770\,N$ ◀

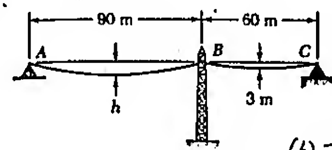
(b) $s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 + \cdots\right]$

$= 37.5m\left[1 + \frac{2}{3}\left(\frac{2m}{37.5m}\right)^2 + \cdots\right] = 37.57m$

LENGTH $= 2s_B = 2(37.57m)$

LENGTH $= 75.14\,m$ ◀

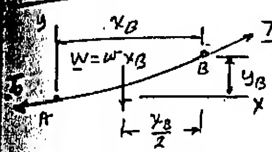
7.108



GIVEN: CABLE OF 0.4 kg/m

FIND: (a) h FOR EQUAL HORIZ. COMPS.

(b) T_m IN EACH CABLE



$W = wx_B$

$+\circlearrowleft \Sigma M_B = 0$:

$T_0 y_B - (wx_B)\frac{x_B}{2} = 0$

(a) HORIZ. COMP. $= T_0 = \dfrac{wx_B^2}{2y_B}$

CABLE AB $\quad x_B = 45\text{ m}$

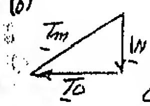
$$T_0 = \frac{w(45\text{ m})^2}{2h}$$

CABLE BC $\quad x_B = 30\text{ m},\ y_B = 3\text{ m}$

$$T_0 = \frac{w(30\text{ m})^2}{2(3\text{ m})}$$

EQUATE $T_0 = T_0$ $\quad \dfrac{w(45\text{ m})^2}{2h} = \dfrac{w(30\text{ m})^2}{2(3\text{ m})}$; $\quad h = 6.75\text{ m}$ ◀

(b)



$T_m^2 = T_0^2 + W^2$

CABLE AB: $w = (0.4\text{ kg/m})(9.81\text{ m/s}^2) = 3.924\text{ N/m}$

$x_B = 45\text{ m},\ y_B = h = 6.75\text{ m}$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(3.924\text{ N/m})(45\text{ m})^2}{2(6.75\text{ m})} = 588.6\text{ N}$$

$W = wx_B = (3.924\text{ N/m})(45\text{ m}) = 176.58\text{ N}$

$T_m^2 = (588.6\text{ N})^2 + (176.58\text{ N})^2$; $\quad$ FOR AB, $T_m = 615\text{ N}$ ◀

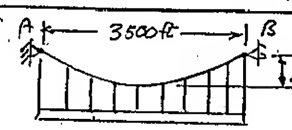
CABLE BC $\quad x_B = 30\text{ m},\ y_B = 3\text{ m}$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(3.924\text{ N/m})(30\text{ m})^2}{2(3\text{ m})} = 588.6\text{ N (checks)}$$

$W = wx_B = (3.924\text{ N/m})(30\text{ m}) = 117.72\text{ N}$

$T_m^2 = (588.6\text{ N})^2 + (117.72\text{ N})^2$; $\quad$ FOR BC, $T_m = 600\text{ N}$ ◀

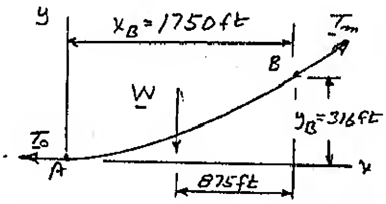
7.109



GIVEN: $w = 9.75\text{ kips/ft}$

FIND: (a) T_m

(b) LENGTH OF CABLE



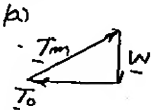
$W = wx_B$

$= (9.75\text{ kips/ft})(1750\text{ ft})$

$W = 17{,}063\text{ kips}$

$+\circlearrowleft \Sigma M_B = 0$: $T_0(316\text{ ft}) - (17063\text{ kips})(875\text{ ft}) = 0$

(a) $\quad T_0 = 47{,}247\text{ kips}$



$T_m = \sqrt{T_0^2 + W^2} = \sqrt{(47{,}247\text{ kips})^2 + (17{,}063\text{ kips})^2}$

$T_m = 50{,}230\text{ kips}$ ◀

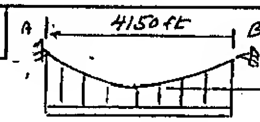
(b) $\quad s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4 + \cdots\right] \qquad \dfrac{y_B}{x_B} = \dfrac{316\text{ ft}}{1750\text{ ft}} = 0.18057$

$= (1750\text{ ft})\left[1 + \frac{2}{3}(0.18057)^2 - \frac{2}{5}(0.18057)^4 + \cdots\right]$

$s_B = 1787.3\text{ ft}$; $\quad$ LENGTH $= 2s_B = 3574.6\text{ ft}$

LENGTH $= 3575\text{ ft}$ ◀

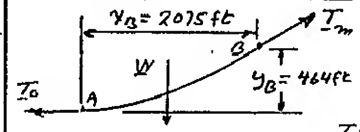
7.110



GIVEN: $w = 11.1\text{ kips/ft}$

FIND: (a) T_m

(b) LENGTH OF CABLE



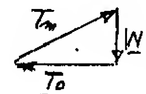
EQ.(7.8) Page 374:

AT B: $\quad y_B = \dfrac{wx_B^2}{2T_0}$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(11.1\text{ kip/ft})(2075\text{ ft})^2}{2(464\text{ ft})}$$

(a) $\quad T_0 = 51{,}500\text{ kips}$

$W = wx_B = (11.1\text{ kips/ft})(2075\text{ ft}) = 23{,}033\text{ kips}$



$T_m = \sqrt{T_0^2 + W^2} = \sqrt{(51{,}500\text{ kips})^2 + (23{,}033\text{ kips})^2}$

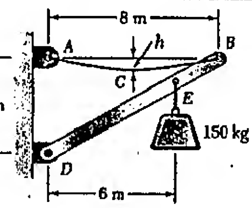
$T_m = 56{,}420\text{ kips}$ ◀

(b) $\quad s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4 + \cdots\right] \qquad \dfrac{y_B}{x_B} = \dfrac{464\text{ ft}}{2075\text{ ft}} = 0.22361$

$s_B = (2075\text{ ft})\left[1 + \frac{2}{3}(0.22361)^2 - \frac{2}{5}(0.22361)^4 + \cdots\right] = 2142.1\text{ ft}$

LENGTH $= 2s_B = 2(2142.1\text{ ft})$ $\qquad$ LENGTH $= 4284\text{ ft}$ ◀

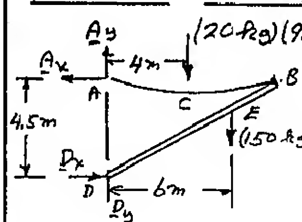
7.111



GIVEN: MASS OF CABLE ABC IS 20 kg

FIND: (a) SAG h

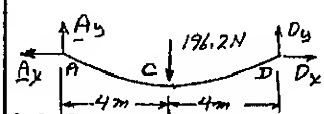
(b) SLOPE OF CABLE AT A



FREE BODY: ENTIRE FRAME

$+\circlearrowleft \Sigma M_D = 0$: $\quad A_x(4.5\text{ m}) - (196.2\text{ N})(4\text{ m}) - (1471.5\text{ N})(6\text{ m}) = 0$

$A_x = 2136.4\text{ N}$



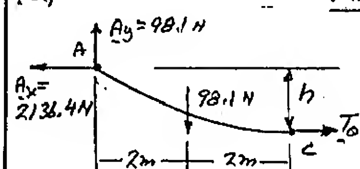
FREE BODY: ENTIRE CABLE

$+\circlearrowleft \Sigma M_D = 0$

$A_y(8\text{ m}) - (196.2\text{ N})(4\text{ m}) = 0$

$A_y = 98.1\text{ N}$

(a)



FREE BODY: PORTION AC

$\Sigma F_x = 0$: $T_0 = A_x = 2136.4\text{ N}$

$+\circlearrowleft \Sigma M_A = 0$

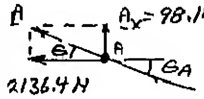
$T_0 h - (98.1\text{ N})(2\text{ m}) = 0$

$(2136.4\text{ N})h - 196.2\text{ N·m} = 0$

$h = 0.09183\text{ m}$

$h = 91.8\text{ mm}$ ◀

(b)



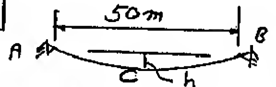
$$\tan\theta_A = \frac{A_x}{A_y} = \frac{98.1\text{ N}}{2136.4\text{ N}}$$

$\tan\theta_A = 0.045918$

$\theta_A = 2.629°$

$\theta_A = 2.63°$ ◀

7.112



GIVEN: CABLE AB LENGTH = 50.5 m, MASS/METER = 0.75 kg/m.

FIND: (a) SAG h. (b) MAXIMUM TENSION

FIRST TWO TERMS OF EQ. 7.10, page 374:

(a) $s_B = \frac{1}{2}(50.5\,m) = 25.25\,m$, $x_B = \frac{1}{2}(50\,m) = 25\,m$

$y_B = h$

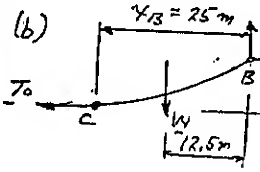
$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]$$

$$25.25\,m = 25\,m\left[1 - \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]; \quad \left(\frac{y_B}{x_B}\right)^2 = 0.01\left(\frac{3}{2}\right)^2 = \sqrt{0.015}$$

$$\frac{y_B}{x_B} = 0.12247; \quad \frac{h}{25\,m} = 0.12247$$

$h = 3.0619\,m$ $\qquad h = 3.06\,m$ ◁

(b)



FREE BODY PORTION CB

$w = (0.75\,kg/m)(9.81\,m) = 7.3575\,N/m$

$W = s_B w = (25.25\,m)(7.3575\,N/m)$

$W = 185.78\,N$

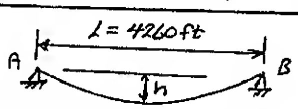
$+\circlearrowleft \Sigma M_B = 0$: $T_0(3.0619\,m) - (185.78\,N)(12.5\,m) = 0$

$T_0 = 758.4\,N$ $\qquad B_x = T_0 = 758.4\,N$

$+\uparrow \Sigma F_y = 0$: $B_y - 185.78\,N = 0$ $\qquad B_y = 185.78\,N$

$T_m = \sqrt{B_x^2 + B_y^2} = \sqrt{(758.4\,N)^2 + (185.78\,N)^2}$; $\quad T_m = 781\,N$ ◁

7.113



GIVEN: IN WINTER $h = 386\,ft$, IN SUMMER $h = 394\,ft$.

FIND: CHANGE IN LENGTH OF CABLE, WINTER TO SUMMER.

EQ. 7.10, page 374: $s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4 + \ldots\right]$

WINTER: $y_B = h = 386\,ft$, $x_B = \frac{1}{2}L = 2130\,ft$

$$s_B = (2130)\left[1 + \frac{2}{3}\left(\frac{386}{2130}\right)^2 - \frac{2}{5}\left(\frac{386}{2130}\right)^4 + \ldots\right] = 2177.59\,ft$$

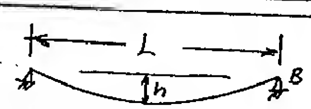
SUMMER: $y_B = h = 394\,ft$, $x_B = \frac{1}{2}L = 2130\,ft$

$$s_B = (2130)\left[1 + \frac{2}{3}\left(\frac{394}{2130}\right)^2 - \frac{2}{5}\left(\frac{394}{2130}\right)^4 + \ldots\right] = 2175.715\,ft$$

$\Delta = 2(\Delta s_B) = 2(2177.59\,ft - 2175.715\,ft) = 2(1.875\,ft)$

CHANGE IN LENGTH $= 3.75\,ft$ ◀

7.114



GIVEN: CABLE LENGTH $= L + \Delta$

FIND: APPROXIMATE SAG. (a) IN TERMS OF L AND Δ.

(b) FOR $L = 100\,ft$, $\Delta = 4\,ft$

EQ. 7.10, page 374: (FIRST TWO TERMS)

(a) $s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]$ $\qquad x_B = L/2$, $s_B = \frac{1}{2}(L + \Delta)$

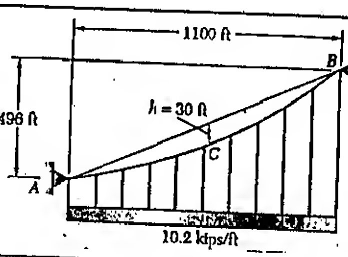
$y_B = h$

$$\frac{1}{2}(L+\Delta) = \frac{L}{2}\left[1 + \frac{2}{3}\left(\frac{h}{L/2}\right)^2\right]$$

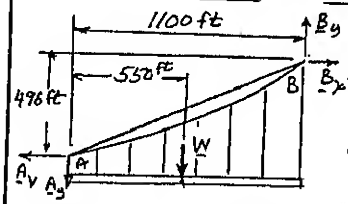
$\frac{\Delta}{2} = \frac{4}{3}\frac{h^2}{L}$; $\quad h^2 = \frac{3}{8}L\Delta$; $\qquad h = \sqrt{\frac{3}{8}L\Delta}$ ◀

(b) $L = 100\,ft$, $\Delta = 4\,ft$. $\quad h = \sqrt{\frac{3}{8}(100)(4)}$; $\qquad h = 12.25\,ft$ ◀

7.115



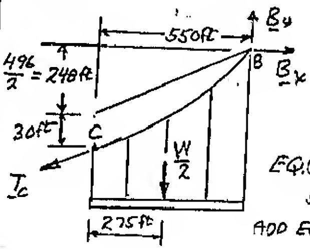
FIND: (a) T_m. (b) SLOPE AT B.



FREE BODY: ENTIRE CABLE

$+\circlearrowleft \Sigma M_A = 0$: $B_y(1100\,ft) - B_x(496\,ft) - W(550\,ft) = 0$

$1100B_y - 496B_x - 550W = 0$ $\qquad$ (1)



FREE BODY: PORTION CB

$+\circlearrowleft \Sigma M_C = 0$: $-B_y(550\,ft) + B_x(278\,ft) + \frac{W}{2}(275\,ft) = 0$

$-550B_y + 278B_x + 137.5W = 0$ $\qquad$ (2)

EQ.(1)×0.5:

$550B_y - 248B_x - 275W = 0$ $\qquad$ (3)

ADD EQS. (2) AND (3)

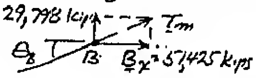
$30B_x - 137.5W = 0$ $\qquad$ (4)

$W = wL = (10.2\,kips/ft)(1100\,ft) = 11{,}220\,kips$

EQ.(4): $30B_x - (137.5)(11{,}220) = 0$ $\qquad B_x = 51{,}425\,kips$

EQ(1): $1100B_y - 496(51{,}425) - 550(11{,}220) = 0$

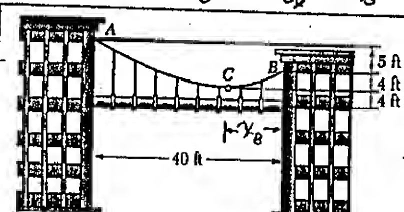
$B_y = 28{,}798\,kips$



$T_m^2 = B_x^2 + B_y^2$; $\quad T_m = 58{,}940\,kips$ ◀

$\theta_B = \tan^{-1}\frac{B_y}{B_x}$; $\quad \theta_B = 29.2°$ ◀

7.116

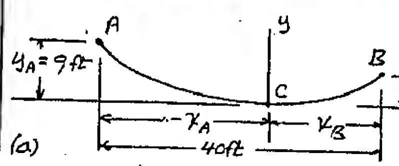


GIVEN: $w = (45 + 5)\,lb/ft$

$w = 50\,lb/ft$

FIND: (a) x_B

(b) T_{max}



(a)

NOTE: $x_B - x_A = 40\,ft$ OR $x_A = x_B - 40\,ft$

USE EQ. 7.8 page 374:

POINT A: $y_A = \frac{w x_A^2}{2T_0}$; $\quad 9 = \frac{w(x_B - 40)^2}{2T_0}$ $\qquad$ (1)

POINT B: $y_B = \frac{w x_B^2}{2T_0}$; $\quad 4 = \frac{w x_B^2}{2T_0}$ $\qquad$ (2)

DIVIDING (1) BY (2): $\frac{4}{9} = \frac{(x_B - 40)^2}{x_B^2}$; $\quad x_B = 16\,ft$

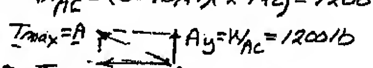
POINT C IS 16 ft TO LEFT OF B ◀

(b) MAXIMUM SLOPE AND THUS T_{max} IS AT A

$x_A = x_B - 40 = 16 - 40 = -24\,ft$

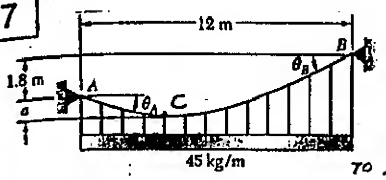
$y_A = \frac{w x_A^2}{2T_0}$; $\quad 9\,ft = \frac{(50\,lb/ft)(-40\,ft)^2}{2T_0}$; $\quad T_0 = 1600\,lb$

$W_{AC} = (50\,lb/ft)(24\,ft) = 1200\,lb$



$T_{max} = 2000\,lb$ ◀

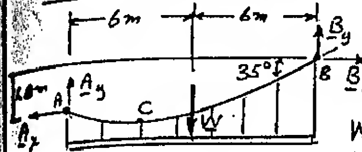
7.117



GIVEN: $\theta_B = 35°$

FIND: (a) T_m

(b) VERTICAL DISTANCE TO LOWEST POINT.



FREE BODY: ENTIRE CABLE

$B_y = B_x \tan 35°$

$W = (45\,kg/m)(12\,m)(9.81\,m/s^2)$

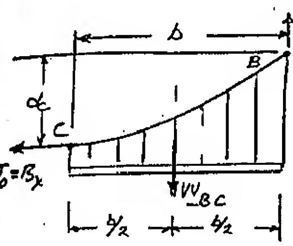
$W = 5297.4\,N$

$+\circlearrowleft \Sigma M_A = 0: \; W(6\,m) + B_x(1.8\,m) - B_y(12\,m) = 0$

$(5297.4)(6) + 1.8 B_x - B_x \tan 35°(12) = 0$

$B_x = 4814\,N \qquad B_y = (4814\,N)\tan 35° = 3370.8\,N$

FREE BODY: PORTION CB



$+\uparrow \Sigma F_y = 0: \; B_y - W_{BC} = 0$

$W_{BC} = B_y = 3370.8\,N$

$W_{BC} = (45\,kg/m)(9.81\,m/s^2)\,b$

$3370.8\,N = (441.45\,N/m)\,b$

$b = 7.6357\,m$

$+\circlearrowleft \Sigma M_B = 0: \; T_0 d_c - W_{BC}(\tfrac{1}{2}b) = 0$

$(4814\,N)d_c - (3370.8\,N)\tfrac{1}{2}(7.6357\,m) = 0$

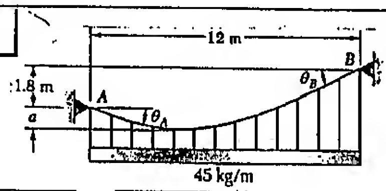
$d_c = 2.6733\,m$

(a) $d_c = 1.8\,m + a; \quad 2.6733\,m = 1.8\,m + a; \quad a = 0.873\,m$

(b) $T_m = B = \sqrt{B_x^2 + B_y^2} = \sqrt{(4814\,N)^2 + (3370.8\,N)^2}$

$T_m = 5877\,N \qquad T_m = 5880\,N$

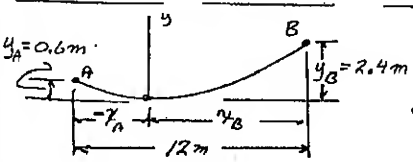
7.118



GIVEN: $a = 0.6\,m$

FIND: (a) T_m

(b) θ_B



NOTE: $x_B - x_A = 12\,m$

OR: $x_A = x_B - 12\,m$

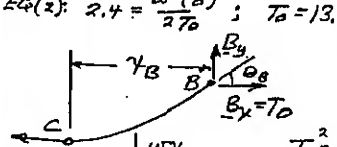
POINT A: $y_A = \dfrac{w x_A^2}{2T_0}; \quad 0.6 = \dfrac{w(x_B - 12)^2}{2T_0}$ (1)

POINT B: $y_B = \dfrac{w x_B^2}{2T_0}; \quad 2.4 = \dfrac{w x_B^2}{2T_0}$ (2)

DIVIDING (1) BY (2): $\dfrac{0.6}{2.4} = \dfrac{(x_B - 12)^2}{x_B^2}; \quad x_B = 8\,m$

(a) EQ(2): $2.4 = \dfrac{w(8)^2}{2T_0}; \quad T_0 = 13.333\,w$

FREE BODY: PORTION CB



$\Sigma F_y = 0 \quad B_y = w x_B$

$B_y = 8w$

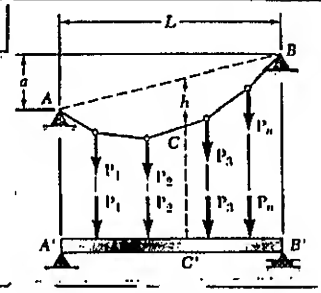
$T_m^2 = B_x^2 + B_y^2; \quad T_m^2 = (13.333w)^2 + (8w)^2$

$T_m = 15.549w = 15.549(45)(9.81)$

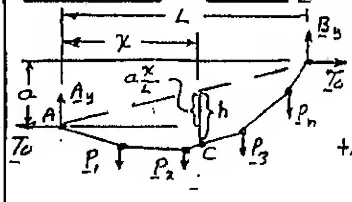
$T_m = 6860\,N$

$\theta_B = \tan^{-1} B_y/B_x = \tan^{-1} 8w/13.333w \qquad \theta_B = 31.0°$

* 7.119



SHOW THAT $M_{C'}$ IN BEAM IS EQUAL TO $T_0 h$ WHERE T_0 IS HORIZ. COMPONENT OF CABLE TENSION



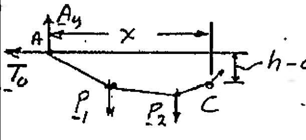
FREE BODY: ENTIRE CABLE

DENOTE BY ΣM_B^L THE SUM OF THE MOMENTS OF ALL LOADS ABOUT B.

$+\circlearrowleft \Sigma M_B = 0: \; -A_y L - T_0 a + \Sigma M_B^L = 0$

$A_y = \frac{1}{L}\Sigma M_B^L - T_0 \frac{a}{L}$ (1)

FREE BODY: PORTION AC

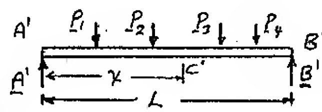


DENOTE BY ΣM_C^x THE SUM OF THE MOMENTS ABOUT C OF LOADS BETWEEN A AND C.

$+\circlearrowleft \Sigma M_C = 0: \; -A_y x + T_0(h - a\frac{x}{L}) + \Sigma M_C^x = 0$ (2)

SUBSTITUTE FOR A_y FROM (1) AND SOLVE (2) FOR $T_0 h$:

$T_0 h = \frac{x}{L}\Sigma M_B^L - \Sigma M_C^x$ (3)

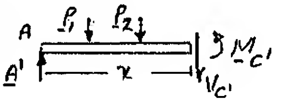


FREE BODY: ENTIRE BEAM

$+\circlearrowleft \Sigma M_{B'} = 0: \; -A'L + \Sigma M_{B'}^L = 0$

$A' = \frac{1}{L}\Sigma M_{B'}^L$ (4)

FREE BODY: PORTION AC'



$+\circlearrowleft \Sigma M_{C'} = 0: \; M_{C'} - A'x + \Sigma M_{C'}^x = 0$

SUBSTITUTE FOR A' FROM (4):

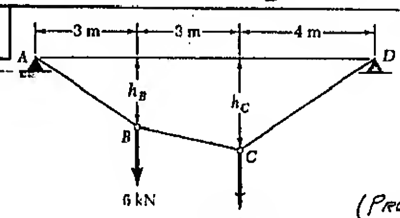
$M_{C'} = \frac{x}{L}\Sigma M_{B'}^L - \Sigma M_{C'}^x$ (5)

COMPARING (3) AND (5) AND NOTING THAT

$\Sigma M_B^L = \Sigma M_{B'}^L \qquad \Sigma M_C^x = \Sigma M_{C'}^x$

WE HAVE: $M_{C'} = T_0 h$ (Q.E.D)

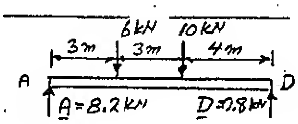
7.120



GIVEN: $T_m = 15\,kN$

FIND: DISTANCES h_B AND h_C

(PROB. 7.94 a)



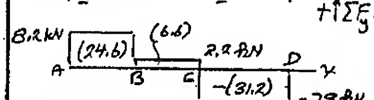
$+\circlearrowleft \Sigma M_B = 0:$

$A(10\,m) - (6\,kN)(7\,m) - (10\,kN)(4\,m) = 0$

$A = 8.2\,kN$

$+\uparrow \Sigma F_y = 0: \; 8.2\,kN - 6\,kN - 10\,kN + B = 0$

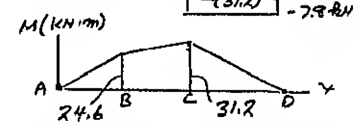
$B = 7.8\,kN$



AT A: $T_m^2 = T_0^2 + A^2$

$15^2 = T_0^2 + 8.2^2$

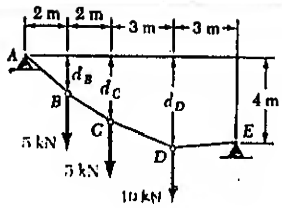
$T_0 = 12.56\,kN$



AT B: $M_B = T_0 h_B; \quad 24.6\,kN\cdot m = (12.56\,kN)h_B; \quad h_B = 1.959\,m$

AT C: $M_C = T_0 h_C; \quad 31.2\,kN\cdot m = (12.56\,kN)h_C; \quad h_C = 2.48\,m$

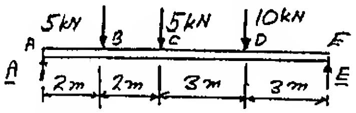
7.121



GIVEN: $d_C = 3\text{ m}$

FIND: d_B AND d_D

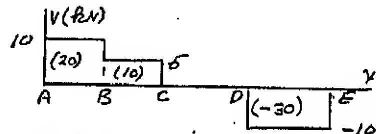
(PROB. 7.97a)

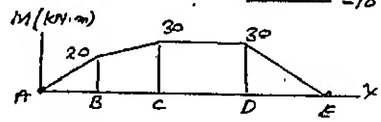


$+\circlearrowleft \Sigma M_B = 0$:

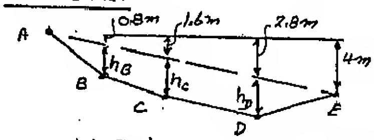
$A(10\text{ m}) - (5\text{ kN})(8\text{ m}) - (5\text{ kN})(6\text{ m}) - (10\text{ kN})(3\text{ m}) = 0$

$A = 10\text{ kN}$





GEOMETRY:



$d_C = 1.6\text{ m} + h_C$

$3\text{ m} = 1.6\text{ m} + h_C$

$h_C = 1.4\text{ m}$

SINCE $M = T_0 h$, h IS PROPORTIONAL TO M, THUS

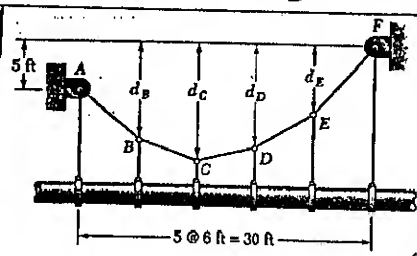
$$\frac{h_B}{M_B} = \frac{h_C}{M_C} = \frac{h_D}{M_D}; \quad \frac{h_B}{20\text{ kN·m}} = \frac{1.4\text{ m}}{30\text{ kN·m}} = \frac{h_D}{30\text{ kN·m}}$$

$h_B = 1.4\left(\frac{20}{30}\right) = 0.9333\text{ m}$ | $h_D = 1.4\left(\frac{30}{30}\right) = 1.4\text{ m}$

$d_B = 0.8\text{ m} + 0.9333\text{ m}$ | $d_D = 2.8\text{ m} + 1.4\text{ m}$

$d_B = 1.733\text{ m}$ ◄ | $d_D = 4.2\text{ m}$ ◄

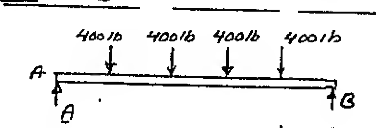
7.122



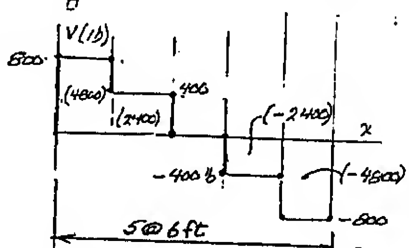
(PROB 7.99b)

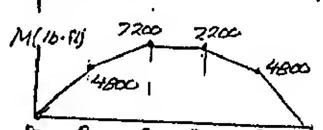
GIVEN: $d_C = 12\text{ ft}$, TENSION IN HANGERS = 400 lb

FIND: d_D



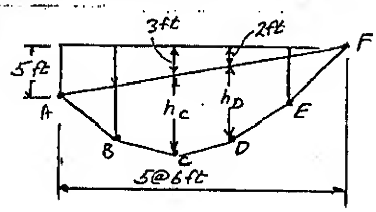
$A = B = \frac{1}{2}(4 \times 400) = 800\text{ lb}$





(CONTINUED)

7.122 CONTINUED



GEOMETRY

$d_C = h_C + 3\text{ ft}$

$12\text{ ft} = h_C + 3\text{ ft}$

$h_C = 9\text{ ft}$

$d_D = h_D + 2\text{ ft}$

AT C: $M_C = T_0 h_C$

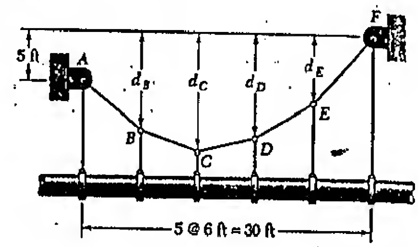
$7200\text{ lb·ft} = T_0(9\text{ ft})$ $\quad T_0 = 800\text{ lb}$

AT D: $M_D = T_0 h_D$

$7200\text{ lb·ft} = (800\text{ lb})h_D$ $\quad h_D = 9\text{ ft}$

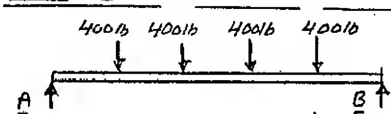
EQ.(1): $d_D = 9\text{ ft} + 2\text{ ft}$ $\quad d_D = 11\text{ ft}$ ◄

7.123



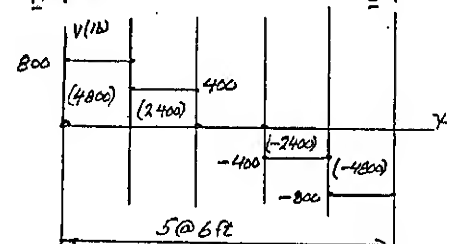
GIVEN: $d_C = 9\text{ ft}$, TENSION IN HANGERS = 400 lb

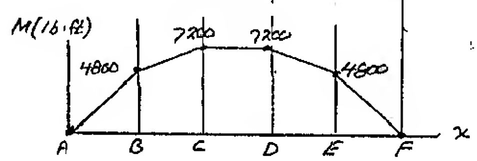
FIND: d_D (PROB. 7.100b)



$A = B = \frac{1}{2}(4 \times 400)$

$A = B = 800\text{ lb}$

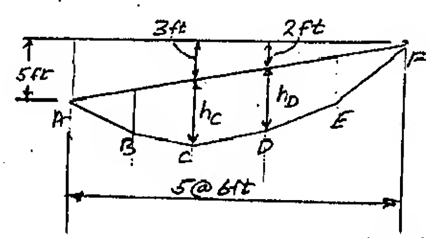




AT ANY POINT: $M = T_0 h$

WE NOTE THAT SINCE $M_C = M_D$, WE HAVE $h_C = h_D$

GEOMETRY



$d_C = h_C + 3\text{ ft}$

$9\text{ ft} = h_C + 3\text{ ft}$

$h_C = 6\text{ ft}$

AND $h_D = 6\text{ ft}$

$d_D = h_D + 2\text{ ft} = 6\text{ ft} + 2\text{ ft}$

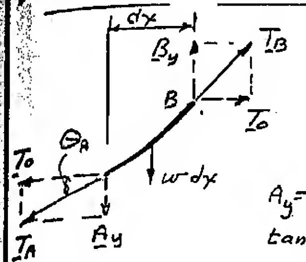
$d_D = 8\text{ ft}$ ◄

* 7.124

FOR A CABLE PROVE THAT

$$\frac{d^2y}{dx^2} = w(x)\, T_0$$

WHERE $w(x)$ IS THE DISTRIBUTED LOAD



FREE BODY: DIFFERENTIAL ELEMENT OF CABLE

$w = w(x) =$ LOAD AS FUNCTION OF x

$$A_y = T_0 \tan\Theta_A \qquad B_y = T_0 \tan\Theta_B$$

$$\tan\Theta_A = \left.\frac{dy}{dx}\right|_A = \frac{dy}{dx}$$

$$\tan\Theta_B = \left.\frac{dy}{dx}\right|_B = \left.\frac{dy}{dx}\right|_A + \frac{d^2y}{dx^2}\,dx$$

OR $\quad \tan\Theta = \frac{dy}{dx} + \frac{d^2y}{dx^2}\,dx$

$+\uparrow \Sigma F_y = 0: \quad -A_y + B_y - w\,dx$

$$-T_0\tan\Theta_A + T_0\tan\Theta_B - w\,dx = 0$$

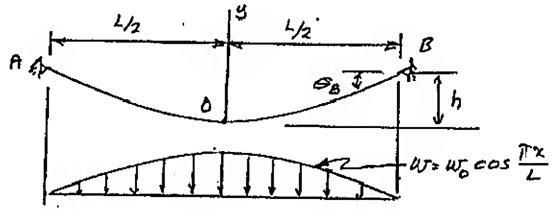
$$-T_0\frac{dy}{dx} + T_0\left(\frac{dy}{dx} + \frac{d^2y}{dx^2}dx\right) - w\,dx = 0$$

$$T_0\frac{d^2y}{dx^2}dx - w\,dx = 0 \qquad \frac{d^2y}{dx^2} = \frac{w}{T_0} \quad (Q.E.D)$$

* 7.125

GIVEN: $w = w_0 \cos(\pi x/L)$, L = SPAN, h = SAG, ORIGIN AT MID-SPAN.

FIND: EQUATION OF CABLE SHAPE, AND T_0 AND T_m



$$\frac{d^2y}{dx^2} = \frac{w}{T_0} \qquad \frac{d^2y}{dx^2} = \frac{w_0}{T_0}\cos\frac{\pi x}{L}$$

$$\frac{dy}{dx} = \frac{w_0 L}{T_0 \pi}\sin\frac{\pi x}{L} + C_1$$

$$y = -\frac{w_0 L^2}{T_0 \pi^2}\cos\frac{\pi x}{L} + C_1 x + C_2$$

BOUNDARY CONDITIONS $\quad x = 0,\ \frac{dy}{dx} = 0,\ \therefore C_1 = 0$

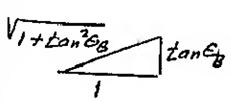
$$x = 0,\ y = 0 = -\frac{w_0 L^2}{T_0\pi^2}\cos 0 + C_2 \qquad C_2 = +\frac{w_0 L^2}{T_0 \pi^2}$$

$$y = +\frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi x}{L}\right) \qquad (1)$$

$$x = \frac{L}{2},\ y = h \qquad h = \frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi}{2}\right) \qquad h = \frac{w_0 L^2}{T_0 \pi^2}$$

EQ(1) $\quad y = h\left(1 - \cos\frac{\pi x}{L}\right)$

$$\frac{dy}{dx} = h\frac{\pi}{L}\sin\frac{\pi x}{L}; \quad \tan\Theta_B = \left.\frac{dy}{dx}\right|_B = h\frac{\pi}{L}\sin\frac{\pi}{2} = h\frac{\pi}{L}$$



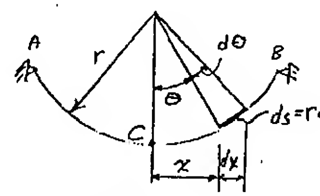
$$\cos\Theta_B = \frac{1}{\sqrt{1+\tan^2\Theta}} = \frac{1}{\sqrt{1+\frac{h^2\pi^2}{L^2}}}$$

$$T_m = \frac{T_0}{\cos\Theta_B} = \left(\frac{w_0 L^2}{T_0 \pi^2}\right)\sqrt{1 + \frac{h^2\pi^2}{L^2}}$$

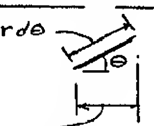
OR: $\quad T_{max} = \frac{w_0 L}{\pi}\sqrt{\frac{L^2}{h^2\pi^2} + 1}$ ◀

* 7.126

IF $w = w_0/\cos\Theta$, PROVE CURVE FORMED BY A CABLE IS A CIRCULAR ARC.



$ds = r\,d\Theta$



$dx = (r\,d\Theta)\cos\Theta$

$$\frac{d\Theta}{dx} = \frac{1}{r\cos\Theta} \qquad (1)$$

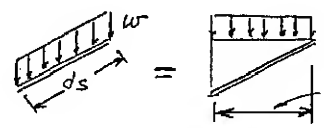
$$\frac{dy}{dx} = \tan\Theta; \quad \frac{d^2y}{dx^2} = +\sec^2\Theta\,\frac{d\Theta}{dx} \qquad (2)$$

SUBSTITUTE FROM (1): $\frac{d^2y}{dx^2} = \frac{\sec^2\Theta}{r\cos\Theta} = \frac{1}{r\cos^3\Theta}$

FROM PROB 7.124:

$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0}; \quad \frac{1}{r\cos^3\Theta} = \frac{w(x)}{T_0}$$

$w(x) = \frac{T_0}{r\cos^3\Theta}$ = LOADING PER HORIZONTAL UNIT



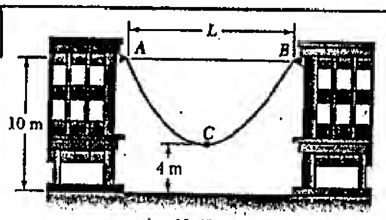
$$w\,ds = w(x)\,dx$$

$$w\,ds = \frac{T_0}{r\cos^3\Theta}\cos\Theta\,ds \qquad w = \frac{T_0}{r\cos^2\Theta}$$

WE NOTE THAT SMALLEST VALUE OF w OCCURS AT $\Theta = 0$. DENOTING SMALLEST VALUE BY w_0, WE FIND

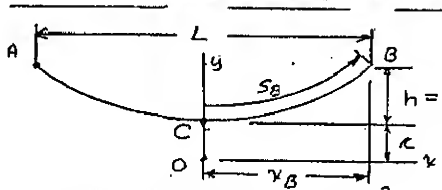
$$w_0 = \frac{T_0}{r} \qquad w = \frac{w_0}{\cos^2\Theta} \qquad (Q.E.D.)$$

7.127



GIVEN: 30-m CABLE, $T_m = 500\text{N}$

FIND: (a) L (b) MASS OF CABLE



$s_B = 15\text{ m}$

$h = 10\text{m} - 4\text{m} = 6\text{m}$

$T_m = 500\text{ N}$

EQ. 7.17: $\quad y_B^2 - s_B^2 = c^2; \quad (6 + c)^2 - 15^2 = c^2$

$$36 + 12c + c^2 - 225 = c^2$$

$$12c = 189 \qquad c = 15.75\text{m}$$

EQ. 7.15: $\quad s_B = c\sinh\frac{x_B}{c}; \quad 15 = (15.75)\sinh\frac{x_B}{c}$

$$\sinh\frac{x_B}{c} = 0.95238 \qquad \frac{x_B}{c} = 0.8473$$

(a) $x_B = 0.8473(15.75) = 13.345\text{m}; \quad L = 2x_B \qquad L = 26.7\text{m}$ ◀

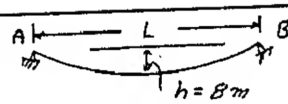
(b) EQ. 7.18: $\quad T_m = w\,y_B; \quad 500\text{N} = w(6 + 15.75)$

$$w = 22.99\text{ N/m}$$

$$W = 2s_B w = (30\text{m})(22.99\text{ N/m}) = 689.7\text{N}$$

$$m = \frac{W}{g} = \frac{689.7\text{N}}{9.81\text{m/s}^2} \qquad \text{TOTAL MASS} = 70.3\text{ kg}$$ ◀

7.128

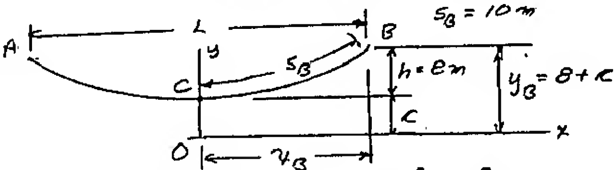


GIVEN: 20-m CHAIN OF MASS 12 kg

FIND: (a) L, (b) T_m

$$\text{MASS/METER} = (12\text{ kg})/(20\text{ m}) = 0.6\text{ kg/m}$$

$$w = (0.6\text{ kg/m})(9.81\text{ m/s}^2) = 5.886\text{ N/m}$$



EQ. 7.17: $y_B^2 - s_B^2 = c^2$; $(8+c)^2 - 10^2 = c^2$

$$64 + 16c + c^2 - 100 = c^2$$

$$16c = 36 \qquad c = 2.25\text{ m}$$

EQ. 7.18: $T_m = w y_B = (5.886\text{ N/m})(8\text{ m} + 2.25\text{ m})$

$$T_m = 60.33\text{ N} \qquad T_m = 60.3\text{ N}$$

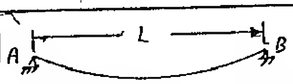
EQ. 7.15: $s_B = c \sinh \frac{x_B}{c}$; $10\text{ m} = (2.25\text{ m}) \sinh \frac{x_B}{c}$

$$\sinh \frac{x_B}{c} = 4.444 \; ; \qquad \frac{x_B}{c} = 2.197$$

$x_B = 2.197(2.25\text{ m}) = 4.944\text{ m}$; $L = 2x_B = 2(4.944\text{ m}) = 9.888\text{ m}$

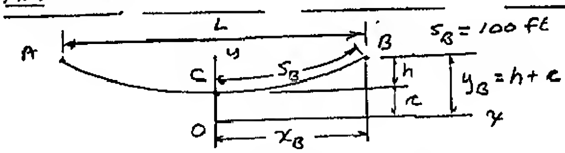
$$L = 9.89\text{ m}$$

7.129



GIVEN: 200-ft TAPE WEIGHS 4 lb, $T_m = 16$ lb

FIND: SPAN L



$w = (4\text{ lb})/(200\text{ ft}) = 0.02\text{ lb/ft} \qquad T_m = 16\text{ m}$

EQ. 7.18: $T_m = w y_B$; $16\text{ lb} = (0.02\text{ lb/ft}) y_B$; $y_B = 800$ ft

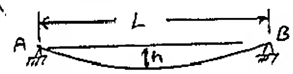
EQ. 7.17: $y_B^2 - s_B^2 = c^2$; $(800)^2 - (100)^2 = c^2$; $c = 793.73$ ft

EQ. 7.15: $s_B = c \sinh \frac{x_B}{c}$; $100 = 793.73 \sinh \frac{x_B}{c}$

$$\frac{x_B}{c} = 0.12566 \; ; \quad x_B = 99.737\text{ ft}$$

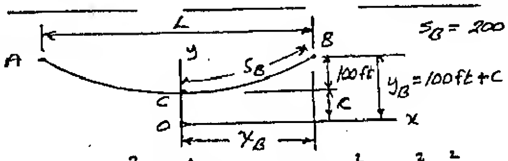
$$L = 2x_B = 2(99.737\text{ ft}) \; ; \qquad L = 199.47\text{ ft}$$

7.130



GIVEN: 400-ft CABLE, $h = 100$ ft, $w = 2.5$ lb/ft.

FIND: SPAN L AND T_m



EQ. 7.17: $y_B^2 - s_B^2 = c^2$; $(100+c)^2 - 200^2 = c^2$

$$10000 + 200c + c^2 - 40000 = c^2 \; ; \quad c = 150\text{ ft}$$

EQ. 7.15: $s_B = c \sinh \frac{x_B}{c}$; $200 = 150 \sinh \frac{x_B}{c}$

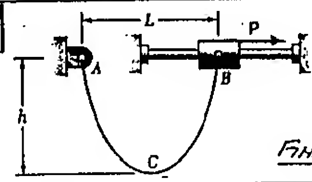
$\sinh \frac{x_B}{c} = \frac{4}{3}$; $\frac{x_B}{c} = 1.0986$; $x_B = (150)(1.0986) = 164.79$ ft

$$L = 2x_B = 2(164.79\text{ ft}) = 329.58\text{ ft} \; ; \qquad L = 330\text{ ft}$$

EQ. 7.18: $T_m = w y_B = (2.5\text{ lb/ft})(100\text{ ft} + 150\text{ ft})$

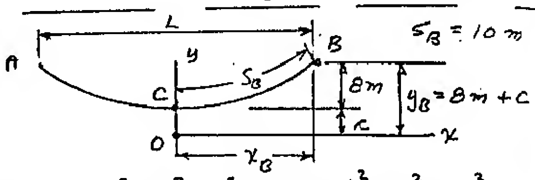
$$T_m = 625\text{ lb}$$

7.131



GIVEN: 20-m CABLE ACB OF UNIT MASS = 0.2 kg/m, h = 8 m

FIND: (a) P, (b) L.



EQ. 7.17: $y_B^2 - s_B^2 = c^2$; $(8+c)^2 - 10^2 = c^2$

$$64 + 16c + c^2 - 100 = c^2$$

$$16c = 36 \qquad c = 2.25\text{ m}$$

EQ. 7.15: $s_B = c \sinh \frac{x_B}{c}$; $10\text{ m} = (2.25\text{ m}) \sinh \frac{x_B}{c}$

$\sinh \frac{x_B}{c} = 4.444$; $\frac{x_B}{c} = 2.197$; $x_B = (2.197)(2.25\text{ m}) = 4.944$

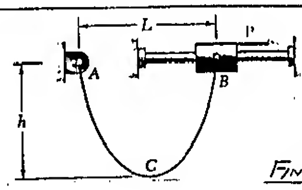
$L = 2x_B = 2(4.944\text{ m}) = 9.888\text{ m}$; $L = 9.89\text{ m}$

NOTE THAT P = HORIZ. COMP. OF CABLE TENSION. $\therefore T_0 = P$

EQ. 7.18: $T_0 = P = wc$: $P = (0.2\text{ kg/m})(9.81\text{ m/s}^2)(2.25\text{ m})$

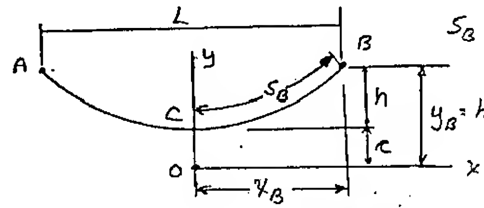
$$P = 4.414\text{ N} \qquad P = 4.41\text{ N}$$

7.132



GIVEN: 20-m CABLE ACB OF UNIT MASS = 0.2 kg/m, $P = 20$ N.

FIND: (a) h, (b) L.



TOTAL CABLE: $W = (0.2\text{ kg/m})(9.81\text{ m/s}^2)(20\text{ m}) = 39.24\text{ N}$

COLLAR AT B: $B_y = \frac{1}{2}W = 19.62\text{ N}$

T_m $\qquad P = 20\text{ N}$ $\qquad T_m = \sqrt{(20\text{ N})^2 + (19.62\text{ N})^2}$

$$T_m = 28.017\text{ N}$$

EQ. 7.18: $T_m = w y_B$; $28.017\text{ N} = (0.2\text{ kg/m})(9.81\text{ m/s}^2) y_B$

$$y_B = 14.280\text{ m}$$

EQ. 7.17: $y_B^2 - s_B^2 = c^2$; $(14.280\text{ m})^2 - (10\text{ m})^2 = c^2$

$$c^2 = 103.92 \qquad c = 10.194\text{ m}$$

$$y_B = h + c$$

$$14.280\text{ m} = h + 10.194\text{ m}$$

$$h = 4.086\text{ m} \qquad h = 4.09\text{ m}$$

EQ. 7.15: $s_B = c \sinh \frac{x_B}{c}$

$$10\text{ m} = (10.194\text{ m}) \sinh \frac{x_B}{c}$$

$$\sinh \frac{x_B}{c} = 0.981 \qquad \frac{x_B}{c} = 0.8678$$

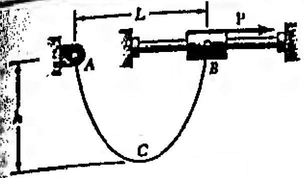
$$x_B = 0.8678(10.194\text{ m}) = 8.847\text{ m}$$

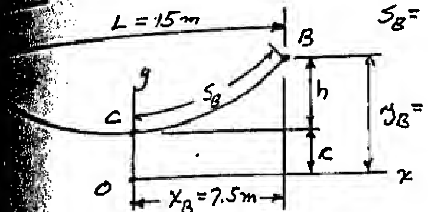
$$L = 2x_B = 2(8.847\text{ m}) = 17.694\text{ m}$$

$$L = 17.69\text{ m}$$

7.133

7.133 GIVEN: 20-m CABLE ACB OF UNIT MASS $= 0.2\ \text{kg/m}$, $L = 15\ \text{m}$
FIND: (a) SAG h, (b) FORCE $\underline{P}$





$s_B = 10\ \text{m}$ $\qquad y_B = h + c$

EQ. 7.17: $s_B = c \sinh x_B/c$

$10 = c \sinh \frac{7.5}{c}$; $\quad c = 5.5504\ \text{m}$

$y_B = c \cosh \frac{x_B}{c} = (5.5504) \cosh \frac{7.5}{5.5504}$

$y_B = 11.437\ \text{m}$; $\quad y_B = h + c$

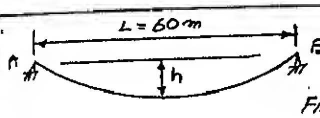
$11.437 = h + 5.5504$; $\qquad h = 5.89\ \text{m}$ ◀

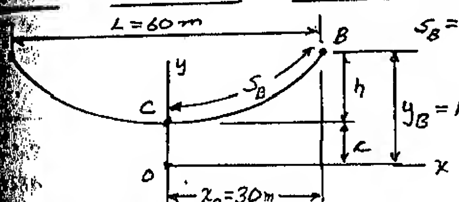
$T_0 = wc = (0.2\ \text{kg/m})(9.81\ \text{m/s}^2)(5.5504\ \text{m})$

$T_0 = 10.89\ \text{N}$; $\quad P = T_0 =$ HORIZ. COMP. OF TENSION

$\underline{P} = 10.89\ \text{N} \rightarrow$ ◀

7.134 GIVEN: $T_m = 300\ \text{N}$, 90-m WIRE
FIND: (a) SAG h, (b) TOTAL MASS OF CABLE





$s_B = 45\ \text{m}$ $\qquad y_B = h + c$

EQ. 7.17: $s_B = c \sinh \frac{x_B}{c}$

$45 = c \sinh \frac{30}{c}$; $\qquad c = 18.494\ \text{m}$

EQ. 7.16: $y_B = c \cosh \frac{x_B}{c}$

$y_B = (18.494) \cosh \frac{30}{18.494}$; $\quad y_B = 48.652\ \text{m}$

$y_B = h + c$

$48.652 = h + 18.494$;

$h = 30.158\ \text{m}$ $\qquad h = 30.2\ \text{m}$ ◀

EQ. 7.18: $T_m = w y_B$

$300\ \text{N} = w(48.652\ \text{m})$; $\qquad w = 6.166\ \text{N/m}$

TOTAL WEIGHT OF CABLE

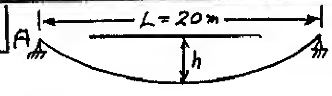
$W = w(\text{LENGTH}) = (6.166\ \text{N/m})(90\ \text{m}) = 554.96\ \text{N}$

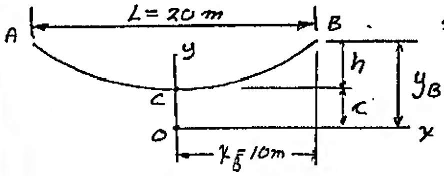
TOTAL MASS OF CABLE

$m = \frac{W}{g} = \frac{554.96\ \text{N}}{9.81\ \text{m/s}} = 56.57\ \text{kg}$

$m = 56.6\ \text{kg}$ ◀

7.135 GIVEN: CABLE OF LENGTH 30 m.
FIND: SAG h





$s_B = 15\ \text{m}$ $\qquad y_B = h + c$

EQ. 7.17: $s_B = c \sinh \frac{x_B}{c}$

$15 = c \sinh \frac{10}{c}$; $\qquad c = 6.1647\ \text{m}$

EQ. 7.16: $y_B = c \cosh \frac{x_B}{c}$

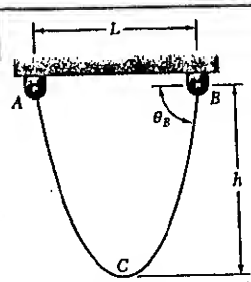
$y_B = (6.1647\ \text{m}) \cosh \frac{10}{6.1647}$; $\quad y_B = 16.2174\ \text{m}$

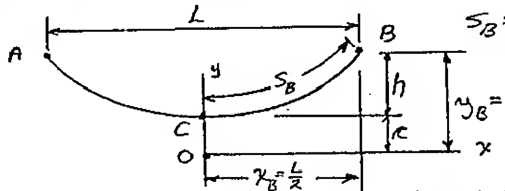
$y_B = h + c$

$16.2174\ \text{m} = h + 6.1647\ \text{m}$

$h = 10.0527\ \text{m}$ $\qquad h = 10.05\ \text{m}$ ◀

7.136 GIVEN: ROPE OF LENGTH 10 ft WITH SPAN L EQUAL TO SAG h.
FIND: (a) SPAN L, (b) ANGLE θ_B.





$s_B = 5\ \text{ft}$ $\qquad y_B = h + c$

NOTE: SINCE $L = h$, $x_B = \frac{L}{2} = \frac{h}{2}$

EQ. 7.16: $y_B = c \cosh \frac{x_B}{c}$

$h + c = c \cosh \frac{h/2}{c}$

$\frac{h}{c} + 1 = \cosh\left(\frac{1}{2}\frac{h}{c}\right)$

SOLVE FOR h/c: $\qquad \frac{h}{c} = 4.933$

EQ. 7.16: $y = c \cosh \frac{x}{c}$

$\frac{dy}{dx} = \sinh \frac{x}{c}$

AT B: $\tan\theta_B = \left.\frac{dy}{dx}\right|_B = \sinh \frac{x_B}{c}$

SUBSTITUTE $x_B = \frac{h}{2}$; $\tan\theta_B = \sinh\left(\frac{1}{2}\frac{h}{c}\right) = \sinh\left(\frac{1}{2} \times 4.933\right)$

$\tan\theta_B = 5.848$ $\qquad \theta_B = 80.3^\circ$ ◀

EQ. 7.17: $s_B = c \sinh \frac{x_B}{c} = c \sinh\left(\frac{1}{2}\frac{h}{c}\right)$

$5\ \text{ft} = c \sinh\left(\frac{1}{2} \times 4.933\right)$

$5\ \text{ft} = c\,(5.848)$

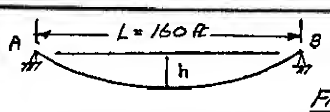
$c = 0.855$

RECALL THAT $\frac{h}{c} = 4.933$

$h = 4.933(0.855) = 4.218$

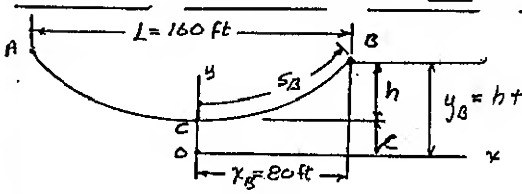
$h = 4.22\ \text{ft}$ ◀

7.137



GIVEN: $w = 2$ lb/ft, $T_m = 400$ lb

FIND: SMALLEST SAG h



EQ. 7.18: $T_m = w y_B$; $400 \text{ lb} = (2 \text{ lb/ft}) y_B$; $y_B = 200$ ft

EQ. 7.16: $y_B = c \cosh \frac{x_B}{c}$

$200 \text{ ft} = c \cosh \frac{80 \text{ ft}}{c}$

SOLVE FOR c: $c = 182.148$ ft AND $c = 31.592$ ft

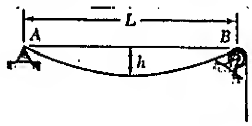
$y_B = h + c$; $h = y_B - c$

FOR $c = 182.148$ ft; $h = 200 - 182.147 = 17.852$ ft ◁

FOR $c = 31.592$ ft; $h = 200 - 31.592 = 168.408$ ft ◁

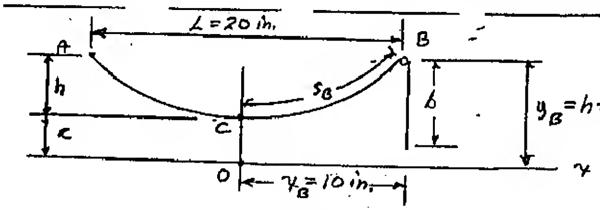
FOR $T_m \le 400$ lb: SMALLEST $h = 17.85$ ft ◀

7.138



GIVEN: 50-in. CORD, $L = 20$ in.

FIND: TWO VALUES OF h FOR EQUILIBRIUM



LENGTH OF OVER HANG: $b = 50 \text{ in.} - 2 s_B$

WEIGHT OF OVER HANG EQUALS MAX. TENSION

$T_m = T_B = w b = w(50 \text{ in.} - 2 s_B)$

EQ. 7.15: $s_B = c \sinh \frac{x_B}{c}$

EQ. 7.16: $y_B = c \cosh \frac{x_B}{c}$

EQ. 7.18: $T_m = w y_B$

$w(50 \text{ in.} - 2 s_B) = w y_B$

$w\left(50 \text{ in.} - 2 c \sinh \frac{x_B}{c}\right) = w c \cosh \frac{x_B}{c}$

$x_B = 10$: $50 - 2c \sinh \frac{10}{c} = c \cosh \frac{10}{c}$

SOLVE BY TRIAL + ERROR:

$c = 5.549$ in. AND $c = 27.742$ in.

FOR $c = 5.549$ in.

$y_B = (5.549 \text{ in.}) \cosh \frac{10 \text{ in.}}{5.549 \text{ in.}} = 17.277$ in.

$y_B = h + c$; $17.277 \text{ in.} = h + 5.549$ in.

$h = 11.728$ in. $\quad h = 11.73$ in. ◀

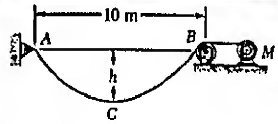
FOR $c = 27.742$ in.

$y_B = (27.742 \text{ in.}) \cosh \frac{10 \text{ in.}}{27.742 \text{ in.}} = 29.564$ in.

$y_B = h + c$; $29.564 \text{ in.} = h + 27.742$ in

$h = 1.8219$ in. $\quad h = 1.822$ in. ◀

7.139 and 7.140

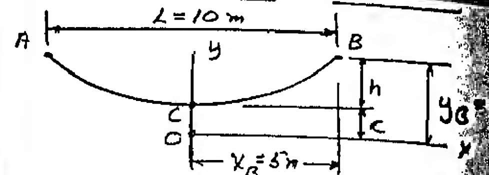


GIVEN: UNIT ... CABLE = 0.4 ...

FIND: MA... IN CABLE W...

PROB. 7.1...

PROB. 7.1...



PROB. 7.139 $\quad h = 5$ m $\quad y_B = 5 \text{ m} + c$

EQ. 7.16: $y_B = c \cosh \frac{x_B}{c}$

$5 \text{ m} + c = c \cosh \frac{5 \text{ m}}{c}$

SOLVE BY TRIAL: $c = 3.0938$ m

EQ 7.18: $T_m = w y_B = w(h + c)$

$= (0.4 \text{ kg/m})(9.81 \text{ m/s}^2)(5 \text{ m} + 3.093\ldots$

$T_m = 31.76$ N $\quad T_m = 31.\ldots$

PROB. 7.140 $\quad h = 3$ m $\quad y_B = 3 \text{ m} + c$

EQ. 7.16: $y_B = c \cosh \frac{x_B}{c}$

$3 \text{ m} + c = c \cosh \frac{10 \text{ m}}{c}$

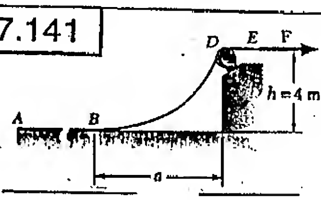
SOLVE BY TRIAL: $c = 4.5945$ m

EQ. 7.18: $T_m = w y_B = w(h + c)$

$= (0.4 \text{ kg/m})(9.81 \text{ m/s}^2)(3 \text{ m} + 4.59\ldots$

$T_m = 29.80$ N $\quad T_m = 29.\ldots$

7.141

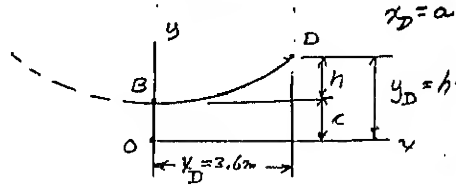


GIVEN: a = ...

UNIT MASS OF C... = 2 kg/m

FIND: FORCE ...

$x_D = a = 3.6$ ...



EQ 7.16: $y_D = c \cosh \frac{x_D}{c}$

$4 \text{ m} + c = c \cosh \frac{3.6 \text{ m}}{c}$

SOLVE BY TRIAL; $c = 2.0712$ m

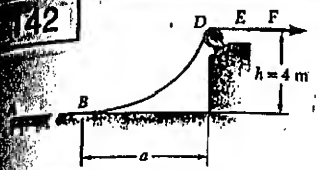
NOTE: $F = T_m$

EQ. 7.18: $F = T_m = w y_D = w(4 \text{ m} + c)$

$F = (2 \text{ kg/m})(9.81 \text{ m/s}^2)(4 \text{ m} + 2.0712 \text{ m})$

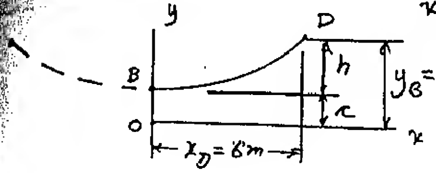
$F = 119.12$ N $\quad F = 119.1$ N →

7.142



GIVEN: $a = 6\,m$, UNIT MASS OF CABLE $= 2\,kg/m$.
FIND: FORCE P

$x_D = a = 6\,m$



$y_B = h + c = 4\,m + c$

$x_D = 6\,m$

EQ. 7.16: $y_D = c\cosh\frac{x_D}{c}$

$4m + c = c\cosh\frac{6m}{c}$

SOLVE BY TRIAL: $c = 5.054\,m$

NOTE: $F = T_m$

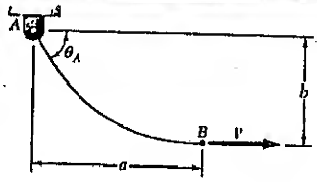
EQ. 7.18: $F = T_m = w\,y_D = w(4m + c)$

$F = (2\,kg/m)(9.81\,m/s^2)(4m + 5.054\,m)$

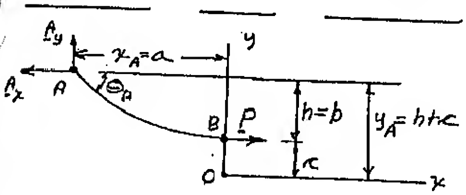
$F = 177.64\,N$

$F = 177.6\,N \rightarrow$

7.143



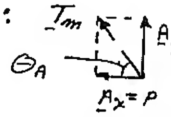
GIVEN: $w = 3\,lb/ft$, $\theta_A = 60°$, $P = 180\,lb$.
FIND: (a) DISTANCES a AND b. (b) LENGTH OF CABLE



EQ. 7.18: $T_o = P = c\,w$

$c = \frac{P}{w} = \frac{180\,lb}{3\,lb/ft}$; $c = 60\,ft$

AT A:



$T_m = \frac{P}{\cos 60°} = \frac{cw}{0.5} = 2cw$

EQ. 7.18: $T_m = w(h + c)$

$2cw = w(h + c)$

$2c = h + c$; $h = b = c$; $b = 60\,ft$

EQ. 7.16: $y_A = c\cosh\frac{x_A}{c}$

$h + c = c\cosh\frac{x_A}{c}$

$(60ft + 60ft) = (60ft)\cosh\frac{x_A}{60}$

$\cosh\frac{x_A}{60m} = 2$; $\frac{x_A}{60m} = 1.3170$

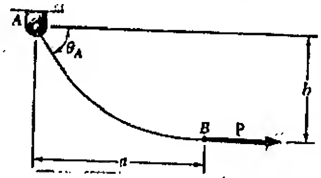
$x_A = 79.02\,ft$ $a = 79.0\,ft$

EQ 7.15: $s_A = c\sinh\frac{x_B}{c} = (60ft)\sinh\frac{79.02\,ft}{60\,ft}$

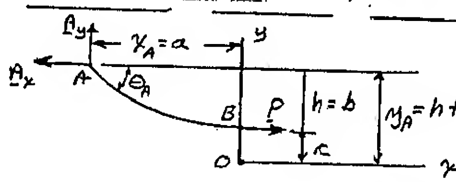
$s_A = 103.92\,ft$

LENGTH $= s_A$ $s_A = 103.9\,ft$

7.144

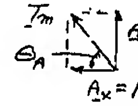


GIVEN: $w = 3\,lb/ft$, $\theta_A = 60°$, $P = 150\,lb$
FIND: (a) DISTANCES a AND b. (b) LENGTH OF CABLE.



EQ. 7.18: $T_o = P = c\,w$ $c = \frac{P}{w} = \frac{150\,lb}{3\,lb/ft} = 50\,ft$

AT A:



$T_m = \frac{P}{\cos 60°} = \frac{cw}{0.5} = 2cw$

EQ. 7.18: $T_m = w(h + c)$

$2cw = w(h + c)$

$2c = h + c$; $h = c = b$; $b = 50\,ft$

EQ. 7.16: $y_A = c\cosh\frac{x_A}{c}$

$h + c = c\cosh\frac{x_A}{c}$

$(50ft + 50ft) = (50ft)\cosh\frac{x_A}{c}$

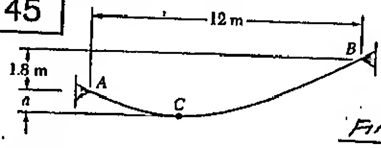
$\cosh\frac{x_A}{c} = 2$; $\frac{x_A}{c} = 1.3170$

$x_A = 1.3170(50ft) = 65.85\,ft$; $a = 65.8\,ft$

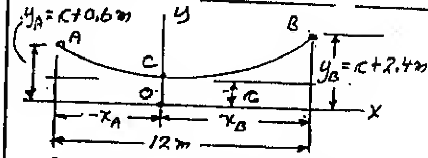
EQ. 7.15: $s_A = c\sinh\frac{x_A}{c} = (50ft)\sinh\frac{65.85\,ft}{50\,ft}$

$s_A = 86.6\,ft$; LENGTH $= s_A = 86.6\,ft$

7.145



GIVEN: $a = 0.6\,m$
UNIT MASS OF CABLE $= 0.45\,kg/m$.
FIND: (a) LOCATION OF C. (b) T_m.



NOTE: $x_B - x_A = 12\,m$
OR, $-x_A = 12m - x_B$

POINT A: $y_A = c\cosh\frac{-x_A}{c}$; $c + 0.6 = c\cosh\frac{12 - x_B}{c}$ (1)

POINT B: $y_B = c\cosh\frac{x_B}{c}$; $c + 2.4 = c\cosh\frac{x_B}{c}$ (2)

FROM (1): $\frac{12}{c} - \frac{x_B}{c} = \cosh^{-1}\left(\frac{c + 0.6}{c}\right)$ (3)

FROM (2): $\frac{x_B}{c} = \cosh^{-1}\left(\frac{c + 2.4}{c}\right)$ (4)

ADD (3)+(4): $\frac{12}{c} = \cosh^{-1}\left(\frac{c + 0.6}{c}\right) + \cosh^{-1}\left(\frac{c + 2.4}{c}\right)$

SOLVE BY TRIAL + ERROR: $c = 13.6214\,m$

EQ(2) $13.6214 + 2.4 = 13.6214\cosh\frac{x_B}{c}$

$\cosh x_B/c = 1.1762$; $x_B/c = 0.58523$

$x_B = 0.58523(13.6214\,m) = 7.9717\,m$

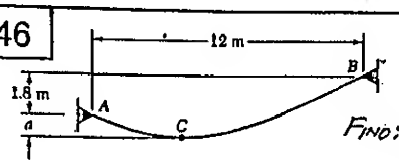
POINT C IS 7.97 m TO LEFT OF B

$y_B = c + 2.4 = 13.6214 + 2.4 = 16.0214\,m$

EQ 7.18: $T_m = w\,y_B = (0.45\,kg/m)(9.81\,m/s^2)(16.0214\,m)$

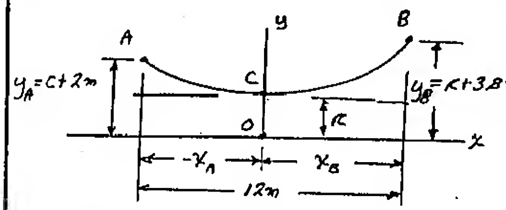
$T_m = 70.726\,N$ $T_m = 70.7\,N$

7.146



GIVEN: $a = 2\,m$, UNIT MASS OF CABLE $= 0.45\ kg/m$.

FIND: (a) LOCATION OF C, (b) T_m



NOTE: $x_B - x_A = 12m$ OR $-x_A = 12m - x_B$

POINT A: $y_A = c\cosh\frac{-x_A}{c}$; $c+2 = c\cosh\frac{12-x_B}{c}$ (1)

POINT B: $y_B = c\cosh\frac{x_B}{c}$; $c+3.8 = c\cosh\frac{x_B}{c}$ (2)

FROM (1): $\frac{12}{c} - \frac{x_B}{c} = \cosh^{-1}\left(\frac{c+2}{c}\right)$ (3)

FROM (2): $\frac{x_B}{c} = \cosh^{-1}\left(\frac{c+3.8}{c}\right)$ (4)

ADD (3)+(4): $\frac{12}{c} = \cosh^{-1}\left(\frac{c+2}{c}\right) + \cosh^{-1}\left(\frac{c+3.8}{c}\right)$

SOLVE BY TRIAL AND ERROR: $c = 6.8154\ m$

EQ.(2): $6.8154m + 3.8m = (6.8154m)\cosh\frac{x_B}{c}$

$\cosh\frac{x_B}{c} = 1.5576$ $\quad \frac{x_B}{c} = 1.0122$

$x_B = 1.0122(6.8154m) = 6.899\ m$

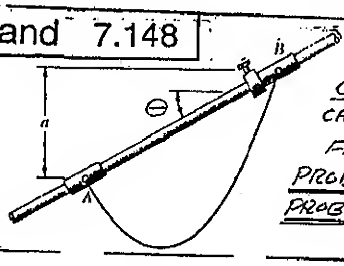
POINT C IS 6.90 m TO LEFT OF B ◀

$y_B = c + 3.8 = 6.8154 + 3.8 = 10.6154\ m$

EQ(7.18): $T_m = w y_B = (0.45\ kg/m)(9.81\ m/s^2)(10.6154\ m)$

$T_m = 46.86\ N$ $\qquad T_m = 46.9\ N$ ◀

*7.147 and 7.148



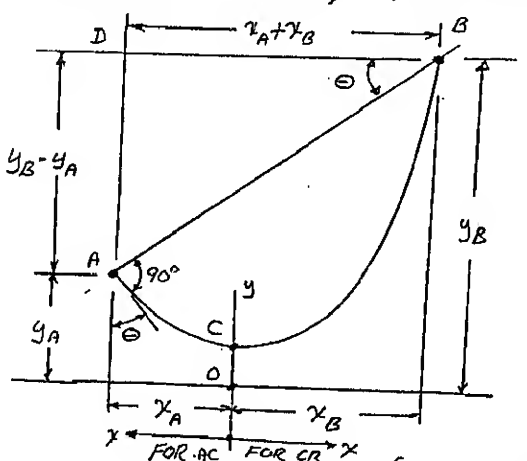
GIVEN: LENGTH OF CABLE AB IS 10 ft.

FIND: DISTANCE a;

PROB. 7.147: FOR $\theta = 30°$.

PROB. 7.148: FOR $\theta = 45°$.

COLLAR AT A: SINCE $\mu = 0$, CABLE $\perp$ ROD



(CONTINUED)

*7.147 and 7.148 CONTINUED

POINT A: $y = c\cosh\frac{x}{c}$; $\frac{dy}{dx} = \sinh\frac{x}{c}$

$\tan\theta = \frac{dy}{dx}\Big|_A = \sinh\frac{x_A}{c}$

$\therefore \frac{x_A}{c} = \sinh^{-1}(\tan\theta)$; $x_A = c\sinh^{-1}(\tan\theta)$ (1)

LENGTH OF CABLE = 10 ft

$10\,ft = AC + CB$

$10 = c\sinh\frac{x_A}{c} + c\sinh\frac{x_B}{c}$

$\sinh\frac{x_B}{c} = \frac{10}{c} - \sinh\frac{x_A}{c}$

$x_B = c\sinh^{-1}\left[\frac{10}{c} - \sinh\frac{x_A}{c}\right]$ (2)

$y_A = c\cosh\frac{x_B}{c}$ $\qquad y_B = c\cosh\frac{x_B}{c}$ (3)

IN Δ ABD: $\tan\theta = \frac{y_B - y_A}{x_B + x_A}$ (4)

METHOD OF SOLUTION:

FOR GIVEN VALUE OF θ, CHOOSE TRIAL VALUE OF c AND CALCULATE:

FROM EQ(1): x_A

USING VALUE OF x_A AND c, CALCULATE:

FROM EQ(2): x_B

FROM EQ(3): y_A AND y_B

SUBSTITUTE VALUES OBTAINED FOR x_A, x_B, y_A, y_B INTO EQ(4) AND CALCULATE θ

CHOOSE NEW TRIAL VALUE OF θ AND REPEAT ABOVE PROCEDURE UNTIL CALCULATED VALUE OF θ IS EQUAL TO GIVEN VALUE OF θ.

PROB. 7.147: GIVEN VALUE: $\theta = 30°$

RESULT OF TRIAL AND ERROR PROCEDURE

$c = 1.803\ m$

$x_A = 2.3745\ m$ $\qquad x_B = 3.6937\ m$

$y_A = 3.606\ m$ $\qquad y_B = 7.109\ m$

$a = y_B - y_A = 7.109m - 3.606m = 3.503\ m$

$a = 3.50\ m$ ◀

PROB. 7.148: GIVEN VALUE: $\theta = 45°$

RESULT OF TRIAL AND ERROR PROCEDURE

$c = 1.8652\ m$

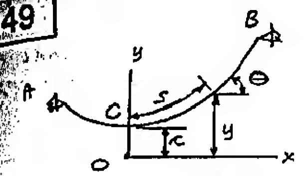
$x_A = 1.644\ m$ $\qquad x_B = 4.064\ m$

$y_A = 2.638\ m$ $\qquad y_B = 8.346\ m$

$a = y_B - y_A = 8.346m - 2.638m = 5.708\ m$

$a = 5.71\ m$ ◀

7.149



GIVEN: UNIFORM CABLE

PROVE: (a) $s = c \tan\theta$.
(b) $y = c \sec\theta$.

(a) EQ. 7.16: $y = c \cosh \frac{x}{c}$

$$\tan\theta = \frac{dy}{dx} = \sinh \frac{x}{c}$$

EQ. 7.15: $s = c \sinh \frac{x}{c}$; $s = c \tan\theta$ ◀

(b) EQ. 7.14: $\cosh^2 \frac{x}{c} - \sinh^2 \frac{x}{c} = 1$

$$\cosh \frac{x}{c} = \sqrt{1 + \sinh^2 \frac{x}{c}} = \sqrt{1 + \tan^2\theta} \qquad (1)$$

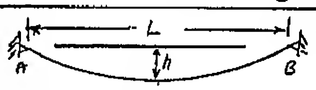
$\sqrt{1+\tan^2\theta}$, $\tan\theta$, 1

$$\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}} \qquad (2)$$

SUBSTITUTE (2) INTO (1): $\cosh \frac{x}{c} = \frac{1}{\cos\theta} \qquad (3)$

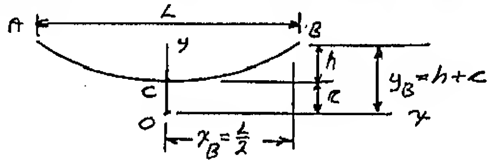
EQ. 7.16: $y = c \cosh \frac{x}{c} = c \frac{1}{\cos\theta}$; $y = c \sec\theta$ ◀

* 7.150



GIVEN: UNIFORM CABLE, w lb/ft

FIND: (a) MAXIMUM SPAN FOR GIVEN VALUE T_m
(b) MAXIMUM SPAN FOR $w = 0.25$ lb/ft AND $T_m = 8000$ lb



(a)

$$T_m = w y_B = w c \cosh \frac{x_B}{c} = w x_B \left(\frac{1}{x_B/c}\right) \cosh \frac{x_B}{c}$$

WE SHALL FIND RATIO (x_B/c) FOR WHICH T_m IS MINIMUM

$$\frac{d T_m}{d(x_B/c)} = w x_B \left[\frac{1}{x_B/c} \sinh \frac{x_B}{c} - \left(\frac{1}{x_B/c}\right)^2 \cosh \frac{x_B}{c}\right] = 0$$

$$\frac{\sinh \frac{x_B}{c}}{\cosh \frac{x_B}{c}} = \frac{1}{x_B/c} \; ; \qquad \tanh \frac{x_B}{c} = \frac{c}{x_B}$$

SOLVE BY TRIAL AND ERROR FOR: $\frac{x_B}{c} = 1.200 \qquad (1)$

$s_B = c \sinh \frac{x_B}{c} = c \sinh(1.200)$: $\frac{s_B}{c} = 1.509$

EQ. 7.17: $y_B^2 - s_B^2 = c^2$; $y_B^2 = c^2\left[1 + \left(\frac{s_B}{c}\right)^2\right] = c^2(1 + 1.509^2)$

$$y_B = 1.810\, c$$

EQ. 7.18: $T_m = w y_B = 1.810\, w c$

$$c = \frac{T_m}{1.810\, w}$$

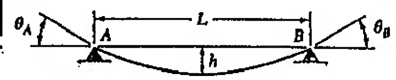
EQ (1): $x_B = 1.509\, c = 1.509\, T_m / 1.810\, w = 0.6630 \frac{T_m}{w}$

SPAN: $L = 2x_B = 2(0.6630) T_m/w$ $\quad L = 1.326 \frac{T_m}{w}$ ◀

(b) FOR $w = 0.25$ lb/ft AND $T_m = 8000$ lb,

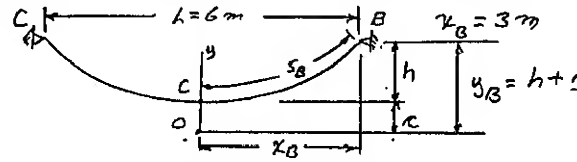
$L = 1.326 \frac{8000 \text{ lb}}{0.25 \text{ lb/ft}} = 42{,}432$ ft ; $L = 8.04$ miles ◀

* 7.151



GIVEN: UNIT MASS = 3 kg/m, $L = 6$ m

FIND: TWO VALUES OF h FOR WHICH $T_m = 350$ N



$x_B = 3$ m

$w = (3 \text{ kg/m})(9.81 \text{ m/s}^2) = 29.43$ N/m

EQ. 7.18: $T_m = w y_B$; 350 N/m $= (29.43$ N/m$) y_B$; $y_B = 11.893$ m

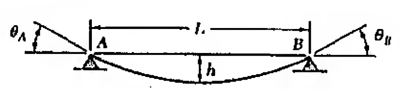
EQ. 7.16: $y_B = c \cosh \frac{x_B}{c}$

$$9.81 \text{ m/s}^2 = c \cosh \frac{3 \text{ m}}{c}$$

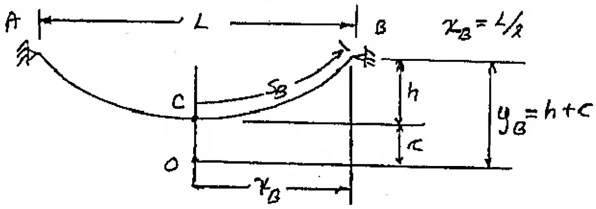
SOLVE BY TRIAL AND ERROR FOR TWO VALUES OF c

$c = 0.9241$ m	$c = 11.499$ m
$h = y_B - c$	$h = y_B - c$
$h = 11.893$ m $- 0.9241$ m	$h = 11.893$ m $- 11.499$ m
$h = 10.97$ m ◀	$h = 0.394$ m ◀

* 7.152



FIND THE h/L RATIO FOR TOTAL WEIGHT EQUAL TO T_m.



$x_B = L/2$

TOTAL WEIGHT: $W = (2 s_B) w$; $\therefore T_m = 2 s_B w$

EQ. 7.15: $s_B = c \sinh \frac{x_B}{c}$

EQ. 7.16: $y_B = c \cosh \frac{x_B}{c}$

EQ. 7.18:

$$T_m = w y_B$$

$$2 s_B w = w y_B$$

$$2c \sinh \frac{x_B}{c} = c \cosh \frac{x_B}{c}$$

$$\tanh \frac{x_B}{c} = \frac{1}{2} \; ; \quad \frac{x_B}{c} = 0.5493 \qquad (1)$$

$$h = y_B - c = c \cosh \frac{x_B}{c} - c = c\left[\cosh(0.5493) - 1\right]$$

$$h = c(1.1547 - 1) = 0.1547\, c$$

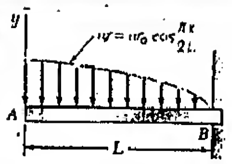
FROM (1): $c = \frac{x_B}{0.5493}$

THUS: $h = (0.1547) \frac{x_B}{0.5493} = 0.2816\, x_B$

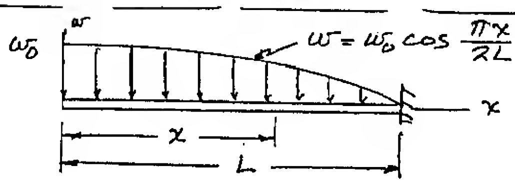
RECALL: $L = 2x_B$; $h = (0.2816) \frac{L}{2}$

$$\frac{h}{L} = 0.1408 \; ◀$$

7.159



WRITE EQUATIONS OF V AND M CURVES.
FIND: M_{max}



$$\frac{dV}{dx} = -w = -w_0 \cos \frac{\pi x}{2L}$$

$$V = -\int w\,dx = -w_0\left(\frac{2L}{\pi}\right)\sin\frac{\pi x}{2L} + C_1 \qquad (1)$$

$$\frac{dM}{dx} = V = -w_0\left(\frac{2L}{\pi}\right)\sin\frac{\pi x}{2L} + C_1$$

$$M = \int V\,dx = +w_0\left(\frac{2L}{\pi}\right)^2\cos\frac{\pi x}{2L} + C_1 x + C_2 \qquad (2)$$

BOUNDARY CONDITIONS

AT $x=0$: $V = C_1 = 0$ $\qquad C_1 = 0$

AT $x=0$: $M = +w_0\left(\frac{2L}{\pi}\right)^2\cos(0) + C_2 = 0$

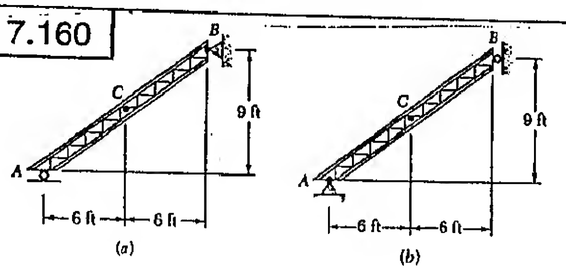
$$C_2 = -w_0\left(\frac{2L}{\pi}\right)^2$$

EQ.(1) $\quad V = -w_0\left(\frac{2L}{\pi}\right)\sin\frac{\pi x}{2L}$ ◀

$$M = w_0\left(\frac{2L}{\pi}\right)^2\left(-1+\cos\frac{\pi x}{2L}\right) \quad ◀$$

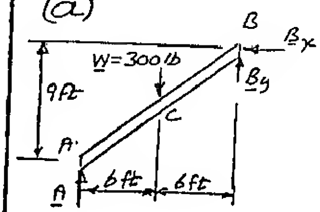
M_{max} at $x=L$: $|M_{max}| = w_0\left(\frac{2L}{\pi}\right)^2|-1+0| = \frac{4}{\pi^2}w_0 L^2$ ◀

7.160



GIVEN: CHANNEL WEIGHS 20 lb/ft
FIND: INTERNAL FORCES AT C FOR EACH SUPPORT

(a)



FREE BODY: AB

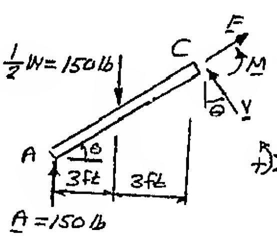
$AB = \sqrt{9^2+12^2} = 15\,ft$

$W = (20\,lb/ft)(15\,ft) = 300\,lb$

$+\circlearrowleft \Sigma M_B = 0$:

$A(12\,ft) - (300\,lb)(6\,ft) = 0$

$A = +150\,lb \qquad \underline{A} = 150\,lb \uparrow$



FREE BODY: AC

(150-lb FORCES FORM A COUPLE)

$\nearrow \Sigma F = 0 \qquad F = 0$ ◀

$\nwarrow \Sigma F \qquad \underline{V} = 0$ ◀

$+\circlearrowleft \Sigma M_C = 0$: $M - (150\,lb)(3\,ft) = 0$

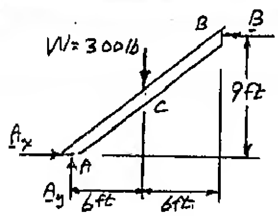
$M = +450\,lb\cdot ft$

$\underline{M} = 450\,lb\cdot ft \circlearrowleft$ ◀

(CONTINUED)

7.160 CONTINUED

(b)



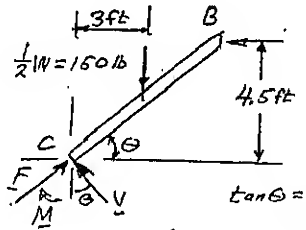
FREE BODY: AB

$+\circlearrowleft \Sigma M_A = 0$:

$B(9\,ft) - (300\,lb)(6\,ft) = 0$

$B = +200\,lb$

$\underline{B} = 200\,lb \leftarrow$



FREE BODY: CB

$+\circlearrowleft \Sigma M_C = 0$: $(200\,lb)(4.5\,ft) - (150\,lb)(3\,ft) - M = 0$

$M = +450\,lb\cdot ft$

$\underline{M} = 450\,lb\cdot ft \circlearrowright$

$\tan\theta = \frac{9}{12} = \frac{3}{4}$; $\sin\theta = \frac{3}{5}$; $\cos\theta = \frac{4}{5}$

$+\nearrow \Sigma F = 0$: $F - \frac{3}{5}(150\,lb) - \frac{4}{5}(200\,lb) = 0$

$F = +250\,lb \qquad \underline{F} = 250\,lb \nearrow$

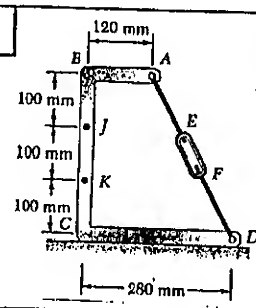
$+\nwarrow \Sigma F = 0$: $V - \frac{4}{5}(150\,lb) + \frac{3}{5}(200\,lb) = 0$

$V = 0 \qquad \underline{V} = 0$

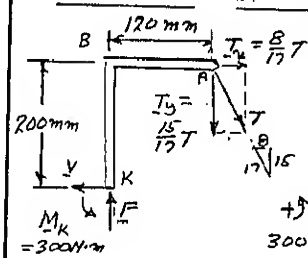
ON PORTION AC INTERNAL FORCES ARE

$\underline{M} = 450\,lb\cdot ft \circlearrowright$, $\underline{F} = 250\,lb \swarrow$, $\underline{V} = 0$ ◀

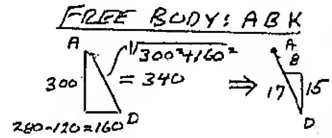
7.161



GIVEN: $M_K = 300\,N\cdot m$
FIND: (a) TENSION IN RODS
(b) INTERNAL FORCES AT J.



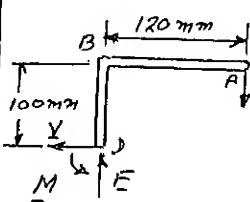
FREE BODY: ABK



$+\circlearrowleft \Sigma M_K = 0$

$300\,N\cdot m - \frac{8}{17}T(0.2\,m) - \frac{15}{17}T(0.12\,m) = 0$

$T = 1500\,N$ ◀



FREE BODY: AJ

$T_x = \frac{8}{17}T = \frac{8}{17}(1500\,N) = 705.88\,N$

$T_y = \frac{15}{17}T = \frac{15}{17}(1500\,N) = 1323.53\,N$

INTERNAL FORCES ON ABJ

$\xrightarrow{+} \Sigma F_x = 0$: $705.88\,N - V = 0$

$V = +705.88\,N \qquad \underline{V} = 706\,N \leftarrow$ ◀

$+\uparrow \Sigma F_y = 0$: $F - 1323.53\,N = 0$

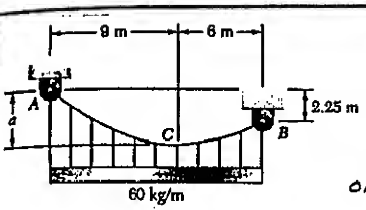
$F = +1323.53\,N \qquad \underline{F} = 1324\,N \uparrow$ ◀

$+\circlearrowleft \Sigma M_J = 0$:

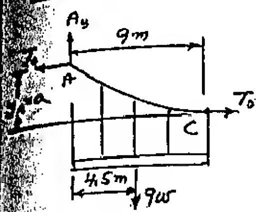
$M - (705.88\,N)(0.1\,m) - (1323.53\,N)(0.12\,m) = 0$

$M = +229.4\,N\cdot m \qquad \underline{M} = 229\,N\cdot m \circlearrowright$ ◀

7.162



FIND:
(a) DISTANCE a
(b) LENGTH ACB
(c) COMPONENTS OF REACTION AT A.

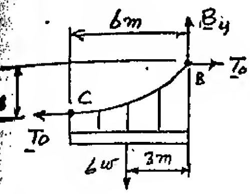


FREE BODY: PORTION AC

$+\uparrow \Sigma F_y = 0$: $A_y - 9w = 0$

$A_y = 9w \uparrow$

$+\circlearrowleft \Sigma M_A = 0$: $T_0 a - (9w)(4.5m) = 0$ (1)



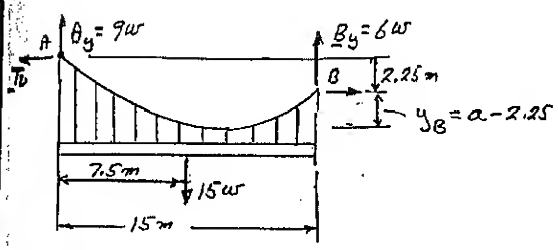
FREE BODY: PORTION CB

$+\uparrow \Sigma F_y = 0$; $B_y - 6w = 0$

$B_y = 6w \uparrow$

$+\circlearrowleft \Sigma M_B = 0$; $T_0 a - 6w(3m) = 0$ (2)

FREE BODY: ENTIRE CABLE



$+\circlearrowleft \Sigma M_A = 0$: $15w(7.5m) - 6w(15m) - T_0(2.25m) = 0$

(a) $T_0 = 10w$

EQ(1): $T_0 a - (9w)(4.5m) = 0$

$10\cancel{w} a = (9\cancel{w})(4.5) = 0$ $\qquad a = 4.05m$

(b) LENGTH = AC + CB

PORTION AC $\quad x_A = 9m,\ y_A = a = 4.05m;\ \frac{y_A}{x_A} = \frac{4.05}{9} = 0.45$

$$S_{AC} = x_B\left[1 + \frac{2}{3}\left(\frac{y_A}{x_A}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_A}\right)^4 + \cdots\right]$$

$S_{AC} = 9m\left(1 + \frac{2}{3}0.45^2 - \frac{2}{5}0.45^4 + \cdots\right) = 10.067m$

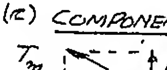
PORTION CB: $x_B = 6m$, $y_B = 4.05 - 2.25 = 1.8m$; $\frac{y_B}{x_B} = 0.3$

$S_{CB} = 6m\left(1 + \frac{2}{3}0.3^2 - \frac{2}{5}0.3^4 + \cdots\right) = 6.341m$

TOTAL LENGTH = 10.067m + 6.341m

TOTAL LENGTH = 16.45m

(c) COMPONENTS OF REACTION AT A.



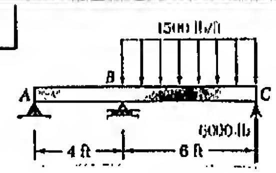
$A_y = 9w = 9(60\,kg/m)(9.81\,m/s^2) = 5297.4N$

$A_x = T_0 = 10w = 10(60\,kg/m)(9.81\,m/s^2) = 5886N$

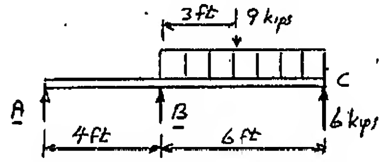
$A_x = 5890N \leftarrow$

$A_y = 5300N \uparrow$

7.163



(a) DRAW V + M DIAGRAMS
(b) FIND $|M_{max}|$



FREE BODY: ENTIRE BEAM

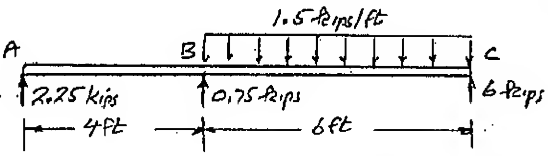
$+\circlearrowleft \Sigma M_A = 0$: $(6\,kips)(10ft) - (9kips)(7ft) + B(4ft) = 0$

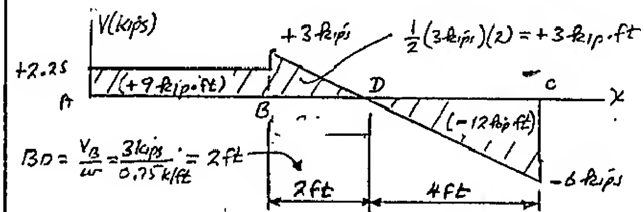
$B = +0.75\,kips$

$B = 0.75\,kips \uparrow$

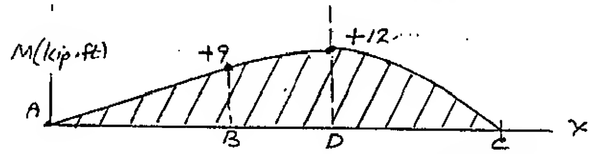
$+\uparrow \Sigma F = 0$: $A + 0.75\,kips - 9\,kips + 6\,kips = 0$

$A = +2.25\,kips \qquad A = 2.25\,kips \uparrow$



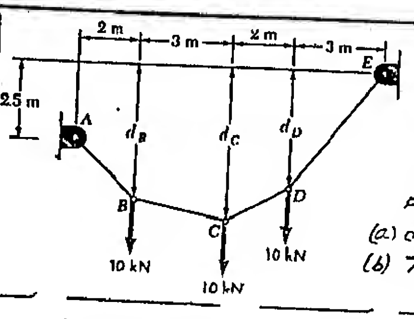


$BD = \frac{V_B}{w} = \frac{3\,kips}{0.75\,k/ft} = 2ft$



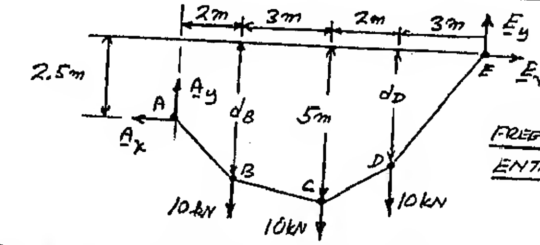
$M_{max} = 12\,kip \cdot ft$

7.164



GIVEN: $d_C = 5\,m$

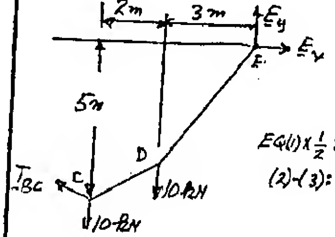
FIND: (a) d_B AND d_D (b) T_m



FREE BODY: ENTIRE CABLE

$+\circlearrowleft \Sigma M_A = 0$: $(10\,kN)(2m) + (10\,kN)(5m) + (10\,kN)(7m) + E_x(2.5m) - E_y(10m) = 0$

$2.5E_x - 10E_y + 140 = 0 \quad (1)$



FREE BODY: PORTION CDE

$+\circlearrowleft \Sigma M_C = 0$: $(10\,kN)(2m) + E_x(5m) - E_y(5m) = 0$

$5E_x - 5E_y + 20 = 0 \quad (2)$

EQ(1) $\times \frac{1}{2}$: $1.25E_x - 5E_y + 70 = 0 \quad (3)$

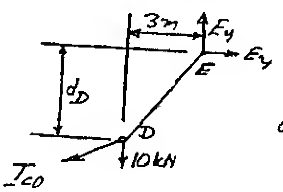
(2)-(3): $3.75E_x - 50 = 0$

$E_x = 13.333\,kN$

EQ(1): $2.5(13.333) - 10E_y + 140 = 0$

$E_y = 17.333\,kN$

$T_m = \sqrt{E_x^2 + E_y^2} = \sqrt{(13.333)^2 + (17.333)^2}$ $\quad T_m = 21.9\,kN$ ◀



FREE BODY: PORTION DE

$+\circlearrowleft \Sigma M_D = 0$: $E_x d_D - E_y(3m) = 0$

$d_D = 3\frac{E_y}{E_x} = 3\frac{17.333\,kN}{13.333\,kN}$

$d_D = 3.90\,m$ ◀

RETURN TO FREE BODY OF ENTIRE CABLE AND WRITE

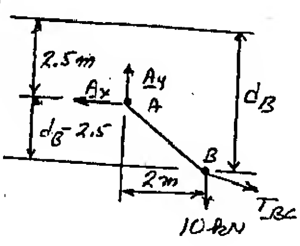
$+\uparrow \Sigma F_y = 0$: $A_y - 3(10\,kN) + E_y = 0$

$A_y - 30\,kN + 17.333\,kN = 0$

$A_y = 12.667\,kN$

$\pm \Sigma F_x = 0$: $-A_x + E_x = 0$ $\quad A_x = 13.333\,kN$

FREE BODY: PORTION AB



$+\circlearrowleft \Sigma M_B = 0$

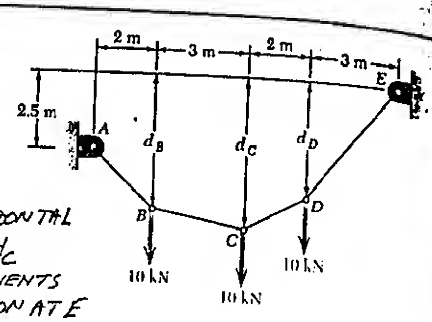
$A_x(d_B - 2.5) - A_y(2m) = 0$

$d_B - 2.5 = 2\frac{A_y}{A_x} = 2\frac{12.667\,kN}{13.333\,kN}$

$d_B - 2.5 = 1.90$

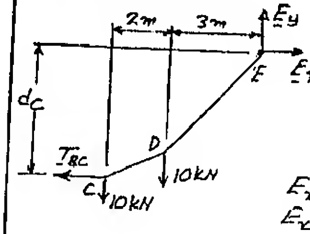
$d_B = 4.40\,m$ ◀

7.165



GIVEN: BC IS HORIZONTAL

FIND: (a) d_C (b) COMPONENTS OF REACTION AT E



FREE BODY: PORTION CDE

$+\uparrow \Sigma F_y = 0$: $E_y - 10\,kN - 10\,kN = 0$

$E_y = 20\,kN$

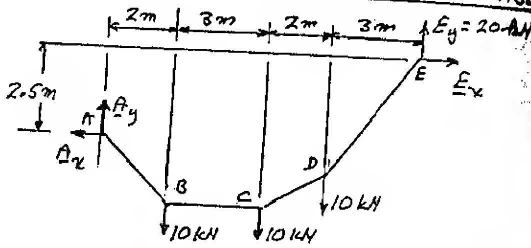
$+\circlearrowleft \Sigma M_C = 0$

$E_x d_C - E_y(5m) + (10\,kN)(2m) = 0$

$E_x d_C - (20\,kN)(5m) + (10\,kN)(2m) = 0$

$E_x d_C = 80\,kN\cdot m \quad (1)$

FREE BODY: ENTIRE CABLE



$+\circlearrowleft \Sigma M_A = 0$:

$(10\,kN)(2m) + (10\,kN)(5m) + (10\,kN)(7m) + E_x(2.5m) - (20\,kN)(10m) = 0$

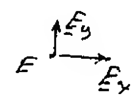
$E_x = 24\,kN$

EQ.(1): $E_x d_C = 80\,kN\cdot m$

$(24\,kN\cdot m)\,d_C = 80\,kN\cdot m$

$d_C = 3.333\,kN$ $\quad d_C = 3.33\,m$ ◀

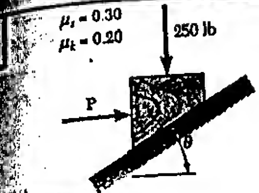
AT E:



$E_x = 24\,kN \rightarrow$ ◀

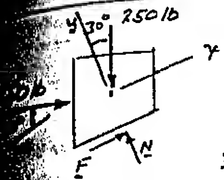
$E_y = 20\,kN \uparrow$ ◀

8.1



GIVEN: $\theta = 30°$, $P = 50\ lb$.

FIND: FRICTION FORCE ACTING ON BLOCK.



ASSUME EQUILIBRIUM

$+\nearrow \Sigma F_y = 0$: $N - (250\ lb)\cos 30° - (50\ lb)\sin 30° = 0$

$N = +241.5\ lb$ $N = 241.5\ lb$

$+\searrow \Sigma F_x = 0$: $F - (250\ lb)\sin 30° + (50\ lb)\cos 30° = 0$

$F = +81.71\ lb$ $F = 81.7\ lb$

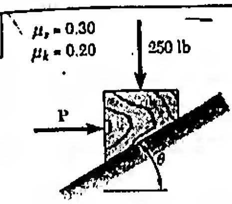
MAXIMUM FRICTION FORCE: $F_m = \mu_s N = 0.3(241.5\ lb) = 72.5\ lb$

SINCE $F > F_m$, BLOCK MOVES DOWN

FRICTION FORCE: $F = \mu_k N = (0.20)(241.5\ lb) = 48.3\ lb$

$F = 48.3\ lb$ ◀

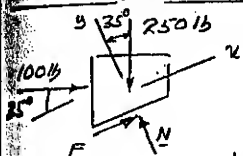
8.2



GIVEN: $\theta = 35°$

$P = 100\ lb$

FIND: FRICTION FORCE ACTING ON BLOCK.



ASSUME EQUILIBRIUM

$+\nearrow \Sigma F_y = 0$: $N - (250\ lb)\cos 35° + (100\ lb)\sin 35° = 0$

$N = 262.15\ lb$ $N = 262.15\ lb$

$+\searrow \Sigma F_x = 0$: $F - (250\ lb)\sin 35° + (100\ lb)\cos 35° = 0$

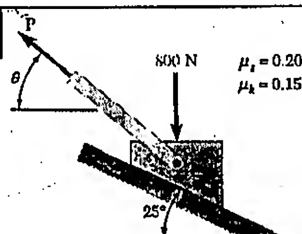
$F = +61.48\ lb$ $F = 61.48\ lb$

MAXIMUM FRICTION FORCE: $F_m = \mu_s N = 0.3(262.15\ lb) = 78.64\ lb$

SINCE $F < F_m$, BLOCK IS IN EQUILIBRIUM

FRICTION FORCE: $F = 61.5\ lb$ ◀

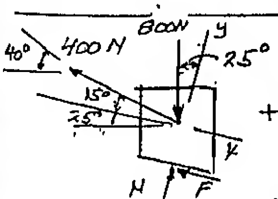
8.3



GIVEN: $\theta = 40°$

$P = 400\ N$

FIND: FRICTION FORCE ACTING ON BLOCK.



ASSUME EQUILIBRIUM

$+\nearrow \Sigma F_y = 0$:

$N - (800\ N)\cos 25° - (400\ N)\sin 15° = 0$

$N = +621.5\ N$ $N = 621.5\ N$

$+\nwarrow \Sigma F_x = 0$

$-F + (800\ N)\sin 25° - (400\ N)\cos 15° = 0$

$F = +48.28\ N$ $F = 48.28\ N$

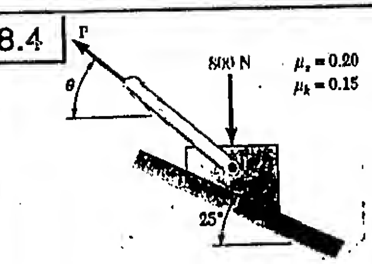
MAXIMUM FRICTION FORCE:

$F_m = \mu_s N = 0.20(621.5\ N) = 124.3\ N$

SINCE $F < F_m$, BLOCK IS IN EQUILIBRIUM

$F = 48.3\ N$ ◀

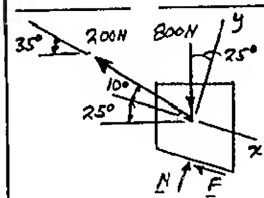
8.4



GIVEN: $\theta = 35°$

$P = 200\ N$

FIND: FRICTION FORCE ACTING ON BLOCK.



ASSUME EQUILIBRIUM

$+\nearrow \Sigma F_y = 0$:

$N - (800\ N)\cos 25° + (200\ N)\sin 10° = 0$

$N = 690.3\ N$ $N = 690.3\ N$

$+\nwarrow \Sigma F_x = 0$:

$-F + (800\ N)\sin 25° - (200\ N)\cos 10° = 0$

$F = 141.13\ N$ $F = 141.13\ N$

MAXIMUM FRICTION FORCE:

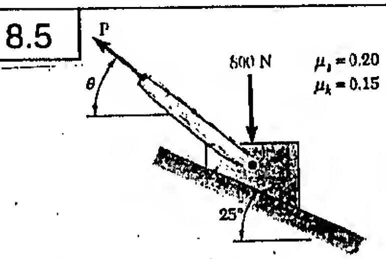
$F_m = \mu_s N = (0.20)(690.3\ N) = 138.06\ N$

SINCE $F > F_m$, BLOCK MOVES DOWN

FRICTION FORCE: $F = \mu_k N = (0.15)(690.3\ N) = 103.54\ N$

$F = 103.5\ N$ ◀

8.5



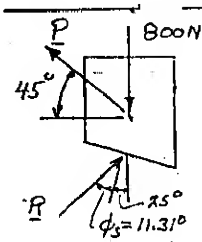
GIVEN: $\theta = 45°$

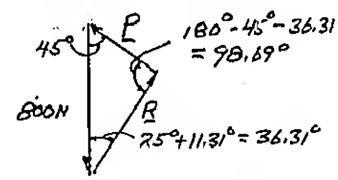
FIND: RANGE OF VALUES OF P FOR EQUILIBRIUM

TO START BLOCK UP THE INCLINE

$\mu_s = 0.20$

$\phi_s = \tan^{-1} 0.20 = 11.31°$



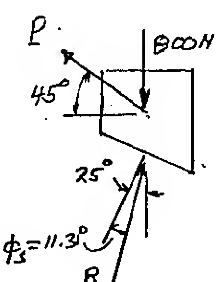


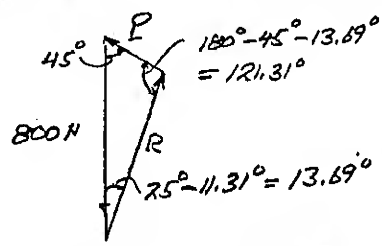
FROM FORCE TRIANGLE

$$\frac{P}{\sin 36.31°} = \frac{800\ N}{\sin 98.69°}$$

$P = 479.2\ N$ ◁

TO PREVENT BLOCK FROM MOVING DOWN





FROM FORCE TRIANGLE

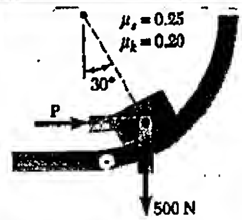
$$\frac{P}{\sin 13.69°} = \frac{800\ N}{\sin 121.31°}$$

$P = 221.61\ N$ ◁

EQUILIBRIUM IS MAINTAINED FOR

$222\ N \le P \le 479\ N$ ◀

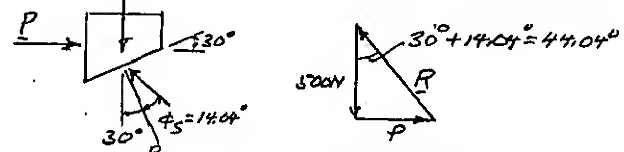
8.6



FIND: RANGE OF VALUES OF P FOR WHICH EQUILIBRIUM IS MAINTAINED.

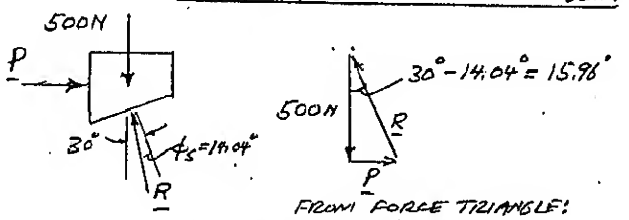
<u>TO START BLOCK UP THE SLOPE</u>

$\mu_s = 0.25$
$\phi_s = \tan^{-1} 0.25 = 14.04°$



FROM FORCE TRIANGLE:
$P = (500\,N) \tan 44.04°$; $P = 483\,lb$ ◁

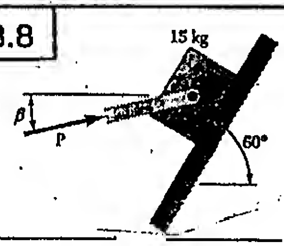
<u>TO PREVENT BLOCK FROM MOVING DOWN</u>



FROM FORCE TRIANGLE:
$P = (500\,N) \tan 15.96°$; $P = 143.0\,lb$ ◁

EQUILIBRIUM MAINTAINED FOR: $143.0\,lb \le P \le 483\,lb$ ◀

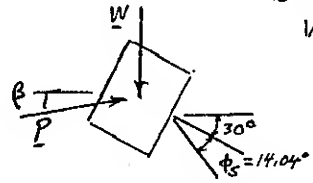
8.8



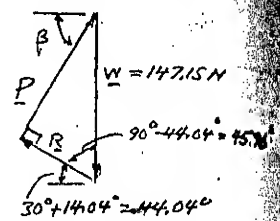
GIVEN: $\mu_s = 0.25$

FIND: (a) SMALLEST VALUE OF P FOR EQUILIBRIUM.
(b) CORRESPONDING VALUE OF β.

TO PREVENT BLOCK FROM MOVING DOWN THE INCLINE
$\mu_s = 0.25$; $\phi_s = \tan^{-1} 0.25 = 14.04°$
$W = (15\,kg)(9.81\,m/s^2) = 147.15\,N$



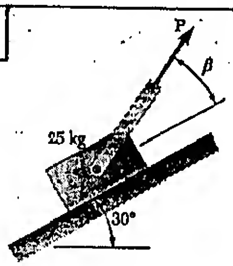
FORCE TRIANGLE FOR SMALLEST P WE CHOOSE $P \perp R$.



$\beta = 45.96°$, $\beta = 46.0°$ ◀

$P = (147.15\,N) \sin 45.96° = 105.78\,N$
$P = 105.8\,N$ ◀

8.7

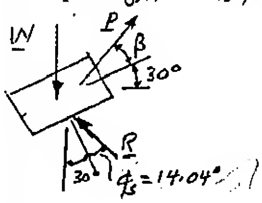


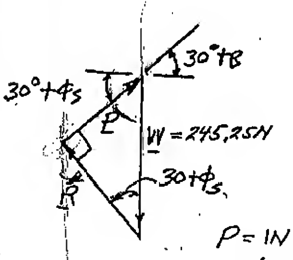
GIVEN: $\mu_s = 0.25$

FIND: (a) SMALLEST VALUE OF P TO START BLOCK UP THE INCLINE.
(b) CORRESPONDING VALUE OF β

TO START BLOCK MOVING UP THE INCLINE
$W = (25\,kg)(9.81\,m/s^2) = 245.25\,N$

$\mu_s = 0.25$
$\phi_s = \tan^{-1} 0.25 = 14.04°$





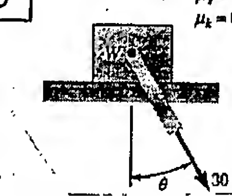
FORCE TRIANGLE FOR SMALLEST P WE CHOOSE $P \perp R$.
$30° + \phi_s = 30° + \beta$
$\therefore \beta = \phi_s = 14.04°$

$\beta = 14.0°$ ◀

$P = W \sin(30° + \phi_s)$
$= (245.25\,N) \sin 44.04°$
$= 170.49\,N$

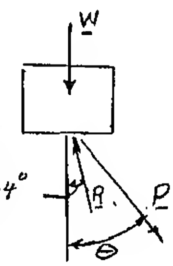
$P = 170.5\,N$ ◀

8.9

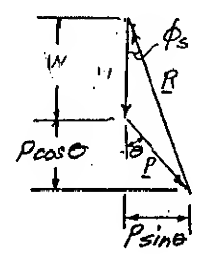


FIND: FOR $\theta < 90°$, THE VALUE OF θ REQUIRED TO START BLOCK MOVING TO RIGHT WHEN (a) $W = 75\,lb$
(b) $W = 100\,lb$

$\mu_s = 0.25$
$\phi_s = \tan^{-1} 0.25$
$\tan \phi_s = 0.25$



FORCE TRIANGLE



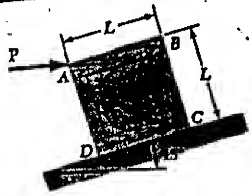
$$\tan\phi_s = \frac{P\sin\theta}{W + P\cos\theta}$$

$0.25(W + P\cos\theta) = P\sin\theta$
$\frac{W}{P} + \cos\theta = 4\sin\theta$

(a) <u>FOR $W = 75\,lb$</u>, $P = 30\,lb$; $\frac{W}{P} = 2.5$
$2.5 + \cos\theta = 4\sin\theta$
SOLVE BY TRIAL & ERROR
$\theta = 51.36°$ $\theta = 51.4°$ ◀

(b) <u>FOR $W = 100\,lb$</u>, $P = 30\,lb$; $W/P = 3.333$
$3.333 + \cos\theta = 4\sin\theta$
$\theta = 67.98°$ $\theta = 68.0°$ ◀

8.17



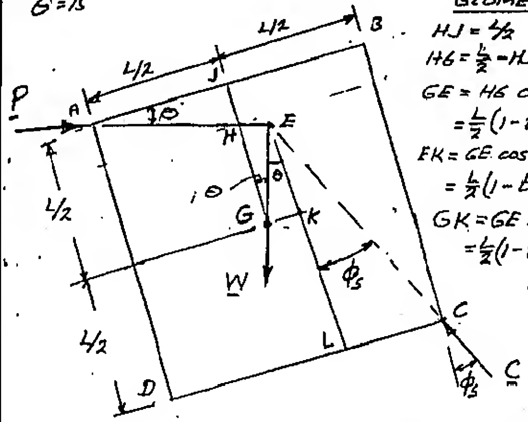
GIVEN: Mass of crate = 30 kg.

FIND: (a) Largest μ_s for which crate can be started up with no tipping. (b) Corresponding magnitude of horizontal force P.

For tipping to be impending reaction is at C.

FREE BODY: CRATE   Three-force body. Reaction C must pass through E where $P + W$ intersect.

(a) $\theta = 15°$



GEOMETRY

$HJ = \frac{L}{2}\tan\theta$

$HG = \frac{L}{2} - HJ = \frac{L}{2}(1-\tan\theta)$

$GE = HG\cos\theta$

$= \frac{L}{2}(1-\tan\theta)\cos\theta$

$EK = GE\cos\theta$

$= \frac{L}{2}(1-\tan\theta)\cos^2\theta$

$GK = GE\sin\theta$

$= \frac{L}{2}(1-\tan\theta)\cos\theta\sin\theta$

$$EL = \frac{L}{2} + EK = \frac{L}{2} + \frac{L}{2}(1-\tan\theta)\cos^2\theta = \frac{L}{2} + \frac{L}{2}\left(\cos^2\theta - \frac{\sin\theta}{\cos\theta}\cos^2\theta\right)$$

$$= \frac{L}{2}(1+\cos^2\theta - \sin\theta\cos\theta)$$

$$EL = \frac{L}{2}(1+\cos^2 15° - \sin 15°\cos 15°) = 0.84151\,L$$

$$LC = \frac{L}{2} - GK = \frac{L}{2} - \frac{L}{2}(1-\tan\theta)\cos\theta\sin\theta$$

$$= \frac{L}{2} - \frac{L}{2}\left(\cos\theta\sin\theta - \frac{\sin\theta}{\cos\theta}\cos\theta\sin\theta\right)$$

$$= \frac{L}{2}(1 + \cos\theta\sin\theta + \sin^2\theta)$$

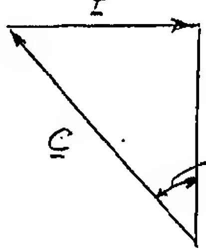
$$LC = \frac{L}{2}(1 - \cos 15°\sin 15° + \sin^2 15°) = 0.40849\,L$$

$$\tan\phi_s = \frac{LC}{EL} = \frac{0.40849\,L}{0.84151\,L} = 0.48543;\quad \phi_s = 25.89°$$

$$\mu_s = \tan\phi_s = 0.485$$

(b) FORCE TRIANGLE

$W = mg = (30\text{ kg})(9.81\text{ m/s}^2)$
$= 294.3\text{ N}$

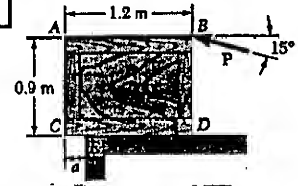


$\theta + \phi_s = 15° + 25.89° = 40.89°$

$P = W\tan(\theta + \phi_s)$
$= (294.3\text{ N})\tan 40.89°$
$= 254.8\text{ N}$

$P = 255\text{ N} \rightarrow$

8.18

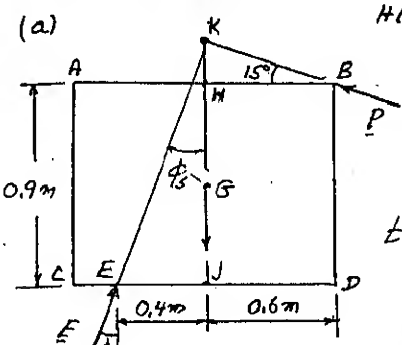


GIVEN: Mass of crate = 50 kg. For $a = 0.2$ m tipping impends.

FIND (a) μ_s (b) Magnitude of P

FREE BODY: CRATE   Three-force body. Reaction E must pass through K where P and W intersect.

(a)



GEOMETRY

$HK = (0.6\text{ m})\tan 15°$
$= 0.16077\text{ m}$

$JK = 0.9\text{ m} + HK$
$= 1.06077\text{ m}$

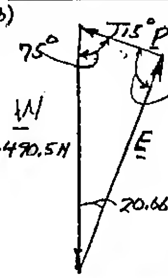
$\tan\phi_s = \frac{0.4\text{ m}}{1.06077\text{ m}} = 0.37708$

$\mu_s = \tan\phi_s = 0.377$

$\phi_s = 20.66°$

FORCE TRIANGLE

(b)



$180° - 75° - 20.66°$
$84.34°$

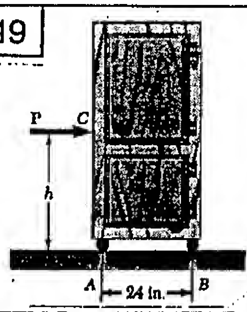
$W = (50\text{ kg})(9.81\text{ m/s}^2) = 490.5\text{ N}$

LAW OF SINES

$$\frac{P}{\sin 20.66°} = \frac{490.5\text{ N}}{\sin 84.34°}$$

$P = 173.91\text{ N}$ $P = 173.9\text{ N}$

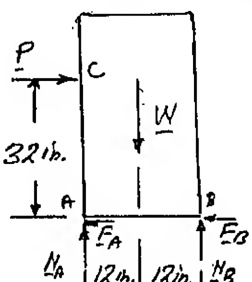
8.19



GIVEN: 120-lb cabinet
$h = 32$ in., $\mu_s = 0.30$

FIND: Force P required to move cabinet, when
(a) All casters are locked
(b) Casters B are locked and casters A are free
(c) Casters A are locked and caster B are free

(a) ALL CASTERS LOCKED



$+\uparrow\Sigma F_y = 0:\quad N_A + N_B - W = 0$

$N_A + N_B = W = 120\text{ lb}$

$F_A + F_B = \mu_s N_A + \mu_s N_B = \mu_s(N_A + N_B)$

$= 0.30(120\text{ lb}) = 36\text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0:\quad P - F_A - F_B = 0$

$P = F_A + F_B$

$P = 36\text{ lb} \rightarrow$

CHECK FOR TIPPING

$+\circlearrowleft \Sigma M_B = 0$

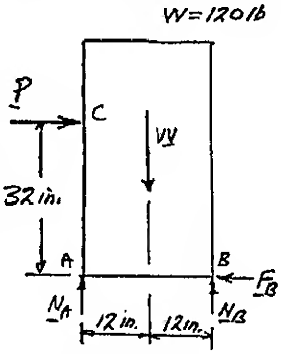
$(120\text{ lb})(12\text{ in.}) - (36\text{ lb})(32\text{ in.}) - N_A(24\text{ in.}) = 0$

$N_A = +12\text{ lb} > 0$ OK

(CONTINUED)

8.19 CONTINUED

(b) CASTERS LOCKED AT B AND FREE AT A



$F_B = \mu_s N_B = 0.3\,N_B$

$\xrightarrow{+}\Sigma F_x = 0$: $\quad P = F_B = 0.3 N_B \quad (1)$

$+\circlearrowleft \Sigma M_A = 0$:

$-P(32\text{ in.}) - (120\text{ lb})(12\text{ in.}) + N_B(24\text{ in.}) = 0$

$-0.3N_B(32\text{ in.}) + N_B(24\text{ in.}) = (120\text{ lb})(12\text{ in.}) = 0$

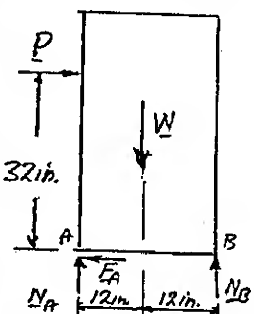
$14.4\, N_B = (120\text{ lb})(12\text{ in.})$

$N_B = 100\text{ lb}$

EQ.(1) $\quad P = 0.3(100\text{ lb}) = 30\text{ lb}$

$P = 30\text{ lb} \rightarrow$

(c) CASTERS LOCKED AT A AND FREE AT B



$F_A = \mu_s N_A = 0.3\,N_A$

$\Sigma F_x = 0$: $\quad P = F_A = 0.3 N_A \quad (2)$

$+\circlearrowleft \Sigma M_B = 0$

$-P(32\text{ in.}) + (120\text{ lb})(12\text{ in.}) - N_A(24\text{ in.}) = 0$

$-0.3N_A(32\text{ in.}) - N_A(24\text{ in.}) + (120\text{ lb})(12\text{ in.}) = 0$

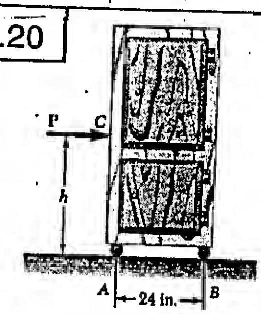
$33.6\, N_A = (120\text{ lb})(12\text{ in.})$

$N_A = 42.85\text{ lb}$

EQ.(2): $\quad P = 0.3(42.857\text{ lb}) = 12.86\text{ lb}$

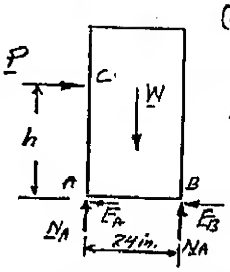
$P = 12.86\text{ lb} \rightarrow$

8.20



GIVEN: 120-lb CABINET
$\mu_s = 0.30$,
ALL CASTERS ARE LOCKED.

FIND: (a) FORCE P TO MOVE CABINET
(b) MAXIMUM h IF CABINET IS NOT TO TIP



(a) $W = 120\text{ lb}$

$+\uparrow \Sigma F_y = 0$: $\quad N_A + N_B - W = 0$

$N_A + N_B = 120\text{ lb}$

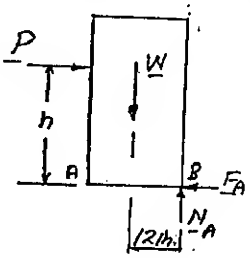
$F_A + F_B = \mu_s N_A + \mu_s N_B = \mu_s(N_A + N_B)$

$F_A + F_B = 0.3(120\text{ lb}) = 36\text{ lb}$

$\xrightarrow{+}\Sigma F_x = 0$: $\quad P - F_A - F_B = 0$

$P = F_A + F_B \qquad P = 36\text{ lb} \rightarrow$

(b) LARGEST ALLOWABLE VALUE OF h.



WHEN TIPPING IMPENDS THERE IS NO REACTION AT A. $N_A = 0$

$+\circlearrowleft \Sigma M_B = 0$:

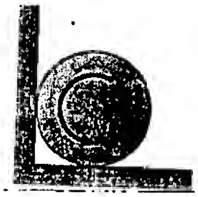
$W(12\text{ in.}) - Ph = 0$

$h = \dfrac{W}{P}(12\text{ in.})$

$= \dfrac{120\text{ lb}}{36\text{ lb}}(12\text{ in.}) = 40\text{ in.}$

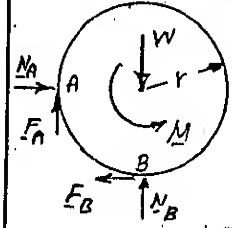
$h = 40\text{ in.}$

8.21



GIVEN: r = RADIUS, W = WEIGHT, μ_s IS SAME AT A AND B.

FIND: LARGEST M IF CYLINDER IS NOT TO ROTATE



SINCE MOTION WILL IMPEND,

$F_A = \mu_s N_A \qquad F_B = \mu_s N_B$

$+\circlearrowleft \Sigma M_B = 0$: $\quad M - rF_A - rN_A = 0$

$M = rN_A + rF_A = rN_A + r\mu_s N_A$

$M = rN_A(1 + \mu_s) \quad (1)$

$\xrightarrow{+}\Sigma F_x = 0$: $\quad N_A - F_B = 0; \quad N_A = \mu_s N_B \quad (2)$

$+\uparrow \Sigma F_y = 0$: $\quad N_B + F_A - W = 0; \quad N_B = W - \mu_s N_A \quad (3)$

SUBSTITUTE FOR N_B FROM (3) INTO (2):

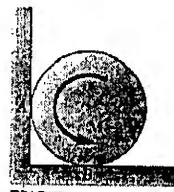
$N_A = \mu_s(W - \mu_s N_A)$

$N_A(1 + \mu_s^2) = \mu_s W \qquad N_A = \dfrac{\mu_s W}{1 + \mu_s^2}$

SUBSTITUTE FOR N_A INTO (1):

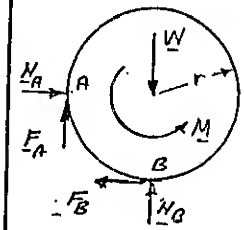
$M = r\dfrac{\mu_s W}{(1 + \mu_s^2)}(1 + \mu_s) \qquad M = Wr\mu_s\left(\dfrac{1 + \mu_s}{1 + \mu_s^2}\right)$

8.22



GIVEN: r = RADIUS
W = WEIGHT

FIND: LARGEST M IF CYLINDER IS NOT TO ROTATE
(a) FOR $\mu_A = 0$, $\mu_B = 0.30$,
(b) FOR $\mu_A = 0.25$, $\mu_B = 0.30$.



SINCE MOTION WILL IMPEND

$F_A = \mu_A N_A \qquad F_B = \mu_B N_B$

$+\circlearrowleft \Sigma M_B = 0$:

$M - rF_A - rN_A = 0$

$M = rN_A + rF_A = rN_A + r\mu_A N_A$

$M = rN_A(1 + \mu_A) \quad (1)$

$\xrightarrow{+}\Sigma F_x = 0$: $\quad N_A - F_B = 0 \qquad N_A = \mu_B N_B \quad (2)$

$+\uparrow \Sigma F_y = 0$: $\quad N_B + F_A - W = 0 \qquad N_B = W - \mu_A N_A \quad (3)$

SUBSTITUTE FOR N_B FROM (3) INTO (2)

$N_A = \mu_B(W - \mu_A N_A)$

$N_A(1 + \mu_A\mu_B) = \mu_B W \qquad N_A = \dfrac{\mu_B W}{1 + \mu_A \mu_B}$

EQ.(1): $\quad M = r\dfrac{\mu_B W}{1 + \mu_A\mu_B}(1 + \mu_A) \qquad M = Wr\dfrac{\mu_B(1 + \mu_A)}{1 + \mu_A\mu_B}$

(a) FOR $\mu_A = 0$ AND $\mu_B = 0.30$:

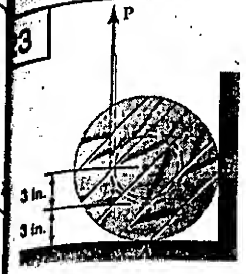
$M = Wr\dfrac{0.30}{1} \qquad M = 0.300\,Wr$

(b) FOR $\mu_A = 0.25$ AND $\mu_B = 0.30$:

$M = Wr\dfrac{(0.30)(1 + 0.25)}{1 + (0.25)(0.30)} = Wr\dfrac{(0.30)(1.25)}{1.075}$

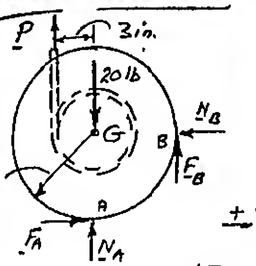
$M = 0.3488\,Wr \qquad M = 0.349\,Wr$

8.23



GIVEN: $W = 20$ lb
AT A AND B, $\mu_s = 0.40$, $\mu_k = 0.30$

FIND: MAGNITUDE OF $\underline{P}$ TO DRAW WIRE AT A CONSTANT RATE.



SINCE SPOOL IS ROTATING
$F_A = \mu_k N_A \qquad F_B = \mu_k N_B$

$+\circlearrowleft \Sigma M_G = 0$:
$P(3in.) - F_A(6in.) - F_B(6in.) = 0$
$3P - 6\mu_k(N_A + N_B) = 0 \qquad (1)$

$\xrightarrow{+} \Sigma F_x = 0$: $F_A - N_B = 0$
$N_B = \mu_k N_A \qquad (2)$

$+\uparrow \Sigma F_y = 0$: $P + N_A + F_B - 20 lb = 0$
$P + N_A + \mu_k N_B - 20 = 0$

SUBSTITUTE FOR N_B FROM (2): $P + N_A + \mu_k^2 N_A - 20 = 0$

$$N_A = \frac{20 - P}{1 + \mu_k^2} \qquad (3)$$

SUBSTITUTE FROM (2) INTO (1): $3P - 6\mu_k(N_A + \mu_k N_A) = 0$

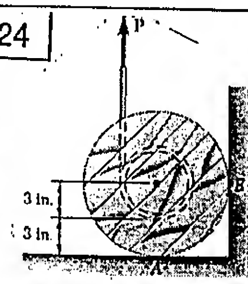
$$N_A = \frac{1}{2}\,\frac{P}{\mu_k(1+\mu_k)} \qquad (4)$$

(3) = (4): $\dfrac{20 - P}{1 + \mu_k^2} = \dfrac{P}{2(\mu_k + \mu_k^2)}$

SUBSTITUTE $\mu_k = 0.30$: $\dfrac{20 - P}{1 + (0.3)^2} \quad \dfrac{P}{2(0.3)(1.03)}$

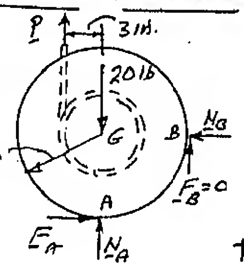
$20 - P = 1.3974P$; $2.3974P = 20$; $\quad P = 8.34$ lb ◀

8.24



GIVEN: $W = 20$ lb
AT A: $\mu_s = 0.40$, $\mu_k = 0.30$
AT B: $\mu_s = \mu_k = 0$

FIND: MAGNITUDE OF $\underline{P}$ TO DRAW WIRE AT A CONSTANT RATE



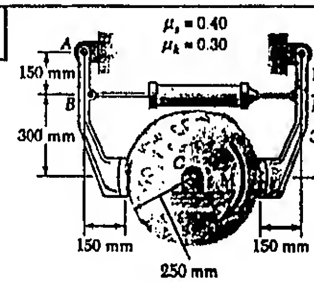
SINCE SPOOL IS ROTATING
$F_A = \mu_k N_A$
$+\circlearrowleft \Sigma M_G = 0$
$P(3in.) - F_A(6in.) = 0$
$P = 2F_A = 2\mu_k N_A \qquad (1)$

$+\uparrow \Sigma F_y = 0$: $P - 20 lb + N_A = 0$
$N_A = 20 - P \qquad (2)$

SUBSTITUTE FOR N_A FROM (2) INTO (1):
$P = 2\mu_k(20 - P)$

SUBSTITUTE $\mu_k = 0.30$: $P = 2(0.3)(20 - P)$
$6.667P = 20 - P$
$7.667P = 20$
$P = 2.609$ lb $\qquad P = 2.61$ lb ◀

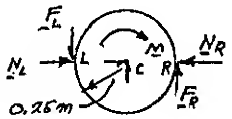
8.25



GIVEN: CYLINDER EXERTS 3 kN ON B AND ON E.

FIND: MAGNITUDE OF $\underline{M}$ REQUIRED FOR CONSTANT ROTATION

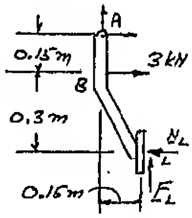
FREE BODY: DRUM



$+\circlearrowleft \Sigma M_C = 0$: $M - (0.25m)(F_L + F_R) = 0$
$M = (0.25m)(F_L + F_R) \qquad (1)$

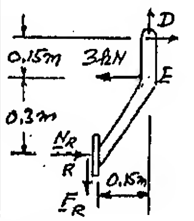
SINCE DRUM IS ROTATING,
$F_L = \mu_k N_L = 0.3N_L \qquad F_R = \mu_k N_R = 0.3N_R$

FREE BODY: LEFT ARM ABL



$+\circlearrowleft \Sigma M_A = 0$
$(3kN)(0.15m) + F_L(0.15m) - N_L(0.45m) = 0$
$0.45 kN \cdot m + (0.3N_L)(0.15m) - N_L(0.45m) = 0$
$0.405N_L = 0.45$;
$N_L = 1.111$ kN
$F_L = 0.3N_L = 0.3(1.111 kN) = 0.3333$ kN (2)

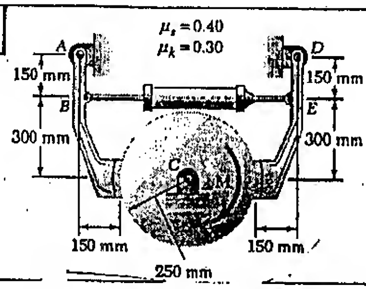
FREE BODY: RIGHT ARM DER



$+\circlearrowright \Sigma M_D = 0$: $(3kN)(0.15m) - F_R(0.15m) - N_R(0.45m) = 0$
$0.45 kN \cdot m - (0.3N_R)(0.15m) - N_R(0.45m) = 0$
$0.495N_R = 0.45$
$N_R = 0.9091$ kN
$F_R = \mu_k N_R = 0.3(0.9091 kN) = 0.2727$ kN $\qquad (3)$

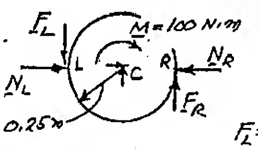
SUBSTITUTE FOR F_L AND F_R INTO (1): $M = (0.25m)(0.3333 kN + 0.2727 kN)$
$M = 0.1515 kN \cdot m \qquad M = 151.5 N \cdot m$ ◀

8.26



GIVEN: $M = 100 N \cdot m$
FIND: SMALLEST FORCE EXERTED BY CYLINDER FOR NO ROTATION OF DRUM.

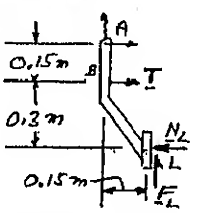
FREE BODY: DRUM



$+\circlearrowleft \Sigma M_C = 0$: $100 N \cdot m - (0.25m)(F_L + F_R) = 0$
$F_L + F_R = 400 N \qquad (1)$

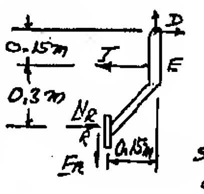
SINCE MOTION IMPENDS
$F_L = \mu_s N_L = 0.4N_L \qquad F_R = \mu_s N_R = 0.4N_R$

FREE BODY: LEFT ARM ABL



$+\circlearrowright \Sigma M_A = 0$: $T(0.15m) + F_L(0.15m) - N_L(0.45m) = 0$
$0.15T + (0.4N_L)(0.15m) - N_L(0.45m) = 0$
$0.39N_L = 0.15T$; $N_L = 0.38462T$
$F_L = 0.4N_L = 0.4(0.384627)$; $F_L = 0.15385T \quad (2)$

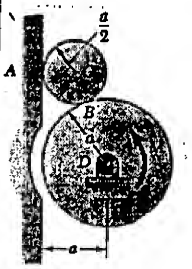
FREE BODY: RIGHT ARM DER



$+\circlearrowright \Sigma M_D = 0$: $T(0.15m) - F_R(0.15m) - N_R(0.45m) = 0$
$0.15T - (0.4N_R)(0.15m) - N_R(0.45m) = 0$
$0.51N_R = 0.15T$; $\quad N_R = 0.29412T$
$F_R = 0.4N_R = 0.4(0.29412T)$; $F_R = 0.11765T \quad (3)$

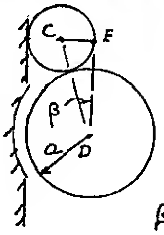
SUBSTITUTE FOR F_L AND F_R INTO (1):
$0.15385T + 0.11765T = 400$
$T = 1473.3 N \qquad T = 1.473$ kN ◀

*8.27



GIVEN: $\mu_s = 0.25$ AT A AND AT B, CYLINDER C WEIGHS W.

FIND: LARGEST COUNTERCLOCKWISE M IF CYLINDER D IS NOT TO ROTATE.



GEOMETRY

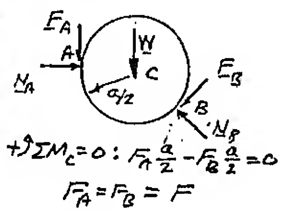
$CE = a/2$

$CD = 3a/2$

$\sin\beta = \frac{CE}{CD} = \frac{a/2}{3a/2} = \frac{1}{3}$

$\beta = 19.471^\circ$

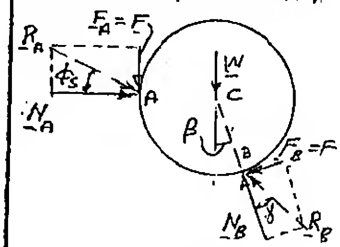
FREE BODY: CYLINDER C



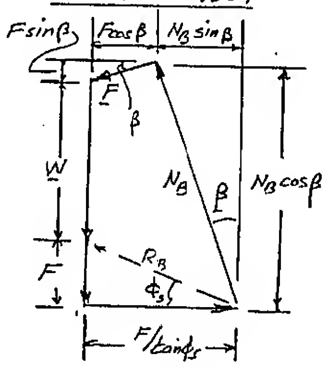
$+\circlearrowleft \Sigma M_C = 0:\ F_A\frac{a}{2} - F_B\frac{a}{2} = 0$

$F_A = F_B = F$

ASSUME MOTION IMPENDS AT A: $\tan\phi_s = 0.25;\ \phi_s = 14.04^\circ$



FORCE POLYGON



ASSUME NO SLIPPING AT B, THAT IS $\gamma < \phi_s$. SEE BELOW FOR VALUE OF γ

VERTICAL COMPONENTS: $N_B\cos\beta = W + F\sin\beta + F$

$$N_B = \frac{W + F(1+\sin\beta)}{\cos\beta} \qquad (1)$$

HORIZONTAL COMPONENTS:

$$\frac{F}{\tan\phi_s} = F\cos\beta + N_B\sin\beta \qquad (2)$$

$(1)\rightarrow(2)$: $\frac{F}{\tan\phi_s} = F\cos\beta + [W + F(1+\sin\beta)]\frac{\sin\beta}{\cos\beta}$

$$\frac{F}{\tan\phi_s} = F\cos\beta + W\tan\beta + F(1+\sin\beta)\tan\beta$$

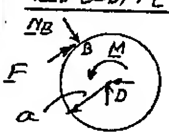
$$F\left[\frac{1}{\tan\phi_s} - \cos\beta - \tan\beta(1+\sin\beta)\right] = W\tan\beta$$

RECALL: $\beta = 19.471^\circ,\ \tan\phi_s = 0.25$

$$F\left[\frac{1}{0.25} - \cos 19.471^\circ - \tan 19.471^\circ(1+\sin 19.471^\circ)\right] = W\tan 19.471$$

$F(2.5857) = 0.35355W \qquad F = 0.13673W$

FREE BODY: CYLINDER D



$+\circlearrowleft \Sigma M_D = 0:\ M - Fa = 0$

$M = Fa = 0.13673Wa$

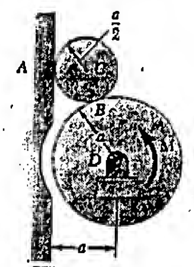
$M = 0.1367Wa \circlearrowleft$ ◀

VALUE OF γ: EQ.(1) $N_B = \frac{W + 0.13673W(1+\sin 19.471^\circ)}{\cos 19.471^\circ} = 1.254W$

$\tan\gamma = \frac{F}{N_B} = \frac{0.13671W}{1.254W} = 0.10902 \quad ; \ \gamma = 6.22^\circ < \phi_s$

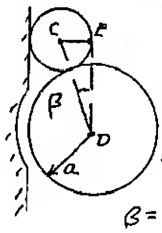
WE FIND NO SLIPPING AT B. OK

*8.28



GIVEN: $\mu_s = 0.25$ AT A AND AT B, CYLINDER C WEIGHS W

FIND: LARGEST CLOCKWISE M IF CYLINDER D IS NOT TO ROTATE



GEOMETRY

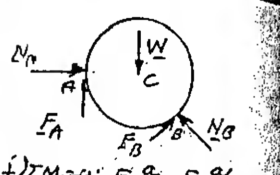
$CE = a/2$

$CD = 3a/2$

$\sin\beta = \frac{CE}{CD} = \frac{a/2}{3a/2} = \frac{1}{3}$

$\beta = 19.471^\circ$

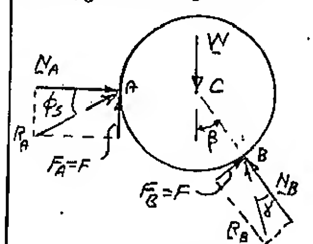
FREE BODY: CYLINDER C



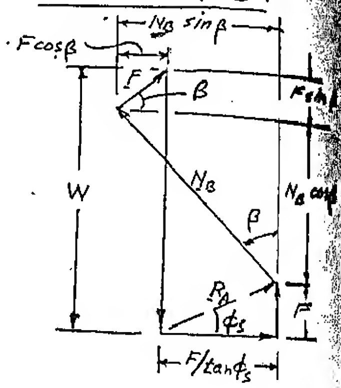
$+\circlearrowleft \Sigma M_C = 0:\ F_B\frac{a}{2} - F_A\,a/2 = 0$

$F_A = F_B = F$

ASSUME MOTION IMPENDS AT A: $\tan\phi_s = 0.25;\ \phi_s = 14.04^\circ$



FORCE POLYGON



ASSUME NO SLIPPING AT B, THAT IS, $\gamma < \phi_s$. SEE BELOW FOR VALUE OF γ.

VERTICAL COMPONENTS: $N_B\cos\beta + F + F\sin\beta = W$

$$N_B = \frac{W - F(1+\sin\beta)}{\cos\beta} \qquad (1)$$

HORIZONTAL COMPONENTS: $\frac{F}{\tan\phi_s} = N_B\sin\beta - F\cos\beta \qquad (2)$

$(1)\rightarrow(2)$: $\frac{F}{\tan\phi_s} = [W - F(1+\sin\beta)]\frac{\sin\beta}{\cos\beta} - F\cos\beta$

$$\frac{F}{\tan\phi_s} = W\tan\beta - F(1+\sin\beta)\tan\beta - F\cos\beta$$

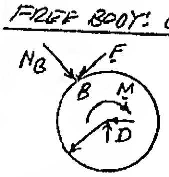
$$F\left[\frac{1}{\tan\phi_s} + \cos\beta + \tan\beta(1+\sin\beta)\right] = W\tan\beta$$

RECALL: $\beta = 19.471^\circ,\ \tan\phi_s = 0.25$

$$F\left[\frac{1}{0.25} + \cos 19.471^\circ + \tan 19.471^\circ(1+\sin 19.471^\circ)\right] = W\tan 19.471^\circ$$

$F(5.4121) = 0.35355W \qquad F = 0.0653W$

FREE BODY: CYLINDER D



$+\circlearrowleft \Sigma M_D = 0:\ M - Fa = 0$

$M = Fa = 0.0653Wa$

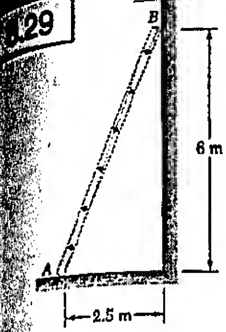
$M = 0.0653Wa \circlearrowright$ ◀

VALUE OF γ: EQ(1): $N_B = \frac{W - 0.0653W(1+\sin 19.471^\circ)}{\cos 19.471^\circ} = 0.9683W$

$\tan\gamma = \frac{F}{N_B} = \frac{0.0653W}{0.9683W} = 0.06743 \qquad \gamma = 3.86^\circ < \phi_s$

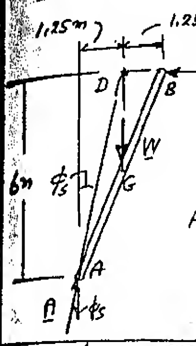
WE FIND NO SLIPPING AT B. OK

8.29



GIVEN: $\mu_s = 0$ AT B

FIND: SMALLEST μ_s AT A FOR WHICH EQUILIBRIUM IS MAINTAINED



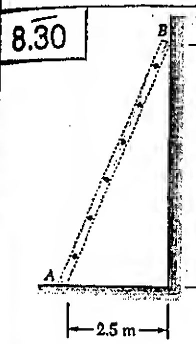
FREE BODY: LADDER

THREE-FORCE BODY. LINE OF ACTION OF $\underline{A}$ MUST PASS THROUGH D, WHERE $\underline{W}$ AND $\underline{B}$ INTERSECT

AT A: $\mu_s = \tan \phi_s = \frac{1.25\,m}{6\,m} = 0.2083$

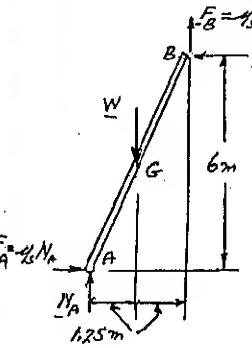
$\mu_s = 0.208$ ◄

8.30



GIVEN: SAME VALUE OF μ_s AT A AND AT B.

FIND: SMALLEST VALUE OF μ_s FOR WHICH EQUILIBRIUM IS MAINTAINED



FREE BODY: LADDER

MOTION IMPENDING

$F_A = \mu_s N_A \qquad F_B = \mu_s N_B$

$+\circlearrowleft \Sigma M_A = 0$:

$W(1.25m) - N_B(6m) - \mu_s N_B(2.5m) = 0$

$N_B = \frac{1.25W}{6 + 2.5\mu_s} \qquad (1)$

$+\uparrow \Sigma F_y = 0: \; N_A + \mu_s N_B - W = 0$

$N_A = W - \mu_s N_B$

$N_A = W - \frac{1.25\mu_s W}{6 + 2.5\mu_s} \qquad (2)$

$\xrightarrow{+} \Sigma F_x = 0: \; \mu_s N_A - N_B = 0$

SUBSTITUTE FOR N_A AND N_B FROM EQS. (1) AND (2).

$$\mu_s W - \frac{1.25\mu_s^2 W}{6 + 2.5\mu_s} = \frac{1.25W}{6 + 2.5\mu_s}$$

$6\mu_s + 2.5\mu_s^2 - 1.25\mu_s^2 = 1.25$

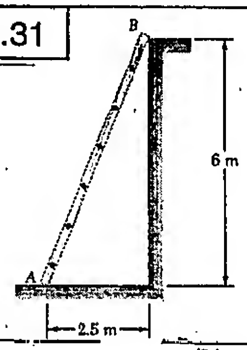
$1.25\mu_s^2 + 6\mu_s - 1.25 = 0$

$\mu_s = 0.2$

AND $\mu_s = -5$ (DISCARD)

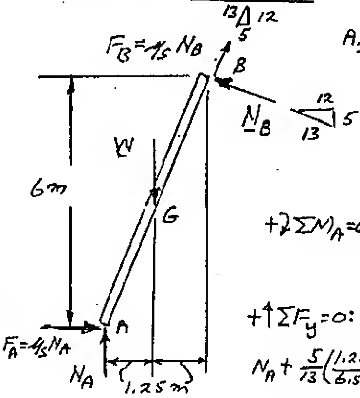
$\mu_s = 0.200$ ◄

8.31



GIVEN: SAME VALUE OF μ_s AT A AND AT B.

FIND: SMALLEST VALUE OF μ_s FOR WHICH EQUILIBRIUM IS MAINTAINED



AB = 6.5 m $\quad$ 6.5, 6, 2.5 $\Rightarrow$ 13, 12, 5

FREE BODY: LADDER

MOTION IMPENDS

$F_A = \mu_s N_A \qquad F_B = \mu_s N_B$

$+\circlearrowleft \Sigma M_A = 0: \; W(1.25m) - N_B(6.5m) = 0$

$N_B = \frac{1.25}{6.5} W \qquad (1)$

$+\uparrow \Sigma F_y = 0: \; N_A + \frac{5}{13} N_B + \frac{12}{13}\mu_s N_B - W = 0$

$N_A + \frac{5}{13}\left(\frac{1.25}{6.5}W\right) + \frac{12}{13}\mu_s\left(\frac{1.25}{6.5}W\right) - W = 0$

$N_A = W\left(1 - \frac{6.25}{84.5} - \frac{15}{84.5}\mu_s\right) \qquad (2)$

$\xrightarrow{+} \Sigma F_x = 0: \; \mu_s N_A - \frac{12}{13} N_B + \frac{5}{13}\mu_s N_B = 0$

SUBSTITUTE FOR N_A AND N_B FROM EQS. (1) AND (2):

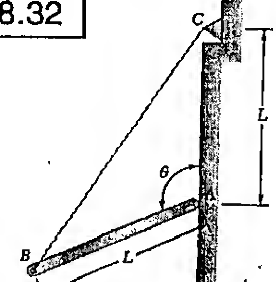
$$\mu_s\left(\frac{78.25 - 15\mu_s}{84.5}\right) = \left(\frac{12 - 5\mu_s}{13}\right)\left(\frac{1.25}{6.5}W\right)$$

$84.5\mu_s - 15\mu_s^2 - 15 = 0$

$\mu_s = 0.18349$ AND $\mu_s = -5.45$ (DISCARD)

$\mu_s = 0.1835$ ◄

8.32

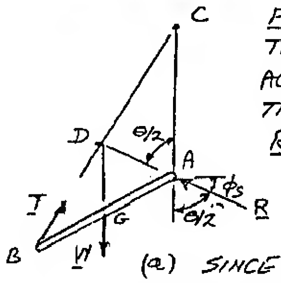


GIVEN: $\mu_s = 0.40$

$\mu_k = 0.30$

FIND: (a) VALUE OF θ FOR IMPENDING MOTION.

(b) CORRESPONDING TENSION IN CORD BC



FREE-BODY DIAGRAM

THREE-FORCE BODY. LINE OF ACTION OF $\underline{R}$ MUST PASS THROUGH D, WHERE $\underline{T}$ AND $\underline{R}$ INTERSECT

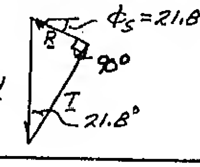
MOTION IMPENDS

$\tan \phi_s = 0.4 \qquad \phi_s = 21.80°$

(a) SINCE BG = GA, IT FOLLOWS THAT BD = DC AND AD BISECTS ∠BAC

$\therefore \; \theta/2 + \phi_s = 90°; \quad \theta/2 + 21.8° = 90° \quad \theta = 136.4°$ ◄

(b)

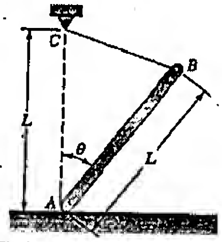


FORCE TRIANGLE (RIGHT TRIANGLE)

$T = W \cos 21.8°$

$T = 0.928\,W$ ◄

8.33

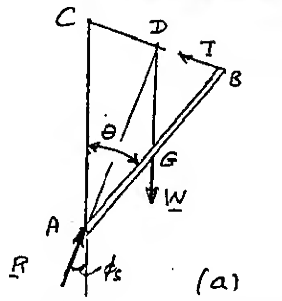


GIVEN: $\mu_s = 0.40$

$\mu_k = 0.30$

FIND: (a) VALUE OF θ FOR IMPENDING MOTION

(b) CORRESPONDING TENSION IN CORD BC



FREE-BODY DIAGRAM

ROD AB IS A THREE-FORCE BODY. THUS, LINE OF ACTION OF $\underline{R}$ MUST PASS THROUGH D, WHERE $\underline{W}$ AND $\underline{T}$ INTERSECT.

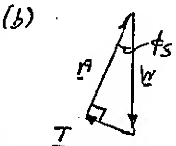
SINCE AG = GB, CD = DB AND THE MEDIAN AD OF THE ISOSCELES TRIANGLE ABC BISECTS THE ANGLE θ.

THUS $\phi_s = \frac{1}{2}\theta$

SINCE MOTION IMPENDS $\phi_s = \tan^{-1} 0.40 = 21.80°$

$\theta = 2\phi_s = 2(21.8°)$ $\qquad \theta = 43.6°$ ◄

(b)



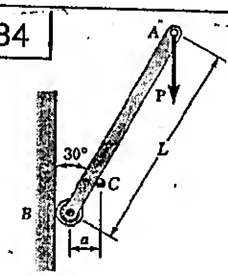
FORCE TRIANGLE

THIS IS A RIGHT TRIANGLE

$T = W \sin\phi_s = W \sin 21.8°$

$T = 0.371\,W$ ◄

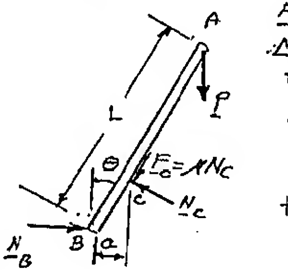
8.34



GIVEN: BETWEEN PIN C AND ROD:

$\mu_s = 0.15$

FIND: RANGE OF VALUES OF L/a FOR WHICH EQUILIBRIUM IS MAINTAINED.



FREE-BODY DIAGRAM: FOR MOTION OF B IMPENDING UPWARD.

$+\circlearrowleft \Sigma M_B = 0$

$PL\sin\theta - N_c\left(\frac{a}{\sin\theta}\right) = 0$

$N_c = \frac{PL}{a}\sin^2\theta \qquad (1)$

$+\uparrow \Sigma F_y = 0$: $N_c \sin\theta - \mu_s N_c \cos\theta - P = 0$

$\therefore N_c(\sin\theta - \mu\cos\theta) = P$

SUBSTITUTE FOR N_c FROM (1), AND SOLVE FOR a/L

$a/L = \sin^2\theta(\sin\theta - \mu_s\cos\theta) \qquad (2)$

FOR $\theta = 30°$ AND $\mu_s = 0.15$:

$a/L = \sin^2 30°(\sin 30° - 0.15\cos 30°)$

$a/L = 0.092524$ $\qquad L/a = 10.808$

FOR MOTION OF B IMPENDING DOWNWARD, REVERSE SENSE OF FRICTION FORCE $\underline{F}_c$. TO DO THIS WE MAKE $\mu_s = -0.15$ IN EQ.(2).

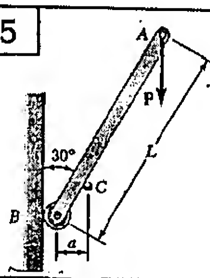
EQ(2): $a/L = \sin^2 30°(\sin 30° - (-0.15)\cos 30°)$

$a/L = 0.15748$ $\qquad L/a = 6.350$

RANGE OF VALUES OF L/a FOR EQUILIBRIUM:

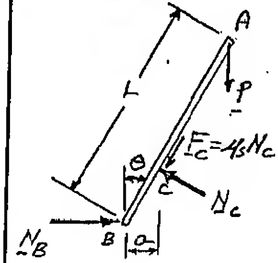
$6.35 \le \frac{L}{a} \le 10.81$ ◄

8.35



GIVEN: BETWEEN PIN C AND ROD: $\mu_s = 0.60$

FIND: RANGE OF VALUES OF L/a FOR WHICH EQUILIBRIUM IS MAINTAINED.



FREE-BODY DIAGRAM: FOR MOTION OF B IMPENDING UPWARD

$+\circlearrowleft \Sigma M_B = 0$

$PL\sin\theta - N_c\left(\frac{a}{\sin\theta}\right) = 0$

$N_c = \frac{PL}{a}\sin^2\theta \qquad (1)$

$+\uparrow \Sigma F_y = 0$: $N_c\sin\theta - \mu_s N_c\cos\theta - P = 0$

$N_c(\sin\theta - \mu_s\cos\theta) = P$

SUBSTITUTE FOR N_c FROM (1), AND SOLVE FOR a/L

$a/L = \sin^2\theta(\sin\theta - \mu_s\cos\theta)$

FOR $\theta = 30°$ AND $\mu_s = 0.60$:

$a/L = \sin^2 30°(\sin 30° - 0.60\cos 30°) \qquad (2)$

$a/L = -0.0049 < 0$

THUS, SLIPPING OF B UPWARD DOES NOT OCCUR

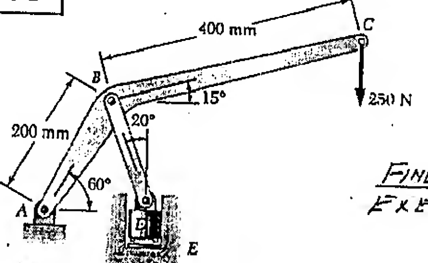
FOR MOTION OF B IMPENDING DOWNWARD, REVERSE SENSE OF FRICTION FORCE F_c. TO DO THIS WE MAKE $\mu_s = -0.60$ IN EQ(2).

$a/L = \sin^2 30°(\sin 30° - (-0.60)\cos 30°)$

$a/L = 0.2459$ $\qquad L/a = 3.923$

RANGE OF L/a FOR EQUILIBRIUM: $L/a \ge 3.92$ ◄

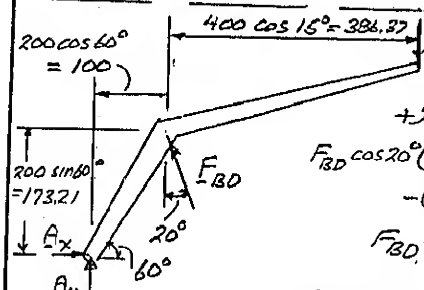
8.36



GIVEN: BETWEEN DIE D AND GUIDE E:

$\mu_s = 0.30$

FIND: FORCE EXERTED ON SEAL



DIMENSIONS IN mm

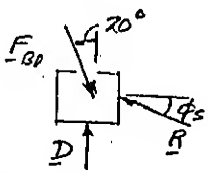
FREE BODY: MEMBER AC

$+\circlearrowright \Sigma M_A = 0$

$F_{BD}\cos 20°(100) + F_{BD}\sin 20°(173.21) - (250N)(100 + 386.37) = 0$

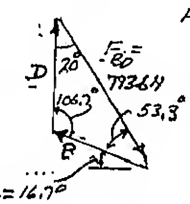
$F_{BD} = 793.6\,N$

FREE BODY: DIE D



$\phi_s = \tan^{-1} 0.30 = 16.70°$

FORCE TRIANGLE



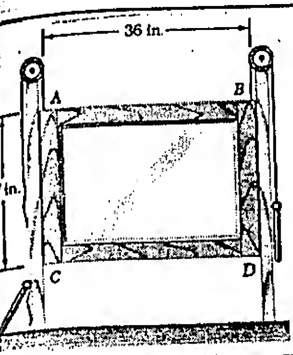
$\frac{D}{\sin 53.3°} = \frac{793.6N}{\sin 106.7°}$

$D = 664.3\,N$

ON SEAL:

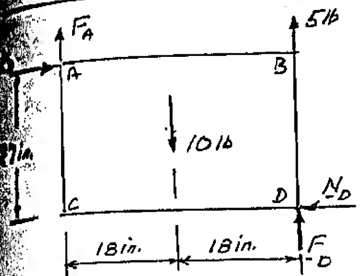
$\underline{D} = 664\,N\downarrow$ ◄

8.37



GIVEN:
10-lb WINDOW SASH
5-lb SASH WEIGHT

FIND: SMALLEST VALUE OF μ_s FOR WHICH WINDOW WILL STAY OPEN.



FREE BODY: SASH

MOTION IMPENDS

$F_A = \mu_s N_A \qquad F_D = \mu_s N_D$

$\Sigma F_x = 0: \quad N_A = N_D$

$+\uparrow \Sigma F_y = 0:$

$F_A + F_D - 10\,lb + 5\,lb = 0$

$\mu_s N_A + \mu_s N_D = 5\,lb$

$2\mu_s N_A = 5\,lb$

$F_A = \mu_s N_A = 2.5\,lb$

$+\circlearrowleft \Sigma M_D = 0:$

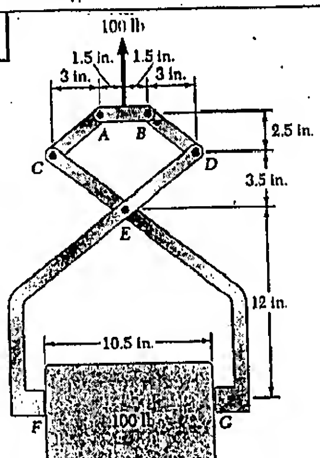
$-N_A(27\,in.) - F_A(36\,in.) + (10\,lb)(18\,in.) = 0$

$-N_A(27\,in.) - (2.5\,lb)(36\,in.) + (10\,lb)(18\,in.) = 0$

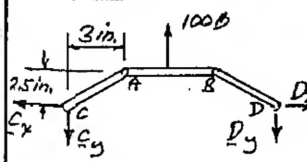
$27 N_A = 90 \qquad N_A = 3.333\,lb$

$\mu_s = \dfrac{F_A}{N_A} = \dfrac{2.5\,lb}{3.333\,lb} = 0.75 \qquad \mu_s = 0.75$ ◀

8.38



FIND: SMALLEST μ_s FOR BLOCK TO BE SUPPORTED.



FREE BODY: MEMBERS CA, AB, BD

BY SYMMETRY: $C_y = D_y = \frac{1}{2}(100\,lb) = 50\,lb$

SINCE CA IS A TWO-FORCE MEMBER

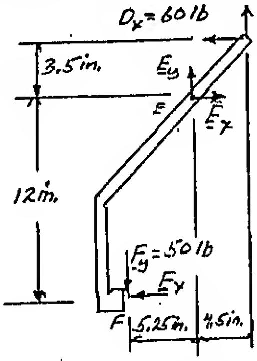
$\dfrac{C_x}{3\,in.} = \dfrac{C_y}{2.5\,in.}; \quad \dfrac{C_x}{3\,in.} = \dfrac{50\,lb}{2.5\,in.}; \quad C_x = 60\,lb$

$\Sigma F_x = 0: \quad D_x = C_x \qquad D_x = 60\,lb$

(CONTINUED)

8.38 CONTINUED

FREE BODY: TONG DEF



$+\circlearrowleft \Sigma M_E = 0:$

$(60\,lb)(3.5\,in.) + (50\,lb)(4.5\,in.) + (50\,lb)(5.25\,in.) - F_x(12\,in.) = 0$

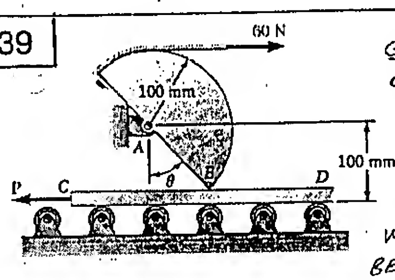
$F_x = +58.125\,lb$

MINIMUM VALUE OF μ_s:

$\mu_s = \dfrac{F_y}{F_x} = \dfrac{50\,lb}{58.125\,lb}; \quad \mu_s = 0.8602$

$\mu_s = 0.86$ ◀

8.39



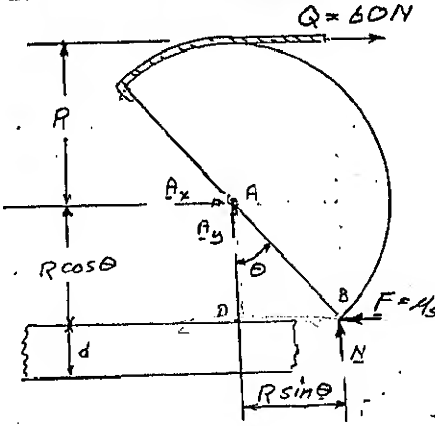
GIVEN: AT B, $\mu_s = 0.45$

d = PLATE THICKNESS

FIND: (a) FORCE P TO MOVE PLATE IF $d = 20\,mm$

(b) LARGEST d FOR WHICH PLATE CANNOT BE MOVED IF $P \to \infty$.

FREE BODY: CAM



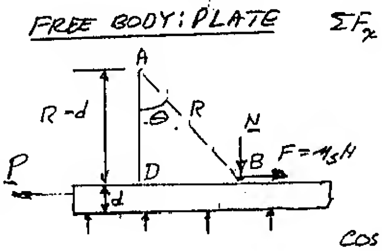
FOR IMPENDING MOTION

$F = \mu_s N$

$+\circlearrowleft \Sigma M_A = 0: \quad QR - NR\sin\theta + (\mu_s N)R\cos\theta = 0$

$N = \dfrac{Q}{\sin\theta - \mu_s\cos\theta} \qquad (1)$

FREE BODY: PLATE $\qquad \Sigma F_x = 0 \quad P = \mu_s N \qquad (2)$



GEOMETRY IN Δ ABD WITH $R = 100\,mm$ AND $d = 20\,mm$

$\cos\theta = \dfrac{R-d}{R} = \dfrac{80\,mm}{100\,mm} = 0.8$

$\sin\theta = \sqrt{1-\cos^2\theta} = 0.6$

(a)

EQ.(1) USING $Q = 60\,N$ AND $\mu_s = 0.45$

$N = \dfrac{60\,N}{0.6 - (0.45)(0.8)} = \dfrac{60}{0.24} = 250\,N$

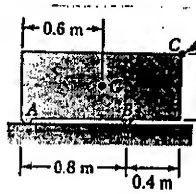
EQ.(2) $P = \mu_s N = (0.45)(250\,N); \quad P = 112.5\,N$ ◀

(b) FOR $P = \infty$, $N = \infty$. DENOMINATOR IS ZERO IN EQ.(1)

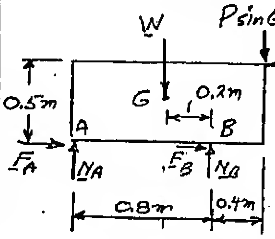
$\sin\theta - \mu_s\cos\theta = 0; \quad \tan\theta = \mu_s = 0.45; \quad \theta = 24.23°$

$\cos\theta = \dfrac{R-d}{R}; \quad \cos 24.23 = \dfrac{100-d}{100}; \qquad d = 8.81\,mm$ ◀

8.40



GIVEN: MASS = 75 kg, P = 500 N, $\mu_s = 0.30$.
FIND: RANGE OF VALUES OF θ FOR BASE TO MOVE



FREE BODY: MACHINE BASE

$m = (75\,kg)(9.81\,m/s^2) = 735.75\,N$

ASSUME SLIDING IMPENDS

$F_A = \mu_s N_A \qquad F_B = \mu_s N_B$

$+\uparrow \Sigma F_y = 0$

$N_A + N_B - W - P\sin\theta = 0$

$(N_A + N_B) = W + P\sin\theta \qquad (1)$

$\overset{+}{\rightarrow} \Sigma F_x = 0: \; F_A + F_B - P\cos\theta = 0$

$\mu_s(N_A + N_B) = P\cos\theta = 0 \qquad (2)$

$\frac{EQ.(2)}{EQ.(1)}: \; \mu_s = \frac{P\cos\theta}{W + P\sin\theta}$

$\mu_s W + \mu_s P\sin\theta = P\cos\theta$

$0.30(735.75\,N) + 0.30(500\,N)\sin\theta = 500\cos\theta$

$500\cos\theta - 150\sin\theta = 220.73$

SOLVE FOR θ: $\theta = 48.28°$

ASSUME TIPPING ABOUT B IMPENDS: $\therefore N_A = 0$

$+\circlearrowleft \Sigma M_B = 0: \; P\sin\theta(0.4\,m) - P\cos\theta(0.5\,m) - W(0.2\,m) = 0$

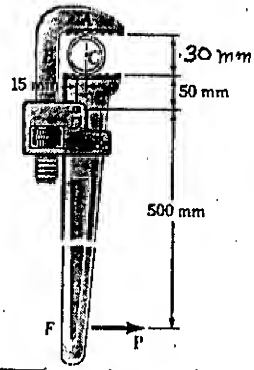
$500\sin\theta(0.4) - 500\cos\theta(0.5) - 735.75(0.2\,m) = 0$

$200\sin\theta - 250\cos\theta = 147.15$

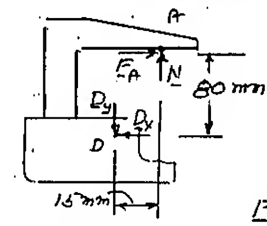
SOLVE FOR θ: $\theta = 78.03°$

RANGE FOR NO MOTION: $48.3° \le \theta \le 78.0°$

8.42



FIND: SMALLEST VALUES OF μ_s AT A AND C FOR WRENCH TO SELF LOCK ON THE PIPE.



FREE BODY: PORTION ABDE

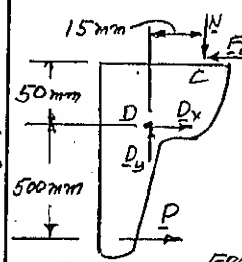
$\Sigma F_x = 0: \; D_x = F_A \qquad (1)$

$+\circlearrowleft \Sigma M_D = 0: \; N(15\,mm) - F_A(80\,mm)$

$F_A = 0.1875\,N$

$\mu_s = F_A/N = 0.1875$

AT A: $\mu_s = 0.188$



FREE BODY PORTION CF

$+\circlearrowleft \Sigma M_D = 0$

$P(500\,mm) - N(15\,mm) + F_C(50\,mm) = 0$

$500P - 15N + 50\mu_s N = 0$

$P = 0.03N - 0.1\mu_s N \qquad (2)$

$\overset{+}{\rightarrow} \Sigma F_x = 0: \; P - \mu_s N + D_x = 0$

USE EQ.(1): $P = \mu_s N - F_A \qquad (3)$

EQUATE (2) AND (3):

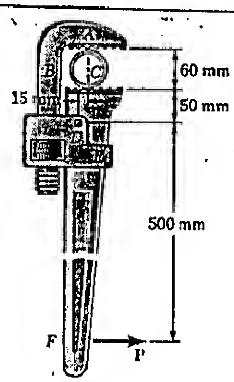
$0.03N - 0.1\mu_s N = \mu_s N - F_A$

FROM EQ(1) SUBSTITUTE $F_A = 0.1875\,N$:

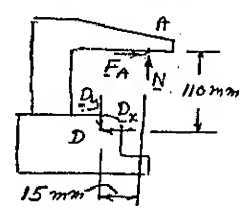
$0.03N - 0.1\mu_s N = \mu_s N - 0.1875\,N$

SOLVE FOR μ_s: $\mu_s = 0.1977$; AT B: $\mu_s = 0.198$

8.41



FIND: SMALLEST VALUE OF μ_s AT A AND C FOR WRENCH TO SELF LOCK ON THE PIPE



FREE BODY: PORTION ABDE

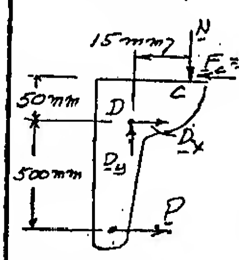
$\Sigma F_x = 0: \; D_x = F_A \qquad (1)$

$+\circlearrowleft \Sigma M_D = 0: \; N(15\,mm) - F_A(110\,mm) = 0$

$F_A = 0.1363\,N$

$\mu_s = F_A/N = 0.1363$

AT A: $\mu_s = 0.136$



FREE BODY: PORTION CF

$+\circlearrowleft \Sigma M_D = 0$

$P(500\,mm) - N(15\,mm) + F_C(50\,mm) = 0$

$500P - 15N + 50\mu_s N = 0$

$P = 0.03N - 0.1\mu_s N \qquad (2)$

$\overset{+}{\rightarrow} \Sigma F_x = 0: \; P - \mu_s N + D_x = 0$

USE EQ.(1) $\quad P = \mu_s N - F_A \qquad (3)$

EQUATE (2) + (3):

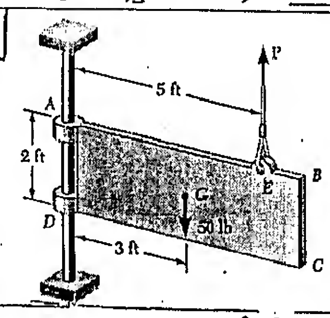
$0.03N - 0.1\mu_s N = \mu_s N - F_A$

FROM EQ(1) SUBSTITUTE $F_A = 0.1363\,N$:

$0.03N - 0.1\mu_s N = \mu_s N - 0.1363\,N$

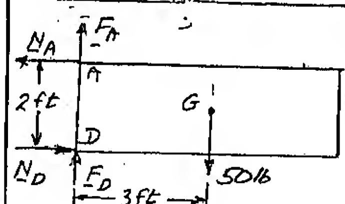
SOLVE FOR μ_s: $\mu_s = 0.1512$; AT C: $\mu_s = 0.151$

8.43



GIVEN: AT A AND AT B $\mu_s = 0.40$

FIND: WHETHER PLATE IS IN EQUILIBRIUM IF (a) P = 0, (b) P = 20 lb.



(a) P = 0

$+\circlearrowleft \Sigma M_D = 0$

$N_A(2\,ft) - (50\,lb)(3\,ft) = 0$

$N_A = 75\,lb$

$\Sigma F_x = 0: \; N_D = N_A = 75\,lb$

$+\uparrow \Sigma F_y = 0: \; F_A + F_D - 50\,lb = 0$

$F_A + F_D = 50\,lb$

BUT: $(F_A)_m = \mu_s N_A = 0.40(75\,lb) = 30\,lb$

$(F_D)_m = \mu_s N_D = 0.40(75\,lb) = 30\,lb$

THUS: $(F_A)_m + (F_D)_m = 60\,lb$

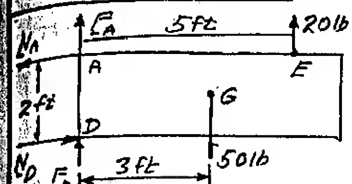
AND $(F_A)_m + (F_D)_m > F_A + F_D$

PLATE IS IN EQUILIBRIUM

(CONTINUED)

8.43 CONTINUED

(b) $P = 20\,lb$



$+\circlearrowleft \Sigma M_D = 0$

$N_A(2ft) - (50\,lb)(3ft) + (20\,lb)(5ft) = 0$

$N_A = 25\,lb$

$\Sigma F_x = 0$: $N_D = N_A = 25\,lb$

$+\uparrow \Sigma F_y = 0$: $F_A + F_D - 50\,lb + 20\,lb = 0$

$F_A + F_D = 30\,lb$

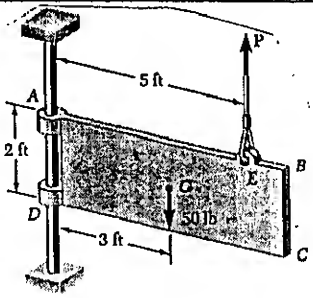
BUT: $(F_A)_m = \mu_s N_A = 0.4(25\,lb) = 10\,lb$

$(F_D)_m = \mu_s N_D = 0.4(25\,lb) = 10\,lb$

THUS: $(F_A)_m + (F_D)_m = 20\,lb$, AND $F_A + F_D > (F_A)_m + (F_D)_m$

PLATE MOVES DOWNWARD ◀

8.44

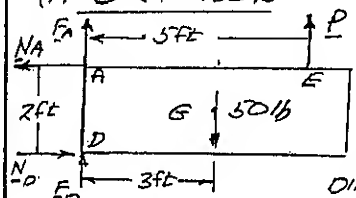


GIVEN: AT A AND AT B: $\mu_s = 0.40$

FIND: RANGE OF VALUES OF P FOR WHICH PLATE WILL MOVE DOWNWARD.

WE SHALL CONSIDER THE FOLLOWING TWO CASES.

(1) $0 < P < 30\,lb$



$+\circlearrowleft \Sigma M_D = 0$:

$N_A(2ft) - (50\,lb)(3ft) + P(5ft) = 0$

$N_A = 75\,lb - 2.5P$

(NOTE: $N_A \geq 0$ AND DIRECTED ← FOR $P \leq 30\,lb$ AS ASSUMED HERE)

$\Sigma F_x = 0$: $N_A = N_D$

$+\uparrow \Sigma F_y = 0$: $F_A + F_D + P - 50 = 0$; $F_A + F_D = 50 - P$

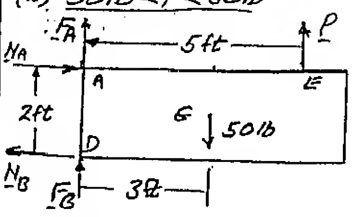
BUT: $(F_A)_m = (F_D)_m = \mu_s N_A = 0.40(75 - 2.5P) = 30 - P$

PLATE MOVES ↓ IF $F_A + F_D > (F_A)_m + (F_D)_m$

OR $50 - P > (30 - P) + (30 - P)$

$P > 10\,lb$ ◁

(2) $30\,lb < P < 50\,lb$



$+\circlearrowleft \Sigma M_D = 0$

$-N_A(2ft) - (50\,lb)(3ft) + P(5ft) = 0$

$N_A = 2.5P - 75$

(NOTE: $N_A >$ AND DIRECTED → FOR $P > 30\,lb$ AS ASSUMED)

$\Sigma F_x = 0$: $N_A = N_D$

$+\uparrow \Sigma F_y = 0$: $F_A + F_D + P - 50 = 0$ $\quad F_A + F_D = 50 - P$

BUT: $(F_A)_m = (F_D)_m = \mu_s N_A = 0.40(2.5P - 75) = P - 30\,lb$

PLATE MOVES ↓ IF $\quad F_A + F_D > (F_A)_m + (F_D)_m$

$50 - P > (P - 30) + (P - 30)$

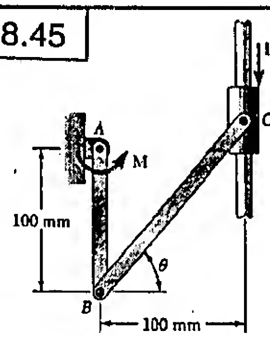
$P < \frac{110}{3} = 36.7\,lb$ ◁

THUS: PLATE MOVE DOWNWARD FOR:

$10\,lb < P < 36.7\,lb$.

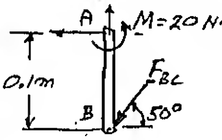
NOTE: FOR $P > 50\,lb$, PLATE IS IN EQUILIBRIUM ◀

8.45



GIVEN: $\mu_s = 0.35$, $\theta = 50°$, $M = 20\,N\cdot m$.

FIND: RANGE OF VALUES OF P FOR EQUILIBRIUM



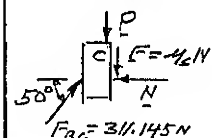
FREE BODY: MEMBER AB

BC IS A TWO-FORCE MEMBER

$+\circlearrowleft \Sigma M_A = 0$: $20\,N\cdot m - F_{BC}\cos 50°(0.1\,m) = 0$

$F_{BC} = 311.145\,N$

MOTION OF C IMPENDING UPWARD



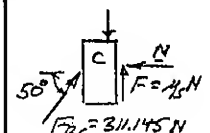
$\xrightarrow{+} \Sigma F_x = 0$: $(311.145\,N)\cos 50° - N = 0$

$N = 200\,N$

$+\uparrow \Sigma F_y = 0$: $(311.145\,N)\sin 50° - P - (0.35)(200\,N) = 0$

$P = 168.351\,N$ ◁

MOTION OF C IMPENDING DOWNWARD



$\xrightarrow{+} \Sigma F_x = 0$: $(311.145\,N)\cos 50° - N = 0$

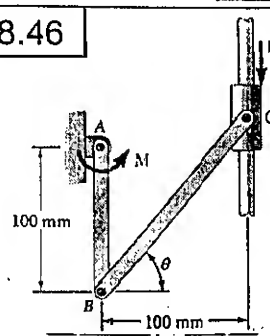
$N = 200\,N$

$+\uparrow \Sigma F_y = 0$: $(311.145\,N)\sin 50° - P + (0.35)(200\,N) = 0$

$P = 308.35\,N$ ◁

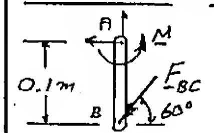
RANGE OF P: $168.4\,N \leq P \leq 308\,N$ ◀

8.46



GIVEN: $\mu_s = 0.40$, $\theta = 60°$, $P = 200\,N$

FIND: RANGE OF VALUES OF M FOR EQUILIBRIUM



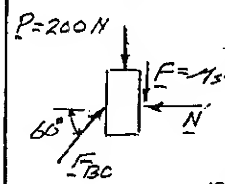
FREE BODY: MEMBER AB

BC IS A TWO-FORCE MEMBER

$+\circlearrowleft \Sigma M_A = 0$: $M - F_{BC}\cos 60°(0.1\,m) = 0$

$M = 0.05F_{BC}$ $\quad$ (1)

MOTION OF C IMPENDING UPWARD



$\xrightarrow{+} \Sigma F_x = 0$: $F_{BC}\cos 60° - N = 0$

$N = 0.5F_{BC}$

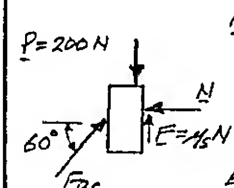
$+\uparrow \Sigma F_y = 0$: $F_{BC}\sin 60° - 200\,N - (0.40)(0.5F_{BC}) = 0$

$F_{BC} = 300.29\,N$

EQ.(1): $M = 0.05(300.29)$

$M = 15.014\,N\cdot m$ ◁

MOTION OF C IMPENDING DOWNWARD



$\xrightarrow{+} \Sigma F_x = 0$: $F_{BC}\cos 60° - N = 0$

$N = 0.5F_{BC}$

$+\uparrow \Sigma F_y = 0$: $F_{BC}\sin 60° - 200\,N + (0.40)(0.5F_{BC}) = 0$

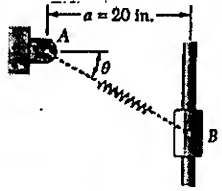
$F_{BC} = 187.613\,N$

EQ.(1): $M = 0.05(187.613)$

$M = 9.381\,N\cdot m$ ◁

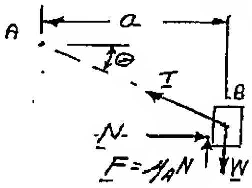
RANGE OF M: $9.38\,N\cdot m \leq M \leq 15.01\,N\cdot m$ ◀

8.47



GIVEN: $k = 15\ lb/in.$, $T = 0$ WHEN $\theta = 0$. $\mu_s = 0.40$.

FIND: RANGE OF W FOR EQUILIBRIUM WHEN (a) $\theta = 20°$, (b) $\theta = 30°$.



TENSION IN SPRING: $AB = \dfrac{a}{\cos\theta}$

$$T = k\Delta = k(AB - a) = k\left(\frac{a}{\cos\theta} - a\right)$$

FOR MOTION IMPENDING DOWNWARD

$\pm\Sigma F_x = 0$: $N - T\cos\theta = 0$

$N = T\cos\theta$

$+\uparrow\Sigma F_y = 0$: $T\sin\theta - W + \mu_A N = 0$

$T\sin\theta - W + \mu_A T\cos\theta = 0$

$W = T(\sin\theta + \mu_A\cos\theta)$

$$W = ka\left(\frac{1}{\cos\theta} - 1\right)(\sin\theta + \mu_A\cos\theta) \quad (1)$$

FOR MOTION IMPENDING UPWARD, $F = \mu_A N$ ACTS DOWNWARD

$\therefore$ IN EQ.(1): $\mu_A \rightarrow -\mu_A$

$$W = ka\left(\frac{1}{\cos\theta} - 1\right)(\sin\theta - \mu_A\cos\theta) \quad (2)$$

(a) $\theta = 20°$, $k = 15\ lb/in.$, $\mu_s = 0.40$

MOTION $\downarrow$, WE USE EQ.(1):

$W = (15\ lb/in.)(20\ in.)\left(\frac{1}{\cos 20°} - 1\right)(\sin 20° + 0.40\cos 20°)$

$W = (300\ lb)(0.06418)(0.34202 + 0.40 \times 0.93969)$

$W = 13.82\ lb$ ◁

MOTION $\uparrow$, WE USE EQ.(2):

$W = (300\ lb)(0.06418)(0.34202 - 0.4 \times 0.93969)$

$W = -0.652\ lb$: NEGATIVE WEIGHT, IMPOSSIBLE ◁

RANGE WHEN $\theta = 20°$: $W \leq 13.82\ lb$ ◀

(b) $\theta = 30°$, $k = 15\ lb/in.$, $\mu_s = 0.40$

MOTION $\downarrow$, WE USE EQ.(1)

$W = (15\ lb/in.)(20\ in.)\left(\frac{1}{\cos 30°} - 1\right)(\sin 30° + 0.40\cos 30°)$

$W = (300\ lb)(0.15470)(0.5 + 0.40 \times 0.86603)$

$W = 39.28\ lb$ ◁

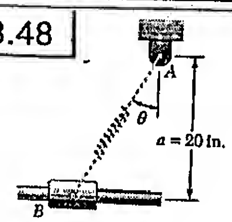
MOTION $\uparrow$, WE USE EQ.(2):

$W = (300\ lb)(0.15470)(0.5 - 0.4 \times 0.86603)$

$W = 7.128\ lb$ ◁

RANGE WHEN $\theta = 30°$: $7.13\ lb \leq W \leq 39.3\ lb$ ◀

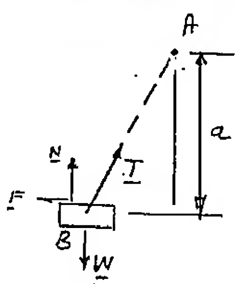
8.48



GIVEN: $k = 15\ lb/in.$, $\mu_s = 0.40$

$T = 0$ WHEN $\theta = 0$.

FIND: RANGE OF W FOR EQUILIBRIUM WHEN (a) $\theta = 20°$, (b) $\theta = 30°$



TENSION IN SPRING

$AB = a/\cos\theta$

ELONGATION OF SPRING

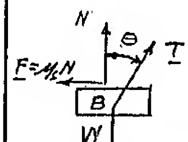
$$\Delta = \frac{a}{\cos\theta} - a$$

$$T = k\Delta = ka\left(\frac{1}{\cos\theta} - 1\right) \quad (1)$$

(CONTINUED)

8.48 CONTINUED

NOTE: ONLY POSSIBLE MOTION IS $\rightarrow$; BUT N CAN BE $\uparrow$ OR $\downarrow$.



$\pm\Sigma F_x = 0$: $T\sin\theta - \mu_s N = 0$

$N = (T\sin\theta)/\mu_s \quad (2)$

$+\uparrow\Sigma F_y = 0$: $N + T\cos\theta - W = 0$

$W = T\cos\theta + N \quad (3)$

(a) $\theta = 20°$, $k = 15\ lb/in.$, $\mu_s = 0.40$

EQ.(1): $T = (15\ lb/in.)(20\ in.)\left(\frac{1}{\cos 20°} - 1\right) = 19.2533\ lb$

EQ.(2): $N = (19.2533\ lb)(\sin 20°)/0.40 = 16.4626\ lb$

IF N ACTS $\uparrow$: THAT IS, $N = +16.4626\ lb$

EQ(3): $W = (19.2533\ lb)\cos 20° + 16.4626\ lb = 34.555\ lb$

COLLAR IN EQUILIBRIUM WHEN: $W \geq 35.6\ lb$ ◀

IF N ACTS $\downarrow$: THAT IS, $N = -16.4626\ lb$

$W = (19.2533\ lb)\cos 20° - 16.4626\ lb = 1.6296\ lb$

COLLAR IN EQUILIBRIUM WHEN: $W \geq 1.630\ lb$ ◀

(b) $\theta = 30°$, $k = 15\ lb/in.$, $\mu_s = 0.40$

EQ(1): $T = (15\ lb/in.)(20\ in.)\left(\frac{1}{\cos 30°} - 1\right) = 46.41\ lb$

EQ.(2): $N = (46.41\ lb)\sin 30°/0.40 = 58.01\ lb$

IF N ACTS $\uparrow$: THAT IS, $N = 58.01\ lb$

EQ(3): $W = (46.41\ lb)\cos 30° + 58.01\ lb = 98.21\ lb$

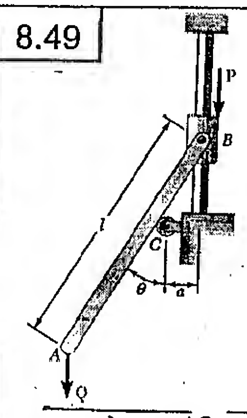
COLLAR IN EQUILIBRIUM WHEN: $W \geq 98.2\ lb$ ◀

IF N ACTS $\downarrow$: THAT IS, $N = -58.01\ lb$

$W = (46.41\ lb)\cos 30° - 58.01\ lb = -17.81\ lb$

NEGATIVE WEIGHT, IMPOSSIBLE

8.49



GIVEN: $l = 600\ mm$

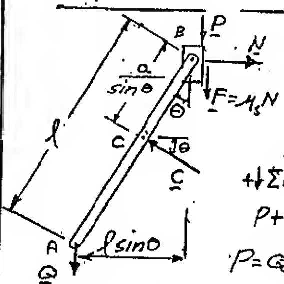
$a = 80\ mm$

$\mu_s = 0.25$

$Q = 100\ N$

$\theta = 30°$

FIND: RANGE OF VALUES OF P FOR EQUILIBRIUM



FOR MOTION OF COLLAR AT B IMPENDING UPWARD, $F = \mu_s N \downarrow$.

$+\circlearrowleft\Sigma M_B = 0$: $Ql\sin\theta - Ca/\sin\theta = 0$

$C = Q(l/a)\sin^2\theta$

$\Sigma F_x = 0$: $N = C\cos\theta = Q(l/a)\sin^2\theta\cos\theta$

$+\downarrow\Sigma F_y = 0$: $P + Q - C\sin\theta - \mu_s N = 0$

$P + Q - Q(l/a)\sin^3\theta - \mu_s Q(l/a)\sin^2\theta\cos\theta = 0$

$$P = Q\left[\frac{l}{a}\sin^2\theta(\sin\theta - \mu_s\cos\theta) - 1\right] \quad (1)$$

SUBSTITUTE DATA:

$$P = (100\ N)\left[\frac{600\ mm}{80\ mm}\sin^2 30°(\sin 30° - 0.25\cos 30°) - 1\right]$$

$P = -46.84\ N$ (P IS DIRECTED $\uparrow$) $\qquad P = -46.8\ N$

FOR MOTION OF COLLAR IMPENDING DOWNWARD $F = \mu_s N \uparrow$

IN EQ.(1) WE SUBSTITUTE $-\mu_s$ FOR μ_s.

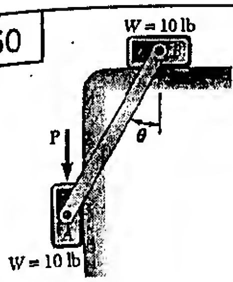
$$P = Q\left[\frac{l}{a}\sin^2\theta(\sin\theta + \mu_s\cos\theta) - 1\right]$$

$$P = (100\ N)\left[\frac{600\ mm}{80\ mm}\sin^2 30°(\sin 30° + 0.25\cos 30°) - 1\right]$$

$P = +34.34\ N$

FOR EQUILIBRIUM: $-46.8\ N \leq P \leq 34.3\ N$

8.50



GIVEN: AT ALL SURFACES $\mu_s = 0.30$, $\theta = 30°$

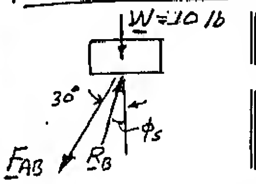
(a) CONFIRM EQUILIBRIUM WHEN $P = 0$

(b) FIND LARGEST P FOR EQUILIBRIUM.

FOR MOTION: BLOCK A MOVES ↓ AND BLOCK B MOVES ←.

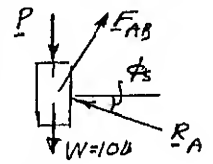
ASSUME MOTION IMPENDS: $\phi_s = \tan^{-1} 0.30 = 16.7°$

FREE BODY: BLOCK B

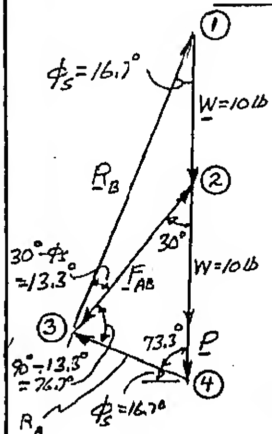


NOTE: AB IS A TWO-FORCE MEMBER

FREE BODY: BLOCK A



FORCE TRIANGLES



BLOCK B: Δ 1,2,3

$$\frac{F_{AB}}{\sin 16.7°} = \frac{10\ lb}{\sin 13.3°}$$

$$F_{AB} = 12.491\ lb$$

BLOCK A: Δ 2,4,3

$$\frac{W+P}{\sin 76.7°} = \frac{F_{AB}}{\sin 73.3°}$$

$$\frac{10\ lb + P}{\sin 76.7°} = \frac{12.491\ lb}{\sin 73.3°}$$

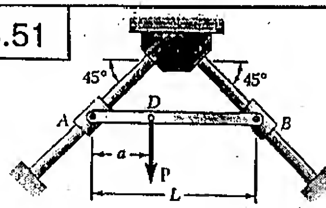
$$10\ lb + P = 12.69\ lb.$$

(b) $P = 2.69\ lb$ ◀

(a) EQUILIBRIUM FOR $P < 2.69\ lb$

∴ EQUILIBRIUM FOR $P = 0$ ◀

8.51



GIVEN: $\mu_s = 0.30$

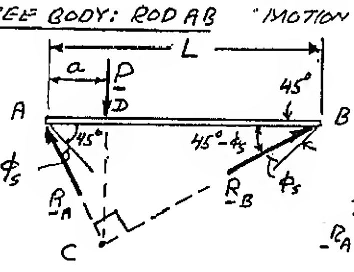
FIND: SMALLEST VALUE OF a/L FOR EQUILIBRIUM

FREE BODY: ROD AB

MOTION IMPENDS

$\phi_s = \tan^{-1} 0.30$

$\phi_s = 16.7°$



THREE-FORCE BODY: P MUST PASS THROUGH E WHERE R_A AND R_B INTERSECT

IN RIGHT TRIANGLE ABC:

$$AC = L \sin(45 - \phi_s)$$

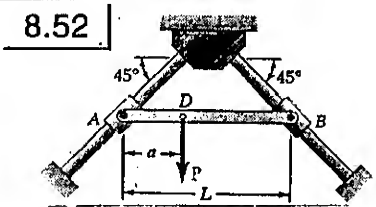
IN RIGHT TRIANGLE ADC:

$$a = AC \cos(45 + \phi_s) = L \sin(45 - \phi_s)\cos(45 + \phi_s)$$

$$\frac{a}{L} = \sin(45° - 16.7°)\cos(45° + 16.7°) = 0.2248$$

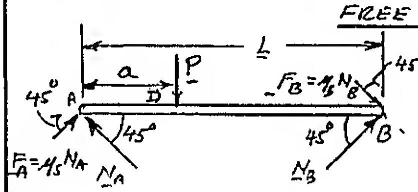
$$\frac{a}{L} = 0.225$$ ◀

8.52



DERIVE: EXPRESSION IN μ_s FOR SMALLEST VALUE OF a/L FOR EQUILIBRIUM

FREE BODY: ROD AB



$$+\circlearrowleft \Sigma M_A = 0$$

$$(N_B - \mu_s N_B)\frac{L}{\sqrt{2}} - Wa = 0$$

$$N_B = \frac{\sqrt{2}\,W a}{(1-\mu_s)L} \quad (1)$$

$$\xrightarrow{+} \Sigma F_x = 0: \ (N_B + \mu_s N_B)\frac{1}{\sqrt{2}} - (N_A - \mu_s N_A)\frac{1}{\sqrt{2}} = 0$$

$$N_A = \frac{1+\mu_s}{1-\mu_s} N_B \quad (2)$$

$$+\uparrow \Sigma F_y = 0: \ (N_A + \mu_s N_A)\frac{1}{\sqrt{2}} + (N_B - \mu_s N_B)\frac{1}{\sqrt{2}} - W = 0$$

$$N_A = \frac{\sqrt{2}\,W - N_B(1-\mu_s)}{1+\mu_s} \quad (3)$$

EQUATE (2) AND (3):

$$\frac{1+\mu_s}{1-\mu_s} N_B = \frac{\sqrt{2}\,W - N_B(1-\mu_s)}{1+\mu_s}$$

$$N_B\left[\frac{1+\mu_s}{1-\mu_s} + \frac{1-\mu_s}{1+\mu_s}\right] = \frac{\sqrt{2}\,W}{1+\mu_s} \quad (4)$$

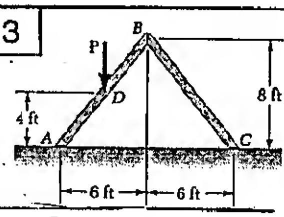
SUBSTITUTE FROM (1):

$$\frac{\sqrt{2}\,Wa}{(1-\mu_s)L}\left[\frac{(1+\mu_s)^2 + (1-\mu_s)^2}{(1-\mu_s)(1+\mu_s)}\right] = \frac{\sqrt{2}\,W}{1+\mu_s}$$

$$\frac{a}{L}\left[\frac{1 + \cancel{2\mu_s} + \mu_s^2 + 1 - \cancel{2\mu_s} + \mu_s^2}{1-\mu_s^2}\right] = \frac{1-\mu_s}{1+\mu_s}$$

$$\frac{a}{L}\left[\frac{2(1+\mu_s^2)}{1-\mu_s^2}\right] = \frac{1-\mu_s}{1+\mu_s}; \quad \frac{a}{L} = \frac{1}{2}\frac{(1-\mu_s)(1-\mu_s^2)}{(1+\mu_s)(1+\mu_s^2)}$$ ◀

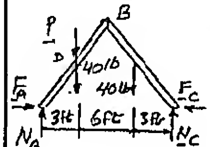
8.53



GIVEN: $\mu_s = 0.40$, EACH BOARD WEIGHS 40 lb.

FIND: (a) LARGEST P FOR EQUILIBRIUM, (b) WHERE MOTION IMPENDS

FREE BODY: ENTIRE FRAME

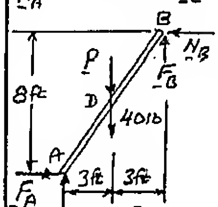


$\Sigma M_C = 0$ YIELDS: $N_A = 40 + \frac{3}{4}P$

$\Sigma M_A = 0$ YIELDS: $N_C = 40 + \frac{1}{4}P$

$\Sigma F_x = 0$ YIELDS: $F_C = F_A$

FREE BODY: BOARD AB



$$+\uparrow \Sigma F_y = 0: \ 40 + \tfrac{3}{4}P - P - 40 + F_B = 0$$

$$F_B = \frac{P}{4}$$

$$+\circlearrowleft \Sigma M_B = 0:$$

$$(P + 40)(3\ ft) - (40 + \tfrac{3}{4}P)(6\ ft) + F_A(8\ ft) = 0$$

$$F_A = 15 + \tfrac{3}{16}P$$

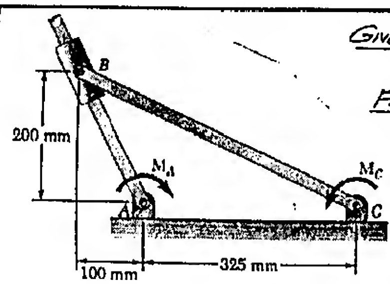
$\Sigma F_x = 0$: $N_B = F_A$

AT POINTS A, B, AND C WE EXPRESS THAT FOR IMPEND MOTION $\mu_s = F/N$.

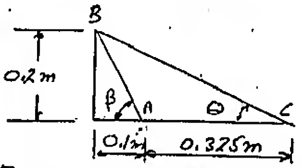
AT POINT A	AT POINT B	AT POINT C
$\mu_s = \frac{F_A}{N_A}$	$\mu_s = \frac{F_B}{N_B} = \frac{F_B}{F_A}$	$\mu_s = \frac{F_C}{N_C} = \frac{F_A}{N_C}$
$0.40 = \dfrac{15 + \frac{3P}{16}}{40 + \frac{3P}{4}}$	$0.40 = \dfrac{P/4}{15 + \frac{3P}{16}}$	$0.40 = \dfrac{15 + \frac{3P}{16}}{40 + P/4}$
$P = -8.889\ lb$	$P = 34.29\ lb$	$P = 11.429\ lb$

(a) $P_{MAX} = 11.43\ lb$ (b) MOTION IMPENDS AT C ◀

8.54



GIVEN: $M_A = 15\ N\cdot m$
$\mu_s = 0.30$
FIND: LARGEST M_C FOR WHICH EQUILIBRIUM IS MAINTAINED

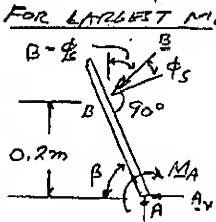


$\tan\beta = \frac{0.2\ m}{0.1\ m}$ $\beta = 63.43°$

$\tan\theta = \frac{0.2\ m}{0.425\ m}$ $\theta = 25.2°$

$\tan\phi_s = \mu_s = 0.3$ $\phi_s = 16.7°$

FOR LARGEST M_C, MOTION OF B IMPENDS ↘.

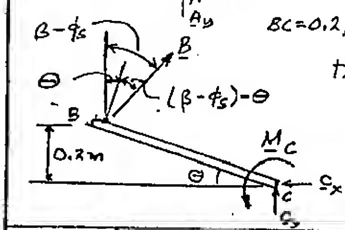


$AB = 0.2/\sin\beta = 0.2/\sin 63.43° = 0.2236\ m$

$+\circlearrowleft \Sigma M_A = 0:\ B\cos\phi_s(AB) - M_A = 0$

$B\cos 16.7°(0.2236\ m) - 15\ N\cdot m$

$B = 70.033\ N$



$BC = 0.2/\sin\theta = 0.2/\sin 25.2 = 0.4697\ m$

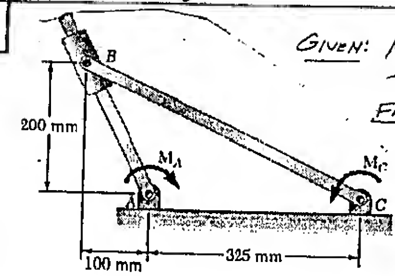
$+\circlearrowleft \Sigma M_C = 0:\ M_C - B\cos(\beta - \phi_s - \theta)\times BC$

$\beta - \phi_s - \theta = 63.43° - 16.7° - 25.2° = 21.53°$

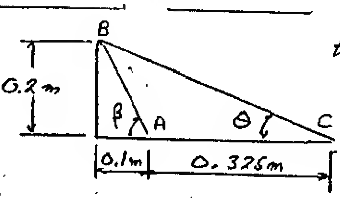
$M_C = (70.033\ N)\cos 21.53°(0.4697\ m)$

$M_C = 30.6\ N\cdot m \circlearrowright$ ◀

8.55



GIVEN: $M_A = 15\ N\cdot m$
$\mu_s = 0.30$
FIND: SMALLEST M_C FOR WHICH EQUILIBRIUM IS MAINTAINED

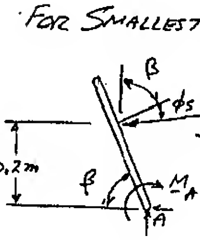


$\tan\beta = \frac{0.2\ m}{0.1\ m}$ $\beta = 63.43°$

$\tan\theta = \frac{0.2\ m}{0.425\ m}$ $\theta = 25.2°$

$\tan\phi_s = \mu_s = 0.3$ $\phi_s = 16.7°$

FOR SMALLEST M_C, MOTION OF B IMPENDS ↖.

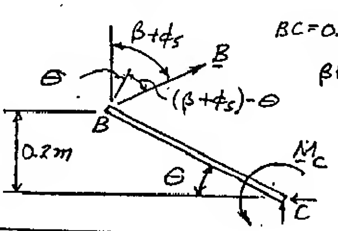


$AB = 0.2/\sin\beta = 0.2/\sin 63.43° = 0.2236\ m$

$+\circlearrowleft \Sigma M_A = 0;\ B\cos\phi(AB) - M_A = 0$

$B\cos 16.7°(0.2236\ m) - 15\ N\cdot m = 0$

$B = 70.033\ N$



$BC = 0.2/\sin\theta = 0.2/\sin 25.2 = 0.4697\ m$

$\beta + \phi_s - \theta = 63.43° + 16.7° - 25.2° = 54.93°$

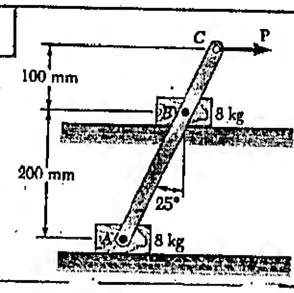
$+\circlearrowleft \Sigma M_C = 0$

$M_C - B\cos(\beta + \phi_s - \theta)\times BC$

$M_C = (70.033\ N)\cos(54.93°)(0.4697\ m)$

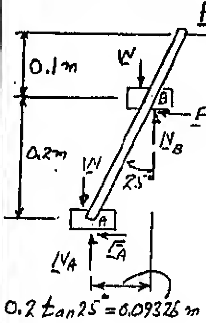
$M_C = 18.90\ N\cdot m \circlearrowright$ ◀

8.56



FIND: VALUE OF P FOR WHICH MOTION OCCURS AND WHAT MOTION IS FOR (a) $\mu_s = 0.40$
(b) $\mu_s = 0.50$

(a) $\mu_s = 0.40$: ASSUME BLOCKS SLIDE TO RIGHT



$W = mg = (8\ kg)(9.81\ m/s^2) = 78.48\ N$

$F_A = \mu_s N_A$ $F_B = \mu_s N_B$

$+\uparrow \Sigma F_y = 0:\ N_A + N_B - 2W = 0$

$N_A + N_B = 2W$

$\xrightarrow{+} \Sigma F_x = 0:\ P - F_A - F_B = 0$

$P = F_A + F_B = \mu_s(N_A + N_B) = \mu_s(2W)$ (1)

$P = 0.40(2)(78.48\ N) = 62.78\ N$

$+\circlearrowleft \Sigma M_B = 0:\ P(0.1\ m) + (N_A - W)(0.09326\ m)$

$N_A - W = -(62.78\ N)(0.1\ m)/(0.09326)\ m$

$N_A - 78.48\ N = -67.32\ N$

$N_A = 11.16\ N > 0$ OK

SYSTEM SLIDES: $P = 62.8\ N$ ◀

(b) $\mu_s = 0.50$: SEE PART a.

Eq.(1) $P = 0.5(2)(78.48\ N) = 78.48\ N$

$+\circlearrowleft \Sigma M_B = 0:\ P(0.1\ m) + (N_A - W)(0.09326\ m) = 0$

$N_A - W = -(78.48\ N)(0.1\ m)/(0.09326\ m)$

$N_A - 78.48\ N = -84.15\ N$

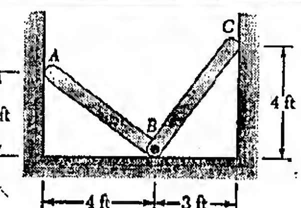
$N_A = -5.67\ N < 0$ UPLIFT, ROTATION ABOUT B

FOR $N_A = 0$: $\circlearrowleft \Sigma M_B = 0:\ P(0.1\ m) - W(0.09326\ m) = 0$

$P = (78.48\ N)(0.09326\ m)/(0.1) = 73.19$

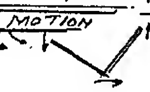
SYSTEM ROTATES ABOUT B: $P = 73.2\ N$ ◀

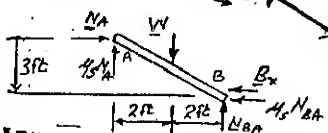
8.57

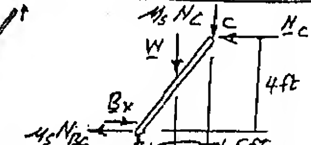


GIVEN: μ_s AT A, B, AND C
FIND: SMALLEST μ_s FOR EQUILIBRIUM

SENSE OF IMPENDING MOTION







$+\circlearrowleft \Sigma M_B = 0:\ 2W - 3N_A - 4\mu_s N_A = 0$

$N_A = 2W/(3 + 4\mu_s)$ (1)

$\Sigma F_y:\ N_{AB} = W - \mu_s N_A$ (3)

$\xrightarrow{+} \Sigma F_x = 0:\ B_x + \mu_s N_{BA} - N_A = 0$

$B_x = N_A - \mu_s N_{BA}$ (5)

$+\circlearrowleft \Sigma M_B = 0:\ 1.5W - 4N_C + 3\mu_s N_C = 0$

$N_C = 1.5W/(4 - 3\mu_s)$ (2)

$\Sigma F_y:\ N_{BC} = W + \mu_s N_C$ (4)

$\xrightarrow{+} \Sigma F_x = 0:\ B_x - N_C - \mu_s N_{BC} = 0$

$B_x = N_C + \mu_s N_{BC}$ (6)

EQUATE (5) AND (6): $N_A - \mu_s N_{BA} = N_C + \mu_s N_{BC}$

SUB FROM (3) AND (4): $N_A - \mu_s(W - \mu_s N_A) = N_C + \mu_s(W + \mu_s N_C)$

$N_A(1 + \mu_s^2) - \mu_s W = N_C(1 + \mu_s^2) + \mu_s W$

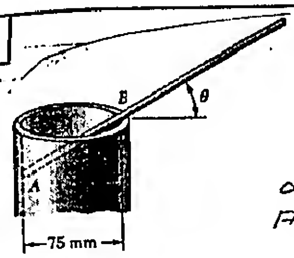
SUB. FROM (1) AND (2): $\frac{2W}{3 + 4\mu_s}(1 + \mu_s^2) - \mu_s W = \frac{1.5W}{4 - 3\mu_s}(1 + \mu_s^2) + \mu_s W$

$\frac{2}{3 + 4\mu_s} - \frac{1.5}{4 - 3\mu_s} = \frac{2\mu_s}{1 + \mu_s^2}$

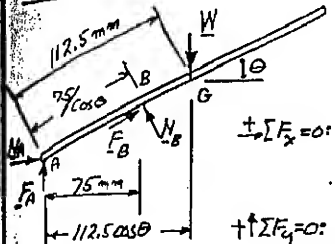
SOLVE FOR μ_s:

$\mu_s = 0.09488$ $\mu_s = 0.095$ ◀

8.58



GIVEN: LENGTH OF ROD = 225 mm,
$\mu_s = 0.20$,
FIND: LARGEST VALUE OF θ FOR ROD TO NOT FALL INTO THE PIPE.



MOTION OF ROD IMPENDS DOWN AT A AND TO LEFT AT B.
$F_A = \mu_s N_A \qquad F_B = \mu_s N_B$

$\overset{+}{\rightarrow}\Sigma F_x = 0: \; N_A - N_B \sin\theta + F_B \cos\theta = 0$
$N_A - N_B \sin\theta + \mu_s N_B \cos\theta = 0$
$N_A = N_B(\sin\theta - \mu_s \cos\theta) \quad (1)$

$+\uparrow\Sigma F_y = 0: \; F_A + N_B\cos\theta + F_B \sin\theta - W = 0$
$\mu_s N_A + N_B \cos\theta + \mu_s N_B \sin\theta - W = 0 \quad (2)$

SUBSTITUTE FOR N_A FROM (1) INTO (2):
$\mu_s N_B(\sin\theta - \mu_s\cos\theta) + N_B\cos\theta + \mu_s N_B \sin\theta - W = 0$

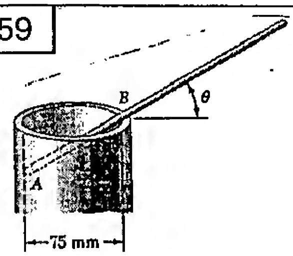
$$N_B = \frac{W}{(1-\mu_s^2)\cos\theta + 2\mu_s \sin\theta} = \frac{W}{(1-0.2^2)\cos\theta + 2(0.2)\sin\theta} \quad (3)$$

$+\circlearrowleft\Sigma M_A = 0: \; N_B(75/\cos\theta) - W(112.5\cos\theta) = 0$

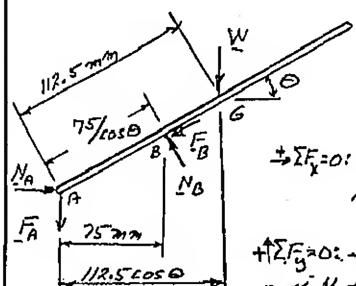
SUBSTITUTE FOR N_B FROM (3), CANCEL W, AND SIMPLIFY TO FIND
$9.6\cos^3\theta + 4\sin\theta\cos^2\theta - 6.6667 = 0$
$\cos^3\theta(2.4 + \tan\theta) = 1.6667$

SOLVE BY TRIAL + ERROR: $\theta = 35.8°$ ◀

8.59



GIVEN: LENGTH OF ROD = 225 mm,
$\mu_s = 0.20$.
FIND: SMALLEST VALUE OF θ FOR ROD TO NOT FALL OUT OF THE PIPE.



MOTION OF ROD IMPENDS UP AT A AND RIGHT AT B
$F_A = \mu_s N_A \qquad F_B = \mu_s N_B$

$\overset{+}{\rightarrow}\Sigma F_x = 0: \; N_A - N_B\sin\theta - F_B\cos\theta = 0$
$N_A - N_B\sin\theta - \mu_s N_B \cos\theta = 0$
$N_A = N_B(\sin\theta + \mu_s\cos\theta) \quad (1)$

$+\uparrow\Sigma F_y = 0: \; -F_A + N_B\cos\theta - F_B\sin\theta - W = 0$
$-\mu_s N_A + N_B\cos\theta - \mu_s N_B \sin\theta - W = 0 \quad (2)$

SUBSTITUTE FOR N_A FROM (1) INTO (2):
$-\mu_s N_B(\sin\theta + \mu_s\cos\theta) + N_B\cos\theta - \mu_s N_B\sin\theta - W = 0$

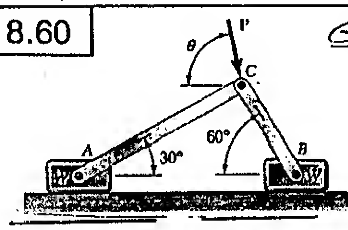
$$N_B = \frac{W}{(1-\mu_s^2)\cos\theta - 2\mu_s\sin\theta} = \frac{W}{(1-0.2^2)\cos\theta - 2(0.2)\sin\theta} \quad (3)$$

$+\circlearrowleft\Sigma M_A = 0: \; N_B(75/\cos\theta) - W(112.5\cos\theta) = 0$

SUBSTITUTE FOR N_B FROM (3), CANCEL W, AND SIMPLIFY
$9.6\cos^3\theta - 4\sin\theta\cos^2\theta - 6.667 = 0$
$\cos^3\theta(2.4 - \tan\theta) = 1.6667$

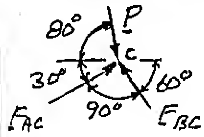
SOLVE BY TRIAL + ERROR: $\theta = 20.5°$ ◀

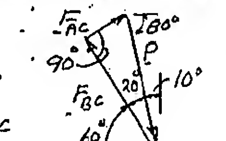
8.60



GIVEN: $\theta = 80°$,
$\mu_s = 0.30$
FIND: LARGEST P FOR EQUILIBRIUM

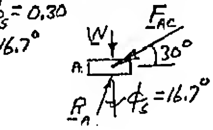
AC AND BC ARE TWO-FORCE MEMBERS

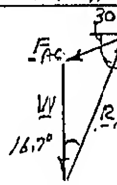




AT JOINT C:
$F_{AC} = P\sin 20° \quad (1)$
$F_{BC} = P\cos 20° \quad (2)$

ASSUME MOTION OF BLOCK A IMPENDS TO LEFT.
$\tan\phi_s = 0.30$
$\phi_s = 16.7°$

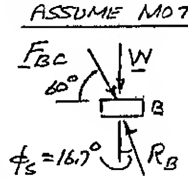


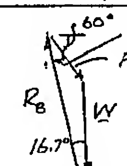


$90° - 30° - 16.7° = 43.3°$
LAW OF SINES
$$\frac{F_{AC}}{\sin 16.7°} = \frac{W}{\sin 43.3°}$$
$F_{AC} = 0.419\,W$

SUBSTITUTE INTO EQ.(1): $F_{AC} = 0.419W = P\sin 20°$; $P = 1.225\,W$ ◀

ASSUME MOTION OF BLOCK B IMPENDS TO RIGHT



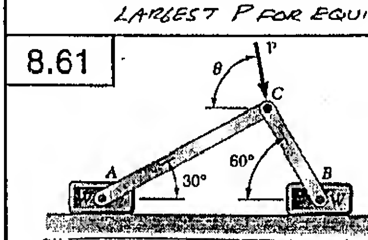


$90° - 60° - 16.7° = 13.3°$
LAW OF SINES
$$\frac{F_{BC}}{\sin 16.7°} = \frac{W}{\sin 13.3°}$$
$F_{BC} = 1.249W$

SUBSTITUTE INTO EQ.(2): $F_{BC} = 1.249W = P\cos 20°$; $P = 1.329W$ ◀

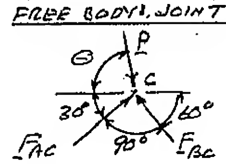
LARGEST P FOR EQUILIBRIUM: $P = 1.225\,W$ ◀

8.61

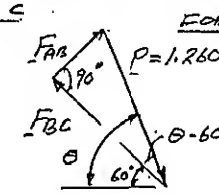


GIVEN: $P = 1.260W$
$\mu_s = 0.30$
FIND: RANGE OF θ, BETWEEN 0 AND 180°, FOR EQUILIBRIUM

AC AND BC ARE TWO-FORCE MEMBERS
FREE BODY: JOINT C



FORCE TRIANGLE



FROM FORCE TRIANGLE:
$F_{AB} = P\sin(\theta - 60°) = 1.26\,W\sin(\theta - 60°) \quad (1)$
$F_{BC} = P\cos(\theta - 60°) = 1.26\,W\cos(\theta - 60°) \quad (2)$

WE SHALL, IN TURN, SEEK θ CORRESPONDING TO IMPENDING MOTION OF EACH BLOCK

FOR MOTION OF A IMPENDING TO LEFT
FROM SOLUTION OF PROB 8.60; $F_{AC} = 0.419\,W$
EQ(1): $F_{AC} = 0.419W = 1.26\,W\sin(\theta - 60°)$
$\sin(\theta - 60°) = 0.33254$
$\theta - 60° = 19.423°$
$\theta = 79.42°$ ◀

(CONTINUED)

8.61 CONTINUED

FOR MOTION OF B IMPENDING TO RIGHT.

FROM SOLUTION OF PROB. 8.60; $F_{BC} = 1.249W$

EQ.(2): $F_{BC} = 1.249W = 1.26W\cos(\theta - 60^\circ)$

$\cos(\theta - 60^\circ) = 0.99127$

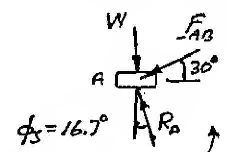
$\theta - 60^\circ = \pm 7.58^\circ$

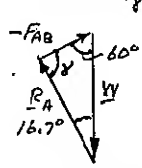
$\theta - 60^\circ = +7.58^\circ$ $\qquad \theta = 67.6^\circ$ ◁

$\theta - 60^\circ = -7.58^\circ$ $\qquad \theta = 52.4^\circ$ ◁

FOR MOTION OF A IMPENDING TO RIGHT

$\gamma = 180^\circ - 60^\circ - 16.7^\circ = 103.3^\circ$





LAW OF SINES

$$\frac{-F_{AB}}{\sin 16.7^\circ} = \frac{W}{\sin 103.3^\circ}$$

$F_{AB} = -0.29528W$

NOTE: DIRECTION OF $+F_{AB}$ IS KEPT SAME AS IN FREE BODY OF JOINT C.

EQ.(1): $F_{AB} = -0.29528W = 1.26W\sin(\theta - 60^\circ)$

$\sin(\theta - 60^\circ) = -0.23435$

$(\theta - 60^\circ) = -13.553^\circ$ $\qquad \theta = 46.4^\circ$ ◁

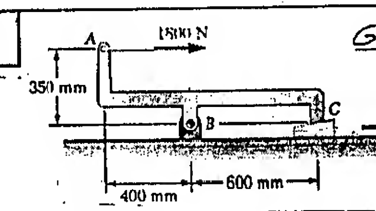
SUMMARY:

A MOVES TO RIGHT	NO MOTION	B MOVES TO RIGHT	NO MOTION	A MOVES TO LEFT

θ: 46.4° 52.4° 67.6° 79.4°

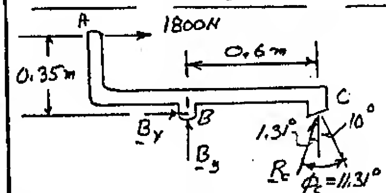
NO MOTION FOR: $46.4^\circ \le \theta \le 52.4^\circ$ AND $67.6^\circ \le \theta \le 79.4^\circ$ ◀

8.63



GIVEN: $\mu_s = 0.20$
10° WEDGE
FIND: FORCE P TO MOVE WEDGE TO THE RIGHT.

$\phi_s = \tan^{-1} 0.20 = 11.31^\circ$



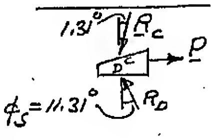
FREE BODY: PART ABC

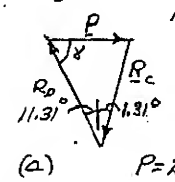
$+\circlearrowleft \Sigma M_B = 0$:

$(1800N)(0.35m) - R_c\cos 1.31^\circ(0.6m) = 0$

$R_c = 1050.3N$

FREE BODY: WEDGE





FORCE TRIANGLE

$\gamma = 90^\circ - 11.31^\circ = 78.69^\circ$

LAW OF SINES

$$\frac{P}{\sin(11.31^\circ + 1.31^\circ)} = \frac{1050.3N}{\sin 78.69^\circ}$$

(a) $\qquad P = 234N$ $\qquad P = 234N \rightarrow$ ◀

(b) RETURN TO PART ABC:

$\xrightarrow{+} \Sigma F_x = 0$: $B_x + 1800N + R_c\sin 1.31^\circ = 0$

$B_x + 1800N + (1050.3N)\sin 1.31^\circ = 0$

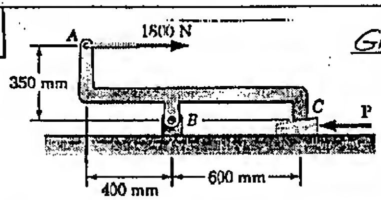
$B_x = -1824N$ $\qquad B_x = 1824N \leftarrow$ ◀

$+\uparrow \Sigma F_y = 0$: $B_y + R_c\cos 1.31^\circ = 0$

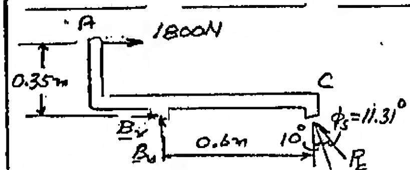
$B_y + (1050.3N)\cos 1.31^\circ = 0$

$B_y = -1050N$ $\qquad B_y = 1050N\downarrow$ ◀

8.62



GIVEN: $\mu_s = 0.20$
10° WEDGE
FIND: FORCE P TO MOVE WEDGE TO LEFT.



$\phi_s = \tan^{-1} 0.20 = 11.31^\circ$

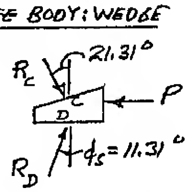
FREE BODY: PART ABC

$+\circlearrowleft \Sigma M_B = 0$

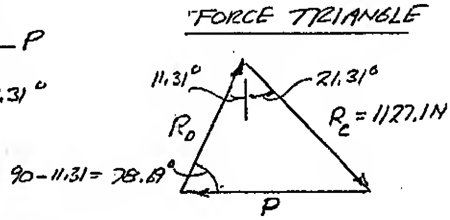
$(1800N)(0.35m) - R_c\cos 21.31^\circ(0.6m) = 0$

$R_c = 1127.1N$

FREE BODY: WEDGE



FORCE TRIANGLE



(a) LAW OF SINES

$$\frac{P}{\sin(11.31^\circ + 21.31^\circ)} = \frac{1127.1N}{\sin 78.69^\circ}$$

$P = 619.6N$; $\qquad P = 620N \leftarrow$ ◀

(b)

RETURN TO PART ABC:

$\xrightarrow{+} \Sigma F_x = 0$: $B_x + 1800N - R_c\sin 21.31^\circ = 0$

$B_x + 1800N - (1127.1N)\sin 21.31^\circ$

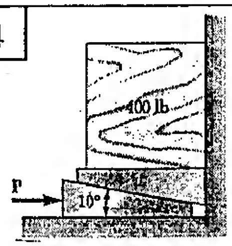
$B_x = -1390.4N$ $\qquad B_x = 1390N \leftarrow$ ◀

$+\uparrow \Sigma F_y = 0$: $B_y + R_c\cos 21.31^\circ = 0$

$B_y + (1127.1N)\cos 21.31^\circ = 0$

$B_y = -1050N$ $\qquad B_y = 1050N\downarrow$ ◀

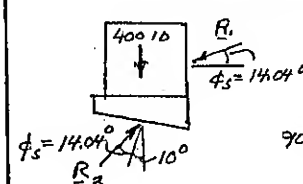
8.64



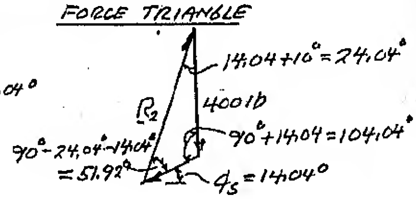
GIVEN: TWO 10° WEDGES
$\mu_s = 0.25$

FIND: SMALLEST P TO MOVE WEDGE

FREE BODY: BLOCK AND TOP WEDGE $\qquad \phi_s = \tan^{-1} 0.25 = 14.04^\circ$



FORCE TRIANGLE

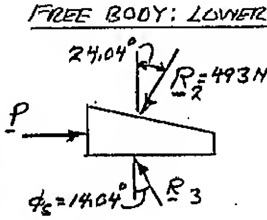


LAW OF SINES

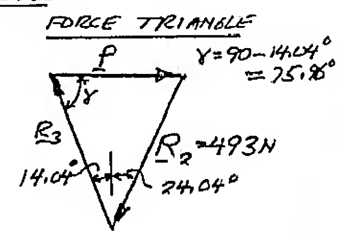
$$\frac{R_2}{\sin 104.04^\circ} = \frac{400\,lb}{\sin 51.92^\circ}$$

$R_2 = 493N$

FREE BODY: LOWER WEDGE



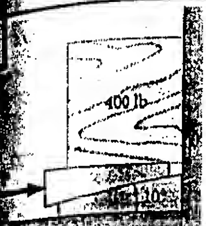
FORCE TRIANGLE



LAW OF SINES

$$\frac{P}{\sin(14.04^\circ + 24.04^\circ)} = \frac{493N}{\sin 75.96^\circ}$$

$P = 313.4N$ $\qquad P = 313N \rightarrow$ ◀



GIVEN: TWO 10° WEDGES
$\mu_s = 0.25$

FIND: SMALLEST P TO MOVE WEDGE

FREE BODY: BLOCK

$\phi_s = \tan^{-1} 0.25 = 14.04°$

FORCE TRIANGLE

400 lb, R_1, $\phi_s = 14.04°$, R_2, $\phi_s = 14.04°$

$90° - 2\phi_s = 61.92°$ $\quad$ $90° + \phi_s = 104.4°$ $\quad$ $\phi_s = 14.04°$

LAW OF SINES

$$\frac{R_2}{\sin 104.4°} = \frac{400\text{ lb}}{\sin 61.92°} \qquad R_2 = 439.8\text{N}$$

FREE BODY: WEDGE

14.04°, R_2, 10°, $\phi_s = 14.04°$, R_3

FORCE TRIANGLE

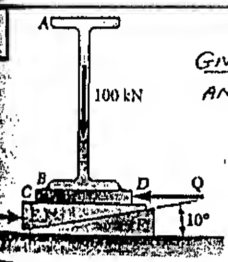
$90° - 24.04°$
$65.96°$

P, R_3, $R_2 = 439.8\text{N}$, $10° + \phi_s = 24.04°$, $\phi_s = 14.04°$

LAW OF SINES

$$\frac{P}{\sin(24.04° + 14.04°)} = \frac{439.8\text{N}}{\sin 61.92°}$$

$P = 297.0\text{ N}$ $\qquad$ $P = 297 \rightarrow$



GIVEN: $\mu_s = 0.60$ BETWEEN STEEL AND CONCRETE
$\mu_s = 0.30$ BETWEEN TWO STEEL SURFACES

FIND: (a) P TO RAISE BEAM
(b) CORRESPONDING Q

FREE BODY: BEAM AND PLATE CD

$\phi_s = \tan^{-1} 0.3 = 16.7°$

100 kN, Q, R_1, 16.7°, $\phi_s = 16.7°$

$Q = (100\text{ kN}) \tan 16.7°$
$Q = 30\text{ kN} \leftarrow$

$R_1 = (100\text{ kN})/\cos 16.7°$
$R_1 = 104.4\text{ kN}$

FREE BODY: WEDGE E

ϕ_s, R_1, ϕ_s, R_2

P, $90° - 10° - \phi_s = 63.3°$, $90° - \phi_s = 73.3°$, $R_1 = 104.4\text{ kN}$, R_2, $10° + 2\phi_s = 43.4°$

$$\frac{P}{\sin 43.4°} = \frac{104.4\text{ kN}}{\sin 63.3°}; \quad P = 80.3\text{ kN} \rightarrow$$

FREE BODY: WEDGE F (TO CHECK THAT IT DOES NOT MOVE.)

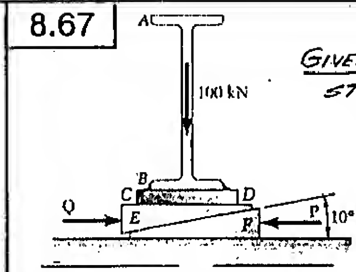
$10° + \phi_s = 26.7°$, R_2, F, R_3

SINCE WEDGE F IS A TWO-FORCE BODY, R_2 AND R_3 ARE COLINEAR

THUS $\theta = 26.7°$.

BUT $\phi_{concrete} = \tan^{-1} 0.6 = 31.0° > \theta$ $\quad$ OK

8.67



GIVEN: $\mu_s = 0.60$ BETWEEN STEEL AND CONCRETE
$\mu_s = 0.30$ BETWEEN TWO STEEL SURFACES

FIND: (a) P TO RAISE BEAM
(b) CORRESPONDING Q.

FREE BODY: WEDGE F

$\phi_s = \tan^{-1} 0.30 = 16.7°$

R_1, 10°, $\phi_s = 16.7°$, P, F, R_2, $\phi_s = 16.7°$

P, R_1, R_2, $10° + \phi_s = 26.7°$, ϕ_s

VERTICAL COMPONENTS $= 100\text{ kN}$

$P = (100\text{ kN}) \tan 26.7° + (100\text{ kN}) \tan \phi_s$
$P = 50.29\text{ kN} + 30\text{ kN}$
$P = 80.29\text{ kN}$ $\qquad$ $P = 80.3\text{ kN} \leftarrow$
$R_1 = (100\text{ kN})/\cos 26.7° = 111.94\text{ kN}$

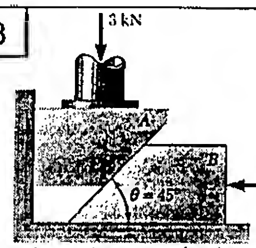
FREE BODY: BEAM, PLATE, AND WEDGE E

100 kN, Q, E, 26.7°, R_1

100 kN, 26.7°, R_1, Q

$Q = R_1 \sin 26.7°$
$= (111.94\text{ kN}) \sin 26.7°$
$Q = 50.29$
$Q = 50.3\text{ kN} \rightarrow$

8.68



GIVEN: $\mu_s = 0.25$ AT ALL SURFACES OF CONTACT.

FIND: SMALLEST P TO RAISE BLOCK A.

FREE BODY: BLOCK A

$\phi_s = \tan^{-1} 0.25 = 14.04°$

3 kN, R_1, ϕ_s, ϕ_s, R_2, 45°

$45° + \phi_s = 45° + 14.04° = 59.04°$

3 kN, R_2, R_1, $90° + \phi_s = 104.04°$

$180° - 104.04° - 59.04° = 16.92°$

LAW OF SINES

$$\frac{R_2}{\sin 104.04°} = \frac{3\text{ kN}}{\sin 16.92°} \qquad R_2 = 10.00\text{ kN}$$

FREE BODY: WEDGE B

$45° + \phi_s = 59.04°$, $R_2 = 10.00\text{ kN}$, P, R_3, $\phi_s = 14.04°$

P, 75.96°, 14.04°, 59.04°, $R_2 = 10.00\text{ kN}$, R_2

$59.04° + 14.04° = 73.08°$

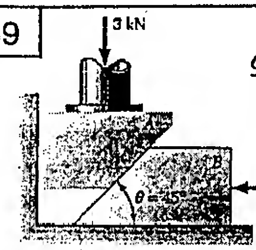
LAW OF SINES

$$\frac{P}{\sin 73.08°} = \frac{10.00\text{ kN}}{\sin 75.96°}$$

$P = 9.86\text{ kN}$

$P = 9.86\text{ kN} \leftarrow$

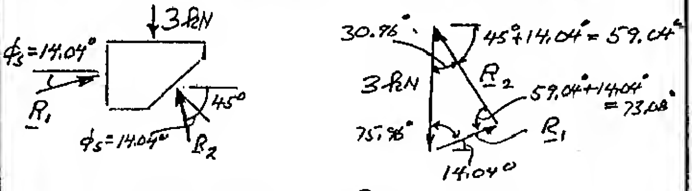
8.69



GIVEN: $\mu_s = 0.25$ BETWEEN ALL SURFACES OF CONTACT

FIND: SMALLEST P FOR EQUILIBRIUM

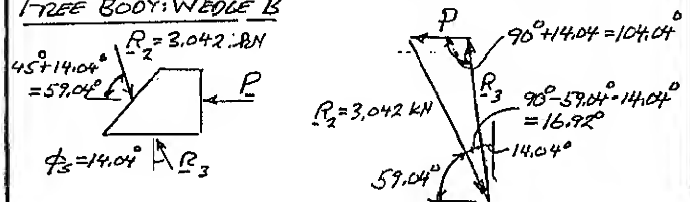
FREE BODY: BLOCK A $\qquad \phi_s = \tan^{-1} 0.25 = 14.04°$



LAW OF SINES $\qquad \dfrac{R_2}{\sin 75.96°} = \dfrac{3\ kN}{\sin 73.08°}$

$R_2 = 3.042\ kN$

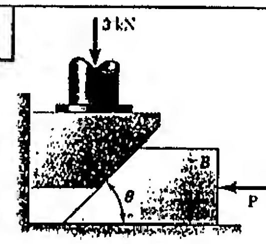
FREE BODY: WEDGE B



LAW OF SINES $\qquad \dfrac{P}{\sin 16.92°} = \dfrac{3.042\ kN}{\sin 104.04°}$

$P = 0.913\ kN \qquad \underline{P} = 913\ N \leftarrow$ ◀

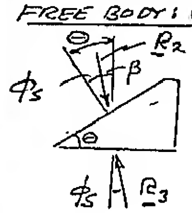
8.70



GIVEN: $P = 0$, $\mu_s = 0.25$

FIND: (a) ANGLE θ FOR IMPENDING MOTION
(b) CORRESPONDING FORCE EXERTED BY WALL.

FREE BODY: WEDGE B $\qquad \phi_s = \tan^{-1} 0.25 = 14.04°$

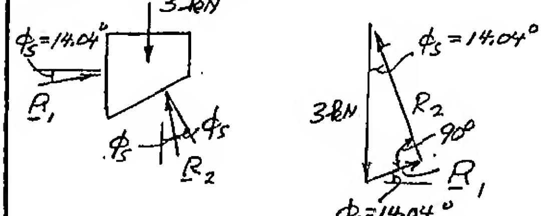


(a) SINCE WEDGE IS A TWO-FORCE BODY, R_2 AND R_3 MUST BE EQUAL AND OPPOSITE. THEREFORE, THEY FORM EQUAL ANGLES WITH VERTICAL $\beta = \phi_s$
AND $\theta - \phi_s = \phi_s$

$\theta = 2\phi_s = 2(14.04°)$

$\theta = 28.1°$ ◀

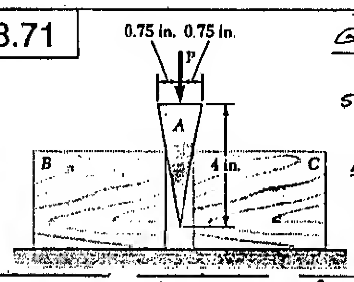
FREE BODY: BLOCK A



$R_1 = (3\ kN) \sin 14.04° = 0.7278\ kN$

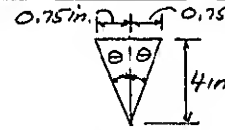
FORCE EXERTED BY WALL: $\underline{R}_1 = 728\ N \angle 14.0°$ ◀

8.71



GIVEN: TWO 100-lb BLOCKS, $\mu_s = 0.35$ AT ALL SURFACES OF CONTACT

FIND: SMALLEST P TO START WEDGE MOVING
(a) IF BOTH B & C CAN MOVE
(b) IF C CANNOT MOVE



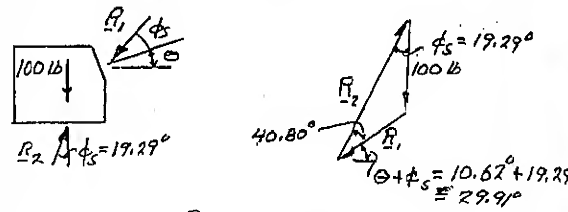
WEDGE ANGLE θ

$\theta = \tan^{-1} \dfrac{0.75\ in.}{4\ in.}$

$\theta = 10.62°$

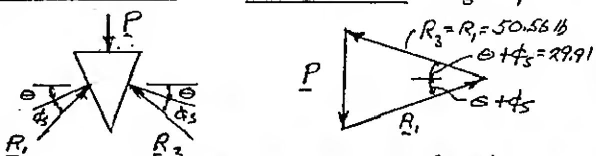
(a)

FREE BODY: BLOCK B $\qquad \phi_s = \tan^{-1} 0.35 = 19.29°$



$\dfrac{R_1}{\sin 19.29°} = \dfrac{100\ lb}{\sin 40.80°}$; $R_1 = 50.56\ lb$

FREE BODY: WEDGE $\qquad$ BY SYMMETRY $R_3 = R_1$

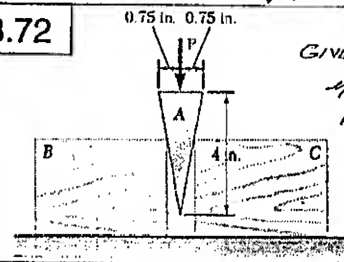


$P = 2R_1 \sin(\theta + \phi_s) = 2(50.56) \sin 29.91°$

$P = 50.42\ lb \qquad \underline{P} = 50.4\ lb \downarrow$ ◀

(b) FREE BODIES UNCHANGED ∴ SAME RESULT. $\quad \underline{P} = 50.4\ lb \downarrow$ ◀

8.72

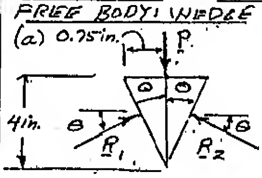


GIVEN: TWO 100-lb BLOCKS
$\mu_s = 0.35$ BETWEEN BLOCKS AND FLOOR, $\mu_s = 0$ AT WEDGE.

FIND: SMALLEST P TO START WEDGE MOVING
(a) IF BOTH B AND C CAN MOVE
(b) IF C CANNOT MOVE

FREE BODY: WEDGE

(a)

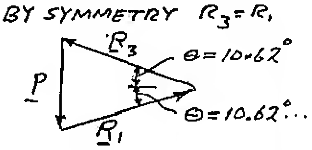


WEDGE ANGLE θ

$\theta = \tan^{-1} \dfrac{0.75\ in}{4\ in} = 10.62°$

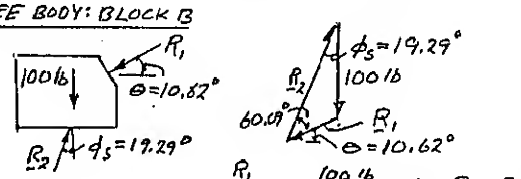
$\phi_s = \tan^{-1} 0.35 = 19.29°$

BY SYMMETRY $R_3 = R_1$



$P = 2R_1 \sin 10.62° \qquad (1)$

FREE BODY: BLOCK B

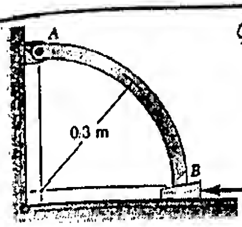


$\dfrac{R_1}{\sin 19.29°} = \dfrac{100\ lb}{\sin 60.09°}$; $R_1 = 38.11\ lb$

EQ.(1): $P = 2R_1 \sin 10.62° = 2(38.11\ lb) \sin 10.62°$; $\underline{P} = 14.05\ lb \downarrow$ ◀

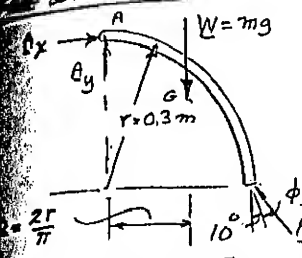
(b) FREE BODIES UNCHANGED, ∴ SAME RESULT. $\quad \underline{P} = 14.05\ lb \downarrow$ ◀

8.73



GIVEN: $10°$ WEDGE
WEIGHT OF AB = 5 kg
$\mu_s = 0.40$ BETWEEN WEDGE AND ROD
$\mu_s = 0.20$ BETWEEN WEDGE AND FLOOR
FIND: SMALLEST P TO MOVE

FREE BODY: ROD AB



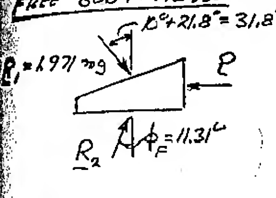
$\phi_s = \tan^{-1} 0.40 = 21.8°$

$+\circlearrowleft \Sigma M_A = 0: R_1 \cos(10° + \phi_s) r - R_1 \sin(10° + \phi_s) r - mg\left(\frac{2r}{\pi}\right) = 0$

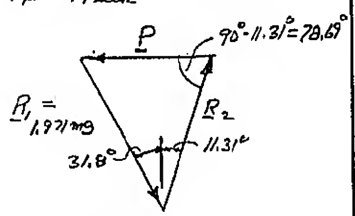
$R_1 = \frac{2mg}{\pi} \cdot \frac{1}{\cos 31.8° - \sin 31.8°}$

$R_1 = \frac{2mg}{\pi(0.3229)} = 1.971\, mg$

FREE BODY: WEDGE



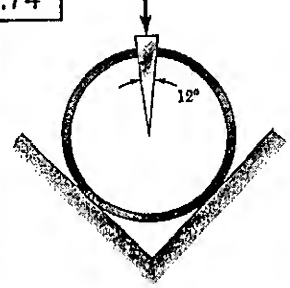
$\phi_F = \phi_{FLOOR} = \tan^{-1} 0.20 = 11.31°$



LAW OF SINES $\quad \frac{P}{\sin(31.8° + 11.31°)} = \frac{1.971\, mg}{\sin 78.69°}$

$P = 1.374\, mg = 1.374(5\text{ kg})(9.81\text{ m/s}^2)$; $\quad P = 67.4\text{ N}$ ◀

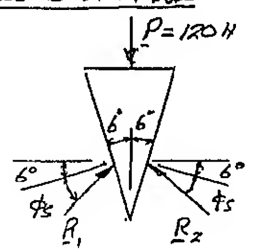
8.74



GIVEN: $\mu_s = 0.30$.
FORCE P = 120 N USED TO INSERT WEDGE

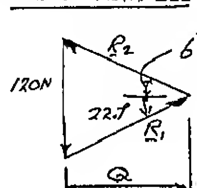
FIND: MAGNITUDE OF FORCES EXERTED ON RING AFTER WEDGE IS INSERTED.

FREE BODY: WEDGE



$\phi_s = \tan^{-1} 0.30 = 16.7°$

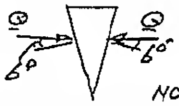
FORCE TRIANGLE



FROM FORCE TRIANGLE: Q = HORIZ. COMPONENT OF R

$Q = \frac{1}{2}(120\text{ N})/\tan 22.7° = 143.4\text{ N}$

FREE BODY: AFTER WEDGE HAS BEEN INSERTED

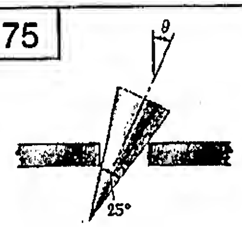


WEDGE IS NOW A TWO-FORCE BODY WITH FORCES SHOWN

$Q = 143.4\text{ N}$ ◀

NOTE: SINCE ANGLES BETWEEN FORCES Q AND NORMAL TO WEDGE IS $6° < \phi_s$, WEDGE STAYS IN PLACE.

8.75

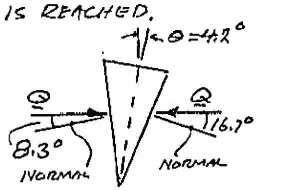


GIVEN: PLATE MOVE TOGETHER

FIND: WHAT HAPPENS TO WEDGE
(a) IF $\mu_s = 0.20$.
(b) IF $\mu_s = 0.30$.

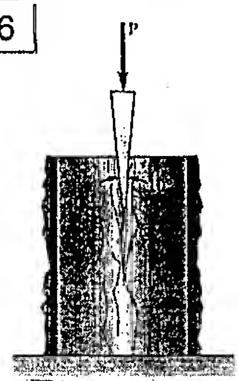
(a) FOR $\mu_s = 0.20$, $\phi_s = 11.31°$, REGARDLESS OF HOW WEDGE IS ORIENTED, ON AT LEAST ONE SIDE THE ANGLE BETWEEN THE FACE AND THE HORIZONTAL WILL BE GREATER THAN ϕ_s. THE WEDGE WILL BE FORCED UP AND OUT FROM BETWEEN THE PLATES. ◀

(b) FOR $\mu_s = 0.30$, $\phi_s = 16.7°$. AS THE PLATES ARE MOVED TOGETHER, θ WILL BECOME SMALLER. AT θ = 4.2°, THE POSITION SHOWN IS REACHED.



AT THIS POSITION THE LARGER ANGLE BETWEEN Q AND THE NORMAL TO THE WEDGE IS 16.7°, THE WEDGE WILL SELF LOCK. ◀

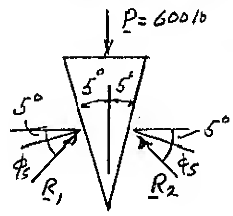
8.76



GIVEN: $\mu_s = 0.35$
FORCE P = 600 lb
REQUIRED TO INSERT WEDGE.

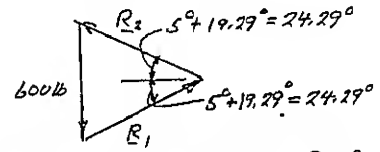
FIND: MAGNITUDE OF FORCES EXERTED ON WOOD BY WEDGE AFTER INSERTION

FREE BODY: WEDGE



$\phi_s = \tan^{-1} 0.35 = 19.29°$

FORCE TRIANGLE

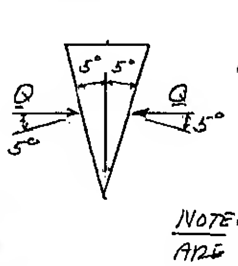


Q = HORIZONTAL COMPONENT OF R_1 & R_2

$Q = \frac{1}{2}(600\text{ lb})/\tan 24.29°$

$Q = 664.7\text{ lb}$

FREE BODY: AFTER WEDGE HAS BEEN INSERTED

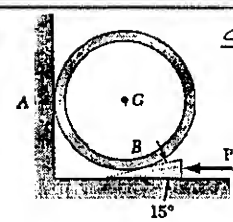


WEDGE IS NOW A TWO-FORCE BODY. FORCE F EXERTED ON WOOD IS EQUAL AND OPPOSITE TO Q.

$F = 664\text{ lb}$ ◀

NOTE: SINCE THE 5° ANGLES SHOWN ARE LESS THAN ϕ_s. WEDGE STAYS IN PLACE

8.77



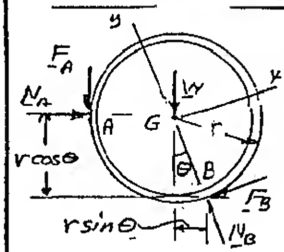
GIVEN: MASS OF PIPE = 50 kg

$\mu_s = 0.20$

AT ALL SURFACES

(a) SHOW THAT SLIPPING OCCURS FIRST AT A

(b) FIND: FORCE P TO CAUSE MOTION



FREE BODY: PIPE

$+\circlearrowright \Sigma M_B = 0:$

$W r\sin\theta + F_A r(1+\sin\theta) - N_A r\cos\theta = 0$

ASSUME SLIPPING AT A: $F_A = \mu_s N_A$

$N_A\cos\theta - \mu_s N_A(1+\sin\theta) = W\sin\theta$

$N_A = \dfrac{W\sin\theta}{\cos\theta - \mu_s(1+\sin\theta)}$

$N_A = \dfrac{W\sin 15^\circ}{\cos 15^\circ - (0.20)(1+\sin 15^\circ)} = 0.36241\ W$

$\xrightarrow{+} \Sigma F_x = 0: \ -F_B - W\sin\theta - F_A\sin\theta + N_A\cos\theta = 0$

$F_B = N_A\cos\theta - \mu_s N_A\sin\theta - W\sin\theta$

$F_B = 0.36241W\cos 15^\circ - 0.20(0.36241W)\sin 15^\circ - W\sin 15^\circ$

$F_B = 0.072482W$

$+\uparrow \Sigma F_y = 0: \ N_B - W\cos\theta - F_A\cos\theta - N_A\sin\theta = 0$

$N_B = N_A\sin\theta + \mu_s N_A\cos\theta + W\cos\theta$

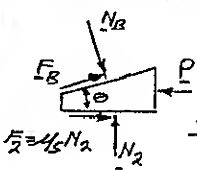
$N_B = (0.36241W)\sin 15^\circ + 0.20(0.36241W)\cos 15^\circ + W\cos 15^\circ$

$N_B = 1.12974\ W$

MAX. AVAILABLE $F_B = \mu_s N_B = 0.22595W$

WE NOTE THAT $F_B < F_{max}$ ∴ NO SLIP AT B ◀

FREE BODY: WEDGE



$+\uparrow \Sigma F_y = 0: \ N_2 - N_B\sin\theta + F_B\sin\theta = 0$

$N_2 = N_B\cos\theta - F_B\sin\theta$

$N_2 = (1.12974W)\cos 15^\circ - (0.07248W)\sin 15^\circ$

$N_2 = 1.07249\ W$

$\xrightarrow{+} \Sigma F_x = 0: \ F_B\cos\theta + N_B\sin\theta + \mu_s N_2 - P = 0$

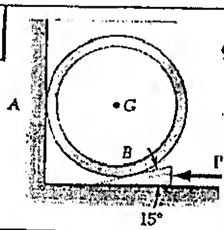
$P = F_B\cos\theta + N_B\sin\theta + \mu_s N_2$

$P = (0.07248W)\cos 15^\circ + (1.12974W)\sin 15^\circ + 0.2(1.07249W)$

$P = 0.5769\ W$

$W = mg: \quad P = 0.5769(50\ kg)(9.81\ m/s^2) \qquad P = 283N \leftarrow$ ◀

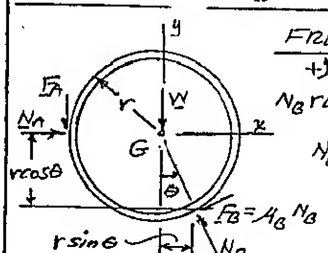
8.78



GIVEN: MASS OF PIPE = 50 kg

$\mu_s = 0.20$

FIND: LARGEST μ_A AT A FOR WHICH SLIPPING WILL OCCUR AT A.



FREE BODY: PIPE

$+\circlearrowright \Sigma M_A = 0$

$N_B r\cos\theta - \mu_B N_B r - (\mu_B N_B\sin\theta)r - Wr = 0$

$N_B = \dfrac{W}{\cos\theta - \mu_B(1+\sin\theta)}$

$N_B = \dfrac{W}{\cos 15^\circ - 0.2(1+\sin 15^\circ)}$

$N_B = 1.4002\ W$

$\xrightarrow{+} \Sigma F_x = 0: \ N_A - N_B\sin\theta - \mu_B N_B\cos\theta = 0$

$N_A = N_B(\sin\theta + \mu_B\cos\theta)$

$= (1.4002W)(\sin 15^\circ + 0.2\times\cos 15^\circ)$

$N_A = 0.63292\ W$

(CONTINUED)

8.78 CONTINUED

$+\uparrow \Sigma F_y = 0: \ -F_A - W + N_B\cos\theta - \mu_B N_B\sin\theta = 0$

$F_A = N_B(\cos\theta - \mu_B\sin\theta) - W$

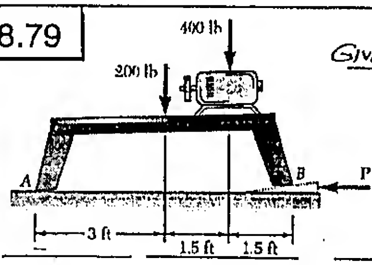
$F_A = (1.4002W)(\cos 15^\circ - 0.2\times\sin\theta) - W$

$F_A = 0.28001W$

FOR SLIPPING AT A: $F_A = \mu_A N_A$

$\mu_A = \dfrac{F_A}{N_A} = \dfrac{0.28001W}{0.63293W} \qquad \mu_A = 0.442$ ◀

8.79

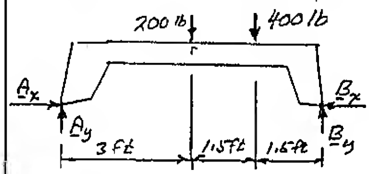


GIVEN: 8° WEDGE

$\mu_s = 0.15$

FIND: (a) FORCE P TO MOVE THE WEDGE.

(b) DOES MACHINE BASE MOVE?



FREE BODY: MACHINE BASE

$+\circlearrowright \Sigma M_B = 0: \ (200\ lb)(3\ ft) + (400\ lb)(1.5\ ft) - A_y(6\ ft) = 0$

$A_y = 200\ lb$

$+\uparrow \Sigma F_y = 0: \ A_y + B_y - 200\ lb - 400\ lb = 0$

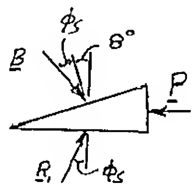
$200\ lb + B_y - 200\ lb - 400\ lb = 0$

$B_y = 400\ lb$

FREE BODY: WEDGE

(ASSUME MACHINE BASE WILL NOT MOVE)

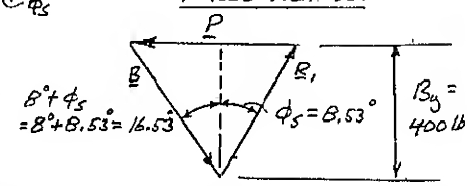
$\mu_s = 0.15, \quad \phi_s = \tan^{-1} 0.15 = 8.53^\circ$



WE KNOW THAT

$B_y = 400\ lb$

FORCE TRIANGLE



$P = (400\ lb)\tan 16.53^\circ + (400\ lb)\tan 8.53^\circ; \quad P = 178.7\ lb$

TOTAL MAXIMUM FRICTION FORCE AT A AND B

$F_m = \mu_s W = 0.15(200\ lb + 400\ lb) = 90\ lb$

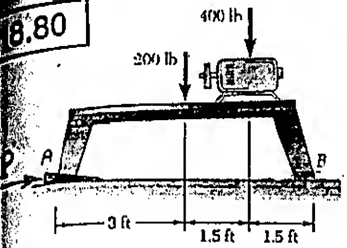
∴ IF MACHINE MOVES WITH WEDGE $P = F_m = 90\ lb$

USING SMALLER P, WE HAVE:

(a) $P = 90\ lb$ ◀

(b) MACHINE BASE MOVES ◀

8.80



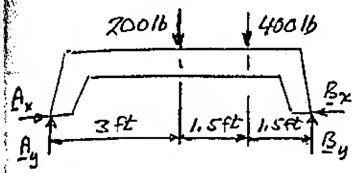
GIVEN: 8° WEDGE

$\mu_s = 0.15$

FIND: (a) FORCE $\underline{P}$ TO MOVE THE WEDGE

(b) DOES MACHINE BASE MOVE?

FREE BODY: MACHINE BASE



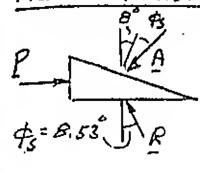
$+\circlearrowleft \Sigma M_B = 0$: $(200\text{ lb})(3\text{ ft}) + (400\text{ lb})(1.5\text{ ft}) - A_y(6\text{ ft}) = 0$

$A_y = 200\text{ lb}\uparrow$

$+\uparrow \Sigma F_y = 0$: $A_y + B_y - 200\text{ lb} - 400\text{ lb} = 0$

$B_y = 400\text{ lb}\uparrow$

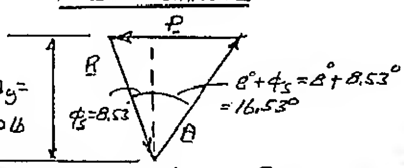
FREE BODY: WEDGE



$\phi_s = \tan^{-1} 0.15 = 8.53^\circ$

WE KNOW THAT $A_y = 200\text{ lb}$

FORCE TRIANGLE



$8^\circ + \phi_s = 8^\circ + 8.53^\circ = 16.53^\circ$

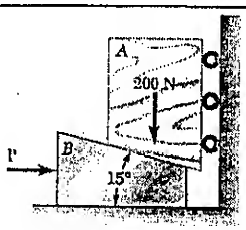
$P = (200\text{ lb})\tan 8.53^\circ + (200\text{ lb})\tan 16.53^\circ$; $\quad P = 89.4\text{ lb}$ ◀

TOTAL MAX. FRICTION FORCE AT A AND B:

$F_m = \mu_s(W) = 0.15(200\text{ lb} + 400\text{ lb}) = 90\text{ lb}$

SINCE $P < F_m$, MACHINE BASE WILL NOT MOVE ◀

* 8.81 and 8.82



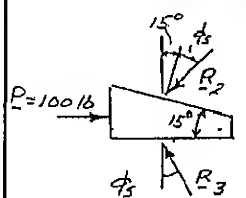
GIVEN: $P = 100\text{ lb}$

FIND: VALUE OF μ_s FOR IMPENDING MOTION

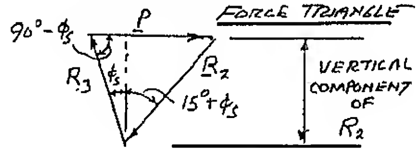
PROB. 8.81: FOR SYSTEM SHOWN

PROB. 8.82: AFTER ROLLERS ARE REMOVED

FREE BODY: WEDGE



FORCE TRIANGLE



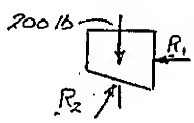
LAW OF SINES

$$\frac{R_2}{\sin(90^\circ - \phi_s)} = \frac{P}{\sin(15^\circ + 2\phi_s)}$$

$$R_2 = P\,\frac{\sin(90^\circ - \phi_s)}{\sin(15^\circ + 2\phi_s)} \qquad (1)$$

PROB 8.81

FREE BODY: BLOCK



$\Sigma F_y = 0$

VERT. COMPONENT OF R_2 IS 200 lb

RETURN TO FORCE TRIANGLE OF WEDGE. NOTE $P = 100\text{ lb}$

$100\text{ lb} = (200\text{ lb})\tan 15^\circ + (200\text{ lb})\tan(15^\circ + \phi_s)$

$0.5 = \tan 15^\circ + \tan(15^\circ + \phi_s)$

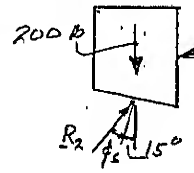
SOLVE BY TRIAL AND ERROR $\quad \phi_s = 6.301^\circ$

$\mu_s = \tan\phi_s = \tan 6.301^\circ$; $\quad \mu_s = 0.110$ ◀

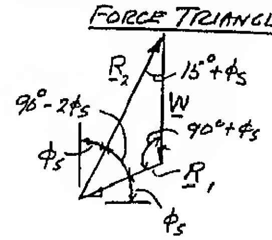
(CONTINUED)

* 8.81 and 8.82 CONTINUED

PROB 8.82: FREE BODY: BLOCK (ROLLERS REMOVED)



FORCE TRIANGLE



LAW OF SINES

$$\frac{R_2}{\sin(90^\circ + \phi_s)} = \frac{W}{\sin(90^\circ - 2\phi_s)}$$

$$R_2 = W\,\frac{\sin(90^\circ + \phi_s)}{\sin(90^\circ - 2\phi_s)} \qquad (2)$$

EQUATE R_2 FROM EQ.(1) AND EQ.(2):

$$P\,\frac{\sin(90^\circ - \phi_s)}{\sin(15^\circ + 2\phi_s)} = W\,\frac{\sin(90^\circ + \phi_s)}{\sin(90^\circ - 2\phi_s)}$$

$P = 100\text{ lb}$; $W = 200\text{ lb}$: $\quad 0.5 = \dfrac{\sin(90^\circ + \phi_s)\sin(15^\circ + 2\phi_s)}{\sin(90^\circ - 2\phi_s)\sin(90^\circ - \phi_s)}$

SOLVE BY TRIAL AND ERROR: $\phi_s = 5.784^\circ$

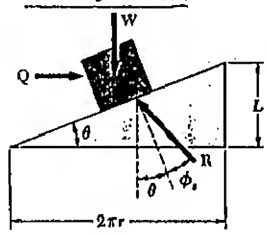
$\mu_s = \tan\phi_s = \tan 5.784^\circ \qquad \mu_s = 0.101$ ◀

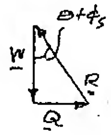
8.83

FOR THE JACK OF SEC. 8.6 (page 418) DERIVE FORMULAS FOR FORCE P FOR CASES LISTED BELOW

FROM SEC. 8.6: $P = \frac{r}{a}Q$

(a) TO RAISE LOAD



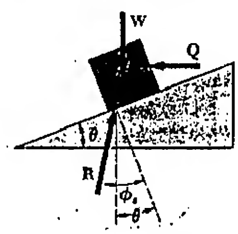


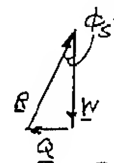
$Q = W\tan(\theta + \phi_s)$

$P = \frac{r}{a}Q$

$P = \frac{Wr}{a}\tan(\theta + \phi_s)$ ◀

(b) TO LOWER LOAD



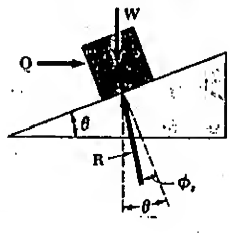


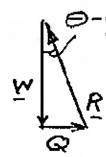
$Q = W\tan(\phi_s - \theta)$

$P = \frac{r}{a}Q$

$P = \frac{Wr}{a}\tan(\phi_s - \theta)$ ◀

(c) TO HOLD LOAD (JACK IS NOT SELF LOCKING)



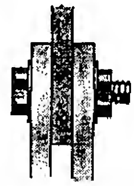


$Q = W\tan(\theta - \phi_s)$

$P = \frac{r}{a}Q$

$P = \frac{Wr}{a}\tan(\theta - \phi_s)$ ◀

8.84



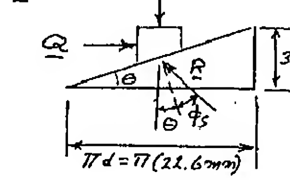
GIVEN: THREAD DIAMETER = 22.6 mm

LEAD = 3 mm, $\mu_s = 0.40$,

TENSION = 210 kN

FIND: REQUIRED TORQUE

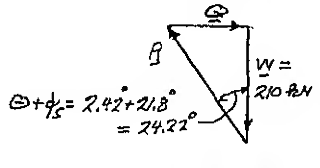
BLOCK-AND-INCLINE ANALYSIS OF BOLT AND NUT:



$\tan\theta = \frac{3\ mm}{\pi(22.6\ mm)}$

$\theta = 2.42°$

$\phi_s = \tan^{-1} 0.40 = 21.8$



$Q = (210\ kN)\tan 24.22°$

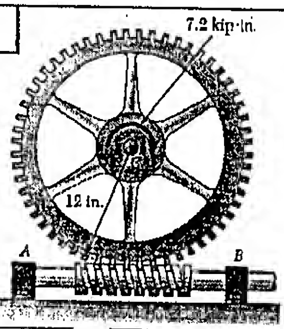
$Q = 94.47\ kN$

TORQUE $= Qr$

$= (94.47\ kN)\left(\frac{22.6 \times 10^{-3}\ m}{2}\right)$

TORQUE $= 1068\ N\cdot m$

8.85

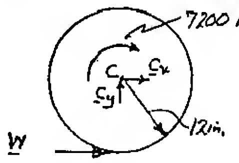


GIVEN: MEAN RADIUS = 1.5 in.,

LEAD = 0.375 in.,

$\mu_s = 0.12$,

FIND: TORQUE APPLIED TO SHAFT REQUIRED TO ROTATE GEAR COUNTERCLOCKWISE.

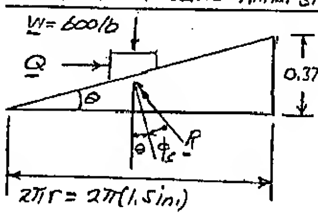


FREE BODY: LARGE GEAR

$+\circlearrowleft \Sigma M_C = 0$: $W(12\ in.) - 7200\ lb\cdot in. = 0$

$W = 600\ lb$

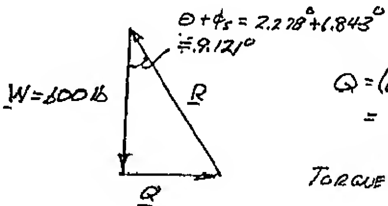
BLOCK-AND-INCLINE ANALYSIS OF WORM GEAR



$\tan\theta = \frac{0.375\ in.}{2\pi(1.5\ in.)}$

$\theta = 2.278°$

$\phi_s = \tan^{-1} 0.12 = 6.843°$



$Q = (600\ lb)\tan 9.121°$

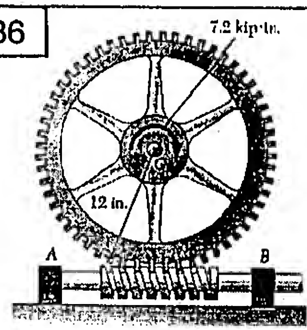
$= 96.33\ lb$

TORQUE $= Qr$

$= (96.33\ lb)(1.5\ in.)$

TORQUE $= 144.5\ lb\cdot in.$

8.86

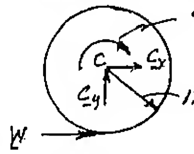


GIVEN: MEAN RADIUS = 1.5 in.

LEAD = 0.375 in.

$\mu_s = 0.12$.

FIND: TORQUE APPLIED TO SHAFT REQUIRED TO ROTATE THE GEAR CLOCKWISE.

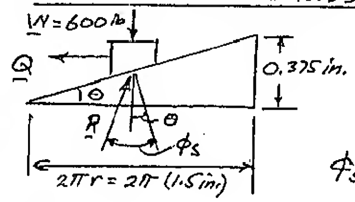


FREE BODY: LARGE GEAR

$+\circlearrowright \Sigma M_C = 0$: $W(12\ in.) - 7200\ lb\cdot in. = 0$

$W = 600\ lb$

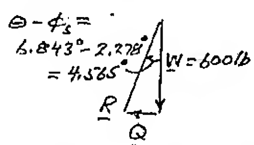
BLOCK-AND-INCLINE ANALYSIS OF WORM GEAR



$\tan\theta = \frac{0.375\ in.}{2\pi(1.5\ in.)}$

$\theta = 2.278$

$\phi_s = \tan^{-1} 0.12 = 6.843°$



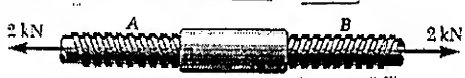
$Q = (600\ lb)\tan 4.565°$

$= 47.91\ lb$

TORQUE $= Qr = (47.91\ lb)(1.5\ in.)$

TORQUE $= 71.9\ lb\cdot in.$

8.87 and 8.88



GIVEN: MEAN RADIUS = 6 mm,

PITCH = 2 mm,

$\mu_s = 0.12$.

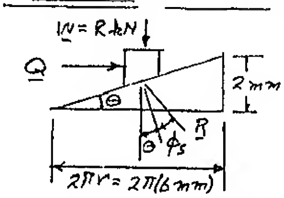
FIND: COUPLE REQUIRED TO ROTATE SLEEVE

PROB 8.87: ROD A, RIGHT-HANDED THREAD, ROD B, LEFT-HANDED

PROB 8.88: RIGHT-HANDED THREAD AT BOTH A AND B

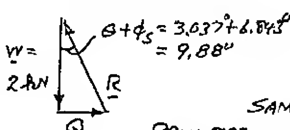
TO DRAW RODS TOGETHER.

SCREW AT A



$\tan\theta = \frac{2\ mm}{2\pi(6\ mm)}$ $\theta = 3.037°$

$\phi_s = \tan^{-1} 0.12 = 6.843°$



$Q = (2\ kN)\tan 9.88° = 348.3\ N$

TORQUE AT A $= Qr$

$= (348.3\ N)(6\ mm) = 2.09\ N\cdot m$

SAME TORQUE REQUIRED AT B

PROB 8.87: TOTAL TORQUE $= 4.18\ N\cdot m$

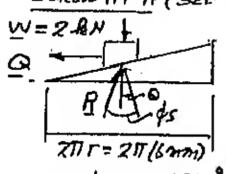
$\therefore$ FOR BOTH THREADS RIGHT HANDED (RODS DO NOT MOVE)

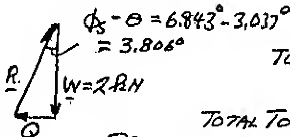
SCREW AT A (SEE ABOVE) TORQUE AT A $= 2.09\ N\cdot m$

SCREW AT B (LOOSENING)

SEE ABOVE: $\theta = 3.037°$

$\phi_s = 6.843°$





$Q = (2\ kN)\tan 3.806° = 133.1\ N$

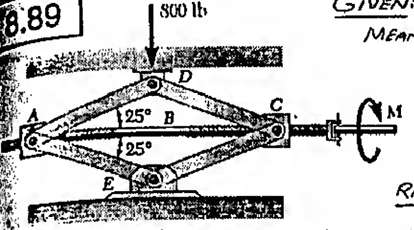
TORQUE AT B $= Qr$

$= (133.1\ N)(6\ mm) = 0.798\ N\cdot m$

TOTAL TORQUE $= 2.09\ N\cdot m + 0.798\ N\cdot m$

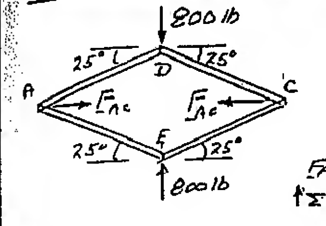
PROB. 8.88: TOTAL TORQUE $= 2.89\ N\cdot m$

8.89



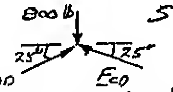
GIVEN: PITCH = 0.1 in., MEAN DIAMETER = 0.375 in., $\mu_s = 0.15$

FIND: COUPLE M REQUIRED TO RAISE AUTOMOBILE.



FREE BODY: PARTS A, D, C, E
TWO-FORCE MEMBERS

JOINT D:

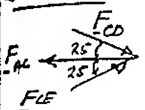


SYMMETRY: $F_{AD} = F_{CD}$

$+\uparrow \Sigma F_y = 0$: $2F_{CD}\sin 25° - 800\text{ lb} = 0$

$F_{CD} = 946.5\text{ lb}$

JOINT C:

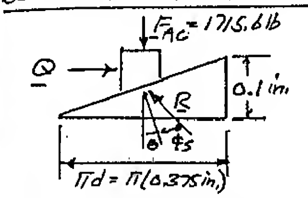


SYMMETRY: $F_{CE} = F_{CD}$

$\overset{+}{\rightarrow} \Sigma F_x = 0$: $2F_{CD}\cos 25° - F_{AC} = 0$

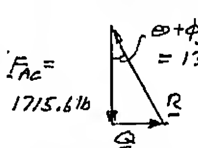
$F_{AC} = 2(946.5\text{ lb})\cos 25°$; $F_{AC} = 1715.6\text{ lb}$

BLOCK-AND-INCLINE ANALYSIS OF ONE SCREW:



$\tan\theta = \dfrac{0.1\text{ in}}{\pi(0.375\text{ in.})}$; $\theta = 4.852°$

$\phi_s = \tan^{-1} 0.15 = 8.531°$



$\theta + \phi_s = 4.852° + 8.531° = 13.383°$

$Q = (1715.6\text{ lb})\tan 13.383°$

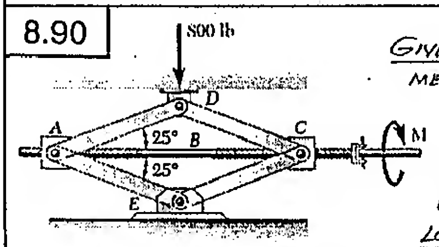
$Q = 408.2\text{ lb}$

BUT, WE HAVE TWO SCREWS

TORQUE $= 2Qr = 2(408.2\text{ lb})\left(\dfrac{0.375\text{ in.}}{2}\right)$

TORQUE $= 153.1\text{ lb}\cdot\text{in.}$ ◀

8.90

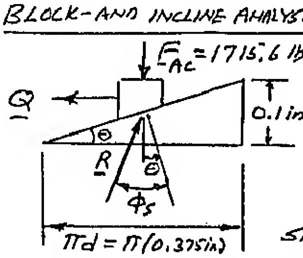


GIVEN: PITCH = 0.1 in., MEAN DIAMETER = 0.375 in., $\mu_s = 0.15$.

FIND: COUPLE M REQUIRED TO LOWER AUTOMOBILE.

SEE SOLUTION OF PROB 8.89 FOR ANALYSIS OF LINKAGE ADCE, $F_{AC} = 1715.6\text{ lb}$

BLOCK-AND INCLINE ANALYSIS OF ONE SCREW:

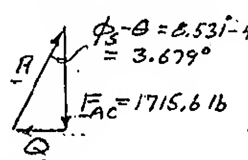


$\tan\theta = \dfrac{0.1\text{ in.}}{\pi(0.375\text{ in})}$

$\theta = 4.852°$

$\phi_s = \tan^{-1} 0.15 = 8.531°$

SINCE $\phi_s > \theta$, THE SCREW IS SELF LOCKING



$\phi_s - \theta = 8.531° - 4.852° = 3.679°$

$Q = (1715.6\text{ lb})\tan 3.679°$

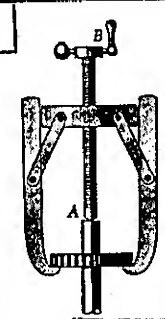
$Q = 110.3\text{ lb}$

FOR TWO SCREWS:

TORQUE $= 2(110.3\text{ lb})\frac{1}{2}(0.375\text{ in})$

TORQUE $= 41.4\text{ lb}\cdot\text{in.}$ ◀

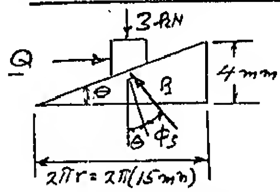
8.91



GIVEN: LEAD = 4 mm, MEAN RADIUS = 15 mm, $\mu_s = 0.10$, FORCE TO BE APPLIED TO GEAR = 3 kN.

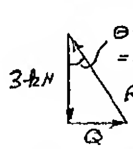
FIND: TORQUE THAT MUST BE APPLIED TO SCREW.

BLOCK-AND-INCLINE ANALYSIS OF SCREW



$\tan\theta = \dfrac{4\text{ mm}}{2\pi(15\text{ mm})}$ $\quad \theta = 2.43°$

$\phi_s = \tan^{-1} 0.10 = 5.71°$



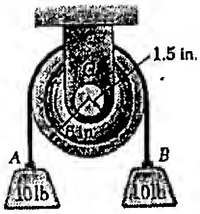
$\theta + \phi_s = 2.43° + 5.71° = 8.14°$

$Q = (3\text{ kN})\tan 8.14° = 429\text{ N}$

TORQUE $= Qr = (429\text{ N})(0.015\text{ m})$

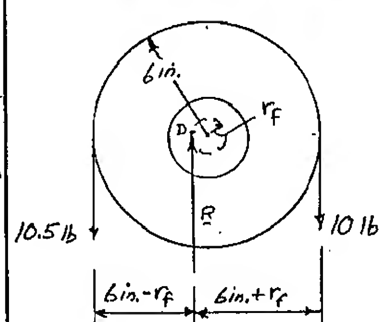
TORQUE $= 6.44\text{ N}\cdot\text{m}$ ◀

8.92



GIVEN: PULLEY WEIGHS 5 lb.

FIND: COEFFICIENT OF STATIC FRICTION IF A 0.5-lb WEIGHT ADDED TO BLOCK A STARTS ROTATION.



$+\circlearrowleft \Sigma M_D = 0$: $(10.5\text{ lb})(6\text{ in.} - r_f) - (10\text{ lb})(6\text{ in.} + r_f) = 0$

$(0.5\text{ lb})(6\text{ in.}) = (20.5\text{ lb})r_f$

$r_f = 0.14634\text{ in.}$

$r_f = r\sin\phi_s$

$\sin\phi_s = \dfrac{0.14634\text{ in.}}{1.5\text{ in}} = 0.09756$

$\phi_s = 5.5987°$

$\mu_s = \tan\phi_s = \tan 5.5987°$

$\mu_s = 0.09803$

$\mu_s = 0.098$ ◀

8.93 through 8.96

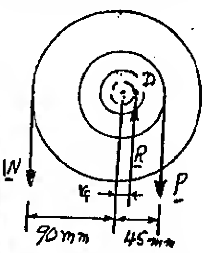
FOR EACH PULLEY: RADIUS OF SHAFT; $r = 10\,mm$

$\mu_s = 0.40, \quad \phi_s = \tan^{-1} 0.40 = 21.8°$

$r_f = r\sin\phi_s \approx r\mu_s = (10\,mm)\,0.40 \doteq 4\,mm$

$W = mg = (20\,kg)(9.81\,m/s^2) = 196.2\,N$

PROB. 8.93: FIND P REQUIRED TO START RAISING LOAD

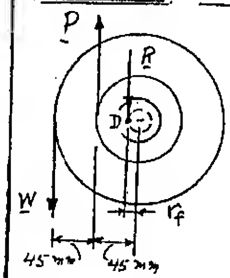


$+\circlearrowright \Sigma M_D = 0: \; P(45 - r_f) - W(90 + r_f) = 0$

$P = W\dfrac{90 + r_f}{45 - r_f} = (196.2\,N)\dfrac{90\,mm + 4\,mm}{45\,mm - 4\,mm}$

$P = 449.8\,N \qquad P = 450\,N \downarrow$ ◀

PROB. 8.94 FIND P REQUIRED TO START RAISING LOAD

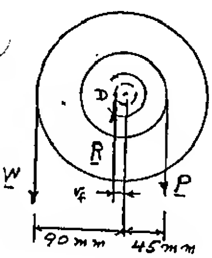


$+\circlearrowright \Sigma M_D = 0: \; P(45 - r_f) - W(90 - r_f) = 0$

$P = W\dfrac{90 - r_f}{45 - r_f} = (196.2\,N)\dfrac{90\,mm - 4\,mm}{45\,mm - 4\,mm}$

$P = 411.54\,N \qquad P = 412\,N \uparrow$ ◀

PROB 8.95 FIND SMALLEST P TO MAINTAIN EQUILIBRIUM

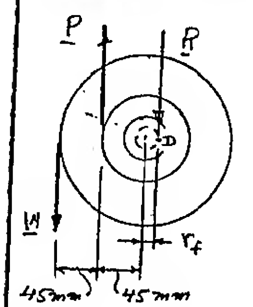


$+\circlearrowright \Sigma M_D = 0: \; P(45 + r_f) - W(90 - r_f) = 0$

$P = W\dfrac{90 - r_f}{45 + r_f} = (196.2\,N)\dfrac{90\,mm - 4\,mm}{45\,mm + 4\,mm}$

$P = 344.3\,N \qquad P = 344\,N \downarrow$ ◀

PROB. 8.96 FIND SMALLEST P TO MAINTAIN EQUILIBRIUM

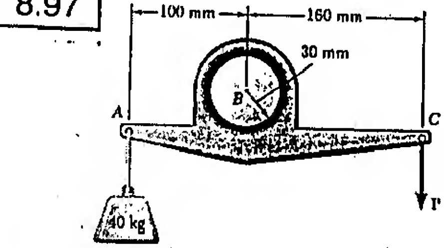


$+\circlearrowright \Sigma M_D = 0: \; P(45 + r_f) - W(90 + r_f) = 0$

$P = W\dfrac{90 + r_f}{45 + r_f} = (196.2\,N)\dfrac{90\,mm + 4\,mm}{45\,mm + 4\,mm}$

$P = 376.4\,N \qquad P = 376\,N \uparrow$ ◀

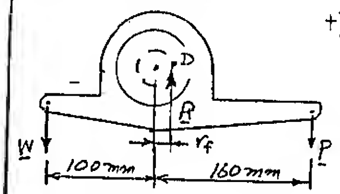
8.97



(a) LEVER MOVES $\circlearrowright$ FOR $P = 275\,N$, FIND μ_s.
(b) FIND SMALLEST P TO PREVENT ROTATION.

(a) IMPENDING MOTION $\circlearrowright$ $\quad r = 30\,mm$

$W = (40\,kg)(9.81\,m/s^2) = 392.4\,N$



$+\circlearrowright \Sigma M_D = 0: \; P(160 - r_f) - W(100 + r_f) = 0$

$r_f = \dfrac{160P - 100W}{P + W}$

$r_f = \dfrac{(160\,mm)(275\,N) - (100\,mm)(392.4\,N)}{275\,N + 392.4\,N}$

$r_f = 7.132\,mm$

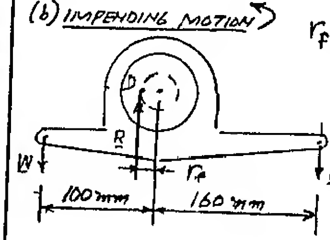
$r_f = r\sin\phi_s \approx r\mu_s \qquad \mu_s = \dfrac{r_f}{r} = \dfrac{7.132\,mm}{30\,mm} = 0.2377$

$\mu_s = 0.24$ ◀

(b) IMPENDING MOTION $\circlearrowleft$

$r_f = r\sin\phi_s \approx r\mu_s = (30\,mm)(0.2377)$

$r_f = 7.132\,mm$



$+\circlearrowright \Sigma M_D = 0:$

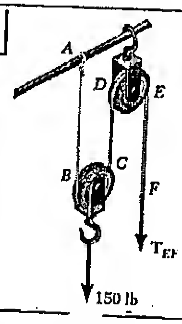
$P(160 + r_f) - W(100 - r_f) = 0$

$P = W\dfrac{100 - r_f}{160 + r_f}$

$P = (392.4\,N)\dfrac{100\,mm - 7.132\,mm}{160\,mm + 7.132\,mm}$

$P = 218.04\,N \qquad P = 218\,N \downarrow$ ◀

8.98



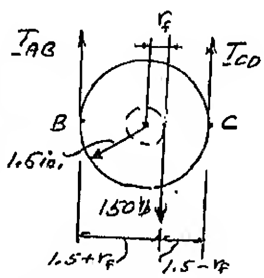
GIVEN: 3-in.-DIAMETER PULLEYS ON 0.5-in.-DIAMETER AXLES.

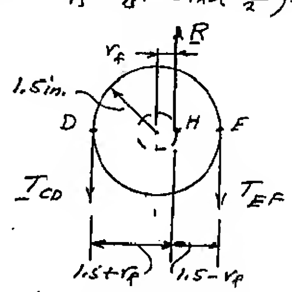
$\mu_s = 0.20$

FIND: TENSION IN EACH PORTION OF ROPE AS LOAD IS LOWERED.

FOR EACH PULLEY: AXLE DIAMETER = 0.5 in

$r_f = r\sin\phi_s \approx \mu_s r = 0.20\left(\frac{0.5\,in.}{2}\right) = 0.05\,in.$





PULLEY BC: $+\circlearrowright \Sigma M_B = 0: \; T_{CD}(3\,in.) - (150\,lb)(1.5\,in. + r_f) = 0$

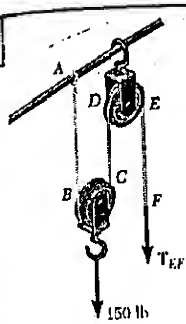
$T_{CD} = \frac{1}{3}(150\,lb)(1.5\,in. + 0.05\,in.); \qquad T_{CD} = 77.5\,lb$ ◀

$+\uparrow \Sigma F_y = 0: \; T_{AB} + 77.5\,lb - 150\,lb = 0; \qquad T_{AB} = 72.5\,lb$ ◀

PULLEY DE: $+\circlearrowright \Sigma M_H = 0: \; T_{CD}(1.5 + r_f) - T_{EF}(1.5 - r_f) = 0$

$T_{EF} = T_{CD}\dfrac{1.5 + r_f}{1.5 - r_f} = (77.5\,lb)\dfrac{1.5\,in. + 0.05\,in.}{1.5\,in. - 0.05\,in.}; \qquad T_{EF} = 82.8\,lb$ ◀

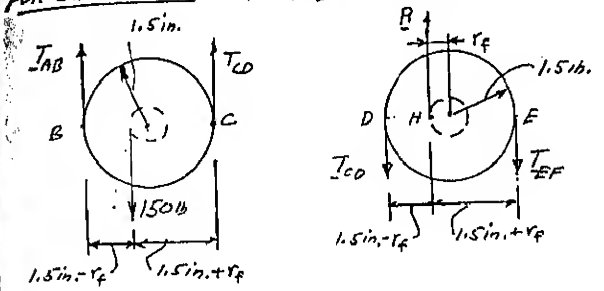
8.99



GIVEN: 3-in.-DIAMETER PULLEYS ON 0.5-in.-DIAMETER AXLES, $\mu_s = 0.20$.

FIND: TENSION IN EACH PORTION OF ROPE AS LOAD IS LOWERED.

FOR EACH PULLEY: $r_f = r\mu_s = \left(\frac{0.5\text{ in.}}{2}\right)0.2 = 0.05\text{ in.}$



PULLEY BC: $+\circlearrowleft \Sigma M_B = 0$: $T_{CD}(3\text{ in.}) - (150\text{ lb})(1.5\text{ in.} - r_f) = 0$

$$T_{CD} = \frac{(150\text{ lb})(1.5\text{ in.} - 0.05\text{ in.})}{3\text{ in.}} \qquad T_{CD} = 72.5\text{ lb}$$

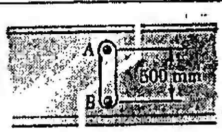
$+\uparrow \Sigma F_y = 0$: $T_{AB} + 72.5\text{ lb} - 150\text{ lb} = 0$ $\qquad T_{AB} = 77.5\text{ lb}$

PULLEY DE:

$$T_{CD}(1.5\text{ in.} - r_f) - T_{EF}(1.5\text{ in.} + r_f) = 0$$

$$T_{EF} = T_{CD}\frac{1.5\text{ in.} - r_f}{1.5\text{ in.} + r_f} = (72.5\text{ lb})\frac{1.5\text{ in.} - 0.5\text{ in.}}{1.5\text{ in.} + 0.5\text{ in.}} \qquad T_{EF} = 67.8\text{ lb}$$

8.100

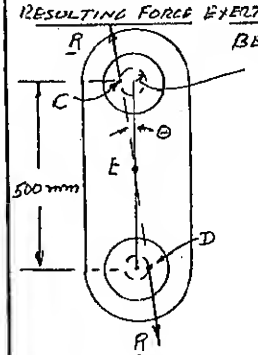


GIVEN: 60-mm-DIAMETER PINS AT A AND B, $\mu_s = 0.20$, LOAD = 200 kN

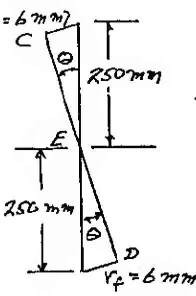
FIND: (a) HORIZ. FORCE AT C REQUIRED TO JUST MOVE THE LINK. (b) ANGLE THAT RESULTING FORCE EXERTED ON LINK WILL FORM WITH VERTICAL.

BEARING: $r = 30\text{ mm}$

$r_f = \mu_s r = 0.20(30\text{ mm}) = 6\text{ mm}$



RESULTANT FORCES $\underline{R}$ MUST BE TANGENT TO FRICTION CIRCLES AT POINTS C AND D.

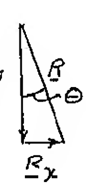


$$\sin\theta = \frac{6\text{ mm}}{250\text{ mm}}$$

$$\sin\theta = 0.024$$

$$\theta = 1.375°$$

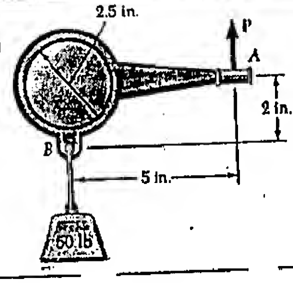
R_y = VERT. COMPONENT = 200 kN



$$R_x = R_y \tan\theta = (200\text{ kN})\tan 1.375° = 4.80\text{ kN}$$

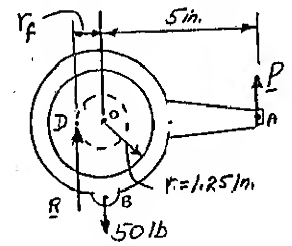
HORIZ. FORCE = 4.80 kN

8.101



GIVEN: $\mu_s = 0.15$,

FIND: FORCE P REQUIRED TO START COUNTERCLOCKWISE ROTATION.



$r_f = \mu_s r = 0.15(1.25\text{ in.}) = 0.1875\text{ in}$

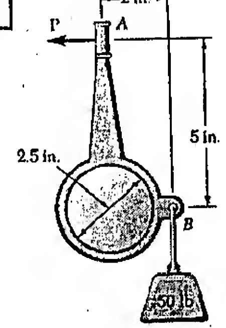
$+\circlearrowleft \Sigma M_D = 0$

$P(5\text{ in.} + r_f) - (50\text{ lb})r_f = 0$

$$P = \frac{50(0.1875)}{5.1875} = 1.807\text{ lb}$$

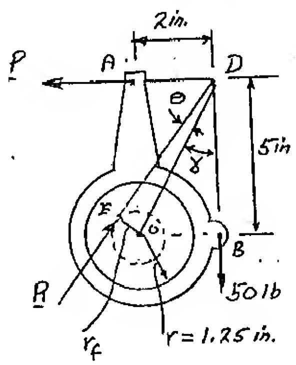
$$\underline{P} = 1.807\text{ lb}\uparrow$$

8.102



GIVEN: $\mu_s = 0.15$.

FIND: FORCE P REQUIRED TO START COUNTERCLOCKWISE ROTATION



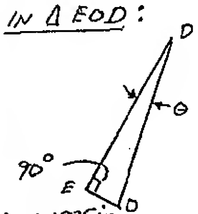
$r_f = \mu_s r = 0.15(1.25\text{ in})$

$r_f = 0.1875\text{ in.}$

$$\tan\gamma = \frac{2\text{ in.}}{5\text{ in.}}$$

$$\gamma = 21.801°$$

IN Δ EOD:

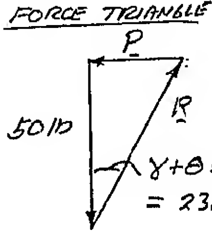


$$OD = \sqrt{(2\text{ in})^2 + (5\text{ in})^2} = 5.3852\text{ in.}$$

$$\sin\theta = \frac{OE}{OD} = \frac{r_f}{OD} = \frac{0.1875\text{ in.}}{5.3852\text{ in.}}$$

$$\theta = 1.995°$$

FORCE TRIANGLE

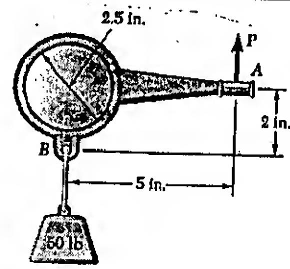


$\gamma + \theta = 21.801° + 1.995° = 23.796°$

$$P = (50\text{ lb})\tan(\gamma + \theta) = (50\text{ lb})\tan 23.796° = 22.049\text{ lb}$$

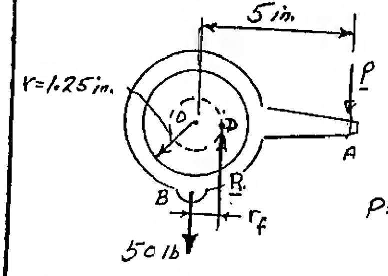
$$\underline{P} = 22.0\text{ lb} \leftarrow$$

8.103



GIVEN: $\mu_s = 0.15$,

FIND: FORCE P REQUIRED TO START CLOCKWISE ROTATION.



$r_f = \mu_s r = 0.15(1.25\text{ in})$

$r_f = 0.1875\text{ in.}$

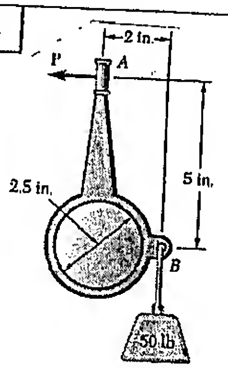
$+\circlearrowright \Sigma M_D = 0$

$P(5\text{ in.} - r_f) - (50\text{ lb})r_f = 0$

$P = \dfrac{50(0.1875)}{5 - 0.1875} = 1.948\text{ lb}$

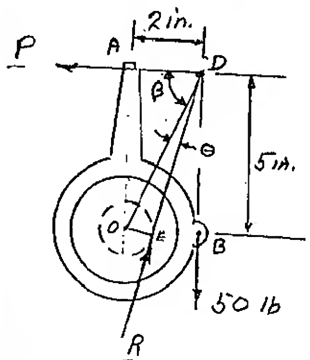
$\underline{P} = 1.948\text{ lb} \downarrow$ ◀

8.104



GIVEN: $\mu_s = 0.15$.

FIND: FORCE P REQUIRED TO START CLOCKWISE ROTATION



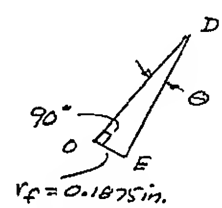
$r_f = \mu_s r = 0.15(1.25\text{ in})$

$= 0.1875\text{ in.}$

$\tan\beta = \dfrac{5\text{ in.}}{2\text{ in.}}$

$\beta = 68.198°$

IN Δ EOD:



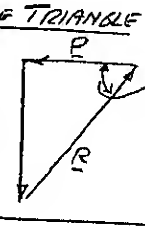
$OD = \sqrt{(2\text{ in.})^2 + (5\text{ in.})^2}$

$OD = 5.3852\text{ in.}$

$\sin\theta = \dfrac{OE}{OD} = \dfrac{0.1875\text{ in.}}{5.3852\text{ in.}}$

$\theta = 1.994°$

FORCE TRIANGLE



$\beta + \theta = 68.198° + 1.994°$

$= 70.193°$

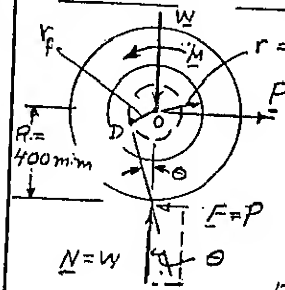
$P = 50/\tan(\beta+\theta) = \dfrac{50\text{ lb}}{\tan 70.192°}$

$\underline{P} = 18.01\text{ lb} \leftarrow$ ◀

8.105

GIVEN: RAILROAD CAR OF MASS 30 Mg ON EIGHT 800-mm-DIAMETER WHEEL WITH 125-mm-DIAMETER AXLES. $\mu_s = 0.020$, $\mu_k = 0.015$.

FIND: HORIZONTAL FORCE REQUIRED (a) TO START CAR MOVING (b) TO KEEP IT MOVING.



$r_f = \mu r$; $R = 400$ mm

$\sin\theta = \tan\theta = \dfrac{r_f}{R} = \dfrac{\mu r}{R}$

$P = W\tan\theta = W\dfrac{\mu r}{R}$

$P = W\mu\dfrac{62.5\text{ mm}}{400\text{ mm}} = 0.15625 W\mu$

FOR ONE WHEEL:

$W = \frac{1}{8}(30\text{ Mg})(9.81\text{ m/s}^2) = \frac{1}{8}(294.3\text{ kN})$

FOR EIGHT WHEELS OF RAILROAD CAR

$\Sigma P = 8(0.15625)\frac{1}{8}(294.3\text{ kN})\mu = (45.984\mu)\text{ kN}$

(a) TO START MOTION: $\mu_s = 0.020$

$\Sigma P = (45.984)(0.020) = 0.9197\text{ kN}$; $\Sigma P = 920\text{ N}$ ◀

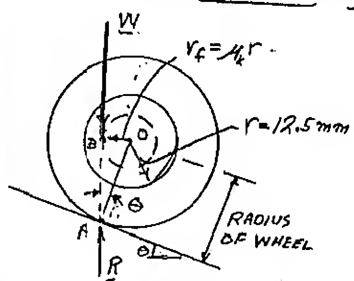
(b) TO MAINTAIN MOTION: $\mu_k = 0.015$

$\Sigma P = (45.984)(0.015) = 0.6897\text{ kN}$, $\Sigma P = 690\text{ N}$ ◀

8.106

GIVEN: SCOOTER IS TO ROLL DOWN A 2 PERCENT SLOPE AT CONSTANT SPEED. AXLES OF WHEELS ARE 25 mm IN DIAMETER, $\mu_k = 0.10$.

FIND: REQUIRED DIAMETER OF WHEELS.



$\tan\theta = \dfrac{2}{100} = 0.02$

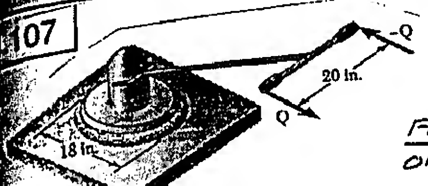
SINCE SCOOTER ROLLS AT CONSTANT SPEED, EACH WHEEL IS IN EQUILIBRIUM. THUS W AND R MUST HAVE COMMON LINE OF ACTION TANGENT TO THE FRICTION CIRCLE.

$r_f = \mu_k r = (0.10)(12.5\text{ mm}) = 1.25\text{ mm}$

$OA = \dfrac{OB}{\tan\theta} = \dfrac{r_f}{\tan\theta} = \dfrac{1.25\text{ mm}}{0.02} = 62.5\text{ mm}$

DIAMETER OF WHEEL $= 2(OA) = 125$ mm ◀

8.107



GIVEN: $\mu_k = 0.25$
50-lb FLOOR POLISHER
FIND: MAGNITUDE OF FORCES $\underline{Q}$

SEE Fig. 8.12 (page 343) AND EQ. 8.9 (page 344)
USING: $R = 9$ in., $P = 50$ lb, AND $\mu_k = 0.25$

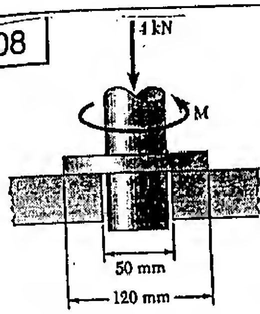
$$M = \frac{2}{3}\mu_k PR = \frac{2}{3}(0.25)(50\text{ lb})(9\text{ in}) = 75\text{ lb}\cdot\text{in.}$$

$\Sigma M_y = 0$ YIELDS: $M = Q(20\text{ in.})$

$$75\text{ lb}\cdot\text{in.} = Q(20\text{ in.})$$

$$Q = 3.75\text{ lb}$$

8.108



GIVEN: COUPLE $\underline{M} = 30$ N·m REQUIRED TO START ROTATION

FIND: μ_s

SEE FIG. 8.12 (page 343) AND EQ. 8.8 (page 344).
USING: $R_1 = 25$ mm $= 0.025$ m
$R_2 = 60$ mm $= 0.060$ m
$P = 4{,}000$ N, $M = 30$ N·m

$$M = \frac{2}{3}\mu_s P \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}.$$

$$30\text{ N}\cdot\text{m} = \frac{2}{3}\mu_s(4000\text{ N})\frac{(0.060\text{ m})^3 - (0.025\text{ m})^3}{(0.060)^2 - (0.025\text{ m})^2}$$

$$30\text{ N}\cdot\text{m} = \frac{2}{3}\mu_s(4000\text{ N})(0.06735\text{ m}); \quad \mu_s = 0.167$$

*** 8.109**

FOR SHAFT AND BEARING ASSUME NORMAL FORCE PER UNIT AREA IS INVERSELY PROPORTIONAL TO r. SHOW THAT M IS 75% OF VALUE GIVEN BY FORMULA (8.9) ON PAGE 344.

USING FIG 8.12 (page 343), WE ASSUME

$$\Delta N = \frac{k}{r}\Delta A: \quad \Delta A = r\,\Delta\theta\,\Delta r$$

$$\Delta N = \frac{k}{r} r\,\Delta\theta\,\Delta r = k\,\Delta\theta\,\Delta r$$

WE WRITE, $P = \Sigma\Delta N$ OR $P = \int dN$

$$P = \int_0^{2\pi}\int_0^R k\,\Delta\theta\,\Delta r = 2\pi R k; \quad k = \frac{P}{2\pi R}$$

$$\Delta N = \frac{P\,\Delta\theta\,\Delta r}{2\pi R}$$

$$\Delta M = r\,\Delta F = r\mu_k\,\Delta N = r\mu_k\frac{P\,\Delta\theta\,\Delta r}{2\pi R}$$

$$M = \int_0^{2\pi}\int_0^R \frac{\mu_k P}{2\pi R}\,r\,dr\,d\theta = \frac{2\pi\mu_k P}{2\pi R}\cdot\frac{R^2}{2} = \frac{1}{2}\mu_k PR$$

FROM EQ(8.9) FOR A NEW BEARING $M_{NEW} = \frac{2}{3}\mu_k PR$

THUS $\frac{M}{M_{NEW}} = \frac{1/2}{2/3} = \frac{3}{4}$ $\qquad M = 0.75 M_{NEW}$

*** 8.110**

ASSUMING BEARING WEAR AS GIVEN IN PROB. 8.109, SHOW THAT MAGNITUDE OF COUPLE TO OVERCOME FRICTION IN A WORN-OUT COLLAR BEARING (SEE Fig 8.12) IS

$$M = \frac{1}{2}\mu_k P(R_1 + R_2)$$

USING Fig 8.12 (page 343), WE ASSUME $\Delta N = \frac{k}{r}\Delta A$

$$\Delta A = r\,\Delta\theta\,\Delta r; \quad \Delta N = \frac{k}{r} r\,\Delta\theta\,\Delta r = k\,\Delta\theta\,\Delta r$$

BUT: $P = \Sigma\Delta N$ OR $P = \int dN$

$$P = \int_0^{2\pi}\int_{R_1}^{R_2} k\,\Delta\theta\,\Delta r = 2\pi(R_2 - R_1)k$$

THUS, $k = \frac{P}{2\pi(R_2 - R_1)}$, AND $\Delta N = \frac{P\,\Delta\theta\,\Delta r}{2\pi(R_2 - R_1)}$

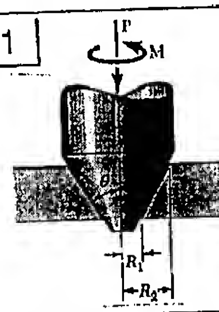
$$\Delta M = r\,\Delta F = r\mu_k\,\Delta N = r\mu_k\frac{P\,\Delta\theta\,\Delta r}{2\pi(R_2 - R_1)}$$

$$M = \int_0^{2\pi}\int_{R_1}^{R_2}\frac{\mu_k P}{2\pi(R_2 - R_1)}\,r\,dr\,d\theta = \frac{2\pi\mu_k P}{2\pi(R_2 - R_1)}\cdot\frac{R_2^2 - R_1^2}{2}$$

SINCE $R_2^2 - R_1^2 = (R_2 - R_1)(R_2 + R_1)$

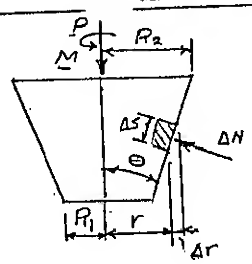
$$M = \frac{1}{2}\mu_k P(R_1 + R_2)$$

*** 8.111**



ASSUME: UNIFORM PRESSURE BETWEEN SURFACES OF CONTACT

SHOW THAT
$$M = \frac{2}{3}\cdot\frac{\mu_k P}{\sin\theta}\cdot\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$



$$\Delta N = k\,\Delta A$$

$$\Delta A = (r\,\Delta\phi)\,\Delta s = (r\,\Delta\phi)\frac{\Delta r}{\sin\theta}$$

THUS:
$$\Delta N = k\,\Delta A = \frac{kr}{\sin\theta}\,\Delta\phi\,\Delta r$$

VERTICAL COMPONENT OF ΔN:
$$(\Delta N)_y = \Delta N\sin\theta = kr\,\Delta\phi\,\Delta r$$

$$P = \Sigma(\Delta N)_y = \Sigma kr\,\Delta\phi\,\Delta r$$

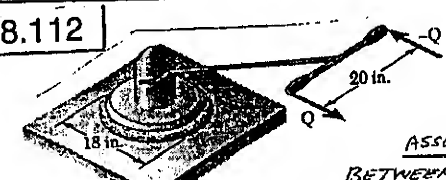
OR, USING INTEGRALS
$$P = \int_0^{2\pi}\int_{R_1}^{R_2} kr\,d\phi\,dr = 2\pi k\frac{R_2^2 - R_1^2}{2}$$

THUS, $k = \frac{P}{\pi(R_2^2 - R_1^2)}$; $\quad \Delta N = \frac{kr}{\sin\theta}\Delta\phi\,\Delta r = \frac{\mu_k P r^2\,\Delta\phi\,\Delta r}{\pi\sin\theta(R_2^2 - R_1^2)}$

INTEGRATING:
$$M = \int_0^{2\pi}\int_{R_1}^{R_2}\frac{\mu_k P r^2\,d\phi\,dr}{\pi\sin\theta(R_2^2 - R_1^2)} = \frac{2\pi}{\pi}\cdot\frac{\mu_k P}{\sin\theta}\cdot\frac{R_2^3 - R_1^3}{3(R_2^2 - R_1^2)}$$

$$M = \frac{2}{3}\cdot\frac{\mu_s P}{\sin\theta}\cdot\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

8.112

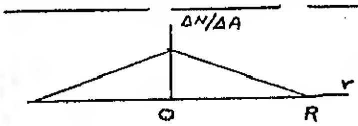


GIVEN: POLISHER OF WEIGHT = 50 lb

$\mu_k = 0.25$

ASSUME: NORMAL FORCE BETWEEN FLOOR AND DISK VARIES LINEARLY FROM A MAXIMUM AT CENTER TO ZERO AT EDGE

FIND: MAGNITUDE Q OF FORCES TO PREVENT MOTION.



$$\frac{\Delta N}{\Delta A} = k\left(1 - \frac{r}{R}\right)$$

$$\Delta A = r\,\Delta\theta\,\Delta r$$

$$\Delta N = k\left(1 - \frac{r}{R}\right) r\,\Delta\theta\,\Delta r$$

$$P = \Sigma\Delta N = \int dN = \int_0^{2\pi}\int_0^{R} k\left(1 - \frac{r}{R}\right) r\,d\theta\,dr = 2\pi k\left[\frac{r^2}{2} - \frac{r^3}{3R}\right]_0^R$$

$$P = k\frac{\pi R^2}{3}$$

THUS: $k = \dfrac{3P}{\pi R^2}$ AND $\Delta N = \dfrac{3P}{\pi R^2}\left(1 - \dfrac{r}{R}\right) r\,\Delta\theta\,\Delta r$

MOMENT OF FRICTION FORCE ON ΔA IS

$$\Delta M = r\,\Delta F = r\mu_k\Delta N = \frac{3P\mu_k}{\pi R^2}\left(1 - \frac{r}{R}\right) r^2\,\Delta\theta\,\Delta r$$

$$M = \Sigma\Delta M = \int dM = \int_0^{2\pi}\int_0^{R} \frac{3P\mu_k}{\pi R^2}\left(r^2 - \frac{r^3}{R}\right) d\theta\,dr = \frac{2\pi}{\pi}\cdot\frac{3P\mu_k}{R^2}\left[\frac{r^3}{3} - \frac{r^4}{4R}\right]_0^R$$

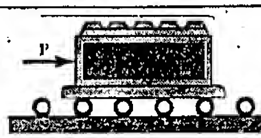
$$M = \tfrac{1}{2}\mu_k P R$$

$\mu_k = 0.25$, $P = 50$ lb, $R = 9$ in

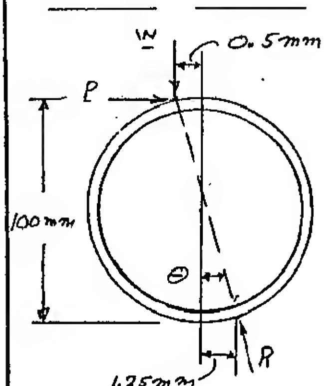
$$M = \tfrac{1}{2}(0.25)(50\text{ lb})(9\text{ in}) = 56.25\text{ lb}\cdot\text{in}$$

BUT: $Q(20\text{ in.}) = M$; $Q = \dfrac{M}{20\text{ in}} = \dfrac{56.25\text{ lb}\cdot\text{in}}{20\text{ in}}$; $Q = 2.81$ lb ◀

8.113



GIVEN: 900-kg BASE; 100-mm DIAMETER PIPES, ROLLING RESISTANCE IS 0.5 m BETWEEN PIPES AND BASE + 1.25 mm BETWEEN PIPES AND CONCRETE FLOOR. FIND: P TO MAINTAIN MOTION



$$\tan\theta = \frac{0.5\text{ mm} + 1.25\text{ mm}}{100\text{ mm}}$$

$$\tan\theta = 0.0175$$

$$P = W\tan\theta$$

$$P = 0.0175\,W$$

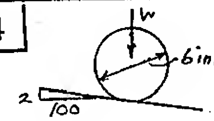
$$W = mg = (900\text{ kg})(9.81\text{ m/s}^2)$$

$$P = (0.0175)(900\text{ kg})(9.81\text{ m/s}^2)$$

$$P = 154.51\text{ N}$$

$P = 154.4$ N ◀

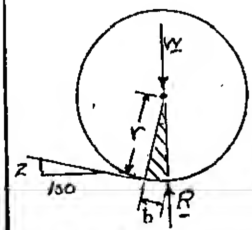
8.114



GIVEN: DISK ROLLS AT CONSTANT VELOCITY

FIND: COEFFICIENT OF ROLLING RESISTANCE

DISK IS IN EQUILIBRIUM



SIMILAR TRIANGLES

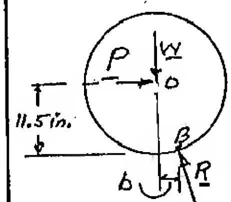
$$\frac{b}{r} = \frac{2}{100}$$

$b = \dfrac{2}{100}r = \dfrac{2}{100}(6\text{ in.})$; $b = 0.060$ in. ◁

8.115

GIVEN: 2500-lb AUTOMOBILE WITH 23-in.-DIAMETER TIRES, COEFFICIENT OF ROLLING RESISTANCE = 0.05 in.

FIND: HORIZONTAL FORCE TO MOVE AUTOMOBILE ON HORIZONTAL ROAD AT CONSTANT SPEED



$+\circlearrowleft\Sigma M_B = 0$:

$$P(11.5\text{ in.}) - Wb = 0$$

$$P(11.5\text{ in.}) = (2500\text{ lb})(0.05\text{ in})$$

$$P = 10.869\text{ lb}$$

$P = 10.87$ lb ◀

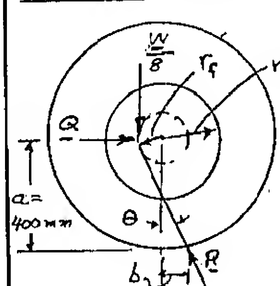
8.116

GIVEN: 30-Mg RAILROAD CAR ON EIGHT 800-mm-DIAMETER WHEELS WITH 125-mm AXLES, $\mu_s = 0.020$, $\mu_k = 0.015$, COEFFICIENT OF ROLLING RESISTANCE 0.5 mm

FIND: HORIZ. FORCE (a) TO START MOTION, (b) TO MAINTAIN MOTION.

$r_f = \mu r$

FOR ONE WHEEL



$$\tan\theta \approx \sin\theta \approx \frac{r_f + b}{a}$$

$$\tan\theta = \frac{\mu r + b}{a}$$

$$Q = \frac{W}{8}\tan\theta = \frac{W}{8}\,\frac{\mu r + b}{a}$$

FOR EIGHT WHEELS OF CAR

$$P = W\frac{\mu r + b}{a}$$

$$W = mg = (30\text{ Mg})(9.81\text{ m/s}^2) = 294.3\text{ kN}$$

$a = 400$ mm, $r = 62.5$ mm, $b = 0.5$ mm

(a) TO START MOTION: $\mu = \mu_s = 0.02$

$$P = (294.3\text{ kN})\frac{(0.020)(62.5\text{ mm}) + 0.5\text{ mm}}{400\text{ mm}}$$

$P = 1.2876$ kN $\qquad$ $P = 1.288$ kN ◀

(b) TO MAINTAIN CONSTANT SPEED $\mu = \mu_k = 0.015$

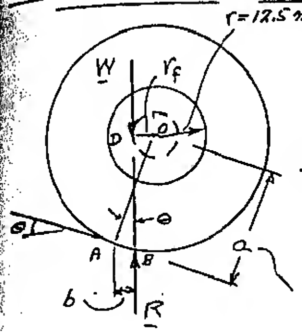
$$P = (294.3\text{ kN})\frac{(0.015)(62.5\text{ mm}) + 0.5\text{ mm}}{400\text{ mm}}$$

$P = 1.0576$ kN $\qquad$ $P = 1.058$ kN ◀

8.117

GIVEN: SCOOTER IS TO ROLL DOWN A 2 PERCENT SLOPE AT CONSTANT SPEED. AXLES OF WHEELS ARE 25 mm IN DIAMETER. $\mu_k = 0.10$, COEFFICIENT OF ROLLING RESISTANCE = 1.75 mm.

FIND: REQUIRED DIAMETER OF WHEELS.



SINCE SCOOTER ROLLS AT CONSTANT SPEED, EACH WHEEL IS IN EQUILIBRIUM. THUS W AND R MUST HAVE COMMON LINE OF ACTION TANGENT TO THE FRICTION CIRCLE.

a = RADIUS OF WHEEL

$\tan\theta = \frac{2}{100} = 0.02$

SINCE b AND r_f ARE SMALL COMPARED TO a,

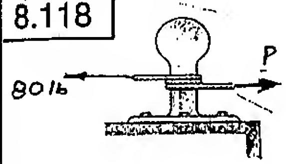
$$\tan\theta \approx \frac{r_f + b}{a} = \frac{\mu_k r + b}{a} = 0.02$$

DATA: $\mu_k = 0.10$, $b = 1.75$ mm, $r = 12.5$ mm

$$\frac{(0.10)(12.5\text{ mm}) + 1.75\text{ mm}}{a} = 0.02$$

$a = 150$ mm; DIAMETER $= 2a = 300$ mm.

8.118



(a) FOR TWO FULL TURNS OF HAWSER AND P = 5000 lb, FIND μ_s

(b) FIND NUMBER OF TURNS, IF P = 20,000 lb.

(a) $\beta = 2$ TURNS $= 2(2\pi) = 4\pi$

$T_1 = 80$ lb $\quad T_2 = 5000$ lb

$$\ln\frac{T_2}{T_1} = \mu_s\beta \qquad \mu_s = \frac{1}{\beta}\ln\frac{T_2}{T_1} = \frac{1}{4\pi}\ln\frac{5000\text{ lb}}{80\text{ lb}}$$

$$\mu_s = \frac{1}{4\pi}\ln 62.5 = \frac{4.1351}{4\pi} \qquad \mu = 0.329$$

(b) $T_1 = 80$ lb, $T_2 = 20{,}000$ lb, $\mu_s = 0.329$

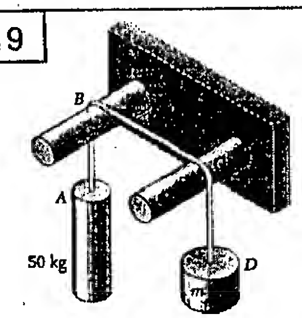
$$\ln\frac{T_2}{T_1} = \mu_s\beta \qquad \beta = \frac{1}{\mu}\ln\frac{T_2}{T_1} = \frac{1}{0.329}\ln\frac{20{,}000\text{ lb}}{80\text{ lb}}$$

$$\beta = \frac{1}{0.329}\ln(250) = \frac{5.5215}{0.329} = 16.783$$

$$\text{NUMBER OF TURNS} = \frac{16.783}{2\pi}$$

NUMBER OF TURNS = 2.67

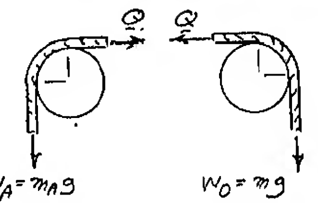
8.119



GIVEN: $\mu_s = 0.40$

FIND: RANGE OF MASS m FOR EQUILIBRIUM

FOR MOTION OF A IMPENDING DOWNWARD



FOR EACH ROD $\beta = \pi/2$, $\mu_s = 0.4$

$$\frac{Q}{m_A g} = e^{\mu_s\beta} \qquad \frac{mg}{Q} = e^{\mu_s\beta}$$

MULTIPLY EQUATIONS MEMBER BY MEMBER

$$\frac{Q}{m_A g}\cdot\frac{mg}{Q} = e^{\mu_s(\beta+\beta)}; \quad \frac{m}{m_A} = e^{0.4(2)\frac{\pi}{2}} = 3.514$$

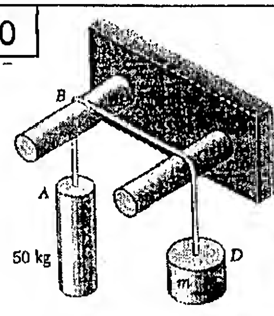
$m = 3.514\, m_A = 3.514(50\text{ kg}) = 175.7$ kg

FOR MOTION OF A IMPENDING UPWARD, WE FIND IN A SIMILAR WAY

$$\frac{m_A}{m} = e^{0.4(2)\frac{\pi}{2}} = 3.514; \quad m = \frac{50\text{ kg}}{3.514} = 14.23\text{ kg}$$

RANGE FOR EQUILIBRIUM: $14.23\text{ kg} \le m \le 175.7\text{ kg}$

8.120

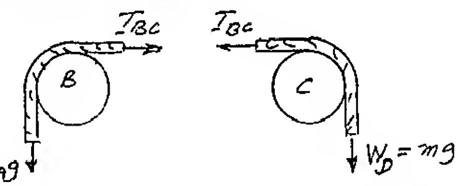


GIVEN: MOTION OF D IMPENDS UPWARD WHEN m = 20 kg.

FIND: (a) μ_s

(b) TENSION IN BC

FOR EACH ROD: $\beta = \pi/2$



$$\text{EQ(1)}: \frac{m_A g}{T_{BC}} = e^{\mu_s\beta} \qquad \text{EQ(2)}: \frac{T_{BC}}{mg} = e^{\mu_s\beta}$$

MULTIPLY EQUATIONS MEMBER BY MEMBER

$$\frac{m_A g}{T_{BC}}\cdot\frac{T_{BC}}{mg} = e^{\mu_s(\beta+\beta)}; \quad \frac{m_A}{m} = e^{\mu_s\pi}$$

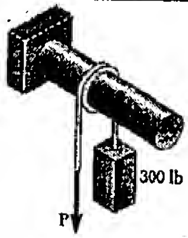
$$\frac{50\text{ kg}}{20\text{ kg}} = e^{\mu_s\pi}; \quad \mu_s\pi = 0.9163; \quad \mu_s = 0.2917$$

$\mu_s = 0.292$

$$\text{EQ (2)} \quad \frac{T_{BC}}{(20\text{ kg})g} = e^{0.2917(\frac{\pi}{2})} = 1.582; \quad T_{BC} = 1.582(20\text{ kg})(9.81\text{ m/s}^2)$$

$T_{BC} = 310$ N

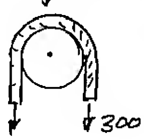
8.121



GIVEN: $\mu_s = 0.15$
ROPE WRAPPED $1\frac{1}{2}$ TIMES AROUND ROD

FIND: RANGE OF P FOR EQUILIBRIUM

$1\frac{1}{2}$ TURNS; $\beta = 1.5(2\pi) = 3\pi$



FOR MOTION OF 300-lb BLOCK IMPENDING UPWARD

$$\frac{P}{300\text{ lb}} = e^{\mu_s\beta} = e^{0.15(3\pi)}$$

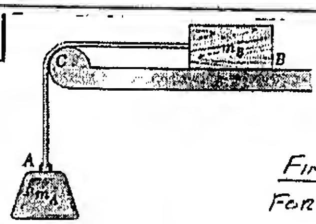
$$\frac{P}{300\text{ lb}} = 4.111 \qquad P = 1233\text{ lb}$$

FOR MOTION OF BLOCK IMPENDING DOWNWARD

$$\frac{300\text{ lb}}{P} = e^{\mu_s\beta} = e^{0.15(3\pi)} = 4.111; \quad P = 73.0\text{ lb}$$

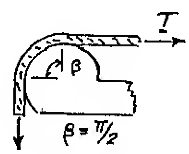
RANGE FOR EQUILIBRIUM: $73.0\text{ lb} \le P \le 1233\text{ lb}$

8.122



GIVEN:
$\mu_s = 0.40$
$m_A = 12$ kg

FIND: SMALLEST m_B FOR EQUILIBRIUM.



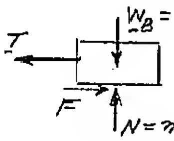
$W_A = m_A g$

$$\frac{m_A g}{T} = e^{\mu_s\beta}$$

$$T = (m_A g)e^{-\mu_s\beta}$$

$$T = (12\text{ kg})g\,e^{-(0.40)\frac{\pi}{2}}$$

$$T = 6.4019\,g$$



$F = \mu_s N = \mu_s m_B g$

$\Sigma F_x = 0: \; T - F = 0$

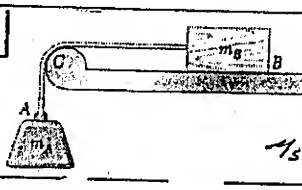
$T = \mu_s m_B g$

$$6.4019\,\cancel{g} = (0.40)\,m_B\,\cancel{g}$$

$$m_B = \frac{6.4019}{0.40}$$

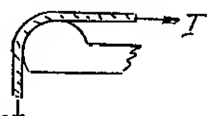
$$m_B = 16.00\text{ kg}$$

8.123



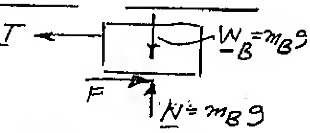
GIVEN: $m_A = m_B$

FIND: SMALLEST μ_s FOR EQUILIBRIUM



$W_A = m_A g$

$$\frac{m_A g}{T} = e^{\mu_s\frac{\pi}{2}}$$



$F = \mu_s N = \mu_s m_B g$

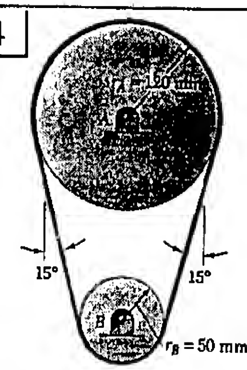
$\Sigma F_x = 0: \; T = F = \mu_s m_B g$

$m_A = m_B = m$:

$$\frac{\cancel{mg}}{\mu_s \cancel{mg}} = e^{\mu_s\frac{\pi}{2}}; \quad \mu_s e^{\mu_s\frac{\pi}{2}} = 1$$

SOLVE BY TRIAL AND ERROR: $\mu_s = 0.475$

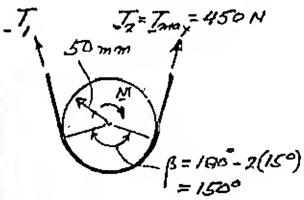
8.124



GIVEN: $\mu_s = 0.40$
$T_{max} = 450$ N

FIND: LARGEST COUPLE THAT CAN BE EXERTED ON DRUM A

BELT WILL SLIP FIRST AT B, SINCE β AT B IS LESS THAN β AT A.



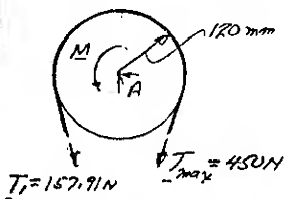
$$\beta = 160° = 150°\frac{\pi}{180°} = \frac{5}{6}\pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_s\beta}$$

$$\frac{450\text{ N}}{T_1} = e^{0.4\left(\frac{5}{6}\pi\right)} = 2.8497$$

$$T_1 = (450\text{ N})/2.8497 = 157.91\text{ lb}$$

TORQUE ON DRUM A:



$+\circlearrowleft \Sigma M_A = 0$

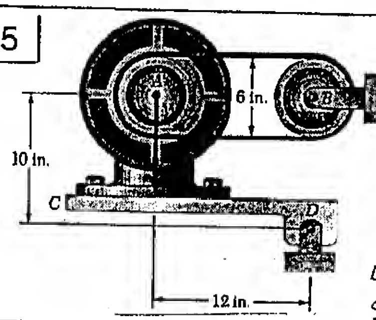
$M - (T_{max} - T_1)(0.120\text{ m})$

$M = (450\text{ N} - 157.91\text{ N})(0.120\text{ m})$

$M = 35.053$ N·m

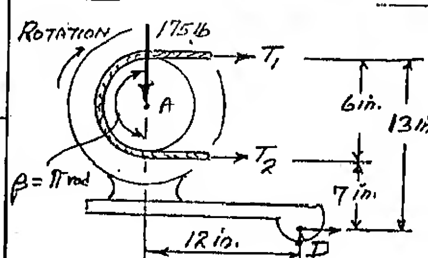
$M = 35.1$ N·m

8.125



GIVEN: MOTOR MOUNT WEIGHS 175 lb.
$\mu_s = 0.40$

FIND: LARGEST TORQUE TRANSMITTED TO $\underline{B}$ WHEN DRUM $\underline{A}$ ROTATES CLOCKWISE.



$$\frac{T_2}{T_1} = e^{\mu_s\beta} = e^{0.40\pi}$$

$$T_2 = 3.5736\,T_1$$

$+\circlearrowleft \Sigma M_D = 0: \; T_1(13\text{ in.}) + T_2(7\text{ in.}) - (175\text{ lb})(12\text{ in.}) = 0$

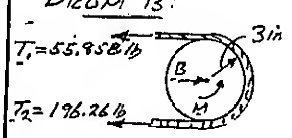
$T_1(13\text{ in.}) + 3.5736\,T_1(7\text{ in.}) - 2100\text{ lb·in.}$

$37.595\,T_1 = 2100$

$T_1 = 55.858$ lb

$T_2 = 3.5736(55.858\text{ lb}) = 196.26$ lb

DRUM B:



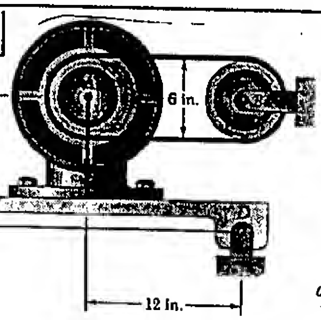
$+\circlearrowleft \Sigma M_B = 0$

$M + (55.858\text{ lb} - 196.26\text{ lb})(3\text{ in.}) = 0$

$M = 421.2$ lb·in.

$M = 421$ lb·in.

8.126

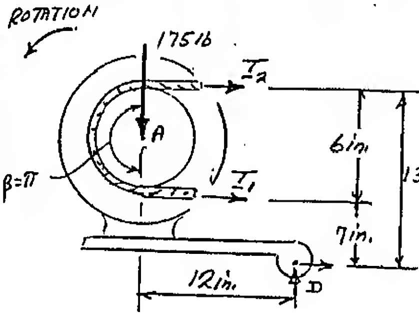


GIVEN: MOTOR MOUNT WEIGHS 175 lb.

$\mu_s = 0.40$

FIND: LARGEST TORQUE TRANSMITTED TO $\underline{B}$ WHEN DRUM $\underline{A}$ ROTATES COUNTERCLOCKWISE

ROTATION



$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.4\pi}$$

$$T_2 = 3.5136\, T_1$$

$$+\circlearrowleft \Sigma M_D = 0: \quad T_1(7\text{ in.}) + T_2(13\text{ in.}) - (175\text{ lb})(12\text{ in.}) = 0$$

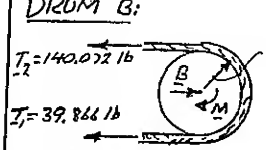
$$T_1(7\text{ in.}) + 3.5136\,T_1(13\text{ in.}) - 2100\text{ lb·in.} = 0$$

$$52.677\,T_1 = 2100$$

$$T_1 = 39.866\text{ lb}$$

$$T_2 = 3.5136(39.866\text{ lb}) = 140.072\text{ lb}$$

DRUM B:



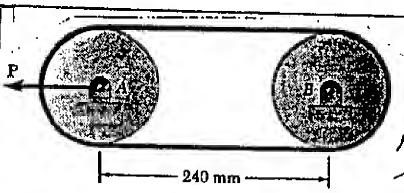
$$+\circlearrowleft \Sigma M_B = 0:$$

$$M + (39.866\text{ lb} - 140.072\text{ lb})(3\text{ in.}) = 0$$

$$M = 300.6\text{ lb·in.}$$

$$M = 301\text{ lb·in.}$$ ◀

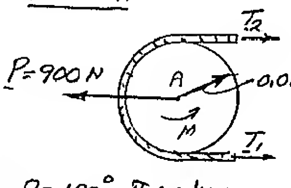
8.127



GIVEN: 60-mm-RADIUS PULLEYS, $P = 900\text{ N}$, $\mu_s = 0.35$.

FIND: (a) LARGEST TORQUE WHICH CAN BE TRANSMITTED.
(b) MAXIMUM TENSION IN BELT.

DRUM A:



$$\frac{T_2}{T_1} = e^{\mu_s \pi} = e^{(0.35)\pi}$$

$$T_2 = 3.0028\,T_1$$

$\beta = 180° = \pi$ radians

$$\xrightarrow{+} \Sigma F_x = 0: \quad T_1 + T_2 - 900\text{ N} = 0$$

$$T_1 + 3.0028\,T_1 - 900\text{ N} = 0$$

$$4.00282\,T_1 = 900$$

$$T_1 = 224.841\text{ N}$$

$$T_2 = 3.0028(224.841\text{ N}) = 675.15\text{ N}$$

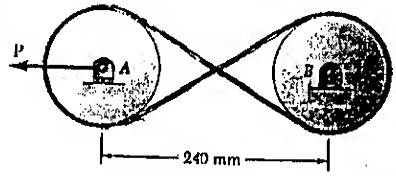
$$T_{max} = 675\text{ N}$$ ◀

TORQUE $\circlearrowleft \Sigma M_A = 0$:

$$M - (675.15\text{ N})(0.06\text{ m}) + (224.84\text{ N})(0.06\text{ m})$$

$$M = 27.0\text{ N·m}$$ ◀

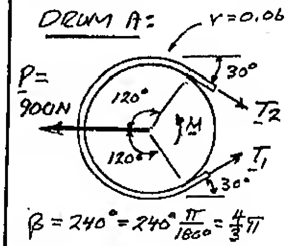
8.128

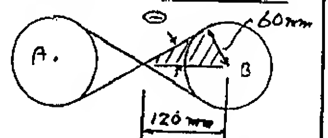


GIVEN: 60-mm-RADIUS PULLEYS, $\mu_s = 0.35$, $P = 900\text{ N}$

FIND: (a) LARGEST TORQUE WHICH CAN BE TRANSMITTED
(b) MAXIMUM TENSION IN BELT.

DRUM A:





$$\sin\theta = \frac{60}{120} = \frac{1}{2}$$

$$\theta = 30°$$

$$\beta = 240° = 240°\frac{\pi}{180°} = \frac{4}{3}\pi$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.35(\frac{4}{3}\pi)}$$

$$T_2 = 4.3322\,T_1$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad (T_1 + T_2)\cos 30° - 900\text{ N}$$

$$(T_1 + 4.3322\,T_1)\cos 30° = 900$$

$$T_1 = 194.90\text{ N}$$

$$T_2 = 4.3322(194.90\text{ N}) = 844.3\text{ N}$$

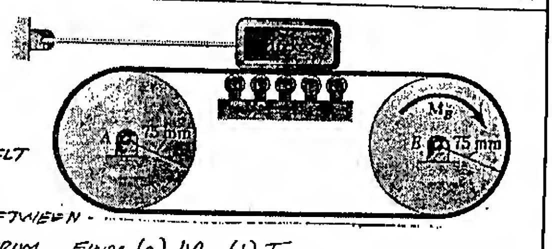
$$T_{max} = 844\text{ N}$$ ◀

TORQUE:

$$+\circlearrowleft \Sigma M_B = 0: \quad M - (844.3\text{ N})(0.06\text{ m}) + (194.9\text{ N})(0.06\text{ m}) = 0$$

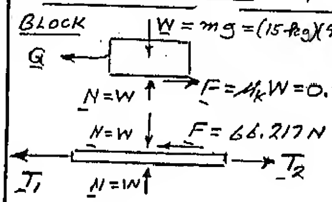
$$M = 39.0\text{ N·m}$$ ◀

8.129



GIVEN: $\mu_k = 0.45$ BETWEEN BELT AND BLOCK, $\mu_s = 0.30$ BETWEEN BELT AND DRUM. FIND: (a) M_B, (b) T_{min} FOR NO SLIPPING.

BLOCK



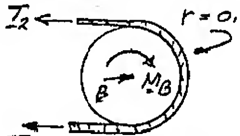
$$W = mg = (15\text{ kg})(9.81\text{ m/s}^2) = 147.15\text{ N}$$

$$F = \mu_k W = 0.45(147.15\text{ N}) = 66.217\text{ N}$$

$$F = 66.217\text{ N}$$

PORTION OF BELT LOCATED UNDER BLOCK

$$\xrightarrow{+} \Sigma F_x = 0: \quad T_2 - T_1 - 66.217\text{ N} = 0 \qquad (1)$$



$\beta = \pi$ radians

DRUM B:

$$\frac{T_2}{T_1} = e^{\mu_s \pi} = e^{0.3\pi} = 2.5663$$

$$T_2 = 2.5663\,T_1 \qquad (2)$$

EQ(1):

$$2.5663\,T_1 - T_1 - 66.217\text{ N} = 0$$

$$1.5663\,T_1 = 66.217\text{ N}$$

$$T_1 = 42.276\text{ N}; \qquad T_{min} = 42.3\text{ N}$$ ◀

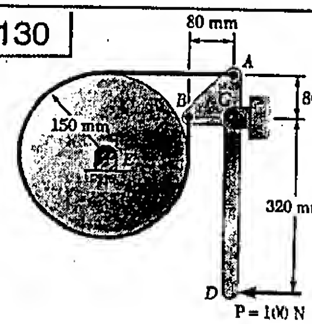
EQ(2):

$$T_2 = 2.5663(42.276\text{ N}) = 108.493\text{ N}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad M_B - (108.493\text{ N})(0.075\text{ m}) + (42.276\text{ N})(0.075\text{ m})$$

$$M_B = 4.966\text{ N·m} \qquad M_B = 4.97\text{ N·m}$$ ◀

8.130

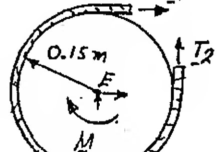


GIVEN: $\mu_k = 0.25$

FIND: MAGNITUDE OF COUPLE APPLIED TO FLYWHEEL FOR CLOCKWISE ROTATION. SHOW THAT RESULT IS SAME FOR COUNTERCLOCKWISE ROTATION

FREE BODY: FLYWHEEL

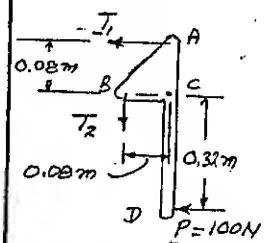
FOR CLOCKWISE ROTATION OF FLYWHEEL T_2 AND T_1 ARE LOCATED AS SHOWN.



$\beta = \frac{3}{4}(360°) = \frac{3}{4}(2\pi) = \frac{3}{2}\pi$ radians

$$\frac{T_2}{T_1} = e^{\mu_k\beta} = e^{0.25(\frac{3}{2}\pi)} = 3.2482$$

$$T_2 = 3.2482\,T_1 \qquad (1)$$



FREE BODY: HANDLE

$+\circlearrowright \Sigma M_C = 0 \qquad (2)$

$(T_1 + T_2)(0.08\,m) - (100\,N)(0.32\,m) = 0$

$(T_1 + 3.2482\,T_1) = 400\,N$

$T_1 = (400\,N)/4.2482 = 94.157\,N$

$T_2 = 3.2482(94.157\,N) = 305.842\,N$

RETURN TO FREE BODY OF FLYWHEEL

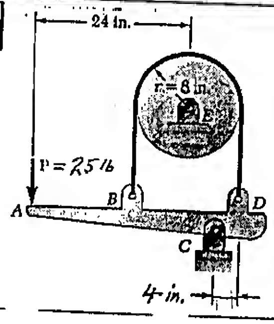
$+\circlearrowright \Sigma M_E = 0: \quad M + (T_1 - T_2)(0.15\,m) = 0$

$M + (94.157\,N - 305.842\,N)(0.15\,m) = 0$

$M = 31.752\,N\cdot m \qquad\qquad M = 31.8\,N\cdot m$ ◀

IF ROTATION IS REVERSED (TO BE $\circlearrowleft$) T_2 AND T_1 ARE INTERCHANGED; EQS. (1) AND (2) ARE NOT CHANGED, THUS VALUES OF T_1, T_2, AND M ARE THE SAME.

8.131



GIVEN: $\mu_k = 0.25$

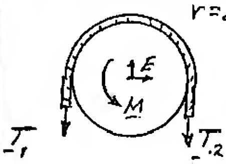
FIND: MAGNITUDE OF COUPLE APPLIED TO DRUM FOR ROTATION

(a) COUNTERCLOCKWISE

(b) CLOCKWISE

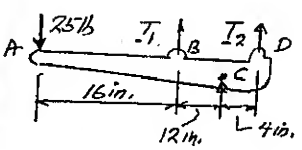
(a) COUNTERCLOCKWISE ROTATION. FREE BODY DRUM

$r = 8\,in. \quad \beta = 180° = \pi$ radians



$$\frac{T_2}{T_1} = e^{\mu_k\beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933\,T_1$$



FREE BODY: CONTROL BAR

$+\circlearrowright \Sigma M_C = 0$

$T_1(12\,in.) - T_2(4\,in.) - (25\,lb)(28\,in.) = 0$

$T_1(12) - 2.1933\,T_1(4) - 700 = 0$

$T_1 = 216.93\,lb$

$T_2 = 2.1933(216.93\,lb) = 475.80\,lb$

(CONTINUED)

8.131 CONTINUED

RETURN TO FREE BODY OF DRUM

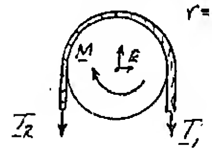
$+\circlearrowright \Sigma M_E = 0: \quad M + T_1(8\,in.) - T_2(8\,in.) = 0$

$M + (216.96\,lb)(8\,in.) - (475.80\,lb)(8\,in.) = 0$

$M = 2070.9\,lb\cdot in. \qquad\qquad M = 2070\,lb\cdot in.$ ◀

(b) CLOCKWISE ROTATION

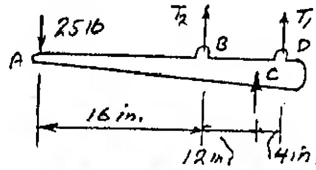
$r = 8\,in. \quad \beta = \pi$ rad



$$\frac{T_2}{T_1} = e^{\mu_k\beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933\,T_1$$

FREE BODY: CONTROL ROD



$+\circlearrowright \Sigma M_C = 0:$

$T_2(12\,in.) - T_1(4\,in.) - (25\,lb)(28\,in.) = 0$

$2.1933\,T_1(12) - T_1(4) - 700 = 0$

$T_1 = 31.363\,lb$

$T_2 = 2.1933(31.363\,lb)$

$T_2 = 68.788\,lb$

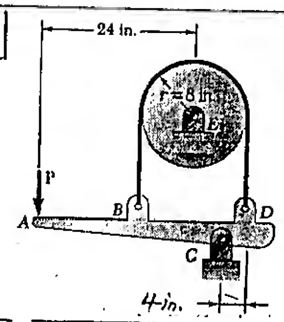
RETURN TO FREE BODY OF DRUM

$+\circlearrowright \Sigma M_E = 0: \quad M + T_1(8\,in.) - T_2(8\,in.) = 0$

$M + (31.363\,lb)(8\,in.) - (68.788\,lb)(8\,in.) = 0$

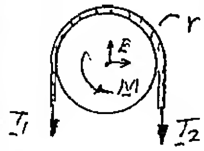
$M = 299.4\,lb\cdot in. \qquad\qquad M = 299\,lb\cdot in.$ ◀

8.132



FIND: MAXIMUM μ_s FOR BRAKE TO BE SELF LOCKING FOR COUNTERCLOCKWISE ROTATION OF DRUM

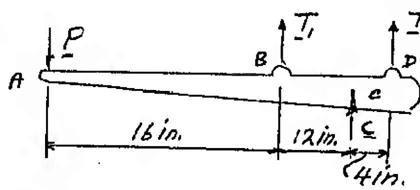
$r = 8\,in. \qquad \beta = 180° = \pi$ radians



$$\frac{T_2}{T_1} = e^{\mu_s\beta} = e^{\mu_s\pi}$$

$$T_2 = e^{\mu_s\pi}\,T_1$$

FREE BODY: CONTROL ROD



$+\circlearrowright \Sigma M_C = 0: \quad P(28\,in.) - T_1(12\,in.) + T_2(4\,in.) = 0$

$28P - 12T_1 + e^{\mu\pi}T_1(4) = 0$

FOR SELF-LOCKING BRAKE $P = 0$

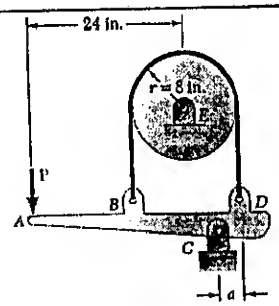
$12T_1 = 4T_1 e^{\mu_s\pi}$

$e^{\mu_s\pi} = 3 \qquad \mu_s\pi = \ln 3 = 1.0986$

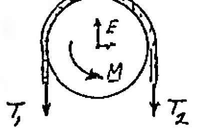
$$\mu_s = \frac{1.0986}{\pi} = 0.3497$$

$\mu_s = 0.350$ ◀

8.133



GIVEN: $\mu_s = 0.30$
ROTATION ↻.
FIND: MINIMUM VALUE OF a FOR WHICH BRAKE IS NOT SELF-LOCKING.

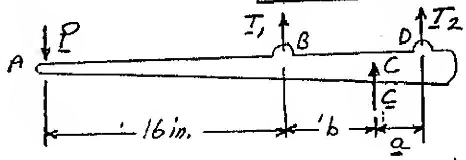


$r = 8$ in., $\beta = \pi$ radians

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.30\pi} = 2.5663$$

$$T_2 = 2.5663\, T_1$$

FREE BODY: CONTROL ROD



$b = 16\text{ in} - a$

$+\circlearrowleft \Sigma M_C = 0$: $P(16\text{ in} + b) - T_1 b + T_2 a = 0$

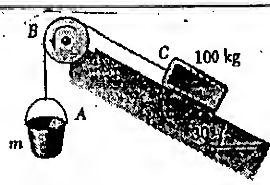
FOR BRAKE TO BE SELF LOCKING, $P = 0$

$T_2 a = T_1 b$; $2.5663 \cancel{T_1} a = \cancel{T_1}(16 - a)$

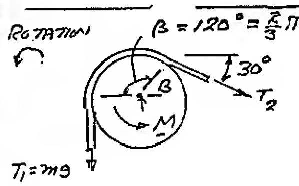
$2.5663 a = 16 - a$

$3.5663 a = 16$ $\qquad a = 4.49$ in. ◀

8.134



GIVEN: $\mu_s = 0.35$
$\mu_k = 0.25$
FIND: SMALLEST m FOR WHICH BLOCK C (a) REMAINS AT REST, (b) STARTS MOVING UP, (c) CONTINUES MOVING UP.



FREE BODY: DRUM

$$\frac{T_2}{mg} = e^{\mu \frac{2}{3}\pi}$$

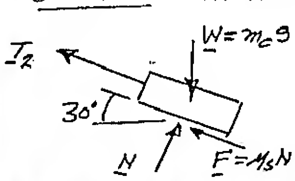
$$T_2 = mg\, e^{2\mu\pi/3} \qquad (1)$$

(a) SMALLEST m FOR BLOCK C TO REMAIN AT REST

CABLE SLIPS ON DRUM

EQ(1) WITH $\mu_k = 0.25$; $T_2 = mg\, e^{2(0.25)\frac{\pi}{3}} = 1.6881 mg$

BLOCK C : AT REST, MOTION IMPENDING →



$+\nearrow \Sigma F = 0$: $N - m_c g \cos 30° $

$N = m_c g \cos 30°$

$F = \mu_s N = 0.35 m_c g \cos 30°$

$m_c = 100$ kg

$+\nwarrow \Sigma F = 0$: $T_2 + F - m_c g \sin 30° = 0$

$1.6881 m\cancel{g} + 0.35 m_c \cancel{g} \cos 30° - m_c \cancel{g} \sin 30° = 0$

$1.6881 m = 0.19689 m_c$

$m = 0.11663 m_c = 0.11663(100\text{ kg})$; $m = 11.66$ kg ◀

(CONTINUED)

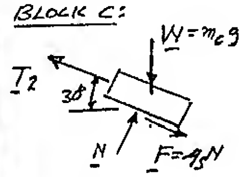
8.134 CONTINUED

(b) SMALLEST m TO START BLOCK MOVING UP

NO SLIPPING AT BOTH DRUM AND BLOCK $\mu_s = 0.35$

EQ(1): $T_2 = mg\, e^{2(0.35)\pi/3} = 2.0814 mg$

BLOCK C:



MOTION IMPENDING ↖ $m_c = 100$ kg

$+\nearrow \Sigma F = 0$: $N - m_c g \cos 30°$

$N = m_c g \cos 30°$

$F = \mu_s N = 0.35 m_c g \cos 30°$

$+\nwarrow \Sigma F = 0$: $T_2 - F - m_c g \sin 30° = 0$

$2.0814 m\cancel{g} - 0.35 m_c \cancel{g} \cos 30° - m_c \cancel{g} \sin 30° = 0$

$2.0814 m = 0.80311 m_c$

$m = 0.38585 m_c = 0.38585(100\text{ kg})$

$m = 38.6$ kg ◀

(c) SMALLEST m TO KEEP BLOCK MOVING UP

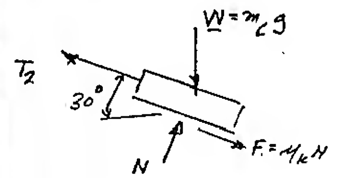
DRUM: NO SLIPPING $\mu_s = 0.35$

EQ(1) WITH $\mu_s = 0.35$

$T_2 = mg\, e^{2\mu_s\pi/3} = mg\, e^{2(0.35)\pi/3}$

$T_2 = 2.0814 mg$

BLOCK C: MOVING UP PLANE, THUS $\mu_k = 0.25$



MOTION UP ↖

$+\nearrow \Sigma F = 0$

$N - m_c g \cos 30° = 0$

$N = m_c g \cos 30°$

$F = \mu_k N = 0.25 m_c g \cos 30°$

$+\nwarrow \Sigma F = 0$: $T_2 - F - m_c g \sin 30° = 0$

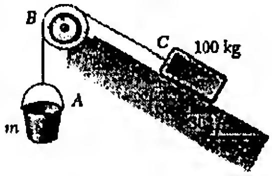
$2.0814 m\cancel{g} - 0.25 m_c \cancel{g} \cos 30° - m_c \cancel{g} \sin 30° = 0$

$2.0814 m = 0.71651 m_c$

$m = 0.34424 m_c = 0.34424(100\text{ kg})$

$m = 34.4$ kg ◀

8.135



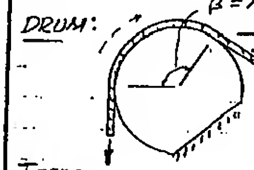
GIVEN: DRUM B IS FIXED.
$\mu_s = 0.35$
$\mu_k = 0.25$

FIND: SMALLEST m FOR WHICH BLOCK C (a) REMAINS AT REST, (b) STARTS MOVING UP, (c) CONTINUES MOVING UP.

(a) BLOCK C REMAINS AT REST, MOTION IMPENDS ↘

$\beta = 120° = \frac{2}{3}\pi$ radians

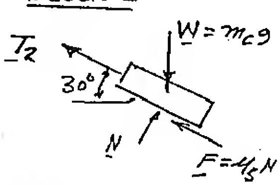
DRUM:



$$\frac{T_2}{mg} = e^{\mu_k \beta} = e^{0.35\left(\frac{2\pi}{3}\right)}$$

$$T_2 = 2.0814\, mg$$

BLOCK C



MOTION IMPENDS ↘

$\nearrow \Sigma F = 0: N - m_c g \cos 30° = 0$

$N = m_c g \cos 30°$

$F = \mu_s N = 0.35\, m_c g \cos 30°$

$+\nwarrow \Sigma F = 0: \quad T_2 + F - m_c g \sin 30° = 0$

$2.0814\, mg + 0.35\, m_c g \cos 30° - m_c g \sin 30° = 0$

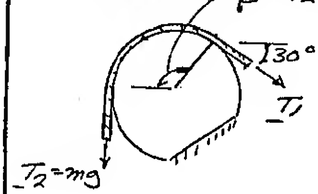
$2.0814\, mg = 0.19689\, m_c$

$m = 0.09459\, m_c = 0.09459\,(100\text{ kg})$

$m = 9.46\text{ kg}$ ◀

(b) BLOCK C STARTS MOVING UP $\quad \mu_s = 0.35$

DRUM: $\quad \beta = 120° = \frac{2}{3}\pi$ radians



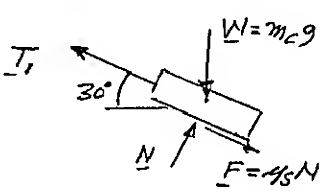
IMPENDING MOTION OF CABLE ↶

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

$$\frac{mg}{T_1} = e^{0.35\left(\frac{2}{3}\pi\right)}$$

$$T_1 = \frac{mg}{2.0814} = 0.48045\, mg$$

BLOCK C $\quad$ MOTION IMPENDS ↖



$+\nearrow \Sigma F = 0: N - m_c g \cos 30° = 0$

$N = m_c g \cos 30°$

$F = \mu_s N = 0.35\, m_c g \cos 30°$

$+\nwarrow \Sigma F = 0: \quad T_1 - F - m_c g \sin 30° = 0$

$0.48045\, m\cancel{g} - 0.35\, m_c \cancel{g} \cos 30° - 0.5\, m_c \cancel{g} = 0$

$0.48045\, m = 0.80311\, m_c$

$m = 1.67158\, m_c = 1.67158\,(100\text{ kg})$

$m = 167.2\text{ kg}$ ◀

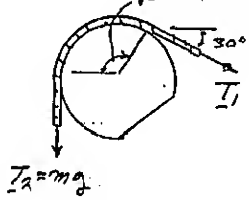
(CONTINUED)

8.135 CONTINUED

(c) SMALLEST m TO KEEP BLOCK MOVING ↖

DRUM: MOTION OF CABLE ↶ $\quad \mu_k = 0.25$

$\beta = 120° = \frac{2}{3}\pi$ radians

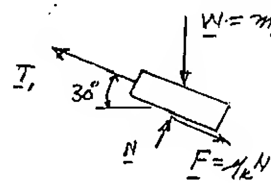


$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\left(\frac{2}{3}\pi\right)}$$

$$\frac{mg}{T_1} = 1.6881$$

$$T = \frac{mg}{1.6881} = 0.59238\, mg$$

BLOCK C: BLOCK MOVES ↖



$+\nearrow \Sigma F = 0: N - m_c g \cos 30° = 0$

$N = m_c g \cos 30°$

$F = \mu_k N = 0.25\, m_c g \cos 30°$

$+\nwarrow \Sigma F = 0: \quad T_1 - F - m_c g \sin 30° = 0$

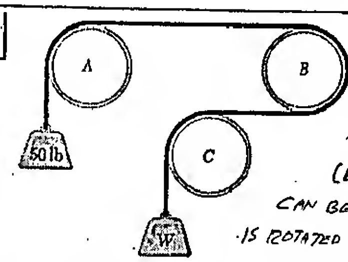
$0.59238\, m\cancel{g} - 0.25\, m_c \cancel{g} \cos 30° - 0.5\, m_c \cancel{g} = 0$

$0.59238\, m = 0.71651\, m_c$

$m = 1.20954\, m_c = 1.20954\,(100\text{ kg})$

$m = 121.0\text{ kg}$ ◀

8.136

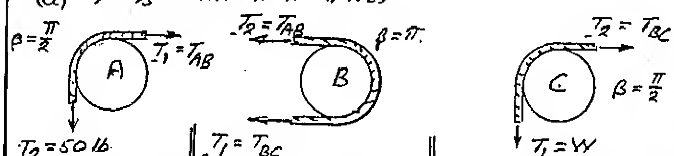


GIVEN: $\mu_s = 0.25$
$\mu_k = 0.20$

FIND: (a) SMALLEST W FOR EQUILIBRIUM
(b) LARGEST W THAT CAN BE RAISED IF PIPE B IS ROTATED WITH A+C FIXED.

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

(a) $\mu = \mu_s = 0.25$ AT ALL PIPES



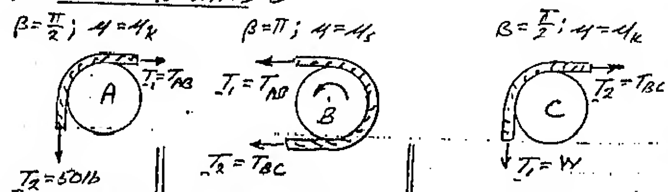
$$\frac{50\text{ lb}}{T_{AB}} = e^{0.25\frac{\pi}{2}} \qquad \frac{T_{AB}}{T_{BC}} = e^{0.25\pi} \qquad \frac{T_{BC}}{W} = e^{0.25\frac{\pi}{2}}$$

$$\frac{50\text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\frac{\pi}{8}} \cdot e^{\frac{\pi}{4}} \cdot e^{\frac{\pi}{8}} = e^{\frac{\pi}{8}+\frac{\pi}{4}+\frac{\pi}{8}} = e^{\frac{\pi}{2}} = 4.8105$$

$\frac{50\text{ lb}}{W} = 4.8015; \quad W = 10.394\text{ lb} \qquad W = 10.39\text{ lb}$ ◀

(b) PIPE B ROTATED ↻

$\beta = \frac{\pi}{2}; \ \mu = \mu_k \qquad \beta = \pi; \ \mu = \mu_s \qquad \beta = \frac{\pi}{2}; \ \mu = \mu_k$



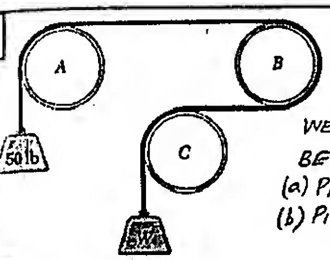
$$\frac{50\text{ lb}}{T_{AB}} = e^{0.2\frac{\pi}{2}} \qquad \frac{T_{BC}}{T_{AB}} = e^{0.25\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\frac{\pi}{2}}$$

$$\frac{50\text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\frac{\pi}{10}} \cdot e^{-\frac{\pi}{4}} \cdot e^{\frac{\pi}{10}} = e^{\frac{\pi}{10}-\frac{\pi}{4}+\frac{\pi}{10}} = e^{-\frac{\pi}{20}} = 0.85464$$

$\frac{50\text{ lb}}{W} = 0.85464; \quad W = \frac{50\text{ lb}}{0.85464} = 58.504\text{ lb}$

$W = 58.5\text{ lb}$ ◀

8.137



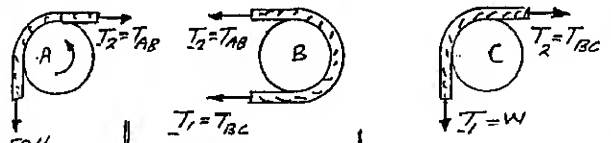
GIVEN: $\mu_s = 0.25$, $\mu_k = 0.20$

FIND: LARGEST WEIGHT W THAT CAN BE RAISED IF ONLY
(a) PIPE A IS ROTATED
(b) PIPE C IS ROTATED

$\frac{T_2}{T_1} = e^{\mu\beta}$

(a) PIPE A ROTATES.

$\beta = \frac{\pi}{2}$; $\mu = \mu_s$ $\qquad$ $\beta = \pi$; $\mu = \mu_k$ $\qquad$ $\beta = \frac{\pi}{2}$; $\mu = \mu_k$



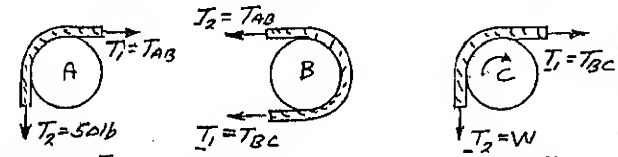
$\frac{T_{AB}}{50 lb} = e^{0.25\frac{\pi}{2}}$ $\qquad$ $\frac{T_{AB}}{T_{BC}} = e^{0.2\pi}$ $\qquad$ $\frac{T_{BC}}{W} = e^{0.2\frac{\pi}{2}}$

$$\frac{T_{AB}}{50 lb} \cdot \frac{T_{BC}}{T_{AB}} \cdot \frac{W}{T_{BC}} = e^{\frac{\pi}{8}} \cdot e^{-\frac{\pi}{5}} \cdot e^{-\frac{\pi}{10}} = e^{\pi(\frac{1}{8} - \frac{1}{5} - \frac{1}{10})} = e^{-\frac{7\pi}{40}} = 0.57708$$

$\frac{W}{50 lb} = 0.57708$; $W = 28.854 lb$: $W = 28.9 lb$ ◀

(b) PIPE C ROTATES.

$\beta = \frac{\pi}{2}$; $\mu = \mu_k$ $\qquad$ $\beta = \pi$; $\mu = \mu_k$ $\qquad$ $\beta = \frac{\pi}{2}$, $\mu = \mu_s$

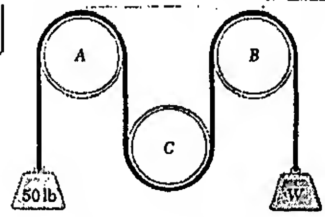


$\frac{50 lb}{T_{AB}} = e^{0.2\frac{\pi}{2}}$ $\qquad$ $\frac{T_{AB}}{T_{BC}} = e^{0.2\pi}$ $\qquad$ $\frac{W}{T_{BC}} = e^{0.25\frac{\pi}{2}}$

$$\frac{50 lb}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\frac{\pi}{10}} \cdot e^{\frac{\pi}{5}} \cdot e^{-\frac{\pi}{8}} = e^{\frac{7\pi}{40}} = 0.57708$$

$\frac{50 lb}{W} = 0.57708$; $W = 28.854 lb$ $\qquad$ $W = 28.9 lb$ ◀

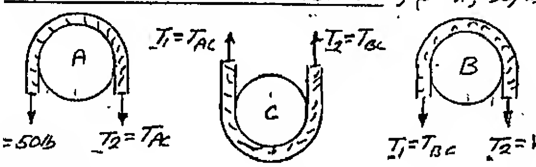
8.138



GIVEN: $\mu_s = 0.25$, $\mu_k = 0.20$.

FIND: (a) SMALLEST WEIGHT W FOR EQUILIBRIUM, (b) LARGEST W WHICH CAN BE RAISED IF PIPE B IS ROTATED WHILE A AND C ARE FIXED.

(a) SMALLEST W FOR EQUILIBRIUM; $\beta = \pi$, $\mu = \mu_s$



$\frac{T_{AC}}{50 lb} = e^{0.25\pi}$ $\qquad$ $\frac{T_{BC}}{T_{AC}} = e^{0.25\pi}$ $\qquad$ $\frac{W}{T_{BC}} = e^{0.25\pi}$

$$\frac{T_{AC}}{50 lb} \cdot \frac{T_{BC}}{T_{AC}} \cdot \frac{W}{T_{BC}} = e^{\pi/4} \cdot e^{\pi/4} \cdot e^{\pi/4} = e^{\frac{3\pi}{4}} = 10.551$$

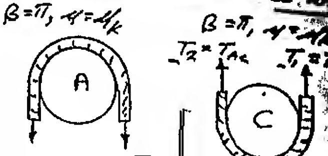
$W/50 lb = 10.551$; $W = 4.739 lb$ $\qquad$ $W = 4.74 lb$ ◀

(CONTINUED)

8.138 CONTINUED

(b) ...

$\beta = \pi$, $\mu = \mu_k$ $\qquad$ $\beta = \pi$, $\mu = \mu_k$

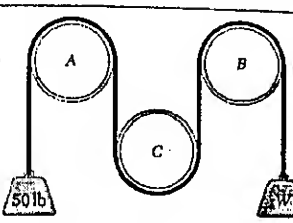


$\frac{50 lb}{T_{AC}} = e^{0.2\pi}$ $\qquad$ $\frac{T_{AC}}{T_{BC}} = e^{0.2\pi}$

$$\frac{50 lb}{T_{AC}} \cdot \frac{T_{AC}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\frac{\pi}{5}} \cdot e^{\frac{\pi}{5}} \cdot e^{-\frac{\pi}{4}} = e^{\pi(\frac{1}{5} + \frac{1}{5} - \frac{1}{4})}$$

$\frac{50 lb}{W} = 1.602$; $W = \frac{50 lb}{1.602} = 31.21 lb$ $\qquad$ $W = 31.2$ ...

8.139



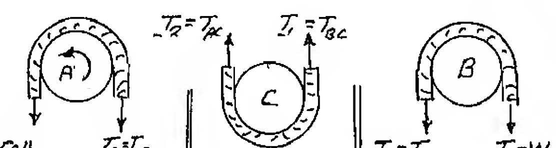
GIVEN: μ_s ... μ_k ...

FIND: LARGEST ... THAT CAN BE RAISED IF ONLY
(a) PIPE A IS ROTATED
(b) PIPE C IS ROTATED

$\frac{T_2}{T_1} = e^{\mu\beta}$

(a) PIPE A ROTATES

$\beta = \pi$, $\mu = \mu_s$ $\qquad$ $\beta = \pi$, $\mu = \mu_k$ $\qquad$ $\beta = \pi$, $\mu = \mu_k$



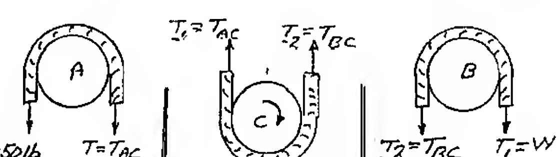
$\frac{T_{AC}}{50 lb} = e^{0.25\pi}$ $\qquad$ $\frac{T_{AC}}{T_{BC}} = e^{0.2\pi}$ $\qquad$ $\frac{T_{BC}}{W} = e^{0.2\pi}$

$$\frac{T_{AC}}{50 lb} \cdot \frac{T_{BC}}{T_{AC}} \cdot \frac{W}{T_{BC}} = e^{\frac{\pi}{4}} \cdot e^{-\frac{\pi}{5}} \cdot e^{-\frac{\pi}{5}} = e^{\pi(\frac{1}{4} - \frac{1}{5} - \frac{1}{5})} = e^{-\frac{3\pi}{20}} = 0.62423$$

$\frac{W}{50 lb} = 0.62423$; $W = 31.21 lb$ $\qquad$ $W = 31.2 lb$ ◀

(b) PIPE C ROTATES

$\beta = \pi$, $\mu = \mu_k$ $\qquad$ $\beta = \pi$, $\mu = \mu_s$ $\qquad$ $\beta = \pi$, $\mu = \mu_k$



$\frac{50 lb}{T_{AC}} = e^{0.2\pi}$ $\qquad$ $\frac{T_{BC}}{T_{AC}} = e^{0.25\pi}$ $\qquad$ $\frac{T_{BC}}{W} = e^{0.2\pi}$

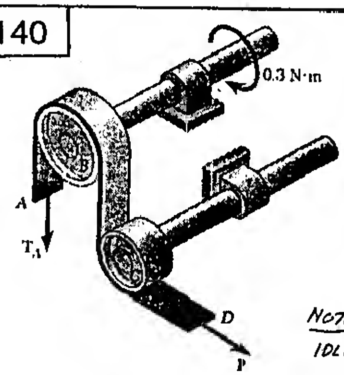
$$\frac{50 lb}{T_{AC}} \cdot \frac{T_{AC}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\frac{\pi}{5}} \cdot e^{-\frac{\pi}{4}} \cdot e^{\frac{\pi}{5}} = e^{\pi(\frac{1}{5} - \frac{1}{4} + \frac{1}{5})} = e^{\frac{3\pi}{20}}$$

$\frac{50 lb}{W} = e^{\frac{3\pi}{20}} = 1.602$

$W = \frac{50 lb}{1.602} = 31.21 lb$

$W = 31.2 lb$ ◀

8.140



GIVEN: $\mu_s = 0.40$, $\mu_k = 0.30$

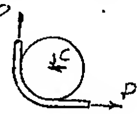
DRUM B, $r = 20$ mm

FIND: SMALLEST P IF SLIPPING IS NOT TO OCCUR ON DRUM B.

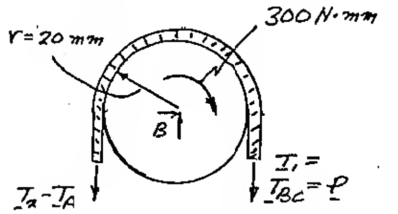
NOTE: DRUM C IS AN IDLER WITH NO FRICTION

DRUM C: IDLER

$T_{BC} = P$



DRUM B



FOR SLIPPING IMPENDING:

$\mu = \mu_s = 0.40$ $\qquad \beta = \pi$ radians

$\frac{T_2}{T_1} = e^{\mu\beta}: \quad \frac{T_A}{P} = e^{0.4\pi} = 3.5136$

$T_A = 3.5136\ P$

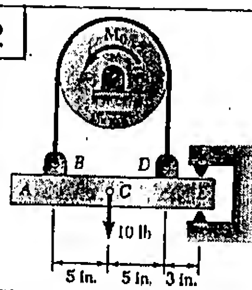
$+\circlearrowleft \Sigma M_B = 0: \quad T_A(20\text{ mm}) - P(20\text{ mm}) - 300\text{ N·mm} = 0$

$(3.5136P - P)(20\text{ mm}) = 300\text{ N·mm}$

$2.5136\ P = 15\text{ N}$

$P = 5.967\text{ N}$ $\qquad P = 5.97\text{ N}$ ◀

8.142



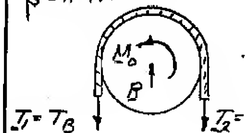
GIVEN: $\mu_s = 0.30$

M_0 ACTS ↺

FIND: (a) M_0 FOR WHICH SLIPPING IMPENDS.

(b) FORCE E EXERTED ON BAR ACE

$\beta = \pi$ rad.

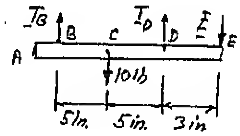


DRUM: SLIPPING IMPENDS

$\mu_s = 0.30$

$\frac{T_2}{T_1} = e^{\mu\beta}: \quad \frac{T_D}{T_B} = e^{0.30\pi} = 2.5663$

$T_D = 2.5663\ T_B$



BAR AE:

$+\uparrow \Sigma F_y = 0: \quad T_B + T_D - E - 10\text{ lb} = 0$

$T_B + 2.5663\ T_B - E - 10\text{ lb} = 0$

$3.5663\ T_B - E - 10\text{ lb} = 0$

$E = 3.5663\ T_B - 10\text{ lb} \qquad (1)$

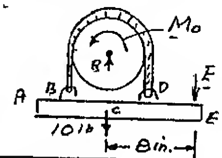
$+\circlearrowleft \Sigma M_D = 0: \quad E(3\text{ in.}) - (10\text{ lb})(5\text{ in.}) + T_B(10\text{ in.}) = 0$

$(3.5663\ T_B - 10\text{ lb})(3\text{ in.}) - 50\text{ lb·in.} + T_B(10\text{ in.}) = 0$

$20.699\ T_B = 80 \qquad T_B = 3.8649\text{ lb}$

EQ.(1): $E = 3.5663(3.8649\text{ lb}) - 10\text{ lb}; \quad E = +3.78347\text{ lb}$

$E = 3.78\text{ lb}\downarrow$ ◀



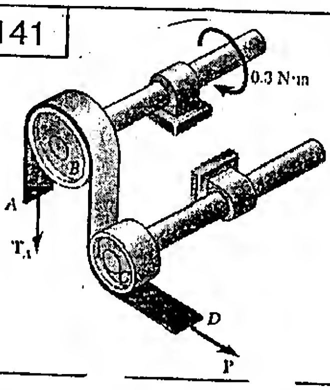
FREE BODY: DRUM AND BAR

$+\circlearrowleft \Sigma M_C = 0: \quad M_0 - E(8\text{ in.}) = 0$

$M_0 = (3.78649\text{ lb})(8\text{ in.}) = 30.27\text{ lb·in.}$

$M_0 = 30.3\text{ lb·in.}$ ↺ ◀

8.141



GIVEN: $\mu_s = 0.40$, $\mu_k = 0.30$, DRUM B, $r = 20$ mm

DRUM C IS FROZEN AND CANNOT ROTATE

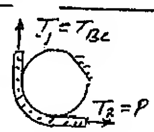
FIND: SMALLEST P IF SLIPPING IS NOT TO OCCUR ON DRUM B.

$\frac{T_2}{T_1} = e^{\mu\beta}$

DRUM C: $\quad \beta = \frac{\pi}{2}$

SLIPPING OCCURS

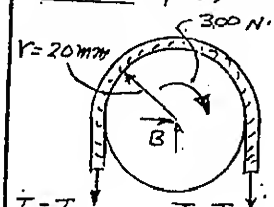
$\mu = \mu_k = 0.30$



$\frac{P}{T_{BC}} = e^{0.3\left(\frac{\pi}{2}\right)} = 1.602$

$P = 1.602\ T_{BC} \qquad (1)$

DRUM B: $\beta = \pi$, $\mu = \mu_s = 0.40$



$\frac{T_2}{T_1} = e^{\mu\beta}: \quad \frac{T_A}{T_{BC}} = e^{0.4\pi} = 3.5136$

$T_A = 3.5136\ T_{BC} \qquad (2)$

$+\circlearrowleft \Sigma M_B = 0:$

$T_A(20\text{ mm}) - T_{BC}(20\text{ mm}) - 300\text{ N·mm} = 0$

SUBSTITUTE FOR T_A FROM EQ.(2):

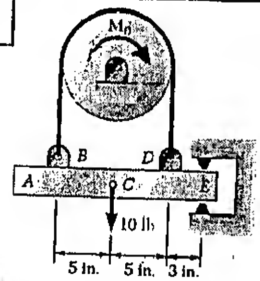
$(3.5136\ T_{BC} - T_{BC})(20\text{ mm}) = 300\text{ N·mm}$

$T_{BC} = 5.967\text{ N}$

EQ(1): $P = 1.602\ T_{BC} = 1.602(5.967\text{ N})$

$P = 9.559\text{ N}$ $\qquad P = 9.56\text{ N}$ ◀

8.143



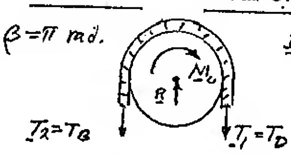
GIVEN: $\mu_s = 0.30$

M_0 ACTS ↻

FIND: (a) M_0 FOR WHICH SLIPPING IMPENDS.

(b) FORCE E EXERTED ON BAR ACE.

$\beta = \pi$ rad.

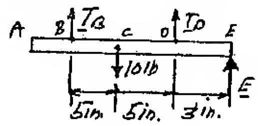


DRUM: SLIPPING IMPENDS

$\mu_s = 0.30$

$\frac{T_2}{T_1} = e^{\mu\beta}; \quad \frac{T_B}{T_D} = e^{0.30\pi} = 2.5663$

$T_B = 2.5663\ T_D$



BAR AE

$+\uparrow \Sigma F_y = 0: \quad T_B + T_D + E - 10\text{ lb} = 0$

$2.5663\ T_D + T_D + E - 10\text{ lb}$

$E = -3.5663\ T_D + 10\text{ lb} \qquad (1)$

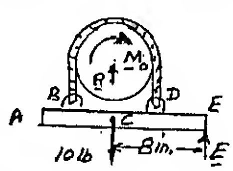
$+\circlearrowleft \Sigma M_B = 0: \quad T_D(10\text{ in.}) - (10\text{ lb})(5\text{ in.}) + E(13\text{ in.}) = 0$

$T_D(10\text{ in.}) - 50\text{ lb·in.} + (-3.5663\ T_D + 10\text{ lb})(13\text{ in.}) = 0$

$-36.362\ T_D + 80\text{ lb·in.} = 0; \quad T_D = 2.200\text{ lb}$

EQ.(1) $\quad E = -3.5633(2.200\text{ lb}) + 10\text{ lb}; \quad E = +2.1538\text{ lb}$

$E = 2.15\text{ lb}\uparrow$ ◀

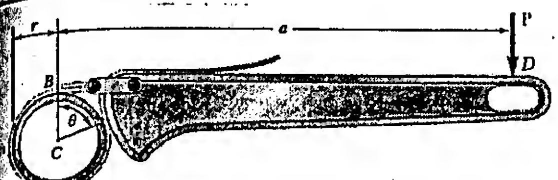


FREE BODY: DRUM AND BAR

$+\circlearrowleft \Sigma M_C = 0: \quad M_0 - E(8\text{ in.}) = 0$

$M_0 = (2.1538\text{ lb})(8\text{ in.})$

$M_0 = 17.23\text{ lb·in.}$ ↻ ◀

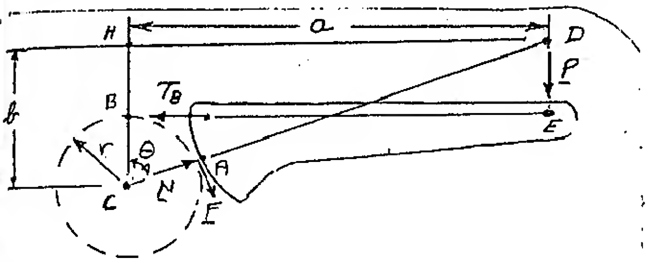


GIVEN: $a = 200$ mm, $r = 30$ mm.
ASSUME VALUE OF μ_s IS THE SAME AT ALL SURFACES OF CONTACT

FIND: SMALLEST VALUE OF μ_s FOR WHICH THE WRENCH IS SELF-LOCKING IF IN
PROB. 8.144 $\theta = 65°$.
PROB. 8.145 $\theta = 75°$.

FOR WRENCH TO BE SELF-LOCKING ($P = 0$), THE VALUE OF μ_s MUST PREVENT SLIPPING OF STRAP WHICH IS IN CONTACT WITH THE PIPE FROM POINT A TO POINT B AND MUST BE LARGE ENOUGH SO THAT AT POINT A THE STRAP TENSION CAN INCREASE FROM ZERO TO THE MINIMUM TENSION REQUIRED TO DEVELOP "BELT FRICTION" BETWEEN STRAP AND PIPE.

FREE BODY: WRENCH HANDLE



GEOMETRY IN ΔCDH: $CH = a/\tan\theta$, $CD = a/\sin\theta$

$DE = BH = CH - BC$

$DE = \dfrac{a}{\tan\theta} - r$

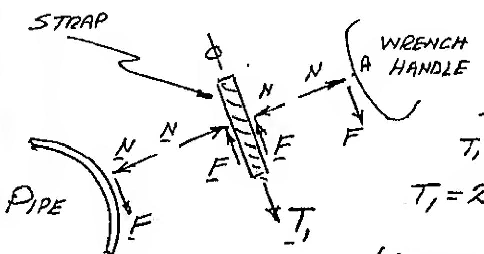
$AD = CD - CA = \dfrac{a}{\sin\theta} - r$

ON WRENCH HANDLE

$+\circlearrowleft \Sigma M_D = 0$: $T_B(DE) - F(AD) = 0$

$$\frac{T_B}{F} = \frac{AD}{DE} = \frac{\frac{a}{\sin\theta} - r}{\frac{a}{\tan\theta} - r} \qquad (1)$$

FREE BODY: STRAP AT POINT A

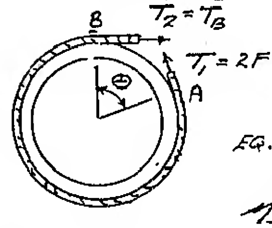


$+\searrow \Sigma F = 0$

$T_1 - 2F = 0$

$$T_1 = 2F \qquad (2)$$

(CONTINUED)



PIPE AND STRAP

$\beta = (2\pi - \theta)$ radians

EQ. (8.13): $\mu_s \beta = \ln \dfrac{T_2}{T_1}$

$$\mu_s = \frac{1}{\beta} \ln \frac{T_B}{2F} \qquad (3)$$

RETURN TO FREE BODY OF WRENCH HANDLE

$\xrightarrow{+} \Sigma F_x = 0$: $N\sin\theta + F\cos\theta - T_B = 0$

$\dfrac{N}{F}\sin\theta = \dfrac{T_B}{F} - \cos\theta$

SINCE $F = \mu_s N$, WE HAVE

$\dfrac{1}{\mu_s}\sin\theta = \dfrac{T_B}{F} - \cos\theta$

OR
$$\mu_s = \frac{\sin\theta}{\frac{T_B}{F} - \cos\theta} \qquad (4)$$

NOTE: FOR A GIVEN SET OF DATA, WE SEEK THE LARGER OF THE VALUES OF μ_s FROM EQS. (3) AND (4)

PROB. 8.144: $a = 200$ mm, $r = 30$ mm, $\theta = 65°$

EQ(1): $\dfrac{T_B}{F} = \dfrac{\frac{200\text{ mm}}{\sin 65°} - 30\text{ mm}}{\frac{200\text{ mm}}{\tan 65°} - 30\text{ mm}} = \dfrac{190.676\text{ mm}}{63.262\text{ mm}} = 3.0141$

$\beta = 2\pi - \theta = 2\pi - 65°\frac{\pi}{180°} = 5.1487$ radians

EQ(3): $\mu_s = \dfrac{1}{5.1487\text{ rad}} \ln \dfrac{3.0141}{2} = \dfrac{0.41015}{5.1487} = 0.0797$ ◁

EQ(4): $\mu_s = \dfrac{\sin 65°}{3.0141 - \cos 65°} = \dfrac{0.90631}{2.1595} = 0.3497$ ◁

WE CHOOSE THE LARGER VALUE: $\mu_s = 0.350$ ◀

PROB. 8.145: $a = 200$ mm, $r = 30$ mm, $\theta = 75°$

EQ.(1): $\dfrac{T_B}{F} = \dfrac{\frac{200\text{ mm}}{\sin 75°} - 30\text{ mm}}{\frac{200\text{ mm}}{\tan 75°} - 30\text{ mm}} = \dfrac{177.055\text{ mm}}{23.590\text{ mm}} = 7.5056$

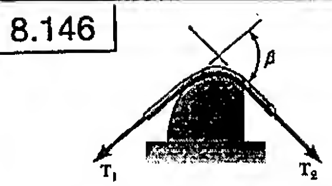
$\beta = 2\pi - \theta = 2\pi - 75°\frac{\pi}{180°} = 4.9742$

EQ.(3): $\mu_s = \dfrac{1}{4.9742\text{ rad}} \ln \dfrac{7.5056}{2} = \dfrac{1.3225}{4.9742} = 0.2659$ ◁

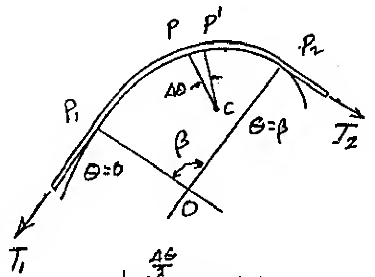
EQ.(4): $\mu_s = \dfrac{\sin 75°}{7.5056 - \cos 75°} = \dfrac{0.96953}{7.2468} = 0.1333$ ◁

WE CHOOSE THE LARGER VALUE: $\mu_s = 0.266$ ◀

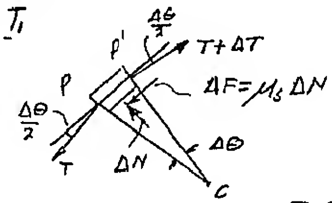
8.146



PROVE THAT EQS. (8.13) AND (8.14) ARE VALID FOR ANY SHAPE SURFACE



NOTE β IS THE ANGLE BETWEEN BOTH TANGENTS AT P_1 + P_2 AND NORMALS AT P_1 + P_2.



NEXT, NOTE THAT THE DERIVATION OF

$$\frac{dT}{T} = \mu_s\, d\theta \qquad (1)$$

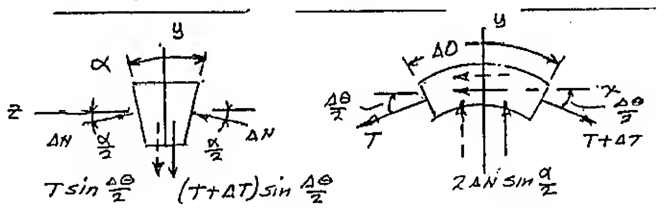
ON PAGES 436 AND 437 DID NOT DEPEND ON THE RADIUS OF CURVATURE BEING CONSTANT. THEREFORE THIS EQUATION MAY BE OBTAINED FROM THE FREE-BODY DIAGRAM SHOWN HERE.

INTEGRATING EQ(1) IN θ FROM 0 TO β AND IN T FROM T_1 TO T_2, WE OBTAIN AGAIN

$$\ln\frac{T_2}{T_1} = \mu_s\beta \quad \text{AND} \quad \frac{T_2}{T_1} = e^{\mu_s\beta}$$

8.147

COMPLETE DERIVATION OF EQ. 8.15



$$\xrightarrow{+} \Sigma F_x = 0:\ (T+\Delta T)\cos\frac{\Delta\theta}{2} - T\cos\frac{\Delta\theta}{2} - 2\mu_s \Delta N = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0:\ 2\Delta N \sin\frac{\alpha}{2} - (T+\Delta T)\sin\frac{\Delta\theta}{2} - T\sin\frac{\Delta\theta}{2} = 0 \qquad (2)$$

SOLVE (1) FOR ΔN AND SUBSTITUTE IN (2):

$$\Delta T \cos\frac{\Delta\theta}{2}\sin\frac{\alpha}{2} - \mu_s(2T+\Delta T)\sin\frac{\Delta\theta}{2} = 0$$

DIVIDE ALL TERMS BY Δθ:

$$\frac{\Delta T}{\Delta\theta}\cos\frac{\Delta\theta}{2}\sin\frac{\alpha}{2} - \mu_s\left(T + \frac{\Delta T}{2}\right)\frac{\sin\frac{\Delta\theta}{2}}{\frac{\Delta\theta}{2}} = 0$$

LET Δθ APPROACH ZERO

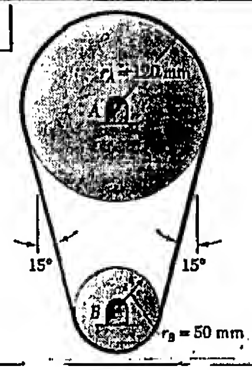
$$\frac{dT}{d\theta}\sin\frac{\alpha}{2} - \mu_s T = 0$$

$$\frac{dT}{T} = \frac{\mu_s}{\sin\frac{\alpha}{2}}\, d\theta$$

INTEGRATE IN θ FROM 0 TO β AND IN T FROM T_1 TO T_2:

$$\ln\frac{T_2}{T_1} = \frac{\mu_s\beta}{\sin\alpha/2} \quad \text{OR,} \quad \frac{T_2}{T_1} = e^{\mu_s\beta/\sin\frac{\alpha}{2}}$$

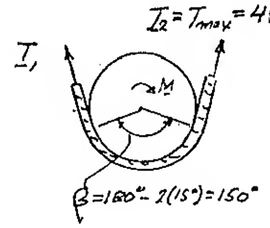
8.148



GIVEN: $\mu_s = 0.40$
$T_{max} = 450$ N
V-BELT WITH $\alpha = 36°$

FIND: LARGEST COUPLE THAT CAN BE EXERTED ON PULLEY A

SINCE β IS SMALLER FOR PULLEY B, THE BELT WILL SLIP FIRST AT B.



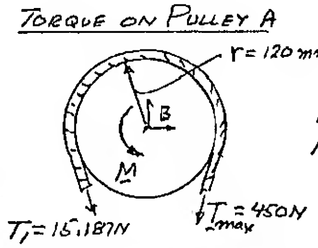
$$\beta = 150°\left(\frac{\pi\ \text{rad}}{180°}\right) = \frac{5}{6}\pi\ \text{rad.}$$

$$\frac{T_2}{T_1} = e^{\mu_s\beta/\sin\frac{\alpha}{2}}$$

$$\frac{450N}{T_1} = e^{(0.4)\frac{5}{6}\pi/\sin 18°} = e^{3.389}$$

$$\frac{450N}{T_1} = 29.63; \quad T_1 = 15.187N$$

TORQUE ON PULLEY A



$$+\circlearrowleft \Sigma M_B = 0$$

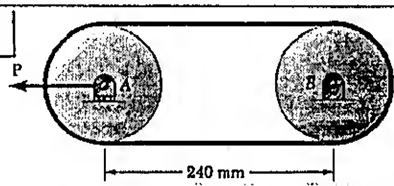
$$M - (T_{max} - T_1)(0.12m) = 0$$

$$M - (450N - 15.187N)(0.12m) = 0$$

$$M = 52.18\ N\cdot m$$

$$M = 52.2\ N\cdot m$$

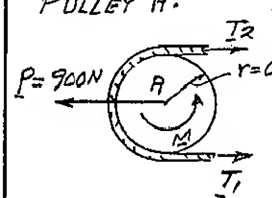
8.149



GIVEN: 60-mm-RADIUS V-BELT PULLEYS WITH $\alpha = 36°$
$P = 900$ N, $\mu_s = 0.35$

FIND: LARGEST TORQUE WHICH CAN BE TRANSMITTED, MAXIMUM TENSION IN V-BELT

PULLEY A: $\beta = \pi$ rad



$$\frac{T_2}{T_1} = e^{\mu_s\beta/\sin\alpha/2}$$

$$\frac{T_2}{T_1} = e^{0.35\pi/\sin 18°}$$

$$\frac{T_2}{T_1} = e^{3.558} = 35.1$$

$$T_2 = 35.1\,T_1$$

$$\xrightarrow{+} \Sigma F_x = 0:\ T_1 + T_2 - 900N = 0$$

$$T_1 + 35.1\,T_1 - 900N = 0$$

$$T_1 = 24.93\ N; \quad T$$

$$T_2 = 35.1(24.93\,N) = 875.03\ N$$

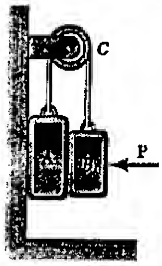
$$+\circlearrowleft \Sigma M_A = 0 \quad M - T_2(0.06\,m) + T_1(0.06\,m) = 0$$

$$M - (875.03N)(0.06m) + (24.93N)(0.06m) = 0$$

$$M = 51.0\ N\cdot m$$

$$T_{max} = T_2 \qquad T_{max} = 875\ N$$

8.150



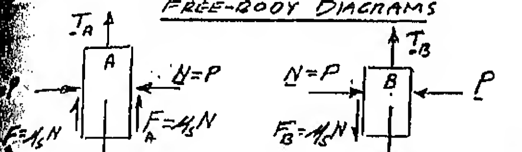
GIVEN: $m_A = 12\,kg$, $m_B = 6\,kg$
$\mu_s = 0.12$

FIND: SMALLEST VALUE OF P FOR EQUILIBRIUM

NOTE: PULLEY CAN FREELY ROTATE

IMPENDING MOTION: BLOCK A ↓ BLOCK B ↑

FREE-BODY DIAGRAMS



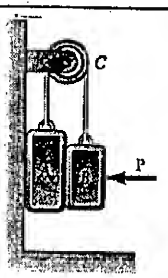
$+\uparrow \Sigma F_y = 0$: $T_A + 2F_A - W_A = 0$
$T_A + 2\mu_s N - m_A g = 0$
$T_A = m_A g - 2\mu_s N$

$+\uparrow \Sigma F_y = 0$: $T_B - F_B - W_B = 0$
$T_B - \mu_s N - m_B g = 0$
$T_B = m_B g + \mu_s N$

LET $T_A = T_B$: $m_A g - 2\mu_s N = m_B g + \mu_s N$
$(m_A - m_B) g = 3\mu_s N$
$(12\,kg - 6\,kg) g = 3(0.12) N$
$N = \frac{6g}{0.36} = 16.667g = 16.667(9.81\,m/s^2) = 163.5\,N$

SINCE $P = N$, WE HAVE $P = 163.5\,N$ ◀

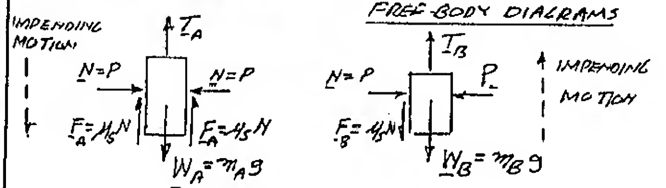
8.151



GIVEN: $m_A = 12\,kg$, $m_B = 6\,kg$
ROTATION OF PULLEY IS PREVENTED.
$\mu_s = 0.12$ AT ALL SURFACES AND BETWEEN CABLE AND PULLEY

FIND: SMALLEST VALUE OF P FOR EQUILIBRIUM

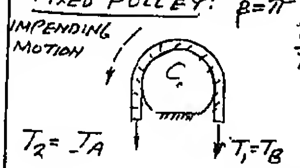
FREE-BODY DIAGRAMS



$+\uparrow \Sigma F_y = 0$: $T_A + 2F_A - W_A = 0$
$T_A + 2\mu_s N - m_A g = 0$
$T_A = m_A g - 2\mu_s N$

$+\uparrow \Sigma F_y = 0$: $T_B - F_B - W_B = 0$
$T_B - \mu_s N - m_B g = 0$
$T_B = m_B g + \mu_s N$

FIXED PULLEY: $\beta = \pi$



$\frac{T_2}{T_1} = e^{\mu_s \beta}$; $\frac{T_A}{T_B} = e^{0.12\pi} = 1.4579$

$T_A = 1.4579\, T_B$

SUBSTITUTE FOR T_A AND T_B

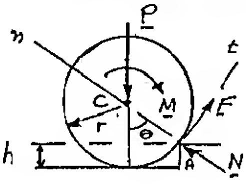
$(m_A g - 2\mu_s N) = 1.4579(m_B g + \mu_s N)$
$(m_A - 1.4579 m_B) g = 3.4579 \mu_s N$
$[12\,kg - 1.4579 \times 6\,kg]\, 9.81\,m/s^2 = 3.4579(0.12) N$
$N = 76.898\,N$

SINCE $P = N$, WE HAVE $P = 76.9\,N$ ◀

8.152

GIVEN: $\mu_s = 0.90$, 12-in.-RADIUS WHEELS, 60% OF WEIGHT IS ON FRONT WHEELS.

FIND: LARGEST h FOR AUTO TO CLIMB CURB
(a) FRONT-WHEEL DRIVE, (b) REAR-WHEEL DRIVE



(a) FRONT-WHEEL DRIVE

ONE FRONT WHEEL: $r = 12\,in.$
$+\nearrow \Sigma F_t = 0$: $F - P\sin\theta = 0$
$+\nwarrow \Sigma F_n = 0$: $N - P\cos\theta = 0$

SLIDING IMPENDS:
$\mu_s = \frac{F}{N} = \frac{P\sin\theta}{P\cos\theta} = \tan\theta$

$\tan\theta = \mu_s = 0.90$; $\theta = 41.987°$

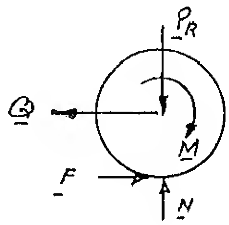
$h = r - r\cos\theta = r(1-\cos\theta) = (12\,in.)(1 - \cos 41.987°)$
$h = 3.0805\,in.$ $h = 3.08\,in.$ ◀

(b) REAR WHEEL DRIVE

EACH REAR WHEEL CARRIES $0.2W$ AND EACH FRONT WHEEL CARRIES $0.3W$. LET Q BE FORCE EXERTED BY CHASSIS ON EACH WHEEL

FREE BODY: REAR WHEEL

$P_R = 0.2W$

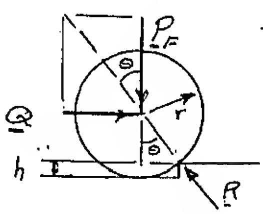


$+\uparrow \Sigma F_y = 0$: $N - 0.2W = 0$
$N = 0.2W$
$F = F_m = \mu_s N = 0.90(0.2W)$
$F = 0.18W$

$\xrightarrow{+} \Sigma F_x = 0$: $F - Q = 0$
$Q = F = 0.18W$

FREE BODY: FRONT WHEEL

$P_F = 0.3W$
$r = 12\,in.$



FRONT WHEEL IS A TWO-FORCE BODY

$\tan\theta = \frac{Q}{P_F} = \frac{0.18W}{0.3W} = 0.6$

$\theta = 30.96°$

$h = r - r\cos\theta = r(1-\cos\theta)$
$= (12\,in.)(1 - \cos 30.96°)$
$= 1.7101\,in$

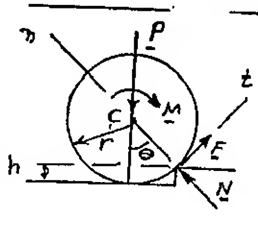
$h = 1.710\,in.$ ◀

NOTE: COMPARING PROBS 8.152 AND 8.153, WE NOTE THAT –
FOR FRONT WHEEL DRIVE THE RESULT IS INDEPENDENT OF WEIGHT DISTRIBUTION
FOR REAR-WHEEL DRIVE THE HEAVIER THE LOAD ON THE REAR WHEELS, THE LARGER THE CURB HEIGHT h WILL BE

8.153

GIVEN: $\mu_s = 0.90$, 12-in.-RADIUS WHEELS, EQUAL WEIGHT ON EACH WHEEL.

FIND: LARGEST h FOR AUTO CLIMB CURB
(a) FRONT-WHEEL DRIVE, (b) REAR-WHEEL DRIVE



(a) FRONT-WHEEL DRIVE

ONE FRONT WHEEL $r = 12$ in.

$+\nearrow \Sigma F_t = 0$: $F - P\sin\theta = 0$

$+\nwarrow \Sigma F_n = 0$: $N - P\cos\theta = 0$

SLIPPING IMPENDS:

$$\mu_s = \frac{F}{N} = \frac{P\sin\theta}{P\cos\theta} = \tan\theta$$

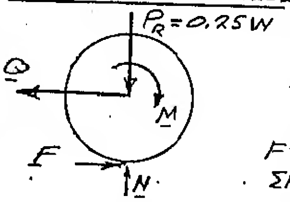
$\tan\theta = \mu_s = 0.90$; $\theta = 41.987°$

$h = r - r\cos\theta = r(1-\cos\theta) = (12\text{ in.})(1-\cos 41.987°)$

$h = 3.0805$ in. $\qquad h = 3.08$ in. ◀

(b) REAR WHEEL DRIVE

FREE BODY: REAR WHEEL



LET Q BE FORCE EXERTED BY CHASSIS ON EACH WHEEL.

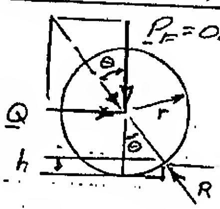
$+\uparrow \Sigma F_y = 0$: $N - 0.25W = 0$

$N = 0.25W$

$F = \mu_s N = 0.90(0.25W) = 0.225W$

$\Sigma F_x = 0$: $Q = 0.225W$

FREE BODY: FRONT WHEEL $\qquad r = 12$ in.



TWO-FORCE BODY

$$\tan\theta = \frac{Q}{P_F} = \frac{0.225W}{0.25W} = 0.9$$

$\theta = 41.987°$

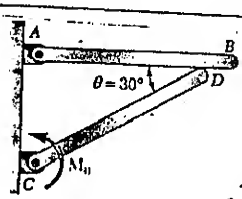
$h = r - r\cos\theta = r(1-\cos\theta)$

$h = (12\text{ in.})(1-\cos 41.987°) = 3.0805$ in.

$h = 3.08$ in. ◀

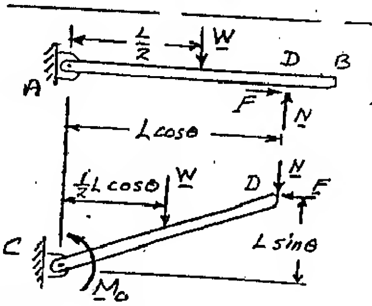
[SEE NOTE AT END OF SOLUTION OF PROB 8.152

8.154



GIVEN: EACH ROD IS OF LENGTH L AND WEIGHT W, $\mu_s = 0.40$

FIND: RANGE OF VALUES OF M_o FOR EQUILIBRIUM



FOR IMPENDING CLOCKWISE MOTION

$+\circlearrowleft \Sigma M_A = 0$

$N(L\cos\theta) - W(\frac{L}{2}) = 0$

$$N = \frac{W}{2\cos\theta}$$

$$F = \mu_s N = \frac{\mu_s W}{2\cos\theta}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad M_o - W(\tfrac{1}{2}L\cos\theta) - \frac{W}{2\cos\theta}(L\cos\theta) + \frac{\mu_s W}{2\cos\theta}(L\sin\theta) = 0$$

$$M_o = \tfrac{1}{2}WL(\cos\theta + 1 - \mu_s\tan\theta) \qquad (1)$$

$M_o = \frac{1}{2}WL(\cos 30° + 1 - 0.40\tan 30°)$

$M_o = 0.81754\,WL \qquad M_o = 0.818\,WL$ ◁

(CONTINUED)

8.154 CONTINUED

FOR IMPENDING COUNTERCLOCKWISE MOTION OF THE RODS, WE CHANGE THE SIGN OF μ_s IN EQ(1).

$M_o = \frac{1}{2}WL(\cos\theta + 1 + \mu_s\tan\theta)$

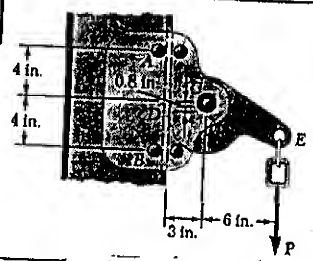
$= \frac{1}{2}WL(\cos 30° + 1 + 0.40\tan 30°)$

$M_o = 1.0484\,WL \qquad M_o = 1.048\,WL$ ◁

RANGE OF M_o FOR EQUILIBRIUM:

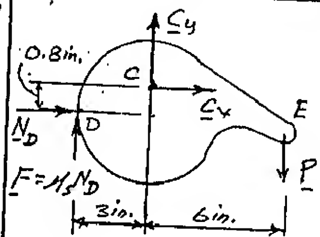
$0.818\,WL \le M_o \le 1.048\,WL$ ◀

8.155



FIND: SMALLEST μ_s BETWEEN RAIL AND CAM AND BETWEEN RAIL AND PINS FOR EQUILIBRIUM

FREE BODY: CAM

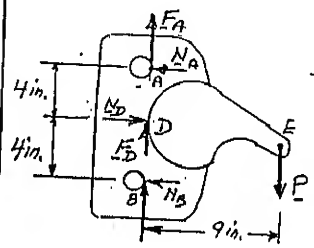


$+\circlearrowleft \Sigma M_C = 0$:

$N_D(0.8\text{ in.}) - \mu_s N_D(3\text{ in.}) - P(6\text{ in.}) = 0$

$$N_D = \frac{6P}{0.8 - 3\mu_s} \qquad (1)$$

FREE BODY: SLEEVE AND CAM



$\overset{+}{\rightarrow}\Sigma F_x = 0$: $N_D - N_A - N_B = 0$

$N_A + N_B = N_D \qquad (2)$

$+\uparrow\Sigma F_y = 0$: $F_A + F_B + F_D - P = 0$

OR $\mu_s(N_A + N_B + N_D) = P \qquad (3)$

SUBSTITUTE FROM (2) INTO (3)

$$\mu_s(2N_D) = P \qquad N_D = \frac{P}{2\mu_s} \qquad (4)$$

EQUATE EXPRESSIONS FOR N_D FROM (1) AND (4)

$$\frac{P}{2\mu_s} = \frac{6P}{0.8 - 3\mu_s}; \qquad 0.8 - 3\mu_s = 12\mu_s$$

$$\mu_s = \frac{0.8}{15} \qquad \mu_s = 0.0533$$ ◀

NOTE: TO VERIFY THAT CONTACT AT PINS A AND B TAKES PLACE AS ASSUMED WE SHALL CHECK THAT $N_A > 0$ AND $N_B = 0$.

FROM (4): $$N_D = \frac{P}{2\mu_s} = \frac{P}{2(0.0533)} = 9.375P$$

FROM FREE BODY OF CAM AND SLEEVE

$+\circlearrowleft \Sigma M_B = 0$ $\quad N_A(8\text{ in.}) - N_D(4\text{ in.}) - P(9\text{ in.}) = 0$

$8N_A = (9.375P)(4) + 9P$

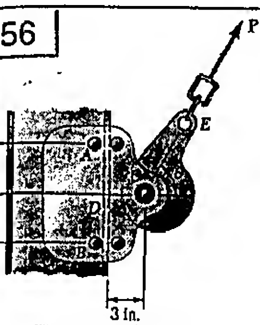
$N_A = 5.8125P > 0$ OK

FROM (2): $N_A + N_B = N_D$

$5.8125P + N_B = 9.375P$

$N_B = 3.5625P > 0$ OK

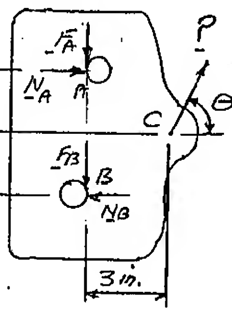
8.156



FIND: LARGEST μ_s BETWEEN RAIL AND PINS A AND B IF SLEEVE IS TO MOVE UP WHEN
(a) $\theta = 60°$,
(b) $\theta = 50°$,
(c) $\theta = 40°$.

NOTE THE CAM IS A TWO-FORCE MEMBER

FREE BODY: SLEEVE



WE ASSUME CONTACT BETWEEN RAIL AND PINS AS SHOWN.

$+\circlearrowleft \Sigma M_C = 0$

$F_A(3in) + F_B(3in.) - N_A(4in) - N_B(4in) = 0$

BUT: $F_A = \mu_s N_A \quad F_B = \mu_s N_B$

WE FIND

$3\mu_s(N_A + N_B) - 4(N_A + N_B) = 0$

$\mu_s = \frac{4}{3} = 1.333$

WE NOW VERIFY THAT OUR ASSUMPTION WAS CORRECT.

$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B + P\cos\theta = 0$

$N_B - N_A = P\cos\theta \qquad (1)$

$+\uparrow \Sigma F_y = 0: \quad -F_A - F_B + P\sin\theta = 0$

$\mu_s N_A + \mu_s N_B = P\sin\theta$

$N_A + N_B = \frac{P\sin\theta}{\mu_s} \qquad (2)$

ADD (1) AND (2): $\quad 2N_B = P\left(\cos\theta - \frac{\sin\theta}{\mu_s}\right) > 0 \quad$ OK

SUBTRACT (1) FROM (2): $\quad 2N_A = P\left(\frac{\sin\theta}{\mu_s} - \cos\theta\right)$

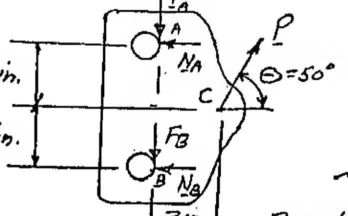
$N_A > 0$ ONLY F $\quad \frac{\sin\theta}{\mu_s} - \cos\theta > 0$

$\tan\theta > \mu_s = 1.333; \quad \theta = 53.13°$

THUS FOR (a) AND (b) CONDITION IS SATISFIED, CONTACT TAKES PLACE AS SHOWN. ANSWER IS CORRECT

(a) AND (b) $\mu_s = 1.333$

BUT FOR (c) $\theta = 50° < 53.13°$ AND OUR ASSUMPTION IS WRONG, N_A IS DIRECTED TO LEFT



$\xrightarrow{+} \Sigma F_x = 0:$

$-N_A - N_B + P\cos 50° = 0$

$N_A + N_B = P\cos 50° \qquad (3)$

$+\uparrow \Sigma F_y = 0:$

$-F_A - F_A + P\sin 50° = 0$

$\mu_s(N_A + N_B) = P\sin 50° \qquad (4)$

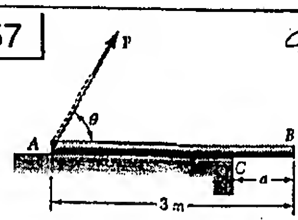
DIVIDE (4) BY (3)

$\mu_s = \tan 50° = 1.192$

(c) $\mu_s = 1.192$

NOTE: FOR $\theta > 53.13°$, μ_s IS INDEPENDENT OF θ.
FOR $\theta < 53.13°$, μ_s DEPENDS ON θ AND IS $\mu_s = \tan\theta$

8.157

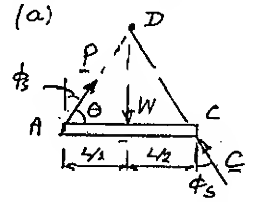


GIVEN: 20-kg TUBE AB, $\mu_s = 0.30$.

FIND: LARGEST θ FOR TUBE TO SLIDE HORIZONTALLY WHEN (a) $a = 0$, (b) $a = 0.75m$.

FOR MAX θ, SLIDING AND ROTATION ABOUT C BOTH IMPEND

(a)



THREE-FORCE BODY

FORCE P MUST PASS THROUGH POINT D WHERE W AND C INTERSECT. SINCE SLIPPING IMPENDS C FORM ANGLE ϕ_s WITH TUBE

$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.30$

$\phi_s = 16.70°$

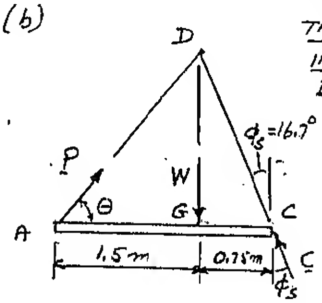
ISOSCELES TRIANGLE

$\theta = 90° - \phi_s = 90° - 16.7° \qquad \theta = 73.3°$

$+\circlearrowleft \Sigma M_C = 0: \quad (P\cos\phi_s)L - W\frac{L}{2} = 0$

$P = \frac{W}{2\cos\phi_s} = \frac{(20 kg)(9.81 m/s^2)}{2\cos 16.7°}, \quad P = 102.4 N$

(b)



THREE-FORCE BODY (See ABOVE)

IN Δ CDG:

$DG = \frac{0.75 m}{\cos 16.7°} = 2.50 m$

IN Δ ADG:

$\tan\theta = \frac{DG}{AG} = \frac{2.5 m}{1.5 m}$

$\tan\theta = 1.667, \quad \theta = 59.04°$

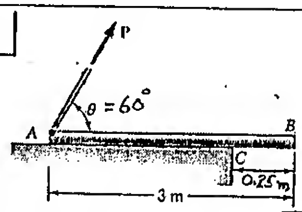
$\theta = 59.0°$

$+\circlearrowleft \Sigma M_C = 0: \quad (P\sin\theta)(2.25m) - W(0.75m) = 0$

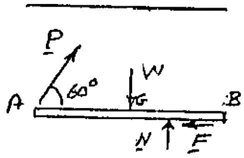
$P = \frac{0.333}{\sin\theta} W = \frac{0.333}{\sin 59.04°}(20 kg)(9.81 m/s^2)$

$P = 76.27 N \qquad P = 76.3 N$

8.158



GIVEN: $\mu_s = 0.30$
20-kg TUBE AB
FIND: (a) SMALLEST P TO MOVE TUBE, (b) WHETHER TUBE SLIDES OR ROTATES.



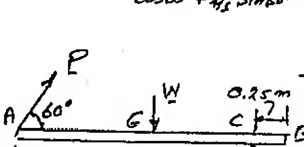
ASSUME SLIDING

$\Sigma F_y: \quad N = W - P\sin 60°$

$F = \mu_s N = \mu_s(W - P\sin 60°)$

$\Sigma F_x: \quad P\cos 60° = F = \mu_s(W - P\sin 60°)$

$P = \frac{\mu_s W}{\cos 60° + \mu_s \sin 60°} = \frac{0.3 W}{\cos 60° + 0.3\sin 60°} = 0.3948 W$



ASSUME ROTATION ABOUT C

$+\circlearrowleft \Sigma M_C = 0$

$(P\sin 60°)(2.75m) - W(1.25m) = 0$

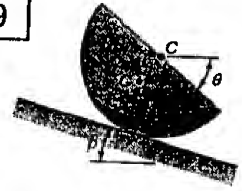
$P = 0.5249 W$

TUBE SLIDES

FOR SLIDING: $P = 0.3948 W = 0.3948(20 kg)(9.81 m/s^2)$

$P = 77.5 N$

8.159



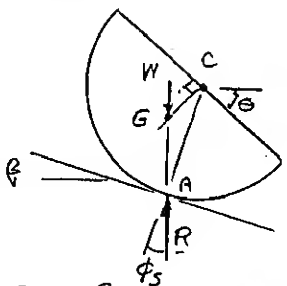
GIVEN: HOMOGENEOUS HEMISPHERE

$\mu_s = 0.30$

FIND: (a) VALUE OF β FOR WHICH SLIDING IMPENDS.

(b) CORRESPONDING VALUE OF θ.

r = RADIUS



WE HAVE A TWO-FORCE BODY

FOR SLIDING TO IMPEND

R FORMS ANGLE ϕ_s WITH INCLINE.

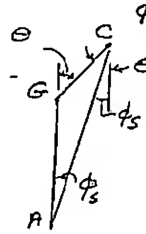
$\phi_s = \tan^{-1} 0.30 = 16.70°$

$\therefore \beta = 16.70°$ ◀

GEOMETRY:

$GC = \frac{3}{8} r$ (See Fig. 5.21)

$AC = r$



TRIANGLE ACG: $\angle ACG = \theta - \phi$

$\angle AGC = 180° - \theta$

LAW OF SINES

$$\frac{\sin(180° - \theta)}{AC} = \frac{\sin \phi_s}{GC}$$

$$\sin(180° - \theta) = \frac{AC}{GC} \sin \phi_s = \frac{r}{3r/8} \sin 16.70°$$

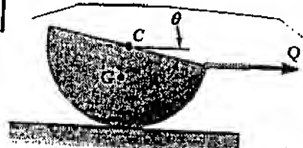
$\sin(180° - \theta) = 0.76629$

$180° - \theta = 50.0°$ AND $130.0°$

$\theta = 130.0°$ AND $50.0°$

$\theta = 130°$ IMPOSSIBLE $\qquad \theta = 50.0°$ ◀

8.160

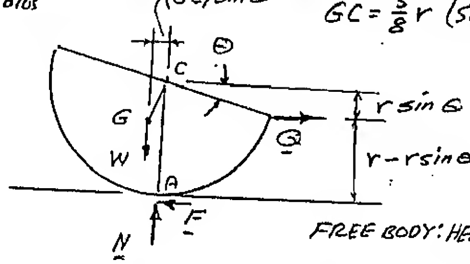


GIVEN: HOMOGENEOUS HEMISPHERE

$\mu_s = 0.30$

FIND: VALUE OF Q FOR WHICH SLIDING IMPENDS.

r = radius $\qquad GC = \frac{3}{8} r$ (See Fig. 5.21)



FREE BODY: HEMISPHERE

$+\uparrow \Sigma F_y = 0$: $\quad N - W = 0$; $\quad N = W$

SLIDING IMPENDS: $F = \mu_s N = \mu_s W$

$\xrightarrow{+} \Sigma F_x = 0$: $\quad Q - F = 0$; $\quad Q = \mu_s W$

$+\circlearrowleft \Sigma M_A = 0$: $\quad W(GC)\sin\theta - Q(r - r\sin\theta) = 0$

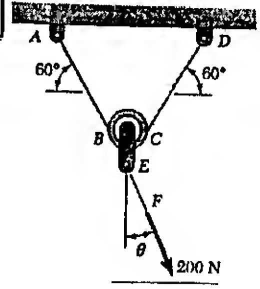
$W\left(\frac{3}{8} r\right) \sin\theta - Qr + Qr\sin\theta = 0$

$$\sin\theta = \frac{Q}{\frac{3}{8}W + Q} = \frac{\mu_s W}{\frac{3}{8}W + \mu_s W}$$

$$\sin\theta = \frac{\mu_s}{\frac{3}{8} + \mu_s} = \frac{0.30}{0.375 + 0.30} = \frac{4}{9}$$

$\theta = 26.39° \qquad \theta = 26.4°$ ◀

8.161



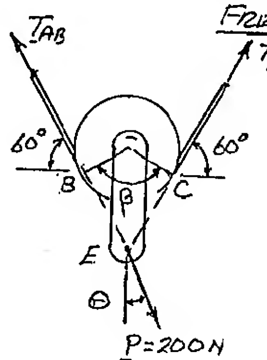
GIVEN: AXLE OF PULLEY IS FROZEN AND CANNOT ROTATE WITH RESPECT TO BLOCK

$\mu_s = 0.30$

FIND: (a) MAXIMUM VALUE OF θ FOR EQUILIBRIUM.

(b) REACTIONS AT SUPPORTS A AND D

FREE BODY: BLOCK AND PULLEY



SINCE 200-N FORCE TENDS TO ROTATE PULLEY ↻, CABLE TENDS TO SLIP RELATIVE TO PULLEY. ∴

$T_1 = T_{CD} \qquad T_2 = T_{AB}$

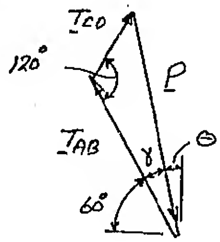
$\beta = 120° = \frac{2\pi}{3}$ rad

$\mu_s = 0.30$

$$\frac{T_1}{T_2} = e^{\mu\beta}$$

$$\frac{T_{AB}}{T_{CD}} = e^{0.30\left(\frac{2\pi}{3}\right)} = e^{0.2\pi} = 1.8745$$

$T_{AB} = 1.8745\, T_{CD} \qquad (1)$



FORCE TRIANGLE

LAW OF COSINES

$P^2 = T_{AB}^2 + T_{CD}^2 - 2T_{AB}T_{CD}\cos 120°$

$= (1.8745\, T_{CD})^2 + T_{CD}^2$

$- 2(1.8745\, T_{CD})T_{CD}(-0.5)$

$= \left[(1.8745)^2 + 1 + 1.8745\right] T_{CD}^2$

$P^2 = 6.3880\, T_{CD}^2$

$T_{CD} = 0.39565\, P \qquad (2)$

(a) MAXIMUM ALLOWABLE VALUE OF θ:

LAW OF SINES: $\quad \dfrac{\sin\gamma}{T_{CD}} = \dfrac{\sin 120°}{P}$; $\quad \sin\gamma = \dfrac{T_{CD}}{P}\sin 120°$

RECALLING EQ(2):

$\sin\gamma = \dfrac{0.39565P}{P} \sin 120° = 0.34264$; $\quad \gamma = 20.04°$

$\theta = 90° - (60° + 20.04°) \qquad \theta = 9.96°$ ◀

(b) REACTIONS AT A AND D. $\quad P = 200$ N

EQ(2): $\quad T_{CD} = 0.39565(200\text{N}) = 79.13$ N

EQ(1) $\quad T_{AB} = 1.8745\, T_{CD} = 1.8745(79.13\text{N}) = 148.33$ N

THUS $\qquad A = 148.3\text{N} \measuredangle 60°$ ◀

$D = 79.1\text{N} \measuredangle 60°$ ◀